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Computable Multi-Country Models of Production and Trade

by

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COMPUTABLE MULTI-COUNTRY MODELS OF PRODUCTION AND TRADE

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The models described here have the following characteristics: (1) there is explicit optimization and equilibrium for a number of individual countries, (2) all commodities and their interdependences are covered, (3) international prices are determined as a result of multi-country market clearing, and (4) the models are designed for empirical implementation and numerical solution. A number of interesting models with one or more of these characteristics have been developed in recent years, but it is difficult to find a model that has all four. Leontief et. al. provide a very broad multi-country analysis, but do not cover individual country optimization. On the other hand, de Melo and Robinson cover optimization for an individual country, but do not cover multi-country equilibrium. The World Bank Model emphasizes inter-country dependence, but has some characteristics that lead its formulators to state that "... cannot be regarded as a typical general equilibrium structure." (See Gupta et. al., p. 183.)

There are also a number of important characteristics possessed by some other recent models that are not possessed by the models described here. The present models are designed for comparative static applications. They allow inferences about future changes, but their static character preclud-

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es explicit dynamics. The current models are limited to relative prices and real phenomena. They do not cover national and international monetary phenomena. They are of little use for most macro questions. One of the important questions that they are designed to answer is: How would production and trade decisions be affected by changes in international prices, or more broadly, by changes in the manner in which international prices are determined?

There are three major sections. The assumptions of the general model are described for a typical country in the first. An individual country optimization problem is placed within a nonlinear programming framework, and the Kuhn-Tucker conditions are interpreted to describe equilibrium in the second. Conditions for multi-country equilibrium are also discussed. Four specific applications of the model are described in the third major section. A final section contains a discussion of further applications and extensions of the model.

1. Production and Consumption for Individual Countries

The current models are constructed using conventional small country assumptions: (1) Each country formulates optimal production and consumption plans without reference to production and consumption in other countries. (2) Each accepts international prices as parameters, and plans accordingly. General assumptions covering consumption, production and trade are described in this section. Specific assumptions are described for four particular models in Section 3. Some variations of the assumptions are outlined in Section 4.

1.1 Classification of Goods

There are J countries. Each produces and consumes some of $n = n_T + n_N$ goods. The n_T goods numbered 1, ... , n_T may enter international trade, i.e., specific quantities may be produced in one country and consumed in another. These are called "international goods" or "traded goods". The n_N goods numbered from n_{T+1} through n must be produced and consumed within the same country. These "home goods" do not enter international trade. There is no reason for home goods to be the same from country-to-country. The assumption that each country has the same number of home goods is a matter of notational convenience. It may be altered whenever circumstances warrent. The term sector is used to denote a production unit for goods. There is a one-to-one relationship between sectors and goods. The terms goods and commodities are used interchangeably.

1.2 Notation

The notation used in this paper is listed in Table 1. Upper case letters are used to denote vectors and a matrix. They are also used for subscripts and superscripts. Lower case letters denote scalars, and vector and matrix components. Only those scalars that do not correspond to vector or matrix components are listed in Table 1. Unless otherwise indicated, each vector has n components, one corresponding to each sector (good). The input-output matrix is (n x n). The subscripts T and N partition international and home goods respectively. For example, P_T is an n_T-component vector of international-good prices, and X_N is an n_N-component vector of home-good outputs. It follows that $X \equiv (X_T, X_H)$. Similarly, A_{NT} is an $(n_N \times n_T)$ matrix of input-output coefficients covering home-good input requirements for the production of international goods.

If more than one country is under consideration, superscripts are used to designate particular countries. For example, X^3 contains gross-output levels for the country numbered 3. The superscript A denotes aggregation over all countries. For example,

$D^A \equiv \Sigma_{j=1}^{j=m} D^j$ gives aggregate excess demands for all countries.

