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ELIAS - A MODEL OF MULTISECTORAL ECONOMIC GROWTH IN A SMALL OPEN ECONOMY

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CONTENTS

Abstract

In this study a computable, multi-period general equilibrium model of a small open economy is presented. The origin of the basic modeling approach is the Norwegian so-called MSG-model, developed by L Johansen in the late 1950's. As the present model is intended to be a tool for analysis of resource allocation problems related to changes in international energy prices or domestic energy policies, energy demand and the substitutability of energy and other factors of production are modeled in some detail. That also applies to foreign trade and the relation between world market and domestic prices of traded goods. Foreign trade is modeled by means of explicit, relative price dependent, import and export functions derived on the basis of the so-called Armington assumption.

In the model the supply side is considerably more elaborated than the demand side. Thus, different vintages of production units are distinguished in each sector, and there is a distinction between the substitutability of capital, labor, fuels and electricity ex ante and the corresponding substitutability ex post. On the other hand, the formation of disposable incomes is treated in a rather crude way, and financial markets are disregarded.

The purpose of this report is to describe the model in some detail, and to indicate the nature of the solution procedure and some problems in connection with the estimation of the numerical values of the model's parameters. In this report, however, no results form model simulations are presented.

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1. Introduction

In this report a computable model of medium term multisectoral economic growth in a small, open economy is presented. The model is based on basic notions of general equilibrium theory; supply and demand factors interact and product and factor prices are assumed to be flexible enough to ensure equilibrium on all markets. However, the supply side is considerably more elaborated than the demand side. Thus, in each production sector different vintages of capital are distinguished and the substitutability of capital, labor, and different kinds of energy is modeled in some detail. Factors such as the formation of disposable incomes and the determination of savings, on the other hand, are treated in a rather crude way. On the supply side the substitutability of energy, labor and capital, as well as the sectoral allocation of energy, labor, and investments, is treated in some detail. Accordingly, the model has been christened ELIAS, Energy, Labor, Investment Allocation and Substitution.

The general modeling approach is to emphasize the interdependence of goods and factor markets, and the role of relative prices in the medium term resource allocation process. Moreover, with two exceptions[1] the structural equations of the model are derived from explicit production and preference functions together with optimization behavior assumptions rather than from econometric analyses of the past behavior of the economy. To a large extent the specification of the model, as well as the estimation of parameter values, is directly based on neoclassical economic theory. Accordingly, the present model primarily is a tool for quantitative analysis within the conceptual framework suggested by general equilibrium theory and the theory of economic growth, and in its present form it is not primarily intended to be a forecasting model.

The origin of the modeling effort presented here is the work of Johansen in the late 1950's.(See Johansen, 1960.) However, Johansen's original so-called MSG-model essentially was a general equilibrium model of a closed economy; net exports entered as exogenously determined final demand components and complementary imports were proportional to domestic production. This property has been retained in later Norwegian versions of the MSG-model. (See for instance Longva et al, 1980.)

The first Swedish MSG-model, developed by Restad (1976), incorporated a current account constraint. On the basis of that constraint and fixed import shares in the domestic production sectors and a given composition of total exports, the "export requirements" were calculated. The compatibility of these "export requirements" and world market demand conditions was then analyzed outside the model. In a later Swedish MSG-model, developed by the author (see Bergman, 1978), explicit price-dependent export and import functions were incorporated. Thus, both the supply side and the demand side of the "rest of the world" were taken into account.

However, although these models are models of small, open economies, neither of them incorporates foreign trade in a theoretically satisfactory way. According to the conventional notion of a "small, open economy", the prices of traded goods should be determined by and be equal to world market prices. In the long run factor prices, and the structure of the production system, in the small economy have to adjust in order to maintain equilibrium between world market prices and domestic production costs. This feature of multisectoral economic growth in open economies is, to a varying extent, lacking in the MSG-models mentioned above. This study is an attempt to incorporate foreign trade in the MSG-model in a way which is consistent with the notion of a "small, open economy"[2] as well as with observations on the actual specialization pattern in the economy.

It is well-known that a multisectoral growth model of an open economy with two homogeneous primary factors of production, linearly homogeneous production functions and parametric world market prices, i.e. a model based on the standard assumptions of the Heckscher-Ohlin model, fails to generate development paths similar to actually observed development paths in real-world economies. Thus, while there are, at most, two producing and exporting sectors in the trade-exposed part of the model economy in equilibrium (or, in general, m producing sectors if there are m homogeneous factors of production) (see Samuelson (1953)), real-world economies exhibit a rather incomplete specialization pattern in the production system. Moreover, in the model economy traded goods of a given type are either domestically produced and exported or imported, while there is a considerable amount of intra-industry trade even within rather disaggregated real-world production sectors.

Of course, there is a number of ways in which the basic assumptions of the Heckscher-Ohlin model can be changed in order to achieve equilibrium allocations with several producing sectors in the trade-exposed part of the economy as well as with intra-industry trade. The approach adopted here is to stay within the general equilibrium framework, but add two factors of significant importance in a medium term perspective: The immobility of the existing capital stock, and the heterogeneity of products of the same type but with different country of origin.[3] The "small open economy" assumptions are retained in the sense that the model-economy is assumed to face a perfectly elastic supply of imports. Moreover, it is assumed that the development of production less export in the rest of the world can be treated as an exogenous magnitude in spite of the fact that the import of the home country is the export of the rest of the world.

Explicit recognition of the immobility of the existing capital stock affects the assumptions about supply conditions in the Heckscher-Ohlin model; a distincton has to be made between production functions ex ante and ex post. If the ex ante supply conditions are represented by a linearly homogeneous neoclassical production function in capital and labor, and capital becomes an immobile resource once it is invested, the ex post production function will exhibit decreasing returns to scale in the only variable factor, labor. Accordingly, unit production costs will be a function of the level of output, and a given production sector can always maintain international competitiveness by a suitable change in the scale of operation. Consequently, complete specialization is avoided.

Explicit distinction between products of the same type but with different country of origin, together with the assumption that these products are not perfect substitutes[4], implies that consumers at home, as well as abroad will demand both imported and domestically produced units of products of a given type. Consequently, there can be both import and export of each type of tradable goods in equilibrium.

This approach to the treatment of the intra-industry trade phenomenon was initially proposed by Armington (1969) and has later on been employed in several numerical general equilibrium models.[5] It should perhaps be noted that the "Armington assumption" implies that each trading sector in each country produces a unique good. Thus, even if the production functions are linearly homogeneous in the variable inputs, the relation mentioned above between the number of homogeneous factors of production and the number of producing sectors in equilibrium does not hold when the Armington assumption is adopted.

The purpose of this report is to describe the model and discuss its implementation on actual data. In Section 2 a one-sector version of the model is presented. There are two reasons for beginning the presentation by considering an aggregated version of the model. One is, of course, that it is easier for the reader to get an overview of the model when it is presented in a condensed form. The second reason is that the mechanisms generating economic growth in the model-economy can be demonstrated without complicating the picture by sectoral disaggregation. Thus, when the complete multi-sectoral model is presented in Section

3, the exposition can be focused on the treatment of intermediate inputs (particularly energy), the allocation of household expenditures between types of consumer goods, and the allocation of gross investments between production sectors. The procedure used for solving the model is briefly described in Section 4, and in Section 5, finally, some problems related to the practical implementation and use of the model.

2. A One-Sector Version of the Model

To begin with we consider a case where only one type of output is produced in the domestic production system. It is convenient to structure the exposition with respect to the three markets dealt with in the model: the commodity market, the labor market, and the capital market. For each one of these we consider the determinants of supply and demand, and specify the equilibrium conditions.

2.1 The Commodity Market

On the supply side there are two sources: import and domestic production. The "home" country is assumed to be small enough to face a completely elastic supply of imports. Thus, any demanded quantity of imports in period t, $M^D(t)$, is supplied at the price $P^M(t) = V(t)P^{WI}(t)$, where $V(t)$ is the exchange rate in period t and $P^{WI}(t)$ is the world market price, in foreign currency units, of the imported commodity in that period.

The domestic producers face an ex ante production function, $f(\tilde{K},\tilde{L},t)$, which is linearly homogeneous in capital, K, and labor, L, and has the usual neoclassical properties.[6] It is assumed that once an investment is carried out, the invested capital becomes an immobile resource, subject to depreciation at a constant annual rate. Thus, assuming that the ex ante production function is a Cobb-Douglas function, the technological constraints on the production system can be characterized by a set of ex post production functions

$$X_\upsilon(t) = K_\upsilon(t)^\alpha L_\upsilon(t)^{1-\alpha} e^{\lambda\upsilon}; \qquad \upsilon=0,1,\ldots,t-1 \qquad (1)$$

where $K_0(0)$ is given as an initial condition and $K(t)$ is exogenously given by

$$K_\upsilon(t) = (1-\delta)^{t-\upsilon-1} I(\upsilon); \qquad \upsilon=0,1,\ldots,t-1; \quad t>0$$

and where δ is the exogenously given rate of depreciation and $I(\upsilon)$ is gross investment in period υ. The exogenously given rate of emboded technical progress is represented by the shift parameter λ in the ex ante production function.

Assuming optimization behavior on the part of the producers, technological constraints and producer behavior can be summarized by a set of profit functions, one for each vintage and time period. These profit functions can be written

$$\Pi_\upsilon(P,W;t) = K_\upsilon(t) e^{\lambda\upsilon} (1-\alpha)^{\frac{1-\alpha}{\alpha}} P^{\frac{1}{\alpha}} W^{\frac{\alpha-1}{\alpha}}.$$

By Hotelling's lemma (see for instance Varian, 1978) the supply from an individual vintage of production units is given by the partial derivative of the profit function with respect to the price of output. Thus, it holds that

$$X_\upsilon(t) = K_\upsilon(t) e^{\lambda\upsilon} \left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{\frac{\alpha-1}{\alpha}}; \qquad (2)$$

and the total supply of domestically produced goods is

$$X(t) = \sum_{\upsilon=0}^{t-1} X_{\upsilon}(t); \tag{3}$$

It is assumed that imports qualitatively differ from domestically produced commodities, and that there is a constant elasticity of substitution, μ, between the two sources of supply in all domestic uses. Thus, domestic users demand as CES-composite, Y, of imports and domestically produced commodities, defined by a time-independent "production" function

$$Y = \{d_d (X-Z)^{\frac{\mu-1}{\mu}} + d_m M^{\frac{\mu-1}{\mu}}\}^{\frac{\mu}{\mu-1}}$$

where Z is export of the domestically produced commodity.

The price index, $P^D(t)$, of the composite good is defined by the unit cost function corresponding to the "production" function defining the composite good. Accordingly

$$P^D = \phi(P, P^M) = \{d_d^{\mu} P^{1-\mu} + d_m^{\mu} P^{M^{1-\mu}}\}^{\frac{1}{1-\mu}}; \tag{4}$$

Thus, the domestic producers receive $P(t)$ for their products and the importers receive $P^M(t)$, while the domestic consumers pay $P^D(t)$ for the composite good they consume.

The demand for the composite good is the sum of consumption and investment demand, i.e. the sum of $C(t)$ and $I(t)$. By Shephard's (see for instance Varian, 1978) lemma the demand for domestically produced goods per unit of composite goods demanded is given by the partial derivative of $\psi(P, P^M)$ with respect to P. Thus, the equilibrium condition for the market for domestically produced goods can be written

$$X(t) = \frac{\partial\psi(P(t), P^M(t))}{P}\{C(t) + I(t) + Z(t);$$

or

$$X(t) = d_d^{\mu}\left(\frac{P^D(t)}{P(t)}\right)^{\mu} \{C(t) + I(t)\} + Z(t); \qquad (5)$$

where Z(t) is the export demand for domestically produced goods.

In a parallel way the equilibrium condition for imported goods becomes

$$M(t) = d_m^{\mu} \left(\frac{P^D(t)}{P^M(t)}\right)^{\mu} \{C(t) + I(t); \qquad (6)$$

Division of eq 5 by eq 6 yields another expression for import demand (using the definition $P^M(t) = V(t)P^{WI}(t)$)

$$M(t) = \left(\frac{d_m}{d_d}\right)^{\mu} \left(\frac{P(t)}{V(t)P^{WI}(t)}\right)^{\mu} \{X(t) - Z(t)\};$$

Assuming that the import to the "rest of the world" from the "home" country can be described by an import function of this kind, and also assuming that the "home" country is small enough to make $X(t) - Z(t) \approx X(t)$ in the rest of the world, the export function of the home country becomes

$$Z(t) = Z^0 \left(\frac{P(t)}{V(t)P^{WE}(t)}\right)^{\varepsilon} e^{\sigma t}; \tag{7}$$

where Z^0 is a constant, ε the elasticity of substitution between domestically produced and imported goods in the "rest of the world", P^{WE} is the price of goods and σ is the rate of growth of production in the rest of the world.[7]

The demand for composite goods for investment purposes is derived from an assumption that a fixed fraction, s, of total factor income is saved, and that the sum of domestic savings and the current account deficit is spent on fixed investment. Thus it holds that

$$sP(t)X(t) - V(t)D(t) = P^D(t)I(t) \; ; \tag{8}$$

The demand for composite goods for consumption purposes is determined by the budget constraint of the consuming sector and the savings behavior assumption introduced above. If the budget constraint of the consuming sector is explicitly incorporated in the model, an equilibrium solution will imply current account equilibrium by Walras' law. Alternatively a current account constraint can be incorporated as a way of implicitly defining the budget constraint of the consuming sector. The latter approach is adopted here, and thus $P^D(t)C(t)$ is implicitly determined by

$$P(t)Z(t) = P^M(t)M(t) + V(t)D(t) \tag{9}$$

where $V(t)D(t)$ is the exogenously given current account surplus.

2.2 The Labor Market

The supply of labor, $L(t)$, is completely inelastic and determined outside the model. The allocation of the labor force over vintages is, however determined within the model, and in such a way that the total demand for labor equals the supply. Thus the labor market part of the model can be described by means of one single equation; the equilibrium condition for the labor market. By Hotelling's lemma this condition can be written

$$L(t) = \sum_{\upsilon=0}^{t-1} - \frac{\partial \Pi_{\upsilon}^{>}(P(t),W(t);t)}{\partial W}$$

or

$$L(t) = \sum_{\upsilon=0}^{y-1} K\ (t)\ e^{\lambda\upsilon}\left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{-\frac{1}{\alpha}} \tag{10}$$

2.3 The Capital Market

In this part of the model the real rate of interest is determined by the process in which new vintages of production units are created. In the one-sector version of the model the creation of new vintages of production units is essentially equivalent to the choice of capital intensity in the new vintage. This is because the gross savings ratio is exogenously given. Thus, the overall rate of capital formation is not related to the real rate of interest.

