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PRODUCTIVITY DECELERATION WHEN TECHNICAL CHANGE ACCELERATES

by

Thomas Lindh

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Abstract

Leif Johansens short run macro production function is used to explore the conditions for a productivity slowdown to take place simultaneously with an accelerated technical change in production. A capacity distribution such that the supply curve is concave at the extensive margin considerably increases the likelihood of a reversed relation. Changes in price expectations may further reinforce this mechanism. Increasing costs of transferring labour to new investment is the key mechanism behind the results. The mechanism emphasizes the decisive role of past investment and future price expectations in shaping the relation between technical change and productivity growth.

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** Department of Economics, Uppsala University, Box 513, S–751 20 Uppsala, Sweden

Section 1.

INTRODUCTION

For a long time economists have been concerned about the slowdown that standard measures of productivity growth indicate in the late 60's or somewhat later. The historically high rate of productivity growth in the first two decades after World War II in the industrialized countries seems to have been replaced by a considerably slower growth rate in almost all these countries. Many explanations have been offered, most often maybe the rise in energy prices in 1973, e.g. see Griliches(1988) and Jorgenson(1988), but declining knowledge production, slower rates of increase in labour skill, declining investment rates, shifts towards service production, and increasing government sectors and distorting taxes have also been appointed as causes, cf. S. Fischer(1988). For countries other than USA the exhaustion of technological gaps may also play a role, cf. Abramowitz(1990). The possibility that the whole slowdown really is due to measurement errors has also been pointed out, for example cf. Baily and Gordon(1988) for a general treatment of mismeasurement in the US, E. Berndt and Wood(1987) for some energy price related measurement issues, and B. Carlsson(1989) for micro–based measurement considerations. But the productivity slowdown still remains largely a puzzle and none of the possible causes has been generally accepted to explain more than minor parts of the slowdown.

One reason why economists continue to put forward new explanations for the productivity slowdown is, I believe, the feeling that productivity should have a close connection with the rate of technological change, in the sense of knowledge accumulation and technological advance in both the production process itself as well as new and better products. One puzzle posed by the productivity slowdown then is that it coincides with a period of technological progress that many intuitively feel must be rather fast and showing little sign of slowdown. Not only has there been a massive computerization in production but new and better materials, e.g. ceramics substituting for metals (Scientific American Oct 1986) as well as great changes in management and organization. The speed and sheer volume in the changes that have taken place in production techniques, in the quality of goods and services and the extension of choice we have experienced during the slowdown period makes it hard to accept that the 50's and 60's just were exceptions in the secular growth trend. Therefore the quest for an explanation to the productivity puzzle continues.

This paper considers one of several possible ways of reconciling an actual acceleration in technical change with a deceleration in the productivity growth rate. The basic idea is that the productivity structure of the inherited capital stock may be such that adjustment to new techniques becomes increasingly costly. The general idea that progress in production possibilities may at least temporarily have negative feed–back effects in the economy has old roots in economic theory. Hicks(1973) have formalized such effects in a neo–austrian framework and refers to Ricardo as the originator of a theory where mechanization causes unemployment and temporary decline in production. Some of Marx's (varying) explanations why the tendency towards a falling rate of profit does not materialize have a similar flavour, see especially ch. 15 sec. 3 in the third volume of "Das Kapital"(1894). The Schumpeterian business cycle theory incorporates similar ideas in the process of "creative destruction" (cf. Schumpeter(1911). Keynes' concept of the "marginal efficiency of capital" seems intended to capture analogue considerations, see ch. 11 in "The General Theory..."(1936). Such effects of course may be captured in many different formal mechanisms but the key idea is in all cases that the existing capital structure somehow prevents or distorts immediate adaptation to new production techniques.

It is a fairly simple exercise to show that decelerating labour productivity may be compatible with acceleration in technical change in a simple standard Solovian growth model given certain parameter settings[1]. In the Solow type of model technical change is however by definition identical to total factor productivity and it is therefore not possible to explain any slowdown in total factor productivity by this mechanism. Of course, the mechanism of increasing adjustment costs due to the capital structure has no place here either, although an assumption of increased depreciation rates or a decreased savings ratio may very well cause labour productivity to fall in the simple Solow model.

Even though it is an old idea, it has turned out to be difficult to model the influence from an irregular capital structure on productivity growth since it is tied to the transitory paths of non–steady state growth. When the importance of induced capital obsolescence mechanisms have been considered in the slowdown debate, as e.g. by Hulten, Robertson and Wykoff(1987), and by Baily(1981), it has been within a framework of parametrization of the capital vintage structures, making it possible to work with quality adjusted capital stocks and measures of capital utilization, instead of an explicitly non–balanced vintage structure.

This study is conceptually based on a simple one good macro model of

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[1] I owe this observation to Stefan Lundgren, at IIE in Stockholm. Let labour productivity $\pi(\kappa,t)$ be a function of capital intensity and time. The proportional rate of growth is $\hat{\pi} = (1-\alpha)\hat{\kappa} + \hat{\theta}$, α labour elasticity of production and $\hat{\theta}$ is the rate of technical change. The time derivative $\dot{\hat{\pi}} = (1-\alpha)\dot{\hat{\kappa}} - \dot{\alpha}\hat{\kappa} + \dot{\hat{\theta}}$, implies $\dot{\hat{\kappa}} < 0$ and $\dot{\alpha} > 0$ is sufficient for some acceleration of technical change to actually result in labour productivity deceleration. Assume the savings ratio s and depreciation rate δ are constant. $\hat{\kappa} = s\pi/\kappa - \delta$ implies $\dot{\hat{\kappa}} = s\frac{\pi}{\kappa}(\hat{\theta} - \alpha\hat{\kappa})$. An elasticity of substitution less than unity imply $\dot{\alpha} > 0$. Choose parameter values $s = 0.2$, $\delta = 0.08$ and assume $\alpha = 0.6$ and the capital/output ratio $\kappa/\pi = 2$, then $\hat{\theta} > 0.012$ will be sufficient. These values are empirically reasonable so it is certainly a possibility.

the vintage type, pioneered by Leif Johansen(1959), Robert Solow(1960,1962) and W. Salter(1960,1965) and extensively studied in a variety of forms in the 60's by i.a. Phelps(1963), Bliss(1968) — putty–clay and perfect foresight— , Sheshinski(1967) — putty–clay and static wage expectations, Solow–Tobin–Yaari–Weizsäcker(1966) — clay–clay and perfect foresight—, Kaldor and Mirrlees(1962) — technological progress function—, etc. just to mention a few representative papers. M. N. Baily(1981) used a putty–putty variant in an early study pointing to obsolescence due to rising energy prices as an important cause of the slowdown which has been followed up by several others seemingly with inconclusive results. An interesting recent paper by J. Benhabib and A. Rustichini(1989) use a variant based on a utility maximizing representative agent with fixed capital objects distributed as a time–indexed sequence. Their model includes an echo effect in investment activity such that earlier investment slumps will cause another slump in the future. This is related to the main idea of this paper. They also estimate the model on aggregated U.S. investment data and find vintage effects supported by the data. A fixed lifelength for capital equipment is assumed, making it difficult to compare with the model in this paper.

The formal production model chosen is version of the short run macro production function of Leif Johansen(1972) with one variable factor, based on aggregation of capacity distributed over fixed input coefficients without explicit reference to vintages, but still closely related to vintage models in general. In this context the concept of labour productivity is more natural than total factor productivity since the marginal rate of substitution between different capital input cannot in general be made independent of the labour input thereby preventing aggregation of capital equipment into a homogeneous stock that is separable from labour in the production function. Without imposing regularity conditions on the structure which we want to avoid the concept of total factor productivity becomes ambiguous.

In a vintage type model productivity varies due to technical change but also by changes in the capital structure. Irregularities of the capital structure will thus provide for variation in the transmission from technical change to productivity growth. The putty–clay context also forces a more explicit recognition of the crucial importance of price expectations for long term investment since every investment is a unique sunk cost. Thereby this model emphasizes the dependence of present economic performance on both the past history and expectations on the future.