1.3 Social Welfare Functions

Each country is assumed to possess a social welfare function of the form

$$(1) \quad \omega = g(C)$$

which is continuous, concave[1] and nondecreasing with the continuous first partial derivatives $\partial\omega/\partial c_i \geqq 0$ for all i. Each country is assumed to organize its production, trade and consumption to maximize (1) subject to the relevant constraints.

1.4 Two-stage Production Functions

The input-output coefficient matrix A describes the intermediate materials input requirements for the production of each output according to the conventional input-output assumptions. The typical element a_{ij} is the quantity of good i required as an input for the production of one unit of good j.

Table 1 Notation

	VECTORS	
C	=	Final-consumption levels.
D	=	Excess demands.
E	=	Export levels.
H	=	Semi-skilled labor-input levels.
K	=	Capital-input levels.
K_{MAX}	=	Maximum allowable capital-input levels.
K_{MIN}	=	Minimum allowable capital-input levels.
L	=	Total (or unskilled) labor input levels.
M	=	Import levels.
P	=	Commodity prices.
Q	=	Efficiency prices.
R	=	Capital-return rates.
S	=	Skilled-labor input levels.
T	=	Exogenous output sources and uses.
U	=	Upper-limit capital premiums.
V	=	Lower-limit capital penalties.
X	=	Gross-output levels.
X_{MAX}	=	Maximum allowable gross-output levels.
X_{MIN}	=	Minimum allowable gross-output levels.
	MATRIX	
A	=	Input-output coefficients.
	SCALARS	
b	=	Net payments surplus.
f_H	=	Semi-skilled labor endowment.
f_L	=	Total (or unskilled) labor endowment.
f_K	=	Capital endowment.
f_S	=	Skilled-labor endowment.
w	=	Wage rate.
α	=	Total (or unskilled) labor-share parameter.
β	=	Skilled labor-share parameter.
γ	=	Production function scale parameter.
δ	=	Limit parameter.
λ	=	Consumption parameter or variable.
μ	=	Skilled wage premium.
η	=	Semi-skilled wage premium.
ρ	=	Marginal capital return.
Θ	=	Exchange rate.
ω	=	Ordinal social welfare.

The quantities of the primary factors required for production are described by the factor production functions

$$(2) \quad x_i \leqq f_i(l_i, h_i, s_i, k_i) \qquad (i = 1, \dots, n)$$

These functions are continuous, positively valued, concave, nondecreasing and homogenous of degree one. Their nonnegative first partial derivatives are the marginal products of the various types of labor and capital. The insertation of a weak inequality rather than a strict equality in (2) allows for inefficient factor use.

1.5 Trade and Commodity Balance

Trade in international goods takes place at fixed international prices with zero international transport costs. The commodity balance equations for a typical country include its trade flows:

$$(3) \quad X + M \geqq AX + E + C + T$$

where $M_N \equiv E_N \equiv 0$ by assumption. Equations (3) simply state that output sources (production plus imports) must be adequate to meet output uses (intermediate inputs plus exports plus consumption plus external uses). Again, the weak inequality allows for inefficient organization. The vector T contains all exogenous sources and uses. Its individual components may be positive, negative or zero.

1.6 Production and Consumption Constraints

A variety of circumstances prevent the attainment of production and consumption levels as large as most countries would desire. Limited factor endowments provide overall, or general, production limits. There are limited quantities of each of the three types of labor, and labor constraints are given in terms of a "job-ladder" specification:

$$\sum_{i=1}^{n} s_i \leqq f_S \tag{4}$$

$$\sum_{i=1}^{n} h_i + \sum_{i=1}^{n} s_i \leqq f_S + f_H \tag{5}$$

$$\sum_{i=1}^{n} l_i + \sum_{i=1}^{n} h_i + \sum_{i=1} s_i \leqq f_S + f_H + f_L \tag{6}$$

where f_S, f_H and f_L are the numbers of people who are skilled, semi-skilled and unskilled respectively. The variables s_i, h_i and l_i are the numbers of people engaged in skilled, semi-skilled and unskilled jobs. The job-ladder specification states that only skilled workers are qualified for skilled jobs (4), skilled and semi-skilled workers are qualified for semi-skilled jobs (5), and all three types of workers are qualified for unskilled jobs (6). If a skilled worker were employed in a semi-skilled job,[2] for example, he would not be available for skilled employment and (4) would be satisfied as a strict inequality. The number of labor classes (rungs in the job ladder) is easily increased or decreased as circumstances warrent.