The choice of capital intensity in new plants depends, among other things, on the expected future prices of output and labor. Price expectations are represented by the following functions

$$\tilde{P}(t) = \zeta^P P(t) + (1-\zeta^P)P^F(t); \tag{11}$$

$$\tilde{W}(t) = \zeta^W W(t) + (1-\zeta^W)W^F(t); \tag{12}$$

where $\tilde{P}(t)$ and $\tilde{W}(t)$ are the prices which producers expect to be the represenative, or average, prices during the life-time of the production unit under construction.

The parameters ζ^P and ζ^W are simply tools for the specification of different expectation formation mechanisms. Thus, when ζ^P and ζ^W are equal to unity, price expectations are "static". The variables $P^F(t)$ and $W^F(t)$ are exogenously determined. They can either be regarded as weighted averages of previous prices and wages, or as some kind of official price and wage forecast. If $\zeta^P = \zeta^W = 0$, and $P^F(t)$ and $W^F(t)$ are given appropriate values, a case with "rational" expectations can be simulated.

New production units are designed on the basis of the ex ante production function, or, equivalently, the ex ante unit cost function. The equilibrium real rate of interest, R(t), should be such that the expected unit production cost in production units under construction should be equal to the expected price of output. Thus, denoting the ex ante unit cost function by κ (W, Q; t), the real rate of interest is determined by the equilibrium condition

$$\tilde{P}(t) = \kappa(\tilde{W}(t), \tilde{Q}(t); t);$$

or, with $\kappa(\cdot)$ written in explicit form,

$$\tilde{P}(t) = \frac{1}{e^{\lambda t}} \alpha^{-\alpha} (1-\alpha)^{\alpha 1} Q(t)^{\alpha} \tilde{W}(t)^{1-\alpha}; \qquad (13)$$

where $\tilde{Q}(t)$ the "user cost of capital", is defined by

$$\tilde{Q}(t) = \tilde{P}^{D}(t)\left(\delta+R(t)\right); \qquad (14)$$

2.4 A Summing-up

The equations of the one-sector version of the model are displayed in Table 1. As can be seen in the table, the complete model can be described by means of 11 equations in 11 unknowns.

Table 1 The Equations of the One-Sector Version of the Model

Supply of Domestically Produced Goods

$$X(t) = \sum_{\upsilon=0}^{t-1} X_{\upsilon}(t) = \sum_{\upsilon=0}^{t-1} K_{\upsilon}(t) \; e^{\lambda\upsilon} \left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{\frac{\alpha-1}{\alpha}};$$

Demand for Imports

$$M(t) = d_m^{\mu} \left(\frac{P^D(t)}{P^M(t)}\right)^{\mu} \{C(t) + I(t)\};$$

Export Demand for Domestically Produced Goods

$$Z(t) = Z^0 \left(\frac{P(t)}{V(t)P^{WE}(t)}\right)^{-\varepsilon} e^{\sigma t};$$

Current Account Constraint

$$P(t)\ Z(t) = P^M(t)\ M(t) + V(t)\ D(t);$$

Savings - investment equilibrium

$$sP(t)\ X(t) - V(t)\ D(t) = P^D(t)\ I(t);$$

Product Market Equilibrium

$$X(t) = d_d^{\mu} \left(\frac{P^D(t)}{P(t)}\right)^{\mu} \{C(t) + I(t)\} + Z(t);$$

Labor Market Equilibrium

$$L(t) = \sum_{\upsilon=0}^{t-1} K_{\upsilon}(t)\ e^{\lambda\upsilon} \left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{-\frac{1}{\alpha}};$$

Capital Market Equilibrium

$$\tilde{P}(t) = e^{-\lambda t}\ \alpha^{-\alpha}\ (1-\alpha)^{\alpha-1}\ \{\tilde{P}^D(t)\ (\delta+R(t))\}^{\alpha}\tilde{W}(t)^{1-\alpha};$$

Definitions

$$P^M(t) = V(t)P^{WI}(t)\ ;$$

$$P^D(t) = \{d_d^{\mu}P(t)^{1-\mu} + d_m^{\mu}\ P^M(t)^{1-\mu}\}^{\frac{1}{1-\mu}};$$

$$\tilde{P}(t) = \zeta^{P} P(t) + (1-\zeta^{P})\ P^{F}(t);$$

$$\tilde{W}(t) = \zeta^{W} W(t) + (1-\zeta^{W})\ W^{F}(t);$$

Endogenous Variables

$P(t)$, $P^{D}(t)$, $W(t)$, $R(t)$, $X(t)$, $C(t)$, $I(t)$, $Z(t)$,

$M(t)$, $\tilde{P}(t)\tilde{P}^{D}(t)$, $\tilde{W}$ (t).

Exogenous Variables

$L(t)$, $D(t)$, $V(t)$, $P^{WI}(t)$, $P^{WE}(t)$, $P^{F}(t)$, $W^{F}(t)$.

As has already been mentioned, the basic elements of the model are contained in the one-sector version. By disaggregating the model into several sectors, however, it becomes a tool for analysis of structural changes, hidden in the growth process generated by the one-sector model. In order to extend the model presented in this section into a multisectoral model, basically three additional features of the economic system have to be incorporated: interindustry transactions, allocation of consumer expenditures between categories of consumer goods, and allocation of investible funds between investing sectors. The difference between the one-sector model discussed so far and the multi-sectoral model presented in the next section is that the latter includes mechanisms which can handle these types of phenomena.

3. The Multisectoral Model

Basically the exposition in this section is structured in the same way as in Section 2. That is, supply and demand factors as well as equilibrium conditions for each one of the commodity, labor, and capital markets are dealt with in separate sub-sections. Before that, however, the necessary background for the exposition is given in three introductory sub-sections. Thus, a general overview of the model is given in 3.1 and in 3.2 the goods and sectors in the model economy are defined. In subsection 3.3 the assumptions about technology are discussed, and the unit cost and profit functions utilized later on in the exposition are derived. Then the commodity, labor and capital markets are dealt with in sub-sections 3.4, 3.5 and 3.6 respectively. In sub-section 3.7 the complete model is summarized. All symbols used in the exposition are defined in an appendix to this section.

3.1 General Characteristics

The model is a multisector, multiperiod model of economic growth in a small, open economy, i.e., an economy facing a perfectly elastic supply of imports at given world market prices. It is assumed that capital once invested in a specific sector cannot be reallocated to some other sector. Moreover, technical change is assumed to be embodied. Consequently there is a number of "vintages" of capital in each production sector at a given point in time.

The total supply of labor is given, and so is the gross savings ratio. For each period the model endogenously allocates the available labor force over sectors and vintages, and total gross investments over sectors. Investments in period t gives productive capacity from period t+1 and on. The allocation mechanisms are derived from optimization behavior assumptions. Thus, producers are assumed to maximize profits subject to technological constraints and the single aggregated household sector is assumed to maximize a utility function subject to a budget constraint.

Technological constraints and the utility functions are exogenously given and explicitly specified. (The choice of specification is discussed in Section 5.) The public sector is treated as a production sector in which the use of inputs is determined on the basis of cost minimization considerations. The demand for public services is exogenously given.

There are no financial assets in the model, and the exchange rate is exogenously given. The budget constraint of the household sector is implicitly determined by a constraint on the current account. Thus, the tax and transfer system is essentially implicit in the model. All product markets and the labor market are treated as if they were competitive, and generally relative prices are assumed to be flexible enough to clear all markets. However, in another version of the model, rigidities in the adjustment of real wages are introduced.

3.2 Goods and Sectors

The model describes an economy with n+3 production sectors producing n+3 goods. There is no joint production, and consequently each good is produced in one sector only. Accordingly, there is no real distinction between domestically produced goods and the domestic production sectors. However, when it is natural to think in terms of "sectors" the index j will be used, while index i will be used when it is natural to think in terms of "goods".

The production sectors are numbered from 0 to n+2, 0 being the fuels production sector and 1 the electricity sector, while n+1 is the housing sector and n+2 the public sector. Foreign trade is ruled out in the last two sectors, as is exports from sector 0. There is also a "book-keeping" sector, n+3, in which different goods are aggregated into one single capital good.

Thus, the economy producers n+3 goods of which n, at least in principle, can be exported. Imports are classified by means of commodity index running from 0 to n. In addition to these n+3 goods, two types of complementary imports are used within the economy. These complementary imports are inputs used in the fuels and electricity producing sectors, respectively, and which cannot be produced within the country. Examples of such inputs are crude oil used in domestic refineries in a country without oil resources, or enriched uranium used in domestic nuclear power plants in a country without uranium enrichment capacity.

3.3 Technology and Producer Behavior

The basic assumption about technology is that there exists a constant return to scale ex ante production function in each sector, i.e. a relation between planned output and the planned use of inputs. This relation is specified in accordance with

$$\tilde{X}_j = \min \{f_j(\tilde{K}_j,\tilde{L}_j,\tilde{X}_{0j},\tilde{X}_{1j};\upsilon),\ \frac{X_{2j}}{a_{2j}},\ldots,\frac{\tilde{X}_{nj}}{a_{nj}}\} \ ; \qquad j=0,1,\ldots,n+2$$

where X_j is output, K_j the use of capital, L_j the use of labor, X_{ij} the use of intermediate input i in sector j and ~ indicates the ex ante nature of the variables. The coefficients $a_{2j},\ldots,a_{nj}$ indicate the minimum input of the corresponding good per unit of output in sector j. The variable υ is a time index, indicating that the ex ante production function shifts over time.

On the basis of the ex ante production function efficiency requires that

$$\tilde{X}_j = f_j(\tilde{K}_j,\tilde{L}_j,\tilde{X}_{0j},\tilde{X}_{1j};\upsilon); \qquad j=0,1,\ldots,n+2$$

$$\tilde{X}_{ij} = a_{ij}\tilde{X}_j; \qquad i=2,3,\ldots,n \ ; \quad j=0,1,\ldots,n+2$$

In parametric form the functions $f_j(\cdot)$ are nested CES - Cobb Douglas functions. Thus,

$$\tilde{X}_j = A_j \ \{a_j\tilde{F}_j^{\rho_j} + b_j\tilde{H}_j^{\rho_j}\}^{\frac{1}{\rho_j}} \ ;$$

$$\tilde{F}_j = \tilde{K}_j^{\alpha_j} \tilde{L}_j^{1-\alpha_j} e^{\lambda_j \upsilon}$$

$$H_j = \{c_j \tilde{X}_{0j}^{\gamma_j} + d_j \tilde{X}_{1j}^{\gamma_j}\}^{\frac{1}{\gamma_j}} e^{\lambda_j^* \upsilon}$$

where $(1-\rho_j)^{-1}$ and $(1-\gamma_j)^{-1}$ are elasticities of substitution, and λ_j and λ_j^* are exogenously given rates of technological progress.

Whereas the ex ante production function is a planning concept, the ex post production function is a relation between actual output and the actual use of inputs. It can be derived from the ex ante production function by assuming that the capital stock and the energy input coefficients are given once a production unit has been taken into operation. Moreover, it is assumed that the once installed capital equipment depreciates at a given, sector specific rate. This means that the ex post production function in period t for vintage υ in sector j can be written

$$X_{\upsilon j}(t) = \min \left\{g_{\upsilon j}(L_{\upsilon j}(t),t), \frac{X_{\upsilon 0j}(t)}{a_{\upsilon 0j}}, \frac{X_{\upsilon 1j}(t)}{a_{\upsilon 1j}} \frac{X_{\upsilon 2j}(t)}{a_{\upsilon 2j}}, \ldots, \frac{X_{\upsilon nj}(t)}{a_{\upsilon nj}}\right\} \qquad j = 0,1,\ldots,n+2$$

where $X_{\upsilon j}(t)$ and $L_{\upsilon j}(t)$ are production and employment, respectively, in vintage υ in sector j, and $a_{\upsilon 0j}$ and $a_{\upsilon 1j}$ are minimum energy input coefficients. The production function $g_{\upsilon j}(.)$ is directly related to the ex ante production function by

$$g_{\upsilon j}(L_{\upsilon j},t) = f_j(K_j,L_j,X_{0j},X_{1j},\upsilon)$$

subject to

$$K = I_j(\upsilon)(1-\delta_j)^{t-\upsilon-1} \quad ; \ t \geq \upsilon + 1$$

$$X_{0j} = a_{\upsilon 0j} X_{\upsilon j}$$

$$X_{1j} = a_{\upsilon 1j} X_{\upsilon j}$$

where $I_j(\upsilon)$ is the gross investment in sector j in period υ. Using the symbols of the ex ante production function, the function $g_{\upsilon j}(\cdot)$ thus becomes

$$g_{\upsilon j}(L_{\upsilon j},t) = \left| \frac{a_j [A_j \; ((1-\delta_j)^{t-\upsilon-1} I_j(\upsilon))^{\alpha_j} \; e^{\lambda_j \upsilon}]^{\rho_j}}{1-b_j A_j^{\rho_j} \; \{(c_j a_{\upsilon 0j}{}^{\gamma_j} + d_j a_{\upsilon 1j}{}^{\gamma_j})^{\frac{1}{\gamma_j}} e^{\lambda_j^* \upsilon}\}^{\rho_j}} \right|^{\frac{1}{\rho_j}} L_{\upsilon j}{}^{(1-\alpha_j)} \equiv$$

$$\equiv A_{\upsilon j}(t) L_{\upsilon j}^{1-\alpha_j}$$

Efficiency requires that in each time period, sector and vintage, it holds that

$$X_{\upsilon j}(t) = A_{\upsilon j}(t) L_{\upsilon j}(t)^{1-\alpha_j} ; \qquad \begin{array}{l} j = 0,1,\ldots,n+2 \\ \upsilon = 0,1,\ldots,t-1 \end{array}$$

$$X_{\upsilon ij}(t) = \begin{cases} a_{\upsilon ij} X_{\upsilon j}(t) & \text{when } i = 0,1 \\ a_{ij} X_{\upsilon j}(t) & \text{when } i = 2,3,\ldots,n; \end{cases}$$

$$\begin{array}{l} j = 0,1,\ldots,n+2 \\ \upsilon = 0,1,\ldots,t-1 \end{array}$$

Observe that energy input coefficients differ across vintages in a given sector, while that is not the case for the non-energy intermediate coefficients. Note also that the input of labor per unit of output is dependent on the level of production. Thus, while the ex ante technology exhibits constant returns to scale, the ex post technology exhibits decreasing returns to scale.

With these equations the description of technology is complete. However, as profit maximization behavior on the part of the producers is assumed throughout, it is more convenient to use a dual representation of technology and producer behavior. That is, to describe the ex ante technology by means of an ex ante unit cost function and the ex post technology by means of profit functions for each vintage of production units. This means that input demand and output supply functions can be determined without explicitly making use of necessary conditions for profit maximum. However, before the derivation of the cost and profit functions, the input and output prices facing the producers have to be defined.