The basic idea about increasing costs for implementing new technique is a two–way mechanism. One way is by an increase in the wage raise required to transfer a certain amount of labour from obsolete equipment at the extensive margin to new best–practice equipment on the intensive margin of production[2]. The other way is by increases in capital costs due to changes in expectations on future returns to capital. The former mechanism starts to function when a given rate of increase in wages tends to free less and less labour per time unit thus raising operating costs in new investment. The latter effect is less well defined since it depends on expectations and how these change, but typically periods of extensive scrapping may induce both an increase in the interest rate due to increased investment demand and a shortening of expected economic lives of capital equipment. Both these effects tend to increase capital costs and thus slows down investment. This is important both for a slowdown in labour productivity growth and total factor productivity growth as

[2] Lawrence Lau has pointed out to me that the same kind of effects could be achieved by replacing the assumption of full employment by a fixed rate of increase in wages. Deceleration in labour productivity will then be caused by a decrease in the growth of labour demand, causing unemployment if labour supply grows at a fixed rate.

measured by deducting factor share–weighted rates of factor growth from the total growth in production.

When the short run supply schedule is concave on the margin of total capacity it can be shown both that a given transfer of labour requires a higher wage raise than in preceding periods and that a given reduction of production costs requires a relatively more extensive scrapping. A concave supply schedule will then be a sufficient condition for productivity slowdown when technical change is constant given that expectations react in the above indicated way. It is far from necessary however, although it is intuitively easier to understand the mechanisms in this case. Such a short run supply schedule is essentially the distribution of input coefficients ranked from the lowest to the highest over accumulated output. One needs only multiply by the ruling wage to get supply as a function of price. It is often referred to as a Salter curve, cf. Salter(1960).

A Salter curve that is concave in the upper portion may be the result of a similar investment slump at the time of installation of the equipment that is now on the margin of obsolescence. Hence we have here an echo mechanism by which earlier slowdowns and speedups tend to show up in the future, too. Not in a perfectly cyclical fashion since lifelengths will change and so there will be no exact periodicity. It is interesting to note here that simulation studies of vintage models calibrated with real data (Bentzel(1978), Melén(1990)) turn up very substantial reductions of the endogenously determined life length of capital equipment in the late 60's, indicating dramatically increased scrapping rates. But the concave portion may also be due to localized changes in the specific techniques used in older vintages which may even have rearranged the order of vintages within the input coefficient ranking.

Expectations are hard to model in this long run context. Assumptions of static expectations about wages and interest rate are obviously unsatisfactory on a growth path with changing growth rates. Perfect foresight are hard to solve for on such a path and also hard to motivate, since there would be no previous experience to learn such expectations from. To avoid these problems at an early stage of the study the choice here is to treat expectations as a black box concept, i.e. as exogenous to the formal model and only discuss their relations to endogenous changes heuristically. Since the determination of interest rates will depend heavily on expectations of future developments of production, so will consumption demand too, being residual to savings. Therefore only the production side of the model economy is explicitly modelled.

The paper is organized as follows. Section 2 attempts to give an intuitive non–formal description of the essential mechanisms of the model. Section 3 develops the formal production model of a putty–clay structure with a representative firm taking decisions on scrapping and investment. A law of motion of the production structure is derived that shows changes in aggregate production to be the product of new capacity flow and the relative difference in input coefficients of the structure. Section 4 derives a basic bench–mark condition for simultaneous deceleration in aggregate productivity growth and acceleration in technical change under simplifying assumptions; fixed discount rates, expected wage rates and life time of capital, fixed labour supply and a Cobb–Douglas type production function characterizing the best–practice technique. In section 5 the restrictions on expectations are relaxed and conditions on how they must move in order to slow down productivity is derived. The plausibility that endogenously determined expectations would move in that direction is discussed informally. In section 6 assumptions of fixed labour supply and unit elasticity of substitution are relaxed and the relation labour productivity to total factor productivity is discussed. Section 7 finally holds some concluding comments. The appendix contains a list of variables for easy reference and some proofs of assertions made in the text.

Section 2.

INTUITION

Before engaging in formal modelling it may be helpful to gain some intuitive insight in the mechanisms of how productivity interacts with technical change in a vintage context. First consider a simplified vintage model. One good is produced by labour and capital in the form of once–and–for–all fixed pieces of equipment which once installed have fixed input/output coefficients, i.e. a conventional putty–clay structure. For simplicity we keep capital costs fixed.

Let the vintage structure of capital be described by a distribution of output capacity over fixed labour input coefficients. In fig. 1a) an arbitrary distribution of output capacity for each input coefficients is depicted and in 1b) the resulting distribution of average labour costs per unit (or input coefficients, with a suitable scale) over accumulated output. The latter we can regard as a short run supply schedule, since equipment will not be used when labour costs exceed returns. Assuming a fixed demand schedule and free competition the price of the good will equal average variable cost at the extensive margin and the present value cost per unit at the intensive margin.

In fig. 2 we have a diagram of two extreme examples of supply schedules together with the fixed demand. Fig. 2a) will be referred to as a backflat supply and 2b) as a backsteep supply. It is assumed that in both cases we have market equilibrium at the same relative price in terms of labour units. We also assume that in both cases the minimal input coefficient is the same. Under these assumptions the economy in 2a) would require more labour than the one in 2b) to carry out the same production and hence would have a much lower aggregate labour productivity.

Given this setup we assume a sudden jump to occur in production possibilities which the economy adapts immediately to. To facilitate comparison we depict the sudden change in production opportunities as an equal reduction in both cases of the minimal input coefficients.

The story behind the diagrams can be taken in steps. At given prices the reduction in the minimal input coefficient will reduce the present value cost per produced unit in best–practice equipment thus making new investment profitable. When the production from new investment arrives in the market real wages will be raised. This partly leads to scrapping of now obsolete equipment and partly offset the reduction in present value cost per unit in front production, thus equilibrating the market. It is then clear that a given change in present value cost will tend to transfer considerably more labour to the front production from the scrapped equipment in the backflat case than in the backsteep case. Hence in the backflat case many production units with low labour productivity are replaced by units using the best available technique, while in the backsteep case relatively few such low productive units are replaced.

Thus, the impact of a given change in technique will have a proportionally greater influence on aggregate productivity in the backflat case. Whether the jump in the rate of technical change will be associated with deceleration in aggregate productivity *growth* of course depends on the rates of change preceding the jump in the front. By introducing another jump of equal size in the input coefficients of the best available technique it is evident that the introduction of this second reduction in average labour cost must imply a transfer of relatively less labour than the preceding reduction in case 2a) where the right end of the distribution is concave, while exactly the converse happens in case 2b). Not only are fewer units replaced in the second transition in the backflat case but the share of each unit in total production has also decreased, thus further reducing the contribution to aggregate productivity growth. Aggregate production per capita would hence increase at a *decreasing* rate in the backflat case. Now it is obvious that a slightly greater reduction of average cost in the second jump might still be associated with a slowdown in the rate of aggregate productivity growth. Of course a slowdown is all the more probable if technical change also decelerates, but the point to be made here is that a slowdown in productivity does not necessarily signal a slowdown in technical change. It should also be noted that if the reduction in average cost is sufficiently large productivity will increase nevertheless. But there is some positive degree of acceleration in technical change that can occur simultaneously with decelerating productivity growth.

The above reasoning is of course much too simplified to be relied on for any definite conclusions. If we allow capital cost adjustments and substitution to work freely there may be both dampening and reinforcing feed–backs from the markets. Note that in case 2a) a considerably larger investment must be made to take advantage of the cost reduction. When capital costs are flexible these would increase and thus dampen investment relatively more in case 2a) than 2b). Hence productivity growth would be further damped. Growth in the labour supply, factor substitution, deterioration and disembodied technical change occurring simultaneously would naturally influence conclusions like many other modifications.

Obviously assumptions on demand and expectations are very important in this context. Since there is no difference in price between investment goods and consumption goods with only one aggregate good the price of capital goods does not enter into capital costs. In a more disaggregated model the movement of relative prices would also be important. Ruling out balanced growth — only example 2b) could possibly be a steady state supply schedule — the direction of change in conclusions will depend on the details of interest determination and expectation formation as well as the elasticity of substitution in the front production. Any factor–saving bias in the evolution of front production possibilities would also be important. So even though the diagrams and the reasoning above make the countervariation in technical change and productivity growth intuitively plausible when the supply schedule is concave at the extensive margin, it is by no means evidence that a formally specified model economy, where indirect effects and price responses are taken into account, would confirm this intuition. In the following we will therefore in very simple formal model specify more rigorously the conditions for second order changes in best practice technique and productivity to be in opposite directions. With these basics clear we can then make some of the extensions mentioned above.