There is only one kind of capital, and consequently, there is only one overall capital constraint:

$$(7) \quad \sum_{i=1}^{n} k_i = f_K$$

Capital, however, also is subject to specific sectoral constraints as described below. The overall capital and labor constraints are weak inequalities. It is always possible to leave some, or all, of a factor endowment unused.

Balance-of-payments requirements provide an overall constraint upon consumption expenditures. Export earnings must be sufficiently large to pay for imports net of exogenous transfers:

$$(8) \quad PE \geqq PM + b$$

where the payments balance b may be positive, negative or zero. A negative value for b might reflect the receipt of foreign aid, and a positive value might reflect net debt repayment.

Specific capital constraints are imposed for traded goods to limit the degree of movement and specialization that might be achieved. These have the form

$$(9) \quad K_T \leqq K_{BT}(1 + \delta)$$

$$(10) \quad K_T \geqq K_{BT}(1 - \delta)$$

where the subscript B denotes predetermined base values. The parameter $0 < \delta < 1$ gives allowable proportional deviations from these values. A single value is used for notational convenience. Different values for increases and decreases, or different values for different sectors are easily introduced whenever circumstances warrent.

The specific limits encompass the influence of effects that cannot be explicitly introduced into the model. One is the time that it takes for change to occur. There are also the factors that cause supply curves to eventually slope upwards. The validity of the model's assumptions may decrease if large movements from the observed values of the variables take place. The applications described in Section 3 are focused upon the directions of movement when prices are altered. Small movements provide a great deal of information in these cases without pushing the model too far. Finally, it should be noted that one-country models of the general type developed here often result in extreme specialization.

2. Optimization and Equilibrium

Each country is assumed to organize its production, trade and consumption to maximize its social welfare (1) subject to (2) through (10). This optimization problem is placed within a Kuhn-Tucker framework in the first part of this section. The resultant "shadow prices" are interpreted in economic terms. One-country equilibrium conditions are derived from the necessary and sufficient Kuhn-Tucker conditions in the second subsection, and some properties of one-country solutions are discussed in the third. Multi-country equilibrium is defined in the fourth, and aspects of existence and computation are discussed in the fifth.

2.1 Kuhn-Tucker Representation

The typical country desires to select nonnegative values for X, L, K, E, and M that maximizes its social welfare subject to its constraints. The appropriate Lagrange function for such constrained optimization is

$$
\begin{aligned}
y = {} & g(C) + \sum_{i=1}^{n} z_i \left[f_i(l_i, h_i, s_i, k_i) - x_i \right] \\
& + Q\left[(I - A)X + M - E - C - T \right] \\
& + w(f_S + f_H + f_L - \sum_{i=1}^{n} s_i - \sum_{i=1}^{n} h_i - \sum_{i=1}^{n} l_i) \\
& + \eta(f_s + f_H - \sum_{i=1}^{n} s_i - \sum_{i=1}^{n} h_i) + \mu(f_S - \sum_{i=1}^{n} s_i) \\
& + \rho(f_K - \sum_{i=1}^{n} k_i) + \Theta(PE - PM - b) \\
& + U(K_{MAX,T} - K_T) + V(K_T - K_{MIN,T})
\end{aligned}
\tag{11}
$$

The weak inequalities (2) through (10) have been written so that each is in the form $\geqq 0$.[3] Each constraint is premultiplied by a Lagrange multiplier, or "shadow price", giving the amount by which the optimal value of the objective function would be increased (or decreased) if whatever is constrained were increased or decreased by one unit. These shadow prices are interpreted within the context of the present model in Table 2. The skilled and semi-skilled wage rates, and the sectoral capital returns are each determined by more than one constraint as indicated at the bottom of Table 2.