In accordance with the assumption that the modeled economy is "small", it is assumed that there is a perfectly elastic supply of imports at world market prices P_i^{WI} (and P_i^c for complementary imports). As these prices should be regarded as c.i.f. prices, tariffs and indirect taxes have to be added in order to get the market prices of imports. Moreover, as P_i^{WI} is a price in foreign currency units, it has to be multiplied by an exchange rate. Thus, the relation between c.i.f. import prices in foreign currency, P_i^{WI}, and domestic market prices of imports, P_i^M, is given by:

$$P_i^M(t) = (1 + \frac{\Delta\phi}{1+\phi_i^0})V(t)P_i^{WI}(t) \; ; \quad i = 0,1,\ldots,n \tag{15}$$ [8]

The variable $V(t)$ is the exogenously given exchange rate. Moreover, $V(t)P_i^{WI}(t)$ where is a major import good is taken to be the numeraire of the price system, and is accordingly set equal to unity. This means that the other $P_i^{WI}(t)$:s represent world market prices relative to the world market price of the numeraire good.

It is assumed that imported and domestically produced goods with the same classification, say i, are not perfect but relatively close substitutes, Thus, equally classified goods from the two sources of supply are aggregated into a composite good in accordance with some "production" function representing their substitutability. This function is assumed to be a CES-function and to apply to all domestic users of the goods in question.

Under these conditions the unit cost of composite good i is solely a function of the price of imported units of good i, $P_i^M(t)$, and the price of domestically produced units of good i, $P_i(t)$. Thus, if the "price" of a composite good, $P_i^D(t)$, is defined as the unit cost of that good, we get

$$P_i^D(t) = \phi_i(P_i(t),\ P_i^M(t)) \equiv \{d_{di}^{\mu_i} P_i(t)^{1-\mu_i} + d_{mi}^{\mu_i} P_i^M(t)^{1-\mu_i}\}^{\frac{1}{1-\mu_i}};$$

$$i = 0,1,\ldots,n \qquad (16)$$

On the basis of Shepherd's lemma (see Varian, 1978) the demand for domestically produced goods of type i per unit of composite good i demanded is given by the partial derivative of $\phi_i(P_i,\ P_i^M)$ with respect to P_i. The demand for imported goods of type i is determined in a parallel way.

As there is only one type of labor in the model economy and a competitive labor market is assumed, there should only be one wage rate. However, in order to make it possible to roughly take labor heterogeneity into account, an exogenous wage structure is imposed. Thus the sectoral wage rates are defined by

$$W_j(t) = \omega_j W(t) \; ; \; j = 0,1, \ldots, n+2 \; ; \qquad (17)$$

where $W_j(t)$ is the sectoral wage rate, $W(t)$ an index of the general wage rate and ω_j a sector specific parameter.

Energy prices might differ across sectors as a result of sector specific energy taxes. Thus, it holds that

$$P_{ij}(t) = \left(1 + \tau_i(t) + \xi_{ij}(t)\right) P_i^D \; ; \qquad i = 0,1, \; j = 0,1, \ldots, n+2 \; ; \qquad (18)$$

where $\tau_i(t)$ is a general tax on energy of type i and $\xi_{ij}(t)$ is a specific tax on the use of that kind of energy in sector j.

Decisions concerning actual production are based on current market prices, i.e., $P_{0j}^D(t)$, $P_{1j}^D(t)$, $P_2^D(t),\ldots,P_n^D(t)$, $W_j(t)$, and the ex post production functions, while decisions concerning the design of new plants are based on expected prices and the ex ante production functions. Expected prices of outputs and inputs respectively, are determined by

$$\tilde{P}_i(t) = \zeta_i^0 P_i(t) + (1-\zeta_i^0) P_i^{OF}(t); \; i=0,1,\ldots,n+2; \qquad (19)$$

$$\tilde{P}_i^D(t) = \zeta_i^I P_i^D(t) + (1-\zeta^I)P_i^{DF}(t); \quad i=0,1,\dots,n; \qquad (20)$$

$$\tilde{P}_i(t) = \zeta_i^C V(t)P_i^C(t) + (1-\zeta_i^C)P_i^{CF}(t); \quad i=0,1; \qquad (21)$$

$$\tilde{W}_j(t) = \zeta^W W_j(t) + (1-\zeta^W)W_j^F(t); \qquad j=0,1,\dots n+2; \; (22)$$

where $P_i^{hF}(t)$, $h=0,I,C,W$, are exogenous variables. By an appropriate choice of values of these variables, and setting $\zeta_i^h=1$, $h=0,I,C,W$, a case with rational expectations can be stimulated, while $\zeta_i^h=1$, $h=0,I,C,W$, implies static expectations. Throughout expectations about indirect taxes and tariffs as well as about the exchange rate are assumed to be static.

The user cost of capital is determined by

$$\tilde{Q}_j(t) = \tilde{P}_{n+3}(t)\left(\delta_j+R(t)\right); \qquad j=0,1,\dots,n+2 \qquad (23)$$

where

$$\tilde{P}_{n+3}(t) = \sum_{i=2}^{n} \tilde{P}_i^D(t)a_{i,n+3}; \qquad (24)$$

where the coefficients $a_{i,\ n+3}$ add up to unity, and where $R(t)$ is the real rate of interest.

The profit function for a production unit of vintage υ in sector j can now be defined as the solution to the problem

$$\Pi_{\upsilon j}(P^*_{\upsilon j},W_j;t) = \max_{X_{\upsilon j},L_{\upsilon j}} \{P^*_{\upsilon j}X_{\upsilon j}-W_jL_{\upsilon j} \| = A_{\upsilon j}(t)L_{\upsilon j}^{1-\alpha_j}\}$$

$$j=0,1,\dots,n+2; \quad \upsilon=0,1,\dots,t-1;$$

where,[9] in a given time period t,

$$P^*_{\upsilon j}(t) = (1-\Theta_j)P_j(t) - \sum_{i=0}^{1} P_{ij}(t)a_{\upsilon ij} -$$

$$- \sum_{i=2}^{n} P^D_i(t)a_{ij} - V(t)P^C_j(t)b_{jj}; \quad \begin{matrix} j=0,1,\ldots,n+2 \\ \upsilon=0,1,\ldots t-1 \end{matrix} \qquad (25)$$

In explicit form the profit function becomes

$$\Pi_{\upsilon j}(P^*_{\upsilon j},W_j;t) = A_{\upsilon J}(t)^{\frac{1}{\alpha_j}} \alpha_j(1-\alpha_j)^{\frac{1-\alpha_j}{\alpha_j}} P^{*\frac{1}{\alpha_j}}_{\upsilon j} W_j^{\frac{\alpha_j-1}{\alpha_j}}; \qquad (26)$$
$$j=0,1,\ldots,n+2$$

The ex ante unit cost function represents the minimum cost of inputs per unit of output at given prices, subject to the ex ante production function in per unit form. As the ex ante technology exhibits constant returns to scale, the ex ante unit cost is independent of the expected level of production, and thus a function of expected input prices only. According to the ex ante production function, non-energy intermediate inputs are used in fixed proportions, while the proportions of other inputs can be varied. Thus, disregarding time indices the ex ante unit cost κ_j is given by the function

$$\kappa_j = \kappa^*_j(\tilde{W}_j,\tilde{Q}_j,\tilde{P}_{0j},\tilde{P}_{1j};\upsilon) + \sum_{i=2}^{n} \tilde{P}^D_i a_{ij} + \tilde{V}\tilde{P}^C_j b_{jj} + \Theta\tilde{P}_j;$$
$$j=0,1,\ldots,n+2 \qquad (27)$$

where the net unit cost function $\kappa^*_j(\cdot)$ represents the minimum cost of labor, capital, fuels and electricity per unit of output. The other terms represent the cost of non-energy intermediate inputs, complementary imports and an ad valorem output tax, respectively.

Thus, the net unit cost function $\kappa_j^*(\cdot)$ is the solution to the problem

$$\text{Min} \frac{1}{\tilde{X}_j} \{W_j\tilde{L}_j + Q_j\tilde{K}_j + P_{0j}\tilde{X}_{0j} + p_{1j}\tilde{X}_{1j}\};$$

subject to

$$\tilde{X}_j = A_j\{a_j\tilde{F}_j^{\rho_j} + b_j\tilde{H}_j^{\rho_j}\}^{\frac{1}{\rho_j}}$$

$$\tilde{F}_j = \tilde{K}_j^{\alpha_j}\tilde{L}_j^{1-\alpha_j}e^{\lambda_j \upsilon}$$

$$\tilde{H}_j = \{c_j\tilde{X}_{0j}^{\gamma_j} + d_j\tilde{X}_{1j}^{\gamma_j}\}^{\frac{1}{\gamma_j}}e^{\lambda_j^* \upsilon}$$

for $j = 0, 1, \ldots, n + 2$.

In explicit form the net unit cost function thus can be written

$$\kappa_j^*(W_j, Q_j, P_{0j}, P_{1j}; \upsilon) = A_j^{-1} \Big| a_j^{\frac{1}{1-\rho_j}} [e^{-\lambda\upsilon}\alpha_j^{-\alpha_j}(1-\alpha_j)^{\alpha_j-1} Q_j^{\alpha_j} W_j^{1-\alpha_j}]^{\frac{\rho_j}{\rho_j-1}} +$$
$$+ b_j^{\frac{1}{1-\rho_j}} [e^{-\lambda^*\upsilon}(c_j^{\frac{1}{1-\gamma_j}} P_{0j}^{\frac{\gamma_j}{\gamma_j-1}} + d_j^{\frac{1}{1-\gamma_j}} P_{1j}^{\frac{\lambda_j}{\lambda_j-1}})^{\frac{\gamma_j-1}{\gamma_j}}]^{\frac{\rho_j}{\rho_j-1}} \Big|^{\frac{\rho_j-1}{\rho_j}} ; \quad j=0,1,\ldots,n+2 \qquad (28)$$

Having now defined the prices, cost and profit functions of the model economy, the derivation of the complete model is quite straightforward. Output supply and input demand functions can be directly derived from the eqs (26) and (28), which are thus the main building blocks of the model.

3.4 The Commodity Markets

3.4.1 Supply of Domestically Produced Goods

By Hotelling's lemma the supply of domestically produced goods from production units of vintage υ in sector j is given by

$$\frac{\partial\Pi_{\upsilon j}(P^*_{\upsilon j}(t),W_j(t);t)}{\partial P^*_{\upsilon j}} = X_{\upsilon j}(t); \qquad \begin{array}{l} j=0,1,\ldots,n+2 \\ \upsilon=0,1,\ldots,t-1 \end{array}$$

Thus, on the basis of this equation and eq (26), total supply of goods produced in sector j is given by

$$X_j(t) = \sum_{\upsilon=0}^{t-1} A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \left|\frac{W_j(t)}{(1-\alpha_j)P^*_{\upsilon j}(t)}\right|^{\frac{\alpha_j-1}{\alpha_j}}; \qquad j=0,1,\ldots,n+2 \qquad (29)$$

There are four types of demand for goods produced in the home country: intermediate, consumption, investment and export demand. The first three types of demand are derived from the demand for composite goods, while export demand is a direct demand for domestically produced goods. Investment demand is implicitly determined by the savings-investment equilibrium condition, while the other types of demand have to be explicitly specified.

3.4.2 Intermediate Demand for Composite Goods

By the technology assumptions the demand for intermediate inputs is given by

$$X_{ij}(t) = \begin{cases} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij} X_{\upsilon j}(t) & \text{when } i=0,1 \\ a_{ij} X_j(t) & \text{when } i=2,3,\ldots,n \end{cases} \qquad j=0,1,\ldots,n+3 \tag{30}$$

3.4.3 Household Demand for Composite Goods

There is only one aggregated household sector in the model. It is assumed that the household sector maximizes a utility function of the form

$$u(t) = \sum_{s=1}^{k} \beta_s \log\left(Q_s(t) - Q_s^0\right); \qquad \sum_{s=1}^{k} \beta_s = 1$$

where $Q_s(t)$ is the consumption of a consumer commodity group, defined as a convex combination of the composite goods $i = 0,1, \ldots, n$ and the domestically produced good $n + 1$. The constraint on the parameters β_s implies that the resulting linear expenditure system will satisfy the budget constraint. The resulting household demand equations for composite goods can be written

$$C_i(t) = \sum_{s=1}^{k} t_{is} \{Q_s^0 + \frac{\beta_s}{P_s^Q(t)} (E(t) - \sum_{s=1}^{k} P_s^Q(t) Q_s^0)\}; \tag{31}$$

$$i = 0,1,\ldots,n+1$$

where $E(t)$ is total household consumption expenditures, and

$$P_s^Q(t) = \sum_{i=0}^{1} P_{ic}(t)t_{is} + \sum_{i=2}^{n} P_i^D(t)t_{is} + P_{n+1}(t)t_{n+1,s} \; ; \quad s=1,2,\ldots,k \qquad (32)$$

$$P_{ic}(t) = \left(1+\tau_i(t) + \zeta_{ic}(t)\right) P_i^D(t); \quad i=0,1 \qquad (33)$$

and where $\sum_{i=0}^{n+1} t_{is} = 1.$

3.4.4 Import and Export Demand

On the basis of Shephard's lemma and the definition of the price of composite good i (see eq (16)), the demand for competitive imports follows directly from the demand for composite goods. Thus, import demand can be written

$$M_i = \frac{\partial\phi_i\left(P_i(t),P_i^M(t)\right)}{\partial P_i^M} \left\{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\right\};$$

or, in explicit form,

$$M_i(t) = d_{mi}^{\mu_i} \left|\frac{P_i^D(t)}{P_i^M(t)}\right|^{\mu_i} \left\{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\right\}; \quad i=0,1,\ldots,n \qquad (34)$$

Observe that $M_i(t)$ is competitive import at constant purchaser's prices. In accordance with the discussion on p. 18, the corresponding import at constant c.i.f. prices is $M_i(t)/(1+\phi_i^0)$, where ϕ_i^0 represent base year tariffs and indirect taxes on imported goods of type i.

The demand for complementary imports follows from the technology assumptions, and can be written:

$$M_j^C(t) = b_{jj}X_j(t) \ ; \quad j = 0,1 \tag{35}$$

Export demand can be regarded as the import demand of the rest of the world. By assuming that there is a constant elasticity of substitution, ε_i, between goods produced in the rest of the world and goods with the same classification produced in the home country, the export demand equations can be written:

$$Z_i(t) = Z_i^0 \left| \frac{P_i(t)}{V(t)P_i^{WE}(t)} \right|^{\varepsilon_i} e^{\sigma_i t}; \qquad i=1,2,\ldots,4 \tag{36}$$

where σ_i is the exogenously given rate of growth of the production of commodity group i in the rest of the world.

3.4.5 Equilibrium Conditions

The derivation of the equilibrium conditions is straightforward. Thus, by Shephard's lemma and the definition of the price of composite good i the equilibrium conditions for the markets for domestically produced goods become

$$X_i(t) = \frac{\partial\phi_i(P_i(t),P_i^M(t))}{\partial P_i} \{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\} + Z_i(t); \qquad i=0,1,\ldots,n \tag{37}$$

or, in explicit form,

$$X_i(t) = d_{di}^{\mu_i}\left(\frac{P_i^D(t)}{P_i(t)}\right)^{\mu_i} \{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\} + Z_i(t); \quad i = 0, 1, \ldots, n \tag{38}$$

$$X_i(t) = C_i(t) \qquad i = n+1,\ n+2 \tag{39}$$

$$X_{n+3}(t) = I(t) \tag{40}$$

Observe that $C_{n+2}(t)$ is the exogenously determined public consumption.