Section 3.

THE REPRESENTATIVE FIRM AND ITS PRODUCTION STRUCTURE

This section contains the formal production model which will be used for a more rigorous analysis of the mechanisms discussed in the preceding section. The model must be simple enough to be tractable yet maintain the essential complexity of a heterogeneous capital structure. The general background should be thought of as an economy in temporary market–clearing equilibrium in continuous time. Formally, however, we will not explicitly model the demand side, but be content to discuss it informally.

Our general modelling strategy aims only at showing the influence from a historically given capital structure and expectations of future price movements on the transmission from changes in the technical potential to changes in economic productivity. After having laid down a formal production framework here we will in the next section derive a basic condition for productivity slowdown. Given that, we will then in section 5 discuss how parametric changes in price expectations will modify the picture. In section 6 we relax two other important simplifying assumptions to be made below, viz. fixed labour supply and unit elasticity of substitution in the best practice technique.

The production model is a conventional putty–clay type where one homogeneous good is produced by one primary factor, labour, using heterogeneous capital equipment. The good can either be immediately consumed or frozen into capital equipment which is impossible to recover for consumption. The good is chosen as numéraire for real prices in the model.

The "one good" assumption abstracts from several real world features, among which the most important probably is the very different characteristics of investment and consumption goods. But one essential difference between these goods is preserved by the freezing assumption, viz. the durability and commitment to the future that investment as a sunk cost carries with it. The investment cost cannot be recovered should outcomes deviate from expectations.

Not only is the equipment as such fixed but we also assume that producers expect that once installed it can only be operated at a fixed labour/output ratio. They also disregard the possibility of disembodied technical change and deterioration of equipment so output from a specific equipment will also be believed to be fixed. We will further adopt the assumption that this belief is true in the current short run.

But there are no restrictions, apart from analytically convenient continuity and differentiability assumptions, on what kind of historic path that actually has generated the structure. The assumptions we will make on the current production possibilities, current technical change etc. refer only to current time and its immediate vicinity. That makes the model more generally valid as an approximation for empirical problems, but of course prevents theoretical conclusions about the long and medium run, limiting us to statements only about the immediate future. Ideally we would of course like to loosen these assumptions, but this must be a later task.

Before proceeding to the formal model we must be specific about what we mean by aggregate productivity growth. As remarked in the introduction the standard definition in terms of total factor productivity growth is not well suited to our heterogeneous capital structure. Productivity here therefore means output per labour unit. In section 6 we will discuss this further and show how the conventional measure of total factor productivity moves in the same direction as labour productivity in our model if the 'stylized fact' of a constant capital/output ratio is assumedd. From here on 'productivity' always refers to labour productivity unless otherwise stated.

Let $\Pi = F/V$ stand for aggregate productivity, where F is aggregate production and V total labour supply. It will prove convenient to work with proportional growth rates, and also more natural since we rarely talk about changes in other terms. To simplify notation we let a circumflex over a variable denote a total logarithmic time differentiation. If we differentiate the proportional growth rate $\hat{\Pi}$ w.r.t. to t we get

(3.1) $$\frac{\mathrm{d}}{\mathrm{d}t}(\hat{\Pi}) = \frac{\mathrm{d}}{\mathrm{d}t}(\hat{F} - \hat{V})$$

which is the change over time in aggregate productivity growth if V varies over time. With V fixed, aggregate productivity will decelerate if growth in aggregate production slows down

(3.2) $$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{F_t}{F}\right] = \frac{F_{tt}}{F} - \frac{F_t^2}{F^2} < 0 \text{ or } \frac{F_{tt}}{F_t} < \frac{F_t}{F} \text{ if } F_t > 0$$

where subindices denote partial derivatives. Obviously this is substantially easier to analyze so we will keep labour supply V fixed here, and discuss the effects of relaxing this assumption only in section 6. Our aim is to show how exogenous technical change, the labour elasticity of best–practice technique and the capital structure will determine F_{tt}/F_t.

Having assumed a description on the basis of a putty–clay approach, there are however alternative formal ways to describe such a structure. A description in terms of capacity ranked by labour/output ratios in the manner of Leif Johansen's(1972) short run macro production function is the more convenient for our modelling strategy. Since the existing structure is given arbitrarily by history, we can dispense with the explicit vintage distribution and use only a description in terms of input coefficients, because we need not derive the structure analytically from any given history of investment.

Let ξ denote labour/output ratios, and assume there is a distribution of capacity over these input coefficients at current time t,

(3.3) $$\Psi(t) = \int_{X(t)}^{\infty} \psi(\xi,t)\mathrm{d}\xi$$

$X(t)$ being the minimal input coefficient at that time. Assuming that new investment is made in equipment with the minimal input coefficient, the density function can be simplied to $\psi(\xi,t) \equiv \psi(\xi)$ since we assumed installed equipment to have both fixed input coefficients and fixed output. We further assume the density to be continuously differentiable without any point masses and the support $\{ \xi : \psi(\xi) > 0 \}$ to be a connected set, i.e. there is a positive capacity for each $\xi \geq X(t)$ up to some maximal input coefficient. $X(t)$ is also assumed to be continously differentiable. That $X(t)$ is monotonously decreasing is a simple consequence of the above assumptions. Keep in mind that this refers to the vicinity of current time and need not have been true over the whole history of the structure.

Assume one uniform wage rate for all uses of labour. It is then obvious that an optimizing representative firm will use all capacity at lower fixed input coefficients before transferring any labour to less productive equipment. Since the capacity density is continuous there will be no unit used at less than full capacity. Labour market equilibrium with an exogenously given labour supply holds when

(3.4) $$V(t) = \int_{X(t)}^{R(V,t)} \xi\psi(\xi)\mathrm{d}\xi$$

where $R(V,t)$ is the maximal input coefficient for capacity actually used. Assume full employment. Then the aggregate production function can be written

(3.5) $$F(V,t) = \int_{X(t)}^{R(V,t)} \xi\psi(\xi)\mathrm{d}\xi$$

The function arguments will be suppressed from here on whenever it is not needed for clarity.

We can differentiate (3.4) and (3.5) w.r.t. V, thus obtaining the standard result

(3.6) $$F_V = \psi(R)R_V \text{ and } 1 = R\psi(R)R_V \Rightarrow F_V = \frac{1}{R}$$

meaning that the aggregate marginal productivity equals the average productivity in the least productive equipment.

Differentiating (3.4) w.r.t. t, using a dot over the variable to denote this, and recalling that V is fixed, we get

(3.7) $$R\psi(R)R_t = X\psi(X)\dot{X}$$

with the obvious interpretation that labour released from existing equipment equals that employed in new equipment. Likewise differentiating (3.5) yields

(3.8) $$F_t = -\psi(X)\dot{X} + \psi(R)R_t$$

The change in production is the added capacity minus the abandoned. These two expressions hold even if V is not fixed but the interpretation then requires the addition of equal terms on both sides for capacity changes due to changes in total labour supply. We can then define the flow of new capacity as

(3.9) $$\varphi(t) \equiv -\psi(X)\dot{X}$$

and by using (3.7) and (3.6) to substitute, rewrite (3.8) as

(3.10) $$F_t(V,t) = \varphi(t)(1 - F_V(V,t)\cdot X(t))$$

where arguments are written out to emphasize dependencies[3] or equivalently, skipping the arguments,

(3.11) $$F_t = \varphi(1 - \frac{X}{R})$$

we can express the parenthesis in terms of the relative gap in input

[3] This partial differential equation actually can be shown to have the vintage description as solution under certain conditions, cf. Pomansky and Trofimov (1990), hence confirming the equivalence of the two descriptions. Such a solution, however, is of little use to us, being only a translation from one formalism to another which could be derived independently.

coefficients. These equations serves as the law of motion for the production system. They quite generally describes the change in production over time and will be our main vehicle in deriving the relation between changes in productivity growth and technical change. But first we need to determine how the capacity flow and minimal input coefficient will move.

We then assume a representative firm in the model that takes all production decisions. The firm first has to make a decision whether to go on using already existing equipment or not. We will call this the scrapping decision since the equipment has no alternative use. But we keep open the possibility that scrapped equipment may be reinstated in production without incurring any extra costs. This will ordinarily not happen but we want to avoid paying specific attention to the restriction imposed by actually truncating the capacity distribution.