Absolute prices are measured in the same units as social welfare. One unit of social welfare serves as numeraire. This is of little significance, however. Only, relative prices are unique. The numeraire is easily changed by multiplying the ordinal social welfare function by a positive constant. If one multiplies by $1/w$, for example, the unskilled wage rate becomes numeraire.

Table 2 Shadow Prices

Symbol	Quantity Constrained	Interpretation
z_i	Output in terms of factor content	Value added
q_i	Output	Domestic efficiency price
w	Unskilled labor	Unskilled wage rate
η	Semi-skilled labor	Semi-skilled wage premium
μ	Skilled labor	Skilled wage premium
Θ	Foreign exchange	Exchange rate
ρ	Unrestricted capital	Marginal capital return
u_i	Sectoral capital in	Positive capital premium
v_i	Sectoral capital out	Negative capital premium
$w_H = w + \eta$		Semi-skilled wage rate
$w_S = w + \eta + \mu$		Skilled wage rate
$r_i = \rho + u_i - v_i$		Sectoral capital return

2.2 Equilibrium Conditions for Individual Countries

Let the Lagrange function (11) be rewritten in general form[4] as

$$\phi(X,Q) = g(X) + \sum_i q_i f_i(X)$$

where ϕ, g and the f_i are functions and the q_i are shadow prices. If g and the f_i are concave,[5] the necessary and sufficient conditions for a constrained maximum are that

$$(12) \qquad \frac{\partial \phi}{\partial x_j} \leqq 0; \quad x_j \geqq 0; \quad x_j \frac{\partial \phi}{\partial x_j} = 0$$

for all j, and

$$(13) \qquad \frac{\partial \phi}{\partial q_i} = f_i(X) \geqq 0; \quad q_i \geqq 0; \quad q_i f_i(X) = 0$$

for all i. Individual country equilibrium conditions are now inferred from (12) and (13).

Taking the labor input derivatives from (11) and using the first conditions of (12) yields the familiar result that the value of the marginal product for each class of labor cannot exceed the corresponding wage:

$$(14) \qquad \begin{array}{l} z_i \dfrac{\partial y}{\partial l_i} \leqq w \;; \quad z_i \dfrac{\partial y}{\partial h_i} \leqq w + \eta = w_H \;; \\[2ex] z_i \dfrac{\partial y}{\partial s_i} \leqq w + \eta + \mu = w_S \end{array}$$

with the equality holding by (12) whenever a particular type of labor is used in a particular sector. Similarly, for capital

$$(15) \qquad z_i \frac{\partial y}{\partial k_i} \leqq \rho + u_i - v_i = r_i$$

Again the equality will hold wherever capital is used.

From (13)

$$u_i = 0 \qquad \text{if} \qquad k_i < k_{max,i}$$

$$v_i = 0 \qquad \text{if} \qquad k_i > k_{min,i}$$

Thus, $u_i v_i = 0$ if $k_{max,i} > k_{min,i}$. One or both of the capital premiums must be zero if the capital limits are distinct. One normally expects a positive capital premiums for capital inputs at their upper limits, and negative premiums for those at their lower limits.[6] The marginal capital return is earned by a sector with a capital input between its upper and lower limits. Since there are no capital limits for home goods, each will earn the marginal capital return in equilibrium.