The budget constraint of the household sector is indirectly defined by a current account constraint. Thus, it is assumed that an "invisible government" instantaneously adjusts taxes so that household consumption expenditure is kept at a level compatible with a target surplus (or deficit), $V(t)D(t)$, on the current account at the given level of public expenditures. Thus, each solution to the model will satisfy the condition

$$\sum_{i=1}^{n} P_i(t)Z_i(t) - \sum_{i=0}^{n} P_i^M(t)M_i(t) -$$

$$\sum_{j=0}^{1} V(t)P_j^C(t)M_j^C(t) = V(t)D(t) \tag{41}$$

It is assumed that a given fraction, s, of the gross national product at market prices, $Y_m(t)$, is saved. Consequently the savings-investments equilibrium condition becomes

$$sY_m(t) - V(t)D(t) - P^D_{n+3}(t)I(t) \tag{42}$$

where $I(t)$ is total gross investment and

$$Y_m(t) = E(t)+P_{n+2}(t)C_{n+2}(t)+P_{n+3}(t)I(t)+\sum_{i=1}^{n} P_i(t)Z_i(t) -$$

$$- \sum_{i=0}^{n} P_i^M(t) \frac{M_i(t)}{i+\phi_i^0} + \sum_{j=0}^{1} V(t)P_j^C(t)M_j^C(t); \tag{43}$$

and where the price of capital goods is given by

$$P_{n+3} = \sum_{i=2}^{n} P_i^D(t) a_{i,n+3}; \tag{44}$$

Substitution of (41) and (43) in (42) yields the following expression for the savings-investment equilibrium condition

$$\frac{s}{1-s} \{E(t)+P_{n+2}(t)C_{n+2}(t)\}-V(t)D(t) = P_{n+3}I(t); \tag{45}$$

3.5 The labor market

By Hotelling's lemma the demand for labor by production units of vintage υ, sector j is given by

$$\frac{\partial \pi_{\upsilon j}(P^*_{\upsilon j}(t), W_j(t), t)}{\partial W_j} = -L_{\upsilon j}(t); \qquad \begin{array}{l} j=0,1,\dots,n+2 \\ \upsilon=0,1,\dots,t-1 \end{array}$$

or in explicit form

$$L_{\upsilon j}(t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \left(\frac{W_j(t)}{(1-\alpha_j)P^*_{\upsilon j}(t)}\right)^{-\frac{1}{\alpha_j}}; \qquad \begin{array}{l} j=0,1,\dots,n+2 \\ \upsilon=0,1,\dots,t-1 \end{array} \tag{46}$$

The equilibrium condition for the labor market becomes

$$L(t) = \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} L_{\upsilon j}(t) \tag{47}$$

where L(t) is the exogenously given supply of labor.

One step towards a full representation of the economy's properties in the short run is to relax the assumption about fully flexible real wage rates. In an alternative specification of the model, real wages are assumed to be flexible only upwards, i.e. a minimum wage rate $\bar{W}(t)$ is incorporated. The labor market equilibrium condition (47) is then replaced by the condition

$$W_j(t) = \begin{cases} \omega_j \bar{W}(t) & \text{when } \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} - \dfrac{\partial \pi_{\upsilon j}(P^*_{\upsilon j}(t), \omega_j \bar{W}(t); t)}{\partial W_j} \leqslant L(t) \\ \omega_j W(t) & \text{when } \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} - \dfrac{\partial \pi_{\upsilon j}(P^*_{\upsilon j}(t), \omega_j \bar{W}(t), t)}{\partial W} > L(t) \end{cases} \tag{47'}$$

$$j=0,1,\ldots,n+2$$

where thus $\bar{W}(t)$ is the exogenously given index of the minimum real wage level, and $W(t)$ is an index of the market clearing real wage level.

3.6 The capital market

In order to make the model complete, it remains to allocate gross investments over sectors, determine the technological coefficients in production units which are to be taken into operation in period t+1, and to determine the real rate of interest. These three aspects of a solution to the model obviously are closely interconected, and the purpose of this sub-section is to discuss in some detail how the links between investment allocation, choice of technology and the real rate of interest are specified in the model. However, it should be stressed that even though the discussion is entirely based on theoretical considerations, the equations in this part of the model are not derived from neoclassical economic theory in the some direct way as the equations in other parts of the model. It should also be stressed that the main role of the equations discussed here is to provide the links between the various periods. That is, the equations (15)-(47) make up a complete model of resource allocation in period t, while the equations dealt with in this sub-section, "creates" the new vintages of production units to be taken into operation in period t+1.

In equilibrium the expected rate of return on marginal investments should be equal to the real rate of interest in all sectors. Thus, defining

$$\tilde{P}^*_j(t) = (1-\tilde{\theta}_j)\tilde{P}_j(t) - \sum_{i=2}^{n} \tilde{P}^D_i(t)a_{ij} - \tilde{V}(t)\tilde{P}^C_j(t)b_{jj};$$

$$j=0,1,\ldots,n+2 \quad (48)$$

total investments should be allocated between the sectors in such a way that

$$\sum_{j=0}^{n+2} I_j(t) = I(t); \quad (49)$$

where $I_j(t)$ is gross investments in sector j in period t, and the real rate of interest, R(t), satisfies the conditions

$$\tilde{P}^*_j(t) = \kappa^*_j(\tilde{W}_j(t),\tilde{Q}_j(t),\tilde{P}_{0j}(t),\tilde{P}_{1j}(t);t)$$

$$j=0,1,\ldots,n+2 \quad (50')$$

where, as before,

$$\tilde{Q}_j(t)=\tilde{P}_{n+3}(\delta_j+R(t)); \qquad j=0,1,...,n+2$$

However, as the model is specified there is no mechanism which could equalize the expected rates of return on investments across sectors. This is because the variables $\tilde{P}_j^*(t)$, which are determined by the equations (19)-(21) and (48), only reflect current prices and a set of exogenous variables. As the current prices of output and input can be determined prior to the solution of the capital market part of the model (by means of equations (15)-(47)) the variables $\tilde{P}_j^*(t)$ are in effect exogenously determined in the present context. Thus the expected rate of return on investments in sector j is independent of the amount of investments in that sector.

It may seem natural to change the specification of the price expectation equations (19)-(21) in such a way that an inverse relation between the amount of investments in sector j and the expected rate of return on investments in that sector was incorporated in the model. However, such a relation would be difficult to specify on theoretical grounds, and perhaps even more on empirical grounds. An alternative approach would be to retain the price expectation formation equations (19)-(21) in their present form, and to design an investment allocation mechanism which tends to equalize the expected rate of return on investments across sectors only in the long run. In that case there are many possible specifications, and of these two will be discussed here.

With exogenously determined expected prices of outputs and inputs, there is, of course, no reason to expect the equations (50') to be satisfied for an arbitrary value of the real rate of interest. However, by introducing a set of, positive or negative, excess profit rates, $r_j^*(t)$, the following equality can be established on the basis of Shephard's lemma

$$\tilde{P}_j^*(t) = \kappa_j^*(\tilde{W}_j(t),\tilde{Q}_j(t),\tilde{P}_{0j}(t),\tilde{P}_{1j}(t);t) +$$

$$+ r_j^*(t)P_{n+3}\frac{\partial\kappa_j^*(\cdot)}{\partial Q_j}; \qquad j=0,1,\ldots,n+2 \quad (50")$$

Next it is assumed that investments in sector j will take place in period t only if $r_j^*(t)\geq 0$, and that investment in sector j is an increasing function of $r_j^*(t)$ when $r_j^*(t)\geq 0$. On the basis of this rule, the following sectoral investment functions can be specified

$$I_j(t) = \begin{cases} \delta_j X_j(t)\dfrac{\partial\kappa_j^*(\cdot)}{\partial Q_j}\left(\dfrac{r_j^*(t) + R(t)}{R(t)}\right)^{\rho_j} & \text{when } r_j^*(t)\geq 0 \\ 0 & \text{when } r_j^*(t)<0 \end{cases} \quad (51')$$

$$j=0,1,\ldots,n+2$$

and again R(t) is determined by the capital market equilibrium condition (49). Observe that the ex ante unit cost function is evaluated at the user cost of capital measure $\tilde{Q}_j(t)$, i.e. the capital intensity of new production units reflects the real rate of interest, R(t).

With this specification investments are made only in sectors where the expected excess rate of return is non-negative. If $r_j^*(t)=0$ gross investments in sector j will be just enough to maintain the capacity in that sector, while additional producers will be attracted whenever $r_j^*(t)>0$. Moreover, ceteris paribus, $P_j^*(t+1)$ will be inversely related to $I_j(t)$. Thus, in the long run the $r_j^*(t)$ tend to be equalized across sectors.

However, the problem with this specification is that there is no reason to expect the $r_j^*(t)$ to tend to zero in the long run. Since the value of $R(t)$ is determined by the capital market equilibrium condition, it might happen that all sectoral excess profit rates attain a common positive value, $r^*(t)$. In that case the expected rate of return on marginal investments will be equalized across sectors but higher than the real rate of interest. This is a strange situation implying that the choice of technology in new production units is based on a capital cost measure which differs from the opportunity cost of capital.

An alternative approach, and the one adopted here, is to compute a set of sectoral internal rates of return, $R_j(t)$, on the basis of the equilibrium condition

$$\tilde{P}_j^*(t) = \kappa^*_j(\tilde{W}_j(t),\tilde{Q}_j(t),\tilde{P}_{0j}(t),\tilde{P}_{1j}(t); \qquad j=0,1,\ldots,n+2 \tag{50}$$

where $\tilde{Q}_j(t)$ is redefined in accordance with

$$\tilde{Q}_j(t) = \tilde{P}_{n+3}(\delta_j R_j(t)); \qquad j=0,1,\ldots,n+2 \tag{52}$$

and let the sectoral investments be determined by[10]

$$I_j(t) = \begin{cases} \delta_j X_j(t) \dfrac{\partial \kappa_j^*(\cdot)}{\partial Q_j} \left(\dfrac{R_j(t)}{r(t)}\right)^{\rho_j} & \text{when } R_j(t) \geq r(t) \\ 0 & \text{when } R_j(t) < r(t) \end{cases} \tag{51}$$

$$j = 0,1,\ldots, n+2$$

where r(t) can be interpreted as a market rate of interest, and $\kappa_j^*(\cdot)$ is evaluated at the prices $\tilde{W}_j(t), \tilde{Q}_j(t), \tilde{P}_{0j}(t), \tilde{P}_{1j}(t)$, i.e. the capital intensity of new production units in a given sector j reflects the expected internal rate of return on investments in that sector. This implies that in each sector the producers take the expected rate of return on investments in that sector as a measure of the real rate of interest.

With this specification there is an inverse relation between $I_j(t)$ and $P_j(t+1)$. Thus, whenever current prices affect the price expectations, there is a tendency towards equalization of the expected internal rates of return on investments across sectors. This means that in the long run the choice of technology in new production units will reflect the opportunity cost of capital, while it will be largely unaffected by current market interest rates.

On the basis of Shephard's lemma, the energy input coefficients in production units of vintage t is determined by

$$a_{tij} = \frac{\partial \kappa^*_j(\tilde{W}_j(t), \tilde{Q}_j(t), \tilde{P}_{0j}(t), \tilde{P}_{1j}(t))}{\partial P_{ij}} \quad ; i=0,1 \quad (53)$$

$$j=0,1,\ldots,n+2$$

With this a solution to the model for period t provides all the data necessary for solving the model for period t+1, i.e. the ex post production function units of vintage t are specified.

3.7 A summing up

The model presented here is designed for analysis of the adjustment of the economy to changes in exogenous conditions such as world market prices or energy prices. In the initial year there is only one vintage of capital in each production sector, but then a new vintage is created in each period. Thus the size of the model increases with the number of periods even though it is solved for only one period at a time.

In Table 2 below, the complete model is summarized. A complete list of the symbols used is given in the appendix to this chapter.

Table 2 The equations of the model

Supply of domestically produced goods

$$X_{\upsilon j}(t) = \frac{\partial \pi_{\upsilon j}(\cdot)}{\partial P^{*}_{\upsilon j}}; \qquad \begin{array}{l} j=0,1,\ldots,n+2 \\ \upsilon=0,1,\ldots,t-1 \end{array}$$

$$X_j(t) = \sum_{\upsilon=0}^{t-1} X_{\upsilon j}(t); \qquad j=0,1,\ldots,n+2$$

Intermediate Demand

$$X_{ij}(t) = \begin{cases} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij} X_{\upsilon j}(t) & \text{when } i=0,1 \\ & j=0,1,\ldots,n+3 \\ a_{ij} X_j(t) & \text{when } i=2,3,\ldots,n \end{cases}$$

Household Demand

$$C_i(t) = \sum_{s=1}^{k} t_{is} \{Q_s^0 + \frac{\beta_s}{P_s^Q(t)} (E(t) -$$

$$- \sum_{s=1}^{k} P_s^Q(t) Q_s^0)\}; \qquad i=0,1,\ldots,n+1$$

Export Demand

$$Z_i(t) = Z_i^0\left(\frac{P_i(t)}{V(t)P^{WE}(t)}\right)^{\varepsilon_i} e^{\sigma_i t}; \qquad i=1,2,\ldots,n$$

Import Demand

$$M_i(t) = \frac{\partial\phi_i(\cdot)}{\partial P_i^M}\left\{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\right\}; \qquad i=0,1,\ldots,n$$

$$M_j^C(t) = b_{jj}X_j(t); \qquad j=0,1$$

Product Market Equilibrium

$$X_i(t) = \frac{\partial\phi_i(\cdot)}{\partial P_i}\left\{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\right\} + Z_i(t); \qquad i=0,1,\ldots,n$$

$$X_i(t) = C_i(t); \qquad i=n+1,n+2$$

$$X_{n+3}(t) = I(t);$$

Savings Investment Equilibrium

$$\frac{s}{1-s}\{E(t) + P_{n+2}(t)C_{n+2}(t)\} - V(t)D(t) = P_{n+3}(t)I(t);$$

Labor Market Equilibrium

$$L(t) = \sum_{\upsilon=0}^{t-1}\sum_{j=0}^{n+2} - \frac{\partial\pi_{\upsilon j}(\cdot)}{\partial W_j};$$

Current Account Constraint

$$\sum_{i=1}^{k} P_i Z_i - \sum_{i=0}^{n} V(t)P_i^{WI}(t)\frac{M_i(t)}{1+\phi_i^0} -$$

$$- \sum_{j=0}^{1} V(t)P_j^C(t)M_j^C(t) = V(t)D(t);$$

Energy Input Coefficients in New Plants

$$a_{tij} = \frac{\partial \kappa_j^*(\cdot)}{\partial P_{ij}}; \qquad i=0,1. \qquad j=0,1,\ldots,n+2$$