The scrapping decision is a very simple one. Because there is no alternative use for the equipment it only depends on whether production can cover operating costs. With a continuous capacity distribution wage costs will be equal to production in the least efficient production unit used. Using w for real wages

(3.12) $$w = F_V = \frac{1}{R}$$

is the scrapping condition, so the usual marginal productivity condition holds for the aggregate.

The firm must also make an investment decision. We assume the firm chooses its investment, k, and the labour supply flow, v, necessary to operate the new equipment in order to maximize the present value of its future profit flow from the investment. The front production function, $f(k,v,t)$, is the maximum capacity given k and v, with standard properties as differentiability and quasi–concavity. It changes over time due to technical change. We will also assume f to be linearly homogeneous in k

and v and $f(k,0,t) = 0$. The firm maximizes the present value $P(k,v)$ of expected profits at each point in time. Let r stand for the interest rate.

(3.13) $$\max_{k,v} P(k,v,t) = \int_t^{\infty} (f(k,v,t) - \tilde{w}(z,t)v(t))\, e^{-\int_t^z \tilde{r}(x,t)\mathrm{d}x}\mathrm{d}z - k$$

subject to $f(k,v,t) \geq \tilde{w}(z,t)v(t)\ \forall z$

where $\tilde{r}(\cdot)$ and $\tilde{w}(\cdot)$ are the currently expected time paths of interest rates and wages. Let $\tilde{\ell}$ stand for the life time the investment is expected to last, i.e. yielding non–negative quasi–rents, and assume the firm not to expect any further use of the equipment. Expectations of monotonously rising wages will then imply a finite $\tilde{\ell}$ and also a connected period of usage. This also guarantees P to be bounded for any finite choice of k and v. To clean up notation somewhat we leave out the argument for current time in expectations and define $\tilde{\nu}(z)w(t) = \tilde{w}(z)$ so we can move the current wage out of the integral in (3.13) and define the shorthand expectational variables.

(3.14) $$D = \int_t^{t+\tilde{\ell}} e^{-\int_t^z \tilde{r}(x)\mathrm{d}x}\mathrm{d}z \quad \text{and} \quad W = \int_t^{t+\tilde{\ell}} \tilde{\nu}(z)e^{-\int_t^z \tilde{r}(x)\mathrm{d}x}\mathrm{d}z$$

The first order conditions for maximization can then be written conveniently as

(3.15) $$f_k = \frac{1}{D} \quad \text{and} \quad f_v = \frac{wW}{D}$$

Note that although only w is an endogenously determined factor price in the model, capital costs generally depend on w too, since $\tilde{\ell}$ depends on the level of current wages. The present value function may be convex due to the dependency of $\tilde{\ell}$ on the capital/labour ratio chosen, so a unique maximum cannot be guaranteed by first order conditions in general[4]. Here

[4] Cf. Bliss(1968), who gives a condition guaranteeing $P(k,v)$ to be locally concave on steady state paths if the elasticity of substitution is sufficiently high. Essentially this is a condition that the ex ante production function must not be too inelastic with respect to changes in capital intensity. In Appendix 3 this is further elaborated.

we simply assume f and expectations to have the properties needed for uniqueness. Assume that the economy is in a state of perfect competition so the present value of expected quasi–rents exactly cover investment costs

(3.16) $\varphi D - wvW - k = 0$

using φ to emphasize that this holds for actually installed capacity.

As stated above we will not write down a formal model for the consumer side of the economy but take that as exogenously given. But a short discussion is appropriate to clarify why it is not needed. First of all we assume labour supply to be exogenous, so only the savings in the economy remain to determine. Since both investment and consumption demand is for the same good all we would really need the consumer side for is the determination of the current interest rate. The savings decision of consumers given market interest rates will in the standard Ramsey model depend mainly on their time preferences and intertemporal elasticities of substitution. Investment demand on the other hand is mainly determined by expectations.

Incorporating e.g. a Ramsey model of the consumer side would determine the ruling interest rate given expectations on future production flow and thereby equalizing saving and investment. The interest rate would be heavily dependent on expectations which we prefer to keep exogenous any way so we loose very little by ignoring the consumer side and take interest as exogenous.

Since we also abstain from formally modelling the capital market it leaves us with the level of investment and production indeterminate, but as will be clear in the next section we will only need their rates of change to make our point.

It will prove convenient later on to work with the cost shares or factor elasticities of the ex ante production function and we therefore define the labour elasticity, the equalities holding in equilibrium

$$(3.17) \qquad \alpha \equiv \frac{wWv}{D\varphi} = \frac{wW}{D}X = \frac{XW}{RD}$$

which allows us to rewrite the law of motion for production, (3.10) and (3.11), in yet another way

$$(3.18) \qquad F_t = \varphi(1 - \alpha\frac{D}{W})$$

We will find both the above formulation and the others useful further on. While (3.10) emphasizes that the movement of production over time is determined by capacity flow times the current quasi–rent earned on new investment, (3.11) expresses this in terms of the relative gap in productivity between the intensive and extensive margin. This last formulation then emphasizes the elasticity of the ex ante production function and the decisive role of expectations, especially the role of wage increase expectations.

Returning to (3.2) we can then reformulate the condition for deceleration as

$$(3.19) \qquad \frac{F_{tt}}{F_t} < \frac{\varphi}{F}(1 - \alpha\frac{D}{W}) \text{ or } \frac{F_{tt}}{F_t} < \frac{\varphi}{F}(1 - \frac{X}{R})$$

Since $D < W$ and $\alpha < 1$ the RHS of the above inequality is always positive and for each given φ/F the more so the larger is the productivity gap between the intensive and extensive margin. We will not elaborate more on this side of the inequality. In the next section we will instead focus on translating the LHS of the condition into rates of technical change and elasticities of the capacity distribution and thereby demonstrate how the capital structure of an economy can dampen the translation of technical change into productivity growth.

Section 4.

THE BASIC CONDITION FOR DECELERATION IN AGGREGATE PRODUCTIVITY

When will the rate of aggregate productivity growth slow down even if the rate of technical efficiency growth increases in the best available techniques? We will derive an answer to that question in our simple model by introducing some further simplifications. Though not as general as could be desired this condition will nevertheless throw light on the interaction between technical change and a heterogeneous capital structure. We shall proceed by relating the rates of change in aggregate productivity growth to the elasticity of a predetermined arbitrary capacity distribution and the rates of change in a parameter of technical change in the ex ante production function.

4.1 Derivation of the basic condition

Given the ex ante production function $f(k,v,t)$ we define the rate of technical change as

$$(4.1) \qquad \hat{\theta} = \frac{f_t}{f}$$

that is the proportional increase with time in production possibilities with k and v fixed. This is a standard definition but note that here it refers to the ex ante production function and not to the aggregate production function and therefore is not identical to total factor productivity growth.

The role of expectations in the model is of course very important but in this section we only aim to achieve a bench–mark condition for deceleration in aggregate productivity growth. Recall that W and D will depend on expected life lengths, in turn implying they are both dependent on current wages and thus would change with wages even if interest is constant and the expected paths of changes invariant. To simplify the derivation of the basic condition we provisionally assume not only that the expectations of changes in interest and wage rates but also the expectational variables W and D are held fixed throughout adjustments and postpone treatment of changes in W and D to the next section. In section 5 we show that under the assumptions behind the basic condition, this will in general be equivalent to fixed expectations functions.

The basic condition for deceleration in this section will be derived under the assumption that the ex ante production function is of Cobb–Douglas type. In the first steps here we will retain a more general front production function in order to avoid duplication in section 6.