Since the production functions (2) are homogeneous of degree one, Euler's theorem gives

$$x_i = \frac{\partial x_i}{\partial l_i} l_i + \frac{\partial x_i}{\partial h_i} h_i + \frac{\partial x_i}{\partial s_i} s_i + \frac{\partial x_i}{\partial k_i} k_i \quad (i=1,\ldots,n)$$

Substituting from (14) and (15) and rearranging terms,[7]

$$(16) \qquad z_i = (\frac{l_i}{x_i})w + (\frac{h_i}{x_i})w_H + (\frac{s_i}{x_i})w_S + (\frac{k_i}{x_i})r_i (i=1,\ldots,n)$$

which establishes that the z_i are unit factor costs.

Taking the derivatives of E and M in (11), using (12), and letting $\Theta \equiv 1$ serve as numeraire, it follows that $Q_T = P_T$. Domestic prices equal their international counterparts for trades goods. Differentiating (11) with respect to X and using (12),

$$(17) \qquad P \leqq PA + Z$$

with the equality holding for each output that is produced. These are the competitive zero-profit conditions. Unit revenues as given by left-hand side of (17) equal unit costs as given by the right-hand side. The vector Z gives unit value added as well as unit factor cost. International prices permeate the entire system. From (17) it is obvious that they are reflected in home-goods and factor prices.

2.3 Dichotomized Optimization

The general model is developed with a view toward multi-country equilibrium. It can be simplified considerably if interest is limited to individual country equilibrium. Normally, it is possible to separate the analysis into two parts. Production is organized to maximize international value added in the first part. This maximum value is then used in the second part to secure a bundle of international goods that maximizes social welfare. If interest is limited to the organization of production, the second part need not be performed.

The first part for the general model of Section 1 can be formulated as follows: find nonnegative X, L, H, S and K that maximize international value added as given by

$$P_T X_T - P_T A_{T.} X$$

subject to (2), (4), (5), (6), (7), (9) and (10). The balance equations for international goods as given by (3) may be dropped along with the social welfare function (1) and the balance-of-payments constraint (8). However, it is necessary to have balance equations for home goods, and it is necessary to specify final consumption levels for home goods. An example of a dichotomized model is given in Section 3.3 below.

2.4 Multi-country Equilibrium

Consider now an international equilibrium in which the market for each international good clears and international prices are determined accordingly. Market-clearing equations obtained by summing the equality forms of (3) over countries:

$$(18) \qquad X^A \equiv \sum_{j=1}^{J} X^j = \sum_{j=1}^{J} (A^j X^j + C^j + T^j)$$

Note that $E^A \equiv M^A$ since one country's export is anothers import. Consistency requires that goods and payments transfers balance. Specifically, that $b^A = P_T T_T{}^A$.

Multi-country equilibrium is more than an aggregation of individual country solutions. The individual country solutions are constructed without direct reference to demands incorporated in (18). Even if the equilibrium international prices were known it would seldom be possible to obtain equilibrium quantities from individual solutions even though all equilibrium price conditions were satisfied. The reason is that country solutions that correspond to a multi-country solution are seldom unique, and specific quantities must be determined by (18).

2.5 Existence and Computation

The Kuhn-Tucker conditions describe the characteristics that equilibria would possess if such equilibria existed. They do not guarantee existence, however. This is a problem that must be investigated for each particular application. Nonetheless, some general considerations are applocable. Except for extraordinary cases, a multi-country solution will exist if optimal solutions exist for each country for all possible prices. Nontrivial country solutions, in turn, will exist if nontrivial points exist which satisfy the country constraints. Existence presents no problems for the models reviewed in Section 3 since these are focused upon deviations from observed points which satisfy the constraints.

The Kuhn-Tucker conditions also provide no direct aid in finding optimal solutions that do exist. Computational methods must be developed for this purpose. Experience with the models of Section 3 suggests that it is more efficient to develop particular procedures for particular models than to develop general procedures.