Investments in New Plants

$$\tilde{P}_j(t) = \kappa_j^*\big(\tilde{W}_j(t),\tilde{Q}_j(t),\tilde{P}_{0j}(t),\tilde{P}_{1j}(t)\big) + \sum_{i=2}^{k} \tilde{P}_i^D(t)a_{ij} +$$

$$+ \tilde{V}(t)\tilde{P}_j^C(t)b_{jj} + \tilde{\Theta}_j\tilde{P}_j(t); \qquad j=0,1,\ldots,n+2$$

$$I_j(t) = \begin{cases} \delta_j X_j(t)\dfrac{\partial \kappa_j(\cdot)}{\partial Q_j}\left(\dfrac{R_j(t)}{R(t)}\right)^{\rho_j} & \text{if } R_j(t) \geq r(t) \\ 0 & \text{if } R_j(t) < r(t) \end{cases} ; \qquad j=0,1,\ldots,n+2$$

$$I(t) = \sum_{j=0}^{n+2} I_j(t);$$

Definitions

$$\psi_i(P_i,P_i^M) = \left\{d_{di}^{\mu_i} P_i^{1-\mu_i} + d_{mi}^{\mu_i} P_i^{M\,1-\mu_i}\right\}^{\frac{1}{1-\mu_i}}; \quad i=0,1,\ldots,n$$

$$\pi_{\upsilon j}(P^*_{\upsilon j},W_j;t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}}\alpha_j(1-\alpha_j)^{\frac{1-\alpha_j}{\alpha_j}} P^{*\,\frac{1}{\alpha_j}}_{\upsilon j} W_j^{\frac{\alpha_j-1}{\alpha_j}};$$

$$j=0,1,\ldots,n+2$$
$$\upsilon=0,1,\ldots,t-1$$

$$\kappa_j^*(W_j,Q_j,P_{0j},P_{1j};\upsilon) = A_j^{-1}\Big|a_j^{\frac{1}{1-\rho_j}}\Big|e^{-\lambda_j\upsilon}\alpha_j^{-\alpha_j}(1-\alpha_j)^{\alpha_j-1}Q_j^{\alpha_j}$$

$$W_j^{1-\alpha_j}\Big|^{\frac{\rho_j}{\rho_j-1}} + b_j^{\frac{1}{1-\rho_j}}\Big|e^{-\lambda_j^*\upsilon}\Big(C_j^{\frac{1}{1-\gamma_j}}P_{0j}^{\frac{\gamma_j}{\gamma_j-1}} +$$

$$+ d_j^{\frac{1}{1-\gamma_j}}P_{1j}^{\frac{\gamma_j}{\gamma_j-1}}\Big)^{\frac{\gamma_j-1}{\gamma_j}}\Big|^{\frac{\rho_j}{\rho_j-1}}\Big|^{\frac{\rho_j-1}{\rho_j}}; \qquad j=0,1,\ldots,n+2$$

$$P^*_{\upsilon j}(t) = (1-\Theta_j)P_j(t) - \sum_{i=0}^{1} P_{ij}(t)a_{\upsilon ij} - \sum_{i=2}^{n} P_i^D(t)a_{ij} -$$

$$- V(t)P_j^C(t)b_{jj}; \qquad j=0,1,\ldots,n+2$$

$$P_i^M(t) = \left(1 + \frac{\Delta\phi_i}{1+\phi_i^0}\right) V(t)P_i^{WI}(t); \qquad i=0,1,\ldots,n$$

$$P_i^D(t) = \phi_i\left(P_i(t),P_i^M(t)\right); \qquad i=0,1,\ldots,n$$

$$P_{ij}(t) = \left(1+\tau_i(t)+\xi_{ij}(t)\right)P_i^D(t); \qquad i=0,1,\ldots,n+2,c$$

$$P_s^\Omega(t) = \sum_{i=0}^{1} P_{ic}(t)t_{is} + \sum_{i=2}^{n} P_i^D(t)t_{is} + P_{n+1}(t)t_{n+1,s};$$

$$s=1,2,\ldots,k$$

$$P_{n+3}(t) = \sum_{i=2}^{n} P_i^D(t)a_{i,n+3};$$

$$W_j(t) = \omega_j W(t);$$

$$\tilde{P}_i(t) = \zeta_i^O P_i(t) + (1-\zeta_i^O)P_i^{OF}(t); \qquad i=0,1,\ldots,n+2$$

$$\tilde{P}_i^D(t) = \zeta_i^I P_i^D(t) + (1-\zeta_i^I)P_i^{DF}(t); \qquad i=0,1,\ldots,n$$

$$\tilde{P}_j^C(t) = \zeta_j^C P_j^C(t) + (1-\zeta_j^C)P_j^{CF}(t); \qquad j=0,1$$

$$\tilde{W}_j(t) = \zeta^W W_j(t) + (1-\zeta^W)W_j^F(t); \qquad j=0,1,\ldots,n+2$$

$$\tilde{Q}_j(t) = \tilde{P}_{n+3}\left(\delta_j+R_j(t)\right); \qquad j=0,1,\ldots,n+2$$

Exogenous variables

$L(t), C_{n+2}(t), D(t), V(t), P_i^{WI}(t), P_i^{WE}(t), P_j^C(t), P_i^{OF}(t), P_i^{DF}(t)$

$P_i^{CF}(t), W_j^F(t), \tau_i(t), \xi_{ij}(t).$

Appendix: List of symbols

A. Endogenous variables in period t

$X_{\upsilon j}(t)$	gross output in sector j=0,1,..., n+2, vintage =0,1,...,t-1 in period t.
$X_j(t)$	total output in sector j=0,1,..., n+2 in period t.
$X_{n+3}(t)$	output of investment goods in period t.
$X_{ij}(t)$	use of commodity i=0,1,..., n+2 in sector j=0,1,..., n+3, in period t.
$I_j(t)$	gross investments in sector j=0,1,..., n+2 in period t.
$I(t)$	total investments in period t.
$C_i(t)$	household consumption of commodity i=0,1,..., n+1 in period t.
$E(t)$	total household consumption expenditures in period t
$Z_i(t)$	export of production sector output i=1,2,..., n in period t
$M_i(t)$	import of goods competing with production sector output i=0,1,..., n, in period t.
$M_j^C(t)$	complementary imports used in sector j=0,1 in period t.
$P^*_{\upsilon j}(t)$	value added per unit of gross output in vintage υ=0,1,..., t-1, sector j=0,1,..., n+2 in period t.
$P_i(t)$	price of production sector output i=0,1,..., n+3 in period t
$P_{ij}(t)$	user price of energy of type i=0,1 in sector j=0,1,..., n+2 in period t.

$P_i^D(t)$ user price of commcdity i=0,1, n in period t

$P_{ic}(t)$ user price of energy of type i=0,1 in the household sector in period t

$P_s^Q(t)$ price of consumer commodity group s=1,2,...,k in period t.

$W_j(t)$ wage rate in sector j=0,1,..., n+2 in sector t.

$W(t)$ index of the level of wages in the economy as a whole in period t.

$r(t)$ the market rate of interest in period t.

$\tilde{P}_j(t)$ expected future price of production sector output j=0,1,..., n+2 in period t.

$\tilde{P}_{ij}(t)$ expected future user price of energy of type i=0,1 sector j=0,1,..., n+2 in period t

$\tilde{P}_i^D(t)$ expected future user price of commodity i=0,1,..., n in period t.

$\tilde{P}_j^C(t)$ expected future price of complementary imports in sector j=0,1 in period t.

$\tilde{W}_j(t)$ expected future wage rate in sector j=0,1,..., n+2 in period t.

$R_j(t)$ expected rate of profit on investments in sector j=0,1,..., n+2 in period t.

a_{tij} expected, and actual, input of energy of type i=0,1, per unit of output in plants designed for sector j=0,1,..., n+2 in period t.

B. Exogenous variables in period t.

$L(t)$ supply of labor in period t.

$C_{n+2}(t)$ public consumption in period t.

$D(t)$ surplus on current account in period t, expressed in foreign exchange.

$V(t)$ exchange rate (units of domestic currency per unit of foreign currency) in period t.

$p_i^{WI}(t)$ c.i.f. price level in period t, expressed in foreign currency, of imported goods competing with domestically produced commodity $i = 0,1,\ldots,n$.

$p_i^{WE}(t)$ f.o.b. price level in period t, expressed in foreign currency, on foreign markets where domestic producers of commodity $i=1,2,\ldots, n$ compete.

$p_j^{C}(t)$ c.i.f. price level in period t, expressed in foreign currency, of complementary imports used as inputs in sector $j=0,1$.

$p_i^{OF}(t)$ predicted future price of production sector output i in period t.

$p_i^{DF}(t)$ predicted future user price of commodity group i in period t.

$p_j^{CF}(t)$ predicted future price of complementary imports in sector $j=0,1$ in period t.

$w_j^{F}(t)$ predicted future wage rate in sector j in period t.

$\tau_i(t)$ ad valorem tax on energy of type $i=0$, in period t.

$\xi_{ij}(t)$ ad valorem tax on energy of type $i=0,1$ used in sector $j=0,1,\ldots, n+2$ in period t.

$\xi_{ic}(t)$ ad valorem tax on energy of type $i=0,1$, used in the household sector in period t.

C. Parameters[11]

s gross savings ratio for the economy as whole.

$a_{\upsilon ij}$ input of energy $i=0,1$ per unit of output in vintage $\upsilon=0.1,\ldots,t-1$, in sector $j=0,1,\ldots, n+2$

a_{ij} input of commodity $i=2,3,\ldots, n$ per unit of output in sector $j=0,1,\ldots, n+2$

b_{jj}	input of complementary imports per unit of output in sector j=0,1
t_{is}	relative weight of commodity i=0,1,..., n+1 in consumer commodity group s=1,2,..., k.
ω_j	index of the relative wage rate in sector j=0,1,..., n+2.
Θ_i	ad valorem indirect tax on commodity i=0,1,..., n+2.
$\Delta\phi_i+\phi_i^0$	custom duty and indirect tax on the c.i.f. value of imports of commodities competing with commodity group i=0,1,..., m.
δ_j	annual rate of depreciation of capital in sector j=0,1,..., n+2.
σ_i	annual rate of change of production of commodity group i=1,2,..., n in the rest of the world.
μ_i, ε_i	elasticity of substitution between domestically produced and imported units of commodity i=0,1,..., n at home and abroad, respectively.
β_s^Q	expenditure allocation parameter in the household demand function for consumer commodity group s=1,2,...,k.
d_{di}, d_{mi}	distribution parameters in the CES-function representing the trade-off between imported and domestically produced goods of type i=0,1,..., n.

ρ_j the elasticity of investments in sector $j=0,1,\ldots, n+2$ with respect to the expected excess profit ratio.

ζ_i^0 the relative weight of the current price of production sector output $i=0,1,\ldots, n+2$ in the formation of expectations about that price.

ζ_i^I the relative weight of the current price of commodity $i=0,1,\ldots, n$ in the formation of expectations about that price.

ζ_j^C the relative weight of the current price of complementary imports to sector $j=0,1$ in the formation of expectations about that price.

ζ^W the relative weight of the current wage rates in the formation of wage expectations.

Z_i^0, Q_s^0, A_j constants in the export, household demand and ex ante production functions, respectively.

4. The solution procedure

The purpose of this section is to give a brief overview of the solution algorithm developed by A. Por for this model. The discussion here does not deal with the mathematical techniques used in various parts of the algorithm. The aim is only to indicate the main steps in the solution procedure, and to give an economic interpretation of that procedure. Before turning to the main topic of this section, however, a few properties of the model should be pointed out, and some of the equations have to be written in a different form.

In accordance with eq. (25) it holds that

$$P^*_{\upsilon j}(t) = (1-\Theta_j)P_j(t) - \sum_{i=0}^{1} P_{ij}(t)a_{\upsilon ij} -$$

$$- \sum_{i=2}^{n} P_i^D(t)a_{ij} - V(t)P_j^C(t)\bar{b}_{jj}$$

$$\upsilon=0,1,\dots,t-1$$
$$j=0,1,\dots,n+2$$

By a slight rearrangement of the terms this can be written so as to define a new variable, $P_j^*(t)$. That is

$$P_j^*(t) = P^*_{\upsilon j}(t) + \sum_{i=0}^{1} P_{ij}(t)a_{\upsilon ij} = (1-\Theta_j)P_j(t) -$$

$$- \sum_{i=2}^{n} P_i^D(t)a_{ij} - V(t)P_j^C(t)\bar{b}_{jj}; \qquad \begin{array}{l}\upsilon=0,1,\dots,t-1\\ j=0,1,\dots,n+2\end{array}$$

By denoting the energy costs per unit of output $e_{\upsilon j}(t)$, i.e.

$$e_{\upsilon j}(t) = \sum_{i=0}^{l} P_{ij}(t)a_{\upsilon ij}; \qquad \begin{array}{l}\upsilon=0,1,\ldots,t-1\\ j=0,1,\ldots,n+2\end{array}$$

the value added per unit of output in vintage υ in sector j can be written

$$P^*_{\upsilon j}(t) = P^*_j(t) - e_{\upsilon j}(t)$$

Thus the value added per unit of output can be subdivided into two parts, where one, the new variable $P^*_j(t)$, applies to all vintages in sector j and the other, the unit energy costs, is vintage specific. Moreover, when the prices of intermediate inputs are given, the values of $e_{\upsilon j}(t)$ are given as well. This means that total output in sector j can be expressed as a function of P^*_j and the wage rate W_j.

The variable $P^*_j(t)$ represents the difference between the producer price of the output in sector j, and the unit cost of non-energy intermediate inputs. Clearly it can happen that $e_{\upsilon j}(t)$ exceeds $P^*_j(t)$. Since negative $P^*_{\upsilon j}(t)$ would imply zero production under conditions of profit maximization behavior, $P^*_{\upsilon j}(t)$ has to be determined in accordance with

$$P^*_{\upsilon j}(t) = \begin{cases} P^*_j(t) - e_{\upsilon j}(t) & \text{if } P^*_j(t) > e_{\upsilon j}(t) \\ 0 & \text{if } P^*_j(t) \leq e_{\upsilon j}(t) \end{cases} \tag{54}$$

If some $P^*_{\upsilon j}(t) = 0$ in equilibrium, the corresponding $X_{\upsilon j}(t)$ is also zero, which can be easily seen when the supply function of production units of vintage υ in sector j is written in the following explicit form.

$$X_{\upsilon j}(t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \left\{\frac{(1-\alpha_j)P^*_{\upsilon j}(t)}{W_j(t)}\right\}^{\frac{1-\alpha_j}{\alpha_j}}$$

Assuming that the prices $P_0(t),\ldots,P_{n+2}$ and the wage rate $W(t)$ are given, and using eq. (54), this supply function can be written

$$X_{\upsilon j}(t) = f_{\upsilon j}\left(P^*_j(t)\right) \qquad \begin{matrix} v=0,1,\ldots,t-1 \\ j=0,1,\ldots,n+2 \end{matrix} \tag{55}$$

where $f_{\upsilon j}$ denotes a nonlinear function of the variable $P^*_j(t)$. Observe that when the domestic producer prices $P_0,\ldots,P_{n+2}$ are given, the domestic user prices $P^D_0(t),\ldots,P^D_n(t)$ are given as well. (See eq. (16).)