Using the formulations (3.11) and (3.18) of the law of motion of the production and differentiating logarithmically, recalling that V is fixed, we then have

(4.2) $$\frac{F_{tt}}{F_t} = \hat{\varphi} - \frac{\alpha D/W}{1-\alpha D/W}\hat{\alpha} \quad \text{or} \quad \frac{F_{tt}}{F_t} = \hat{\varphi} - \frac{1}{R/X-1}(\hat{X} - \hat{R})$$

This we want to relate to the rate of technical change and the capacity distribution. Rearranging (3.7), equating labour flow in new investment to that released from abandoned equipment, using the definitions (3.9) of capacity flow and (3.17) of the labour share, we have

(4.3) $$\varphi\alpha\frac{D}{W} = -\psi(R)R_t \Rightarrow \hat{\varphi} + \hat{\alpha} = \frac{\psi'(R)R_t}{\psi(R)}t + \frac{R_{tt}}{R_t}$$

which relates capacity flow at the intensive margin to the changes in the capacity distribution at the extensive margin. Define the elasticity of the capacity density function as

(4.4) $$e(\xi) = \psi'(\xi)\xi/\psi(\xi)$$

With the understanding that $R_t/R = \hat{R}$ only when V is fixed we can rewrite the RHS of the implication (4.3)

(4.5) $$\hat{\varphi} + \hat{\alpha} = e(R)\hat{R} + \hat{\hat{R}} + \hat{R} = -(1+e(R))\hat{w} + \hat{\hat{w}}$$

where we have used that $\hat{\hat{R}} = \frac{R_{tt}}{R_t} - \hat{R}$ and $w = 1/R$. Note how the second equality here confirms the intuition from section 2 that wage increases will dampen productivity growth when the capacity distribution is of a certain form. This will be still clearer further on. We now want to relate wage changes to the exogenous rate of technical change so we differentiate the ex ante production function logarithmically to get

(4.6) $\quad \hat{\varphi} = (1-\alpha)\hat{k} - \alpha\hat{v} + \hat{\theta} = \hat{k} - \alpha\sigma\hat{w} + \hat{\theta}$

where σ is the elasticity of substitution and the second equality is a simple consequence of the definition of σ implying $(\hat{k} - \hat{v})/\hat{w} = \sigma$ in this particular case when W is fixed. To simplify at this point we now assume a Cobb–Douglas specification of the ex ante production function

(4.7) $\quad f(k,v,t) = \theta k^{1-\alpha} v^{\alpha}$

which implies $\sigma = 1$, α constant and, since D fixed, $\hat{\varphi} = \hat{k}$. Then follows

(4.8) $\quad \alpha\hat{w} = \hat{\theta}$ and $\hat{\hat{w}} = \hat{\hat{\theta}}$

and the second term on the RHS of (4.2) vanishes and we get by (4.5)

(4.9) $\quad \frac{F_{tt}}{F_t} = \hat{\varphi} = -(1 + e(R))\hat{\theta}/\alpha + \hat{\hat{\theta}}$

We can then state the main result in this section, namely that productivity will decelerate if

(4.10) $\quad -(1 + e(R))\hat{\theta}/\alpha + \hat{\hat{\theta}} < (1-\alpha D/W)\frac{\varphi}{F}$

giving us a simple bench–mark form of the deceleration condition. The RHS of this inequality is always positive. Clearly if the elasticity of the capacity distribution is positive a proportional acceleration in the rate of technical change may be perfectly compatible with deceleration in aggregate productivity growth as long as $\hat{\hat{\theta}}$ is not substantially greater than $\hat{\theta}$. The condition may still hold for a range where $e(R)$ is not too negative and $\hat{\hat{\theta}}$ remains positive.

Note that $e(R) > -1$ will make higher rates of technical change contribute to deceleration. In fact, as long as technical change is positive this will be a sufficient condition to guarantee that productivity deceleration can take place at the same time as some positive degree of acceleration in technical change. For any given value of $e(R)$ above -1 it will be the case that the higher $\hat{\theta}$ is, the higher must $\hat{\hat{\theta}}$ be to reverse deceleration in productivity.

We can also note that our intuitive reasoning in section 2 that a concave supply schedule would promote productivity deceleration is borne out since that is exactly equivalent to a positive elasticity in the capacity distribution. Using $\bar{\xi}$ for an independently varying maximal input coefficient and $\bar{F}(\bar{\xi})$ for production as a function of this we have

(4.11) $\bar{F}'(\bar{\xi}) = \psi(\bar{\xi})$ and $\bar{F}''(\bar{\xi}) = \psi'(\bar{\xi})$

Hence, inverting this to obtain the supply schedule in terms of input coefficients as a function of total production and discarding the bars

(4.12) $\xi'(F) = \frac{1}{F'}$ and $\xi''(F) = -\frac{1}{F'^3}F'' = -\frac{e(\xi)}{\xi\psi(\xi)^2}$

proving this assertion for all open sets where $\bar{F}'(\bar{\xi}) \neq 0$. Due to our assumption of a connected support of ψ it therefore holds for the whole supply schedule. Hence our intuition is confirmed but we can also conclude that concavity of the supply schedule is a stronger condition than needed.

The elasticity $e(R)$ is positive whenever $\psi(R)$ is increasing, e.g. if past investment has been insufficient to raise capacity flow at the same rate as input coefficients decreased. This is not what we would regard as a normal case in a growing economy, but it does seem probable that it may happen in the real world for different reasons. Recessions, wars and shocks to the economy like the oil crisis of 1973, may very well cause such downturns or at the very least dampen the capacity flow in relation to technical change. Some decades afterwards these economic downturns would then echo through deceleration in aggregate productivity growth. And furthermore they would continue to echo. The elasticity of the density would be exactly reproduced from the extensive margin to the intensive under our bench–mark assumptions. Under more general assumptions it would be modified in different directions but still influence the evolution of productivity growth.

There are other possibilities, too, if we allow for localized disembodied technical change or deterioration to have shaped humps in the capacity distribution. Similar possibilities have been considered by e.g. Atkinson and Stiglitz(1969) in a short but very suggestive paper. If disembodied technical change, as could be expected, affect different techniques in a non–uniform way this may increase capacity for different input coefficients such that humps are created in the capacity distribution, cf. Pomansky and Trofimov(1990). On one side of the hump the capacity elasticity will of course be positive.

Under our assumptions here $e(R) = e(X)$ and the first part of (4.5) will hold when we substitute X for R everywhere. Under the Cobb–Douglas assumption the equation we then get will hold also if V is variable, cf. appendix 2.

4.2 The steady state example

In order to give some more feeling of the meaning of condition (4.10) we will take a look on how it works out in the special case when the vintage economy actually has evolved along a steady state path[5]. A steady state growth path exists only if technical change in the ex ante production function is Harrod–neutral, i.e. purely labour–augmenting. In the Cobb–Douglas case this of course is of no consequence since this

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[5] The properties of this type of putty–clay model in steady state are well known. Those interested in the details are referred to Bliss(1968) for a general perfect foresight treatment. A treatment with stationary wage expectations can be found in e.g. Sheshinski (1967).

specification is consistent both with capital–augmenting and labour–augmenting technical change but to take notational advantage of this we let $\theta = e^{\alpha\gamma t}$

(4.13) $$f(k,ve^{\gamma t}) = k^{1-\alpha}(ve^{\gamma t})^{\alpha}$$

where γ is the fixed rate of change in labour efficiency. With zero labour supply growth both φ and F will grow at the exponential rate γ. Furthermore the lifelength ℓ will be fixed as will $v(t) = v_0$. It follows that

(4.14) $$\frac{X}{R} = \alpha\frac{D}{W} = e^{-\gamma\ell}$$

Thus the RHS of the basic condition (4.10) becomes

(4.15) $$\frac{\varphi(1-\alpha D/W)}{\varphi(1-e^{-\gamma\ell})}\gamma = \gamma$$

by calculating the integral $F = \int_{t-\ell}^{t} \varphi(t)e^{-\gamma(t-\tau)}d\tau$. Since clearly $\ell = V/v_0$ we can write the production function as

(4.16) $$F(V,t) = \frac{\varphi_0}{\gamma}e^{\gamma t}(1-e^{-\gamma V/v_0})$$

where we use φ_0 to denote capacity flow at an initial time zero. Then follows

(4.17) $$F_V(V,t) = \frac{\varphi_0}{v_0}e^{\gamma t}e^{-\gamma V/v_0} = \frac{1}{Xe^{\gamma V/v_0}} = \frac{1}{R}$$

$$R_V = \frac{\gamma R}{v_0} \qquad \psi(R) = \frac{1}{RR_V} = \frac{v_0}{R^2\gamma}$$

$$\psi'(R) = -\frac{2v_0}{R^3\gamma} \quad \text{and} \quad e(R) = -2$$

Since we also have

(4.18) $$\hat{\theta} = \alpha\gamma$$

it is easily seen that (4.10) in a steady state structure cannot hold since

(4.19) $$\frac{F_{tt}}{F_t} = \gamma = \frac{F_t}{F}$$

which confirms the obvious that since technical change is fixed by parameters there will be no change in aggregate productivity growth thereby corroborating our calculations.