3. Four Specific Models

A specific model is derived from the general model by giving particular form to the functions and constraints. Four specific models are described here. Each has been or is being implemented with a body of data, and has been or is being solved. Input-output data play a principal role. Consequently, applications are limited to situations for which comparable input-output data are available. Considerable data rectification is necessary before a multi-country model can be implemented. Input-output data are defined to be country specific, and must be converted to a basis that is comparable among countries [see Henderson (1970)].

3.1 Model I: A Linear Format

This model was implemented for five EEC countries (Belgium-Lux, France, Germany, Italy and the Netherlands) for 1965 using EEC input-output data. There are fifty-six international goods and three home goods. The particular forms for the constraints are

(I-1) $$\omega = \lambda P C_B$$

(I-3) $$X + M \geqq AX + E + \lambda C_B + T$$

(I-4/7) $$\sum_{j=1}^{n} a_{m+1,j}\, x_j \leqq f$$

(I-8) $$P_T E_T \geqq P_T M_T + P_T T_T$$

(I-9) $$X_R \leqq 1.25 X_{BR}$$

where the subscript B denotes the 1965 base values given by the input-output data.

Final consumption is assumed to take place in fixed proportions with $C = \lambda C_B$. Thus, all of consumption is described by the single variable λ which can also be interpreted as giving the gains from expanded trade since $\lambda = 1$ corresponds to the base data C_B. The fixed vector T contains base-year values for net trade, investment and inventory change. A single primary factor is assumed to be used according to constant coefficients with f being the base-year endowment. Thereby, the production functions (2) are encompassed by (I-4/7). Natural-resource based sectors as denoted by the subscript R are assumed to have upper output limits equal to 125 percent of their 1965 values. Zero is the lower limit for each output.

Relations (I-1) and (I-8) are nonlinear in that the involve products of prices and quantities. The entire system is linear, however, if international prices are taken as exogenous. A grand linear programming model was formulated for all countries considered simultaneously with the aggregate of (I-1) being maximized. The vectors E_T and M_T were eliminated by substitution from (I-3) into (I-8). A solution was obtained using normal linear-programming methods. The shadow prices Q_T were different from the P_T initially assumed. There were then inserted into (I-1) and (I-8) and a new solution computed. The process was repeated until the two sets of prices converged. Convergence was rapid, and the solution method proved quite efficient.

3.2 Model II: An Expansion of the EEC

A similar application of the model was based upon 1970 EEC data for eight countries. The U.K., Denmark and Ireland were the additional three, with $n_T = 22$ and $n_N = 1$. The effects of the entry of the three new countries were analyzed by comparing a solution with them included and a solution with them excluded. This model differs from model I in that (1) two factors, labor and capital, were distinguished, and (2) there are upper and lower capital limits for each international good. The relations that differ from Model I are:

(II-4/6) $$\sum_{j=1}^{n} a_{m+1,j} \leqq f_L$$

(II-7) $$\sum_{j=1}^{n} a_{m+2,j} \leqq f_K$$

(II-9) $$X_T \leqq 1.25X_{BT}$$

(II-10) $$X_T \geqq 0.75X_{BT}$$

The remaining relations are the same as the corresponding relations for Model I.

It was possible for one of the original five countries to be harmed by the entry of the new three. This could occur as a result of a change in relative prices. The applications suggest that none was harmed, but the gains were differentially distributed.

The successive linear-programming solution method used for Model I is also appliable for Model II. However, it is more costly because Model II has 405 constraints by contrast with 110 for Model I. Efforts have been made with some success to find a particular solution procedure for Model II.