On the basis of eq. (31) it holds that

$$C_i(t) = E(t)\left(\sum_{s=1}^{k} \frac{t_{is}\beta_s}{P^Q_s(t)}\right) + \sum_{s=1}^{k} t_{is}Q^0_s - \\ - \sum_{s=1}^{k} \frac{t_{is}\beta_s}{P^Q_s(t)} \left(\sum_{s=1}^{k} P^Q_s(t)Q^0_s\right) \tag{56}$$

Thus, when the prices are given, household demand for commodity group i is a linear function of household expenditure $E(t)$.

Moreover, when prices are given, the "input" of domestically produced goods per unit of composite goods demanded is also given for all types of composite goods. Thus we can define

$$d_i(t) = d_{di}^{\mu_i}\left(\frac{P^D_i(t)}{P_i(t)}\right)^{\mu_i}$$

and treat $d_i(t)$ as a constant as long as the prices are kept unchanged. The equilibrium condition for the markets for domestically produced goods can now be written (setting $d_i(t)=1$ for $i=n+1,n+2$)

$$X_i(t)= d_i(t)\{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\} + Z_i(t);$$

$$i=0,1,\ldots,n+2$$

and on the basis of eq. (30) this becomes

$$X_i(t) = d_i(t) \{\sum_{j=0}^{n+3} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij}X_{\upsilon j}(t) + C_i(t)\} +$$

$$+ Z_i(t); \qquad i=0,1 \qquad (57)$$

for the energy sectors and

$$X_i(t) = d_i(t) \{\sum_{j=0}^{n+3} a_{ij}X_j(t) + C_i(t)\} +$$

$$+ Z_i(t);$$

$$i=2,3,\ldots,n+2 \qquad (58)$$

for the other production sectors.

As the total production in a given sector is the sum of the production in the individual vintages of production units, substitution of eq. (56) in eq. (57) yields, after simple reorganization of the terms.

$$\sum_{\upsilon=0}^{t-1} f_{\upsilon j}(P_j^*(t)) - d_i(t)\{\sum_{j=0}^{1} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij} f_{\upsilon j}(P_j^*(t))\} =$$

$$= d_i(t)\{\sum_{j=2}^{n+3} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij} f_{\upsilon j}(P_j^*(t)) + C_i(t)\} + Z_i(t);$$

$$i=0,1 \qquad (59)$$

Note that when prices are given, the left-hand side is a non-linear function of $P_0^*(t)$ and $P_1^*(t)$, while the right-hand side is linearly dependent on $E(t)$ (through $C_i(t)$ by eq. 56 and non-linearly dependent on $P_2^*(t), P_3^*(t), \ldots, P_{n+2}^*(t)$.

On the basis of eqs. (35), (39), (40), (41) and (45) it holds that

$$\frac{s}{1-s}\left\{E(t) + P_{n+2}(t)X_{n+2}\right\} = \sum_{i=1}^{n} P_i(t)Z_i(t) - \sum_{i=0}^{n} P_i^M(t)M -$$

$$- \sum_{j=0}^{1} V(t)P_j^C(t)b_{jj}X_j(t) + P_{n+3}(t)X_{n+3}(t); \qquad (60)$$

When prices are given $Z_i(t)$ can be treated as constants (by eq. (36)) while $M_i(t)$ will be linearly dependent on $X_i(t)$ (by eqs. (34), (36) and (37)). Thus, with all prices and the wage rate given eq. (60) represents a linear dependency among the variables $X_0(t), X_1(t=, \ldots, X_{n+3}(t)$ and $E(t)$.

The model is solved for one period at a time, and for each period the solution procedure can be subdivided into two blocks. The first block deals with the commodity and labor markets, while the second, which does not affect the first, deals with the capital market. For the first block the solution procedure involves three main steps. In the following each one of these steps will be briefly described.

Step I

In this step an initial guess about the producer prices and the wage rate is made. These values are denoted $\hat{P}_0(t), \hat{P}_1(t), \ldots, \hat{P}_{n+2}$ and $\hat{W}(t)$. When $t=0$ these values are exogenously given, while the solution for the previous period is taken as the initial guess for $t>0$. Using these prices, and on the basis of eq. (16), values for the domestic user prices, denoted $\hat{P}^D_0(t), \hat{P}^D_1(t), \ldots, \hat{P}^D_n(t)$, are computed, while a value for the price of capital goods, $\hat{P}_{n+3}(t)$, is computed in accordance with eq. (44). Then it is possible to establish an initial guess $\hat{P}^*_0(t), \hat{P}^*_1(t), \ldots, \hat{P}^*_{n+2}(t)$ on the basis of eqs. (25) and (54). When these values are determined it is possible to represent $X_0(t)$ and $X_1(t)$ as linear functions of $E(t)$.

Step II

II.1.: The values for $W(t)$, $P_0(t), \ldots, P_{n+3}$, $P^*_0(t), P^*_1(t), \ldots, P^*_{n+2}(t)$ are kept fixed

As $X_0(t)$ and $X_1(t)$ are now (with prices kept fixed) represented as linear functions of $E(t)$, values for the variables $X_2(t), X_3(t), \ldots, X_{n+3}(t)$ and $E(t)$ can be determined by a linear equation system derived from eqs. (41), (58) and (60). The computed output levels are denoted by $\hat{X}_2(t), \hat{X}_3(t), \ldots, \hat{X}_{n+3}(t)$.

II.2: <u>The values for $W(t)$, $P_0(t)$, $P_\theta(t), \ldots, P_{n+3}(t)$ $P_0^*(t), P_1^*(t)$ are kept fixed</u>

On the basis of step II.1, a set of values $\hat{\hat{P}}_2^*(t)$, $\hat{\hat{P}}_3^*(t), \ldots, \hat{\hat{P}}_{n+2}^*$ are determiend by eq. (55), i.e.

$$\hat{X}_i(t) = \sum_{\upsilon=0}^{t-1} f_{\upsilon i}\left(\hat{\hat{P}}_i^*(t)\right); \qquad i=2,3,\ldots,n+2$$

II.3.: <u>The values for $W(t), P_0(t), P_0^*(t), P_1^*(t)$ are kept fixed</u>

Using the values $\hat{\hat{P}}_2^*(t), \hat{\hat{P}}_3^*(t), \ldots, \hat{\hat{P}}_{n+2}^*(t)$ thus computed and the equality (derived from eqs. (16) and (25) and the definition of $P_1^*(t)$)

$$P_j^*(t) = (1-\theta_j)P_j(t) - \sum_{i=2}^{n} \phi_i\left(P_i(t), P_i^M(t)\right) a_{ij} -$$

$$- V(t)P_j^C(t)b_{jj}$$

new values for the producer prices can be computed. These values are denoted $\hat{\hat{P}}_2(t), \hat{\hat{P}}_3(t), \ldots,$ $\hat{\hat{P}}_{n+2}(t)$. If these are sufficiently close to the previous values the procedure continues. Otherwise, it returns to step II.1, i.e. on the basis of $\hat{\hat{P}}_2{}^*(t), \hat{\hat{P}}_3^*(t), \ldots, \hat{\hat{P}}_{n+2}^*$ new values for $X_2(t)$, $X_3(t), \ldots X_{n+2}$ are computed.

II.4.: <u>The values for $W(t)$, $P_0(t)$, $P_1(t)$ are kept fixed</u>

Taking the values of $P_2^*(t), P_3^*(t), \dots, P_{n+2}^*$ and $E(t)$ thus computed as given, eq. (60) represent a non-linear equation system with two equations in the two unknowns $P_0^*(t)$ and $P_1^*(t)$. By solving this system, i.e. clearing the energy markets, new iterative values, P_0^* (t) and $P_1^*(t)$, for these variables are obtained. If these values are close to the previous ones, the procedure continues. Otherwise, it returns to step II.1, where thus the new values for $P_0^*(t)$ and $P_1^*(t)$ change the linear dependency between, on the one hand, $X_0(t)$ and $X_1(t)$, and, on the other hand, $E(t)$. If the values for $P_0^*(t), \dots,$ $P_1^*(t), \dots, P_{n+2}^*(t)$ converge during the steps II.1 - II.4 means that a set of market clearing prices for the non-energy markets, at given wage rates and energy prices, has been found. However, the "net prices" for energy, i.e. the values for $P_0^*(t)$ and $P_1^*(t)$, which bring about the supply of energy which is demanded at the prices ruling at this stage of the procedure, might be inconsistent with the ruling producer prices of energy, i.e. $P_0(t)$ and $P_1(t)$.

II.5.: <u>The value for $W(t)$ is kept fixed</u>

The consistency of the values of $P_0^*(t)$ and $P_1^*(t)$ determined in step II.4, and the initial values $\hat{P}_0(t)$ and $\hat{P}_1(t)$ are checked by means of the equality

$$P_j^*(t) = (1-\Theta_j)P_j(t) - \sum_{i=2}^{n} \phi_i \left(P_i(t), P_i^M(t), \right) a_{ij} - $$
$$- V(t)P_j^C(t)b_{jj}; \qquad j=0,1$$

and the values for $P_2(t)$, $P_3(t),\ldots,P_n(t)$ computed in step II.3. If the equation is approximately satisfied, the procedure continues. Otherwise, it returns to step II.1, using the values for $P_0(t)$ and $P_1(t)$ which satisfy the above equation at the values $P_0^*(t)$ and $P_1^*(t)$ computed in II.4. When step II.5 finally is completed, a market clearing price system, at given wage rates, has been found.

Step III

With given wage rates and commodity prices, the demand for labor is given by eq. (46) added over vintages and sectors. If the demand for labor thus computed is sufficiently close to the supply of labor the procedure stops. Otherwise, W(t) is adjusted (the sign of the revision of W(t) being the same as the sign of the excess demand for labor) and the procedure is repeated from step II.1. When step III is completed a solution for the resource allocation in period t (except the sectoral allocation of investments) is obtained,i.e. the first block in the solution procedure is completed.

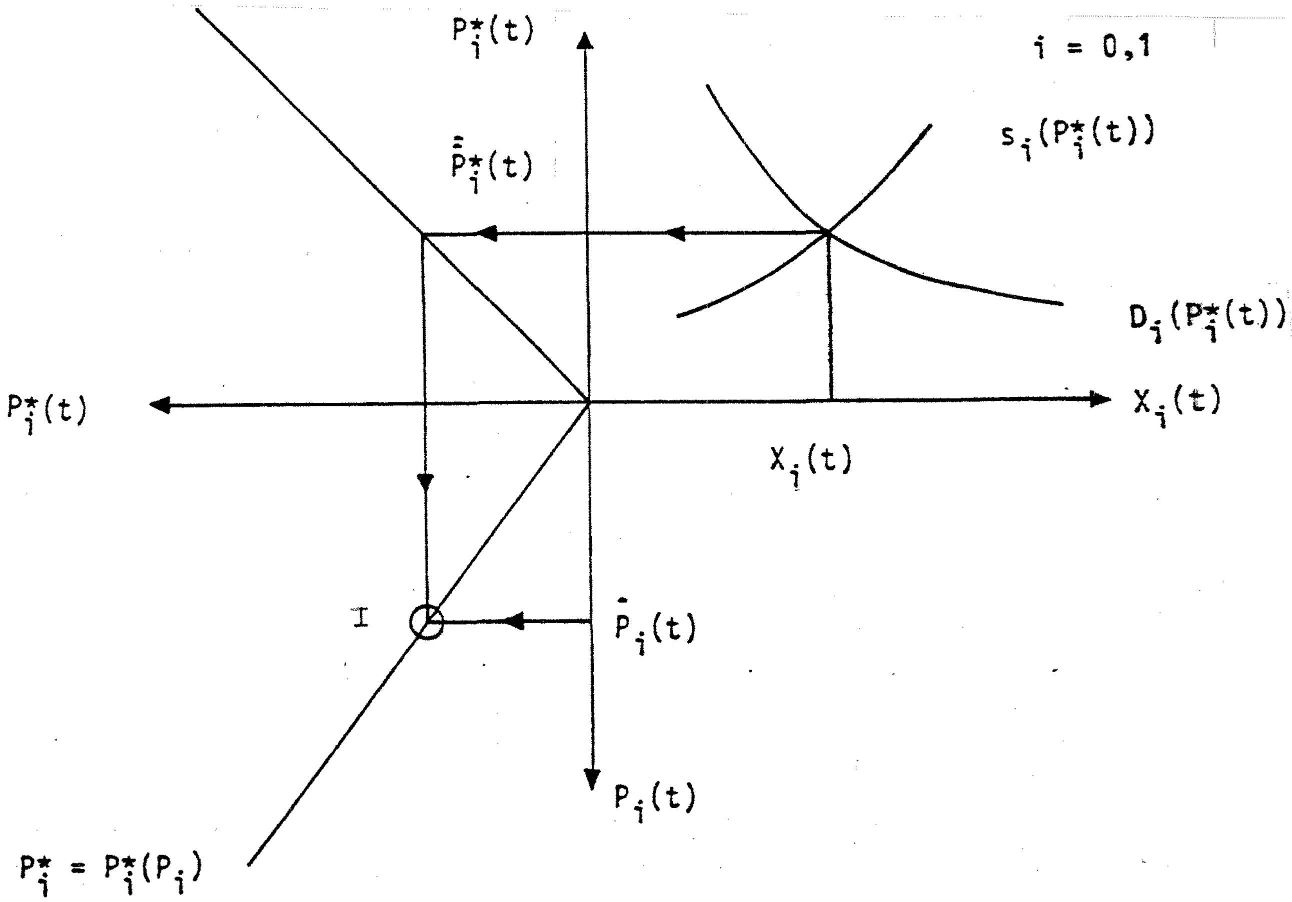
Before turning to the second block in the solution procedure, a few additional remarks should be made about the steps II 1 - II 5 in the first block should be made. The first point to note about these steps is that the two energy sectors are treated in a different way than the other production sectors. The reason for this is that the different vintages in a sector do not differ in terms of the use of non-energy intermediate inputs. Thus, provided prices the production in the energy sectors and total household expenditures are given, the commodity market equations for sectors 2, 3 ..., n+3 can be treated as an input-output model, i.e., a system of linear equations with equal number of equations and unknowns. In the corresponding equations for the energy sectors, however, all vintages in all sectors appear explicitly. Accordingly, the energy markets cannot be cleared unless the allocation of total production over vintages is simultaneously determined.