If we introduce growth in γ, that is $\hat{\hat{\theta}} > 0$, from the current moment then it is easily verified that

$$(4.20) \qquad -\hat{R} = \gamma + \dot{\gamma}t = \frac{F_t}{F}$$

and $\hat{\hat{R}} = \hat{\hat{\theta}} > 0$ so by using (4.5) we can conclude that we would have acceleration and not deceleration in aggregate productivity growth.

$$(4.21) \qquad \frac{F_{tt}}{F_t} = -\hat{R} + \hat{\hat{\theta}} > \frac{F_t}{F}$$

Hence acceleration in the proportional rate of technical change in a steady state structure will always accelerate aggregate productivity as an immediate effect under our assumptions in this section. Fig 3 depicts the steady state production function $F(V)$, the capacity distribution $\psi(\xi)$, and the corresponding supply schedule $\xi(F)$ in terms of labour input. Note that the latter is strictly convex everywhere.

Just to provide a contrast, a concave production function with an underlying capacity density with positive elasticity in the back end have been depicted in Fig 4. The corresponding supply curve is slightly concave in its upper part, illustrating the intuition of section 2 that a back flat supply would be conducive to productivity slowdown. Note that the concavity is hardly discernible in spite of the rather obvious stretch of positive elasticity in the end of the capacity distribution.

From (4.21) we see that the steady state structure is a boundary case, since if $e(R)$ is only slightly greater than -2 and the capacity distribution otherwise have the steady state elasticity except for a small neighbourhood to R we could have deceleration even for some small $\hat{\hat{\theta}} > 0$.

4.3 Summary

Summarizing this section it has been shown that locally on the time path a vintage economy follows it is possible that $\hat{\hat{\theta}} > 0$ at the same time as $\dot{\hat{\Pi}} = \dot{\hat{F}} < 0$. This is derived under very simplified assumptions and the next two sections will consider what happens when some of these assumptions are relaxed. The most notable feature of the condition is that it clearly shows that the above state is most likely when the elasticity of capacity $e(R) > 0$ or when the supply curve (distribution of average labour costs) is concave in its upper end. But as the steady state example shows, it might very well happen even for elasticities close to the steady state value of –2. Note however that boundary value of $e(R)$ will be dependent on the predetermined structure and therefore in general will vary around –2, because the RHS of the basic condition also depends on the structure.

Of course, if $e(R)$ is less than the boundary value we may have retarding technical change at the same time as accelerating productivity growth. The discussion here has been focussed on the reverse case, partly because the inspiration to this study comes from the productivity slowdown debate, partly because it would be very tedious to keep repeating all statements only reversing directions. It should be noted that the symmetry is around the boundary value of $e(R)$, which will be less than –1, and not around $e(R) = 0$. A convex supply schedule hence does not imply that retarding technical change can occur simultaneously with acceleration in productivity.

Relaxations of the assumptions in this section will modify conclusions, but those modifications have to be very substantial in order to eliminate the possibility of productivity slowdown at the same time as technical change accelerates. It therefore seems that the result as such will turn out to be fairly robust, decelerating productivity growth is a distinct possibility even if technical change accelerates. Counter–intuitive though it may seem such responses could be expected fairly often if this model is a not too distant approximation to reality.

More importantly, even if this case does not arise in the real world due to economic forces excluded here, the model still demonstrates the fundamental importance of the capital structure. It is clear that the shape of the capacity distribution even in cases when it does not cause productivity deceleration influences the transmission of technical change into production in very significant ways. When $e(R) > -1$ it even makes a high basic rate of technical change work *against* an accelerated productivity.

In the next two sections some of our restrictive assumptions will be relaxed. First we rather comprehensively will discuss the impact of changes in expectations in section 5 and then more cursorily labour supply growth, more general ex ante production functions and total factor productivity in section 6.

Section 5.

EXPECTATIONS

On the steady state path expectations should be based on perfect foresight because then both life length of the equipment and the interest rate will be constants, and the growth rate of real wages will be the same as the constant rate of increase in labours technical efficiency so the firm should not be expected to systematically deviate in its expectations from these values. Any firm placed in the structurally stationary environment necessary for steady state growth should by simple adaptive rules be able to learn these constant parameters. Theoretical studies of the learning of rational expectations[6] clearly indicate that conditional on some prior coordination among agents such a stationary parametric environment ought to be learnable. Anyway agents should not be expected to remain on a steady state path where outcomes systematically deviates from the expected, so if we want to assume a steady state we should endow agents with perfect foresight.

In the non–balanced state we have a radically different situation. If we are to assume perfect foresight when the rate of technical change is allowed to vary in irregular ways we have to endow the economic agents with a degree of sophistication and precognitive abilities that takes on distinctly occult dimensions. Not only would they need parapsychological foresight to correctly predict the path of technical change for some decades ahead but they also would need a degree of scientific sophistication not available to any Nobel prize winner by being able to correctly trace out the complicated dynamic growth paths resulting from future irregular technical change, even within our simplifying assumptions. Allowing for uncertainty and risk behaviour would not really

[6] I.a. Bray(1982), Marcet–Sargent(1989a and 1989b) and many others, for a more comprehensive list see Lindh(1990)).

improve the situation much, since the rational expectations hypothesis suffers from essentially the same difficulties. The set of future technique states and its probability distribution would be just as hard, or harder, to predict as any specific path. The problem facing long term fixed investment is not to choose among some well defined possible pay–off states. It is a genuine uncertainty about what the state space even might look like, not to mention the complexities of the pay–off path, dependent as it is on the actions of other agents.

However, disregarding that lack of realism it could still be the case that perfect foresight was acceptable as a reasonable approximation to close the model. Because of the complexity of the dynamic path we would probably have to resort to numerical simulation with its lack of generality in order to analyze the model path when technical change becomes irregular.

We could then settle for some rule of thumb forming of expectations, but even such rules may exhibit very complex behaviour if they are allowed to adapt to exogenous influences of an irregular non–stationary character. If we are to have rational economic agents in the model their expectations should adapt when prediction fails. Therefore assumptions of static expectations, although easy to handle, are also clearly unsatisfactory. Agents exposed to a highly irregular history of price changes would be rather thick–headed if they assumed current prices — or rates of price change — to be constant over the life of a long term investment.

The choice here to avoid these difficulties has therefore been to treat expectations as black boxes that may change their output signal in response to changes in other variables but without trying to specify in detail how changes in current variables are interpreted and adapted to by the firms. Up to now we have therefore treated the expectations variables

W and D as fixed. In this section we will try to say something about how changing expectations may influence productivity movements.

First we will establish how exogenously given changes in W and D will affect the growth rate of productivity and find a sufficient condition for the first order changes in W/D to slow down productivity growth. In the next step we use a parametrization of price expectations to establish the expected price changes this condition will hold for. Finally we informally discuss when endogenously determined price expectations are likely to fulfill the sufficient condition for slowing down productivity growth.