3.3 Model III: A Small-country Version

In this case, unlike the other three, international prices are assumed to be exogenous. Solution was limited to the first part of a dichotomized form (see 2.3 above). The model was applied for thriteen developing and developed countries. Optimum solutions were compared to determine what patterns of specialization would be consistent with international prices. The relations for this model are

(III-1) $$\omega = P_T X_T - P_T A_T . X$$

(III-2) $$x_i = \gamma_i l_i^{\alpha_i} s_i^{\beta_i} k_i^{1-\alpha_i-\beta_i} \qquad (i=1,\ldots,n)$$

(III-4/5) $$\sum_{j=1}^{n} s_j \leqq f_s$$

(III-6) $$\sum_{j=1}^{n} l_j + \sum_{j=1}^{n} s_j \leqq f_s + f_L$$

(III-7) $$\sum_{j=1}^{n} k_j \leqq f_K$$

(III-9) $$X_T \leqq 1.25\ X_{BT}$$

(III-10) $$X_T \geqq 0.75\ X_{BT}$$

where $A_{T\cdot}$ is an $(n_T \times n)$ partition of the input output matrix giving the international-good input requirements for each good. Relations (3) and (8) are implicit in this formulation. Output rather than capital limits were used in this case. Some minor details, but no major conclusion, would be altered by a shift to capital limits. It was assumed that final consumption for home goods would remain at base-period levels.

One of the more interesting results of this application was the conclusion that most developing countries have a surplus of skilled labor given free market prices. The skill-intensive sectors normally are protected, import-substitute industries. The sectors that would thrive under free trade are not skill intensive. Computational methods were developed which made solution of Model III quite easy.

3.4 Model IV: Nonlinear with Many Countries

This is by far the most general of the four models. It is being applied for twenty-six (maybe more) countries with $n_T = 12$ and $n_N = 1$. O.E.C.D. information provides the major data source. The basic nonlinear relations are

$$\text{(IV-1)} \qquad \omega = \sum_{i=1}^{n} \lambda_i \ln C_i$$

$$\text{(IV-2)} \qquad x_i = \gamma_i l_i^{\alpha_i} k_i^{1-\alpha_i} \qquad (i=1,\ldots,n)$$

The remaining relations follow the standard form with sector capital levels for international goods limited to twenty-five percent deviations from base-period values. The data are not adequate to allow breakdown of labor by skill class. In general the complexity and dimensions of applications are limited by data rather than ability to model or compute.

The application of Model IV has just begun. Consequently, there are few results to report. Despite the fact there are only twelve international goods the overall dimensions of the model are quite large. There are 78 relations per country, 26 countries and twelve international market clearing equations to give a total of 2040 relations. Experimentation is underway to develop a compound iteration procedure whereby prices are assumed and adjusted until an equilibrium set of international prices is obtained. The major difficulty arises because of discontinuities in the supply functions for individual countries as a result of upper and lower limits.

4. Further Applications and Extensions

The general model can be applied and extended in many different ways. The introduction of more detailed commodity and labor classes is an obvious extension. The introduction of more complex utility and production functions is another. Extensions of this type are desirable, but usually must await the availability of better data.

Comparative statics in the large through comparison of alternative optimal solutions provides a vehicle for many applications. Policy measures often can be incorporated by adding, substracting and altering particular constraints. The imposition of an import quota provides an example of a policy that can be handled through the introduction of an additional constraint. The optimal value of the shadow price for such a constraint provides the "tariff equivalent" of the quota. The opportunity for such applications is almost endless.

For another illustrative application consider increased unskilled labor migration among the countries within the model. This could be incorporated by replacing the unskilled labor constraints (6) which are defined for each individual country by one aggregate constraint covering all countries.

Notes

[1] The concavity of the social welfare functions allows application of the Kuhn-Tucker conditions in Section 2. Since (1) is ordinal, the weaker requirement that (1) possess a positive montone transformation that is concave is sufficient for this purpose.

[2] It is implicitly assumed that a skilled worker would not take a semi-skilled job if a skilled job were available.

[3] This ensures that shadow prices are nonnegative.

[4] This general notation is convenient here. It is not used elsewhere in the paper.

[5] It also is assumed that the Slater condition is satisfied.

[6] Occassionally, one or the other will be zero.

[7] It is possible to treat (14) and (15) as equalities since labor quantity will be zero whenever the strict inequality holds.

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