With this background it is easy to describe step II in the solution procedure in economic terms. In II.1 initial prices of energy are fixed. These prices can be regarded as prices facing the users of energy. Then in II.2 and II.3 market clearing prices for non-energy goods are determined; by an iterative process equality between the prices accepted by the producers and the prices facing the consumers established, and throughout this process the markets for non-energy goods are cleared. When step II.3 is completed, the supply and demand functions for energy are also determined. The situation is illustrated by Figure 1 below where both the supply of and demand for fuels and electricity are expressed as functions of $P_i^*(t)$,[12] $i = 0,1$, respectively.

In step II.4, then, the market clearing values of $P_i^*(t)$, $i=0,1$, are determined and in II.5 these values are compared with those obtained on the basis of the previous energy prices, i.e. $\hat{P}_0(t)$ and $\hat{P}_1(t)$. In Figure 1 the two sets of estimates of $P_0^*(t)$ and $P_1^*(t)$ coincide as they should in a final solution to the model.

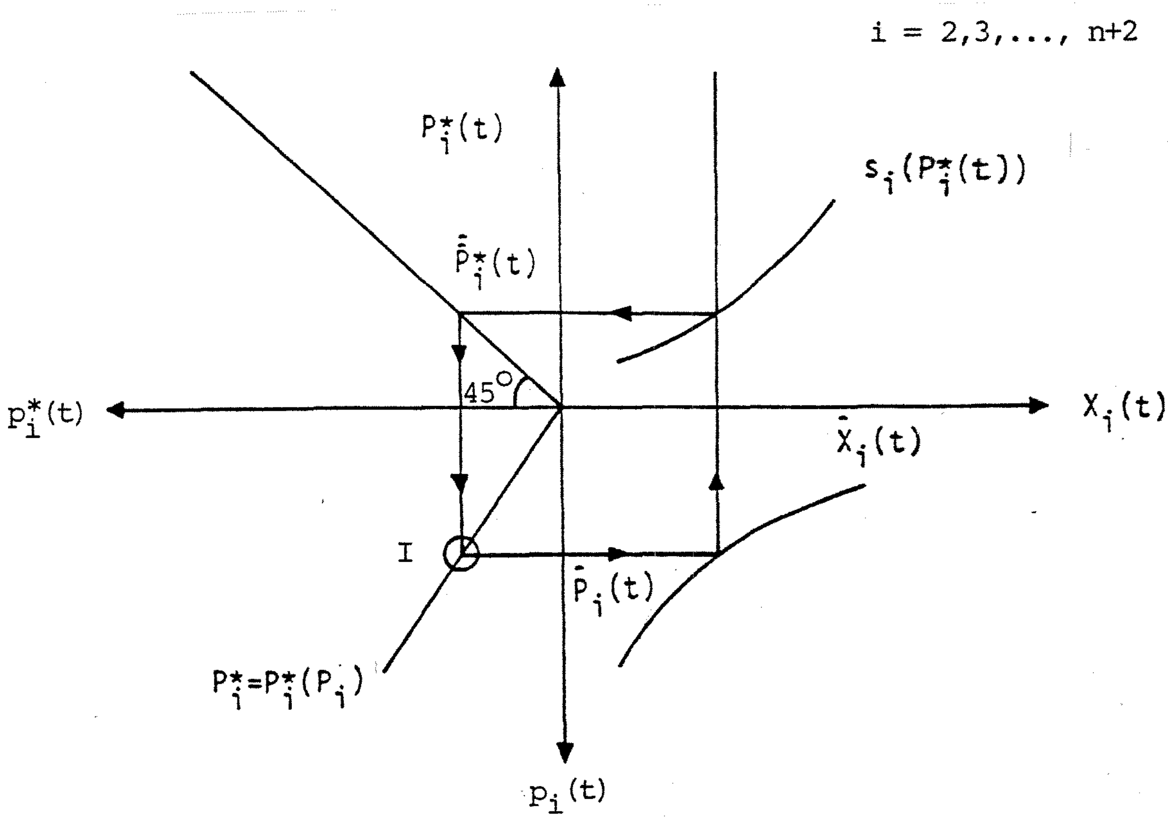
Generally speaking, the solution procedure implies that producers always satisfy the demand which is established at the prevailing prices. However,

Figure 1. Market Clearing in an Energy Market



once they realize that the market clearing output levels lead to marginal production costs, which differ from the prices, the prices are revised in such a way that equality between price and marginal cost is attained. But when prices change demand changes as well and the equality between price and marginal cost may again not be realized. The process continues until all commodity markets are cleared at prices which are compatible with profit maximization in all production units. At an intermediate step in the process, however, the prices needed to induce producers to supply the demanded quantity differ from the prices the consumers are faced with. Figure 2 illustrates how a solution for the non-energy sectors is attained, i.e., the steps II.2 and II.3.

Figure 2. Market clearing in a non-energy market



Step III in the solution procedure does not have to be described in detail. It can be regarded as a Walrasian tatônnement process in which the excess demand for labor at a given wage rate is computed, and then the wage rate is gradually adjusted until the excess demand is eliminated. In this process the sign of the wage revision is the same as the sign of the excess demand.

The solution procedure for the second block, the allocation of investible funds over investing sectors, is much less complicated than the one described above. Once a solution for the first block is obtained, the variables $R_j(t)$ and $\partial K_j^*(\cdot)/\partial Q_j$ can be computed. Since $I(t)$ is determined in the first block the second block can be represented by the single equilibrium condition

$$I(t) = \sum_{j=0}^{n+2} I_j(t)$$

where $I_j(t)$ is a function of the market interest rate, $r(t)$. However, $I_j(t)$ is a discontinuous function of $r(t)$. Thus it may happen that no solution exists for the sectoral investment rule given by eq. (51). In order to obtain a solution in all cases, the rule is somewhat argumented in the solution procedure. Thus, if no solution can be obtained on the basis of eq. (51), the set of sectors competing for investible funds, i.e., the sectors for which $R_j(t)$ is greater than or equal to the market rate of interest at the point of discontinuity, is augmented with the sector with the highest $R_j(t)$ of those not in that set. This procedure is repeated until a solution is obtained.

5. **Implementation, parameter estimation, and model behavior**

5.1 General remarks

When building a model of resource allocation in a national economy, the modeler is faced with several trade-off problems. One of these is that attempts to give a relatively detailed representation of various resource allocation mechanisms tend to involve variables which are difficult, or even impossible, to observe. For instance, explicit incorporation of the notion of "putty-clay" technology facilitates a realistic representation of factor substitution and technical change in a medium term perspective, but requires an explicit representation of the ex ante technology and price expectation formation process, i.e. phenomena which are very difficult to observe. Conversely, models of resource allocation can be entirely based on observable data only if quite dramatic simplifications of the resource allocation mechanisms are accepted.

Obviously the model presented here is based on a number of simplifying assumptions. Yet it represents an attempt to model the process of factor substitutions and technical change in a relatively detailed and, hopefully, realistic way. The price of this realism is the incorporation of a number of variables which generally are difficult or impossible to observe. In addition the model contains variables for which only one or a few observations can be obtained. This applies, for instance, to the intersectoral deliveries of goods and services. Consequently most parameters in the model cannot be estimated by means of standard econometric techniques.

A substitute technique is the so-called "reference equilibrium method", which has been employed in the implementation of the model presented in this report. This approach to parameter estimation <u>starts</u> from the assumption that the model is a true representation of the economy. It is also, implicitly, assumed that the variables dealt with in the model can be measured correctly. If these assumptions are accepted, a statistical description of the state of the economy at one single point in time can be taken as a solution to the model.

With this point of departure the parameter estimation procedure becomes simple and straightforward. To begin with, all the parameters of the model are expressed as functions of the exogenous and endogenous variables, and then these functions are evaluated at the observed values of the model's variables at a particular point in time.

In formal terms the model is written

$$f_i(x,y,\alpha) = 0; \qquad i=1,\dots,m$$

where the vector z denotes the endogenous variables, the vector y the exogenous variables and the vector α the parameters. At a given point in time the endogenous and the exogenous attain the values $\bar{x}$ and $\bar{y}$ respectively, and the model becomes a system of equations in which the vector contains all the unknowns, i.e.

$$f_i(\bar{x},\bar{y},\alpha) = 0; \qquad i=1,2,\dots,m.$$

In simple cases, such as when the parameters of an input-output model are to be estimated, one consistent set of observations on exogenous and endogenous variables is sufficient for unique determination of all the parameters of the model. Generally, however, extraneous information about functional forms and some parameter values is needed as well.

When this procedure is adopted model validation obviously becomes very difficult. The statistical significance of individual parameter estimates cannot be ascertained in the "usual" way, and lack of data often prevents a full ex post comparison between model predictions and the actual development of the economy. In addition, models of the type presented here often are used for simulation of the effects of policies and other changes in exogenous conditions which have not been experienced before. Thus, one may wonder about the proper interpretation of results obtained from a model, in which the parameters have been estimated by means of the reference equilibrium method.

The easiest answer to this question, of course, is to maintain that the principal aim of the study is methodological, and, consequently, that the numerical results should be regarded only as illustrations of a method. This position no doubt implies the minimum commitment for the author, and may in some cases be the only reasonable one, but it also implies some waste of the resources put into model development. After all, a considerable amount of data <u>is</u> available, and efficient use of these data might turn the model into a useful tool for quantitative analysis of resource allocation problems in a real economy.

The model presented in this report has been implemented on Swedish data. The implementation represents a serious effort to make efficient use of available data. The resulting numerical model, which is briefly described in the following subsection, is intended to be used for quantitative analysis of resource allocation problems related to Swedish energy policy. The nature and purpose of the quantitative analysis in which the model will be used is best described by a quotation from Leif Johansen[13]: "The data and the quantitative analysis do serve the purpose of illustrating the method and the model. But, at the same time, if I were required to make decisions and take actions in connection with relationships covered by this study, I would (in the absence of more reliable results, and without doing more work) rely to a great extent on the data and the results presented in the following chapters. Thus, the quantitative analysis does not solely serve the purpose of illustrating a method. I do believe that the numerical results also give a rough description of some important economic relationships in reality".

5.2 Functional Forms and Parameter Values

As mentioned above, the model has been implemented on the basis of Swedish data. The main data source is an input-output table for 1979, aggregated up to seven production sectors. The sector classification, which can be seen in Table 1, is chosen in order to be useful for analyses of the economic impact of national energy policies and alternative assumptions about world market oil prices.

Table 1. Production Sector Definitions

	Sector	SNI
0	Fossil fuels production	353, 354
1	Electricity production	4
2	Mainly import competing industries	11, 13, 31, 32, 33, 3412, 3419, 342, 355, 361, 362
3	Mainly exporting energy intensive industries	12, 2, 3411, 351, 37101 37102
4	Other mainly exporting industries	352, 356, 3699, 37103, 372, 38, 39
5	Sheltered industries and service production	3691, 3692, 5, 6, 7 8, 9 (priv.)
6	The public sector	
7	The capital goods sector (book keeping sector)	

The aim has been to identify the smallest possible number of sectors which is compatible with a meaningful analysis of the issues under study. There are several reasons to keep the number of sectors as small as possible. One is simply that the costs for solving and storing the model are quite sensitive to its size. Another, and more important reason, is that no complete input-output table for 1979 is available. Thus, the estimates of the intersectoral flows of non-energy[14] goods and services have to be based on the 1975 input-output data.[15] As the input-output relations between large aggregates of sectors tend to be more stable than the corresponding relations between more disaggregated sectors, a relatively high level of aggregation accordingly, was preferable in this case. A third reason is that the possibilities of getting good estimates of world market prices, technical change, etc. for a large number of sectors from published sources are quite limited, and if such assumptions cannot be differentiated between individual sectors, there is no point in a far-reaching disaggregation of the production system.

Tables 2, 3, and 4 summarize some base-year data about the production sectors, and in Table 5 the ten aggregates of household consumption goods dealt with in the model are defined.

Given the specification of the model in terms of the general characteristics of supply and demand functions, there is still some freedom to choose functional forms for the ex ante production functions, the household utility function and the "production functions" defining the composite goods in the home country and in the rest of the world. In the following the choices actually made will be briefly discussed. It should, however, be pointed out that other choices can be made without major revisions of the solution algorithm.

Table 2. Base year sectoral employment, energy use and value added shares

Sector	L_j/L	$X_{0j}/(X_0+M_0)$	$X_{1j}/X_1+M_1)$	$Y_j/Y)$
0	0.0009	0.0251	0.0025	0.0223
1	0.0095	0.0814	0.0249	0.0255
2	0.1226	0.0595	0.0620	0.1721
3	0.0472	0.1006	0.1557	0.0612
4	0.1303	0.0470	0.0862	0.1562
5	0.4258	0.4213	0.5346	0.3514
6	0.2637	0.0309	0.1138	0.2113
7	-	-	-	-
$\sum$	1.0000	0.7658[a]	0.9795[a]	1.0000

[a] The difference between this value and unity is made up of the shares of household consumption and exports.

Table 3. Base year input coefficients for capital, labor, fuels, and electricity

Sector	K_j/X_j	L_j/X_j	X_{0j}/X_j	X_{1j}/X_j
0	0.2174	0.0003	0.0512	0.0023
1	6.3451	0.0031	0.1850	0.0255
2	0.7361	0.0047	0.0159	0.0075
3	1.5517	0.0044	0.0649	0.0452
4	0.6229	0.0052	0.0132	0.0109
5	2.6974	0.0093	0.0639	0.0366
6	1.5461	0.0118	0.0096	0.0159
7	-	-	-	-

Table 4. Base year shares of export and import, and export and import shares in individual production sectors

Sector	Z_i/Z	M_i/M	Z_i/X_i	$M_i/(X_i+M_i)$
0	0	0.1946	0	0.6060
1	0.0030	0.0059	0.0210	0.0494
2	0.1530	0.2454	0.1277	0.2024
3	0.2283	0.1056	0.4597	0.2086
4	0.4963	0.3745	0.4342	0.2888
5	0.1194	0.0740	0.0566	0.0417
6	-	-	-	-
	1.0000	1.0000		

All import measures inclusive of complementary imports.

Table 5. Definition of household consumption goods

	Consumption good	SNA[a]
1.	Food	1.1
2.	Beverages and tobacco	1.2-1.4
3.	Clothing	2, (8.1.2)
4.	Gasoline	6.2.2
5.	Medical products and personal care	(4.5.1), 5.1, 8.1.1, (8.1.2)
6.	Gross rents, fuel and power	3.1.1, 3.2
7.	Transport	(6.1), (6.2.1), 6.2.3, 6.3
8.	Recreation	5.2, (6.1), (6.2.1), 7.1, 8.2.1, (8.2.2)
9.	Furniture and other furnishing	4.1, 4.2.1, (4.3.1), 4.3.2, (4.4.1), (4.5.1)
10.	Other	(4.3.1), (4.4.1),(4.5.1) 4.5.2, 4.6, 5.3, 6.4, 7.2-7.4, (8.1.2), (8.2.2) 8.2.3-8.3.2, 8.5, 8.6

[a] SNA numbers in ( ) means "part of".