First we must modify the basic condition. Relaxing our previous assumptions of fixed present value expectations means that we must rewrite equation (4.2)

$$(5.1) \qquad \frac{F_{tt}}{F_t} = \hat{\varphi} - \frac{\alpha D/W}{1 - \alpha D/W}(\hat{\alpha} + \hat{D} - \hat{W}) = $$
$$= \hat{\varphi} - \frac{\alpha D/W}{1 - \alpha D/W}(\hat{X} - \hat{R}) \Rightarrow \hat{R} - \hat{X} = \hat{\alpha} + \hat{W} - \hat{D}$$

introducing another source of difference between the rates of decrease in the front and rear end of the capacity distribution, i.e. the relative productivity gap becomes variable even if the elasticity of substitution is unity. The implied relation between changes in minimal and maximal input coefficients and expectations also follows directly from definition (3.17). The previous equation (4.5) becomes

$$(5.2) \qquad \hat{\varphi} + \hat{\alpha} + \hat{D} - \hat{W} = e(R)\hat{R} + \hat{\hat{R}} + \hat{R} = -(1 + e(R))\hat{w} + \hat{\hat{w}}$$

and, since the wage/rental ratio is wW, (4.6) becomes

$$(5.3) \qquad \hat{\varphi} = (1-a)\hat{k} - \alpha\hat{v} + \hat{\theta} = \hat{k} - \alpha\sigma(\hat{w} + \hat{W}) + \hat{\theta}$$

by definition $\sigma = (\hat{k} - \hat{v})/(\hat{w} + \hat{W})$. Adopting the Cobb–Douglas assumption about the ex ante production function will no longer make the second term in (5.1) vanish so the condition for deceleration in aggregate productivity (4.2) will now read

(5.4) $$\frac{F_{tt}}{F_t} = \hat{\varphi} + \frac{\alpha D/W}{1 - \alpha D/W}(\hat{R} - \hat{X}) < (1-\alpha\frac{D}{W})\frac{\varphi}{F}$$

rearranging signs a little. By (5.1) and (5.2) then

(5.5) $$\frac{F_{tt}}{F_t} = (1 + e(R))\hat{R} + \hat{\hat{R}} + \frac{1}{1-\alpha D/W}(\hat{R} - \hat{X}) < (1-\alpha\frac{D}{W})\frac{\varphi}{F}$$

After some algebraic manipulation, using (5.3) and noting that the Cobb–Douglas assumption now implies $\hat{\varphi} - \hat{k} = -\hat{D}$ and thus

(5.6) $$-\hat{w} = \hat{W} - \hat{D}/\alpha - \hat{\theta}/\alpha$$

the LHS of the inequality can be written

(5.7) $$-(1 + e(R))\hat{\theta}/\alpha + \hat{\hat{R}} + \left[\frac{1}{1-\alpha D/W}(\hat{W} - \hat{D}) + (1+ e(R))(\hat{W} - \hat{D}/\alpha)\right]$$

To determine how changing expectations influence the condition of deceleration we have to determine the sign of the bracketed expression which we will refer to as A, as well as how $\hat{\hat{R}}$ relate to $\hat{\hat{\theta}}$.

Rearranging A it can be shown that $A < 0$ if and only if

(5.8) $$\hat{W}(1 + (1-\alpha D/W)(1+e(R))) < (1 + (1-\alpha D/W)(1+e(R))/\alpha)\hat{D}$$

Assuming $\hat{W} < \hat{D}$ turn out to be a sufficient condition for (5.8) to hold except when $-\alpha/(1-\alpha D/W) < 1 + e(R) < 0$. Hence it will certainly hold for all $\alpha < 1$ and $e(R) > -1$, i.e. in the critical region of a distribution where the rate of technical change contributes to deceleration.

We have thus established a sufficient condition for A to be negative, although it clearly is not necessary. It then remains to see how $\hat{\hat{R}}$ relates to $\hat{\hat{\theta}}$. Using (5.6) and noting that $\hat{R} = -\hat{w} < 0$ by assumption we can by some manipulation establish that $\hat{\hat{R}} < \hat{\hat{\theta}}$ either if

(5.9) $$\frac{\alpha\hat{\hat{W}} - \hat{\hat{D}}}{\alpha\hat{W} - \hat{D}} > \hat{\hat{\theta}} \text{ and } \alpha\hat{W} - \hat{D} > 0$$

or alternatively

(5.10) $$\frac{\alpha\hat{\hat{W}} - \hat{\hat{D}}}{\alpha\hat{W} - \hat{D}} < \hat{\hat{\theta}} \text{ and } \alpha\hat{W} - \hat{D} < 0$$

$\alpha\dot{\hat{W}} - \dot{\hat{D}} > 0$ is necessary for $\dot{\hat{R}} < \dot{\hat{\theta}}$ to hold in the case of (5.9) and sufficient in the case of (5.10). Hence if $\alpha\hat{W} - \hat{D}$ is growing fast enough the second order change will also contribute to deceleration in productivity growth. Note that it is a sufficient condition when $\alpha\hat{W} - \hat{D} < 0$. We will return to the meaning of this later on when expectations have been further discussed.

Recall the definition of W and D from (3.14)

$$D = \int_t^{t+\tilde{\ell}} \mathrm{e}^{-\int_t^z \tilde{r}(x)\mathrm{d}x} \mathrm{d}z \quad \text{and} \quad W = \int_t^{t+\tilde{\ell}} \tilde{\nu}(z) \mathrm{e}^{-\int_t^z \tilde{r}(x)\mathrm{d}x} \mathrm{d}z$$

where $\tilde{\nu}(z)w(t) = \tilde{w}(z)$. To simplify notation somewhat the tilde will be skipped hereafter in this section since it is clear that we only treat expectations of these paths here.

Without specifying how expectations are formed definite conclusions cannot be drawn. Note however that W and D will change with w, it is only ν and r that are independent expectations functions, ℓ must satisfy the consistency criterion that $w(t)\nu(t+\ell)v(t) = \varphi(t)$ so even if the expectations functions stay fixed W and D will change with the changes in ℓ induced by changes in wages and front capacity flow. Furthermore, the forms of the functionals $W(\nu,r)$ and $D(r)$ are such that the ratio W/D cannot change quite arbitrarily. We can note that e.g. a decrease in life length will in itself always decrease W by more than it decreases D, since the integrand is everywhere greater in the former than in the latter and we hence cut off more mass in the former when we change the integration limits equally. Things are complicated, however, because the integrand of W may increase overall and hence add mass to W. Likewise an increase in interest rates can be expected to take away more from W than from D, since its effect is scaled up in the former. To conclude anything definite we want to take account of the proportional effects and hence scale down changes in W more than we do in D. A priori it is then not at all clear in what direction W/D will move.

To get some intuitive handle to judge the likelihood of a decreasing ratio W/D, we therefore parametrize the expectations functions, by assuming that a constant average rate of interest is used as well as a constant expected exponential rate of wage increases. I.e. we write

$$(5.11) \qquad D = \int_0^{\ell} e^{-rz} dz = \frac{1-e^{-r\ell}}{r} \quad \text{and} \quad W = \int_0^{\ell} e^{(\beta-r)z} dz = \frac{1-e^{(\beta-r)\ell}}{r-\beta}$$

where β is the proportional rate of wage increase expected, $t = 0$ has been chosen since the integrands in both cases will equal one at t, so we have no loss of generality. With this parametrized form we will use the partial derivatives to show how a decrease in W/D corresponds to movements in average expectations on wage changes and interest rates such that the latter is greater than the former. We have thereby produced a sufficient condition for changes in expectations to contribute to deceleration through a negative term A in condition (5.4).

First we see that the consistency criterion now will read

$$(5.12) \qquad e^{\beta\ell} = \frac{\varphi}{w} = \frac{R}{X}$$

Keeping β and r fixed thus implies

$$(5.13) \qquad \beta\dot{\ell} = \hat{R} - \hat{X} = \hat{W} - \hat{D} = (e^{(\beta-r)\ell} - e^{-r\ell})\dot{\ell}$$

implying that ℓ must stay constant if $\beta > 0$, so we see that the assumption about fixed W and D made earlier actually is equivalent to fixed expectations in this case with parametrization and Cobb–Douglas technique. It is not difficult to see that this conclusion would hol generally for fixed expectations functions, except in very special cases like static wage expectations.

It is obvious that $\beta > 0$ guarantees $W/D > 1$. Differentiating W and D w.r.t. time and dividing through to get proportional rates we have

(5.14) $\hat{W} - \hat{D} = \left[\frac{W_\ell}{W} - \frac{D_\ell}{D}\right]\dot{\ell} + \frac{W_\beta}{W}(\dot{\beta} - \dot{r}) - \frac{D_r}{D}\dot{r}$

By (5.12) we have

(5.15) $\beta\dot{\ell} + \dot{\beta}\ell = \hat{W} - \hat{D}$

so we can solve for $\dot{\ell}$ in terms of $\dot{\beta}$ and $\dot{r}$. It will prove convenient to proceed by expressing the proportional rate of change of the interest rate as a constant times the proportional rate of change of the wage increase parameter. I.e.