Clearly the knowledge about ex ante production functions is very limited. This would suggest that a flexible functional form such as the translog production function or a generalized Leontief cost function should be used for the representation of the ex ante technology. However, in view of the difficulties to obtain relevant price-data for the estimation of these functions, they were not too attractive. Instead it seemed resonable to choose a functional form with a small number of parameters, and in which each one of the parameters has a simple economic interpretation. In other words, if one is forced to use a lot of "guesstimates" the number of guesses should be kept at a minimum and concern economically meaningful magnitudes.

From these points of view the nested CES-Cobb-Douglas function (see p. 11) was attractive, but the only possible choice. The distribution parameters (α_j) in the Cobb-Douglas part of the function were estimated by means of income distribution data for the base-year, i.e. for 1979, and the input-output coefficients (a_{ij}) are estimated by means of the ratios of intersectoral flows (X_{ij}) and gross output (X_j) that year. The ex ante elasticity of substitution between the capital-labor composite and the fuels electricity composite was set equal to 0.75 in all sectors except the energy sectors. This figure is compatible with the results presented in Pindyck (1980)[16], but of course subject to significant uncertainty. Lacking better information, finally, the same values were assumed for the elasticity of substitution between fuels and electricity. A selection of the adopted parameter values are displayed in Table 6.

The choice of a linear expenditure system for the representation of household demand is motivated by the fact that such systems have been estimated on Swedish data. The linear expenditure system used here is estimated by J. Dargay and A. Lundin (1981). It differs from other systems in that it treats fuels as a separate household consumption good (see Table 5), which is an advantage in the types of studies the present model will be used. However, as all linear expenditure systems it does not take substitution effects into account, which clearly is a disadvantage.

The "production functions" defining the composite goods consumed in the home country and the composite goods consumed in the rest of the world were all specified as CES-functions. The CES-specification is convenient since it leads to import and export functions which are relatively easy to estimate. However, the assumption that the same "production function" applies to all domestic users of good i is generally not plausible. If goods produced in different countries actually are qualitatively different, the substitutability between imported and domestically produced goods of the same "type" should, in general, differ between domestic users. In particular, for industrial users the substitutability should reflect the properties of technology, while it should reflect the properties of preferences for users in the household sector.[17]

However, whereas the model would remain fundmentally the same if different composite goods were defined for different domestic users, the estimation problems would increase significantly. In view of these problems the simplest possible specification was chosen. Thus, all domestic users of a given composite good, say i, are assumed to use the same type of composite good, i.e., use the same "production function" to define the composite good.

The numerical values of the parameters of the import and export functions have been chosen partly on the basis of econometric evidence, partly on the basis of theoretical considerations. Thus, one source of information is Hamilton (1980), another is Restad (1981). However, neither of these sources, or other possible sources, use a sector classification which can be aggregated into the one used in this study. Moreover, many of the estimated export price elasticities have absolute values which are so low that they are hard to accept on theoretical or even common sense grounds. Consequently, a considerable amount of judgement and "fingerspitzgefühl" has gone into the estimates of ε_i and μ_i displayed in Table 6.

Table 6. Estimated values of some key parameters

Sector	α_j	$(1-\rho_j)^{-1}$	$(1-\gamma)^{-1}$	ε_j	μ_j
0	0.8362	0.25	0.25	0.0	0.5
1	0.7555	0.25	0.25	-1.0	0.5
2	0.3770	0.75	0.75	-2.0	4.0
3	0.3076	0.75	0.75	-5.0	0.5
4	0.2053	0.75	0.75	-4.0	2.0
5	0.3600	0.75	0.75	-2.0	0.5
6	0.0436	0.75	0.75	-	-
7					

It should be noted that the export functions and the current account constraint are specified in such a way that the "home country" has some autonomy in the pricing of its exports. Whether or not a significant deviation between domestic production cost, i.e., the variable $P_i(t)$, and the corresponding world market price, i.e., the variable $V(t)P_i^{WE}(t)$, would exist in equilibrium depends on the absolute values of ε_i as well as on the properties of the supply functions. As deviations between domestic production costs and world market prices are not consistent with the notion of "a small open economy", some constellations of parameter values would necessitate a respecification of the foreign trade part of the model. Otherwise the model would indicate terms of trade gains of the optimum tariff type from domestic energy taxation. Such a respecification would then be carried out along the following lines.

The total output in sector j is assumed to consist of an aggregate of a large number of goods. The price $P_j(t)$ is taken to be the price index of this aggregate. Some of the goods in the aggregate are exported at given world market prices. The price $P_j^{WE}(t)$ is taken to be the price index of the aggregate of goods exported from sector j. Using the same symbols as before, the value of the total output from sector j can be written in two identical ways in accordance with

$$P_j(t)X_j(t) = V(t)P_j^{WE}(t)Z_j(t) + P_j^N(t) \{X_j(t) - Z_j()\};$$

where thus $P_j^N(t)$ is the price index of the non-exported part of the output from sector j.

The current account constraint should now be written

$$\sum_{i=1}^{n} P_i^{WE}(t) Z_i(t) - \sum_{i=0}^{n} P_i^{WI}(t) \frac{M_i(t)}{1+\theta_i} - \sum_{j=0}^{n} P_j^{C}(t) M_j^{C}(t) = D(t)$$

while the unit cost of the composite goods used within the country should be defined

$$P_i^D(t) = \phi_i\left(P_i^N(t), P_i^M(t)\right); \qquad i=0,1,\ldots,n.$$

rather than

$$P_i^D(t) = \phi_i\left(P_i(t), P_i^M(t)\right); \qquad i=0,1,\ldots,n.$$

Thus, the model can be made perfectly consistent with the usual "small economy" assumptions also in the case where the absolute values of the parameters ε_i are rather small. However, unless the energy tax variables are given quite high values, "optimum tariff effects" do not seem to be a problem at the parameter values displayed in Table 6.

5.3 Some numerical results

To conclude the description of the ELIAS-model, a few results from model-simulations are presented. These results primarily serve the purpose of indicating the functioning of the model. Thus, there is no point in carrying out an extensive discussion about underlying assumptions about exogenous variables; it is sufficient to say that the "Base case" presented below is based on the assumptions and results of the most recent long-term economic survey published by the Ministry of Economic Affairs.

Table 7. The calculated impact of increasing labor supply.
Annual percentage growth rates 1979-91.

	Base case	Labor Plus case
Private consumption[a]	1.3	2.5
Public consumption[ab]	0.9	0.9
Gross investments[a]	2.9	3.6
Exports[a]	4.0	4.9
Imports[a]	1.7	2.4
GDP[a]	2.2	3.0

[a] In constant 1979 prices.
[b] Exogenously determined.

To begin with two "comparative dynamics" experiments are presented. Thus, in Table 7 the impact of an increasing supply of labor is displayed. In the Base case the labor force is assumed to decline by 0.2 percent per annum, while it is assumed to grow by 0.8 percent per annum in the Labor Plus case.

The next "comparative dynamics" experiment concerns the role of expectation formation and the functioning of the vintage capital and putty-clay properties of the technology. Thus, in the Base case static expectations were assumed, i.e., the producers/investors were assumed to expect current relative prices to prevail in the future as well. In the Oil Price Foresight case, on the other hand, the producers/investors are assumed to foresee an annual 2 percent increase in the real price of imported oil correctly. As a result, total oil consumption declines by 0.1 percent per annum in the Oil Price Foresight case, while it increases by 1.1 percent per annum in the Base case.

In Table 8 below the development of sectorial oil input coefficients in these two cases is displayed. A distinction is made between the average sectoral oil input coefficients and the oil input coefficients in new plants i.e, plants designed in the period in question and taken into operation the following period. The latter set of coefficients are expressed as percentages of the average sectoral oil input coefficient.

It should be noted that the assumptions made in the Oil Price Foresight case lead to approximately the same, but relatively low, oil input coefficients in new plants in all periods, while these coefficients tend to decline over time in the Base case where expected oil prices gradually increase. Also it should be noted that the development of average oil input coefficients over time partly reflects the properties of the new technology, and partly reflects the amount of gross investments in the sector in question.

In Table 9, finally, some comparative static experiments are presented. The results are presented as elasticities computed around the 1991 Base case equilibrium values.

Table 8. The calculated development of sectoral oil input coefficients 1979-91 under various assumptions about oil price expectation formation

	Base case		Oil Price Foresight case	
	Average sectoral oil input coefficient	Oil input coefficient in new plants %	Average sectoral oil input coefficient	Oil input coefficient in new plants %
a. Sector 2: Mainly import competing industries				
1979	0.0159	98	0.0159	75
1982	0.0158	86	0.0142	75
1985	0.0149	84	0.0129	76
1988	0.0140	84	0.0118	78
1991	0.0132	84	0.0108	81
	-1.5 p.a.		-3.2 p.a.	

b. Sector 3: Mainly exporting energy intensive industries

	Average sectoral oil input coefficient	Oil input coefficient in new plants %	Average sectoral oil input coefficient	Oil input coefficient in new plants %
1979	0.0650	100	0.0650	76
1982	0.0650	94	0.0610	79
1985	0.0636	92	0.0570	81
1988	0.0618	91	0.0535	83
1991	0.0599	90	0.0505	84
	-0.7 p.a.		-2.1 p.a.	

Table 8 (cont.)

	Base case		Oil Price Foresight case	
	Average sectoral oil input coefficient	Oil input coefficient in new plants %	Average sectoral oil input coefficcient	Oil input coefficient in new plants %
c.	Sector 4: Other mainly exporting industries			
1979	0.0132	100	0.0132	77
1982	0.132	90	0.0116	80
1985	0.0125	89	0.0104	84
1988	0.0118	88	0.0095	86
1991	0.0110	89	0.0087	87
	-1.5 p.a.		-3.4 p.a.	

d. Sector 5. Sheltered industries and serice production

	Average sectoral oil input coefficient 0.0159	Oil input coefficient in new plants 75 %	Average sectoral oil input coefficient	Oil input coefficient in new plants %
1979	0.0639	100	0.0639	77
1982	0.0639	94	0.0603	78
1985	0.0630	93	0.0571	80
1988	0.0618	94	0.0542	83
1991	0.0606	92	0.0517	84
	-0.4p.a.		-1.8 p.a.	

Table 9. Computed elasticities of selected macro-economic variables with respect to oil and export price changes

	Elasticity with respect to changes in		
	Oil prices		Export price in sector 4[c]
	Import price change	Domestic oil tax change	
Private consumption[a]	-0.17	0.01	0.10
Gross investments[a]	-0.16	-0.01	0.13
Exports[a]	0.24	0	-0.04
Imports[a]	-0.11	0	0.22
GDP[a]	0.01	0	0
National income[b]	-0.19	0.01	0.24
Terms of trade	-0.37	0	0.24
Oil consumption	-0.04	-0.03	0.01

[a] Constant 1979 prices.
[b] Evaluated at equilibrium factor prices.
[c] The variable .

As can be seen in the table the Armington assumption in conjunction with the exogenously given current account deficit (or surplus) leads to very strong terms-of-trade effects of world market price changes. The significant increase in exports which is necessary to restore equilibrium in the case of a world market oil price increase, suggests that rigidities in the adjustment process might cause additional income losses. It should also be noted that, due to the technology assumptions, the short-run price elasticity of the demand for oil is very low.

NOTES

[1] These exceptions are the determination of the gross savings ratio and the sectoral allocation of investments.

[2] For a different approach to the same end, see Norman-Wergeland (1977).

[3] By "type" is simply meant number in some commodity classification system such as the SITC.

[4] The less than perfect substitutability of aggregated commodity groups with different countries of origin can be shown to be a result of aggregation over individual products which do have perfect substitutes produced in other countries. See Bergman and Por (forthcoming).

[5] See for instance Dervis et al (1979) for a brief survey. For some econometric evidence, see Frenger (1980).

[6] The "~" over the variables indicate their ex ante nature.

[7] P^{WE} might differ from P^{WI} because the former corresponds to a fob price while the latter is a cif price.

[8] Let M_i^* be imports at constant cif prices, and define $\phi_i = \phi_i^0 + \Delta\phi_i$, where ϕ_i^0 is base year tariffs and indirect taxes on imports, while $\Delta\phi_i$ reflects changes in that parameter thereafter. Imports at constant market (or purchaser's) prices can then be written $M_i = (1+\phi_i^0)M_i^*$. At a given point in time the domestic market value of imports can be written

$$(1+\phi_i^0+\Delta\phi_t)VP^{WI}M_i^* = VP^{WI}[(1+\Phi_i^0)M_i^*] + \frac{\Delta\phi_i}{1+\phi_i^0}\ VP_i^{WI}[(1+\phi)M_i^*] =$$

$$= (1 + \frac{\Delta\phi_i}{1+\phi_i^0})VP_i^{WI}M_i \equiv P_i^M M_i .$$

But as $M_i^* = M_i/(1+\phi_i^0)$ this becomes

$$P_i^m M_i = (\frac{1+\phi_i^0+\Delta\phi_i}{1+\phi_i^0})VP_i^{WI}M_i \text{ or } P_i^M = (1 + \frac{\Delta\phi_i}{1+\phi_i^0}\ VP_i^{WI}$$

[9] Provided $P^*_{\upsilon j}(t) > \sum_{i=0}^{1} P_{ij}(t)a_{\upsilon ij}$. Otherwise $P^*_{\upsilon j}(t) = 0$

[10] Alternatively sectoral investments can be exogenously determined. That can be a reasonable approach particularly in the public sector.

[11] The difference between "exogenous variables" and "parameters" is that the former may have different values in different periods, while the latter are constant over time.

[12] If Figure 1 is assumed to apply for i=0, it presupposes that a solution for i=1 already has been obtained.

[13] See Johansen (1960), p.3.

[14] Estimates of the 1979 energy flows are, however, available.

[15] For 1975 a complete input-output table is available. However, as only one year had elapsed since the 1973/74 oil price increases, it is quite likely that the energy input coefficients which can be observed in the 1975 input-output table do not represent equilibrium values. To some extent the same problem apply to the 1979 data, but in view of the errors in the national accounts up to 1978 that have been discovered, the 1979 data was regarded as the best choice.

[16] On the basis of pooled time-series data from ten countries, Pindyck estimated, among other things, the elasticity of substitution between capital and labor (σ_{KL}), between capital and energy (σ_{KE}) and between labor and energy (σ_{LE}) for the industrial sector. His results should be regarded as estimates of the long run elasticities of substitution, and can thus be used to characterize the properties of the ex ante technology. With our assumptions included in paranthesis, Pindyck's results were the following:

σ_{KL}: 0.77-0.82(1.00), σ_{KE}: 0.61-0.86(0.75),

σ_{LE}: 0.93-0.97(0.75).

[17] Available econometric evidence does <u>not</u> support the hypothesis that the same "production function" can be used to define the composite good i for all sectors. See Frenger (1980).

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