(5.16) $\hat{r} = \delta\hat{\beta}$ so $\dot{r} = \delta\frac{r}{\beta}\dot{\beta}$

so we have

(5.17) $\hat{W} - \hat{D} = \underbrace{\left[\frac{W_\ell}{W} - \frac{D_\ell}{D}\right]}_{a}\dot{\ell} + \underbrace{\left[\frac{W_\beta}{W}\left(\frac{\beta - \delta r}{\beta}\right) - \frac{\delta r}{\beta}\frac{D_r}{D}\right]}_{b}\dot{\beta}$

Solving from (5.15) and (5.17) we then have

(5.18) $\hat{W} - \hat{D} = \frac{a\ell - b\beta}{a - \beta}\dot{\beta}$ if $a \neq \beta$

It is proved in appendix 4 that the denominator is strictly negative when $\beta < r$, and the sign of the numerator will be the opposite to that of $1-\delta$. Hence the numerator is positive when $\delta > 1$ and therefore we have

(5.19) $\hat{W} - \hat{D} < 0$ if $\hat{r} > \hat{\beta}$ and $\beta < r$

From (5.15) and (5.17) we have

(5.20) $\dot{\ell} = \frac{\ell - b}{a - \beta}\dot{\beta}$ and $a < \beta$, $a\ell > b\beta$

so life length decreases as expected wage change increases. It follows by differentiation that

(5.21) $\dot{W} = e^{(\beta - r)\ell}\dot{\ell} + (\dot{\beta} - \dot{r})W_\beta$ and $\dot{D} = e^{-r\ell}\dot{\ell} + \dot{r}D_r$

If (5.19) holds and $\dot{\beta} > 0$ then both W and D must decrease since it is clear that $W_\beta > 0$ and $D_r < 0$ and $\dot{\beta} < \dot{r}$. Therefore it is clear in this parametric case that when W/D is decreasing both W and D decreases separately, too.

We can then proceed to the question how reasonable price expectations should move in response to increases in the rate of technical change. The critical ratio W/D depends on expectations of wage increases and expectations on the path of interest rates, but also on the expected life time of capital equipment which is determined here by the equality of best practice labour productivity to the wage expected to prevail at the date of scrapping. It is reasonable to assume that acceleration in technical change should persuade the firms to adjust expectations of future wage increases upwards. It also seems reasonable that interest rates would tend to rise as well since we know that they would in general do so if we moved from one steady state to another with higher rate of technical change.

Recall that β and r here only are parametrical representations of the truly expected wage and interest paths. A decreasing ratio W/D therefore does not require that the interest rate change exceeds the rate of change in wage increases proportionally, $\hat{r} > \hat{\beta}$, at the current time nor over the whole life of the current investment, but only that this holds in some average sense. It is a textbook result from the Ramsey model of optimal growth that the impact of technical change on interest rates will be positive and dependent on the intertemporal substitution elasticity of consumption, cf. e.g. Blanchard–Fischer(1989), and also that a positive time preference imply an interest rate higher than the rate of technical change, which on the steady state path will equal the rate of wage increase. We cannot assert that this holds also for non–steady state paths, but it at least suggests that the above condition may hold.

It can be argued that the condition $\hat{r} > \hat{\beta}$ should hold in circumstances when labour is thought to be abundantly available in comparison to investment capital. To the extent that factor price expectations are based on recent experience the above view of the future would make sense in a situation when demand for new investment have been rising and the labour supply for new investment have been abundant. Referring back to fig. 2 in section 2 we see that in the backflat case extensive scrapping have made vast amounts of labour available in the near past at a relatively modest cost in terms of wage increases, while at the same time ample room for profitable investment have been provided. Without extending our model framework to specification of the determinants of consumption and saving it cannot be claimed with certainty that this means that interest rates have risen relatively more than the rate of wage increases, but it certainly seems plausible. At least if the memories of investors are not too short. If investors then base their expectations on extrapolation of the trends, a reinforcement of aggregate productivity deceleration by a negative term A would be very likely at least in the initial stages of a slowdown.

So far we have only established that the term A in condition (5.7) plausibly may reinforce deceleration. What about $\dot{\hat{R}}$ and conditions (5.9) and (5.10)? Observe that W/D is bounded downwards, more exactly our assumptions require $X/R < 1$ implying $W/D > 1/\alpha$, so the ratio cannot fall at an accelerated rate indefinitely, sooner or later its fall must be retarded. Given that it falls, i.e. that $\hat{W} - \hat{D}$ is negative, it seems not unreasonable to conjecture it should be getting less and less negative the closer we come to the bound, that is to guess that $\dot{\hat{W}} - \dot{\hat{D}} > 0$. Both W and D falls given our general assumptions on the direction of change in price expectations, $\hat{W}$ must then fall faster than $\hat{D}$ initially if we start from W/D constant, hence $\dot{\hat{W}} - \dot{\hat{D}} < 0$ initially. Both terms are negative so $\alpha\dot{\hat{W}} - \dot{\hat{D}}$ may well be positive, and as the ratio comes closer to its lower bound that becomes likelier. Of course this only justifies a loose conjecture that the magnitude of any positive contribution from $\dot{\hat{R}}$ becomes smaller and smaller. So far I have not been able to conclude anything more specific than that.

It is interesting to note here the empirical investigations of vintage structures at the industry level by F. Førsund and L. Hjalmarsson(1987), more specifically the Swedish dairy, cement and pulp industries and the Norwegian aluminum industry. Just looking at their diagrams over changes in input coefficients confirm that a more or less pronounced flattening of the structures took place during the 70's. G. Eliasson and T. Lindberg(1988) show a similar flattening of the distribution of rates of return on capital at the micro level in Swedish industry. Both these findings would be consistent with a reduction in the ratio W/D in this model. It at least suggests that expectations may have been contributing to slow down productivity growth in this decade.

Summarizing this section, it clearly is possible that expectations may reinforce a deceleration if W/D is falling and also otherwise. However, such reinforcement may be counter–acted by second order changes in that fall, viz. if $\alpha\dot{\hat{W}} < \dot{\hat{D}}$. Since it seems plausible that the decreases in W and D slows down because W/D must be bounded from below, a fair guess may be that the magnitude of any counter–action is rather small and decreasing in comparison with the first order changes.

Although no definite assertion can be made, the arguments in the typical back–flat supply case points to the conjecture that expectations are likely to have a retarding effect on productivity growth, in exactly those circumstances when characteristics of the capacity distribution would promote a slowdown. That is to say that changes in expectations are likely to reinforce echo effects.

Section 6.

EXTENSIONS OF THE MODEL

In this section we will discuss how relaxations of some of the simplifying assumptions will affect the results. First we examine the effect of a growing labour supply. Then we let the elasticity of substitution in the ex ante production function differ from unity. Finally the relation between labour productivity and total factor productivity is discussed.

6.1 Labour supply growth

Growth in labour supply would reduce the amount of scrapping that is necessary to free labour for any given addition of capacity in the front and thus reduce capacity at the extensive margin less for a given investment. On the other hand it would tend to lower capital intensity by keeping wages down and thus lower the labour productivity of new facilities. At the same time the average would be taken over a larger labour supply, tending to dampen productivity growth.

Starting from (3.1)

$$\frac{\mathrm{d}}{\mathrm{d}t}(\hat{\Pi}) = \frac{\mathrm{d}}{\mathrm{d}t}(\hat{F} - \hat{V})$$

we first assume $\hat{V}$ to be fixed so the above is equivalent to

$$(6.1) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{F}}{F}\right) = \frac{\ddot{F}}{F} - \hat{F}^2$$

Since, when F is twice continuously differentiable,

$$(6.2) \qquad \ddot{F} = \frac{\mathrm{d}}{\mathrm{d}t}(F_t + F_V\dot{V}) = F_{tt} + (2F_{tV}\dot{V} + F_{VV}\dot{V}^2 + F_V\ddot{V})$$

where the parenthesis, call it A, divided by F is the new contribution to changes in aggregate productivity aside from $\frac{F_{tt}}{F} - \hat{F}^2$ due to changes in labour supply. Using

$$(6.3) \qquad \dot{\hat{V}} = \frac{\ddot{V}}{V} - \hat{V}^2 = 0 \quad \Leftrightarrow \quad \ddot{V} = \dot{V}\hat{V}$$

and the second order partial derivatives of F, the first from the law of motion (3.11) and the other two directly from the marginal productivity condition (3.6)

$$(6.4) \qquad F_{tV}=\frac{vR_V}{R^2} \quad F_{Vt}=-\frac{R_t}{R^2} \quad F_{VV}=-\frac{R_V}{R^2}$$

we can write

$$(6.5) \qquad A=\frac{\dot{V}}{R}((2v-\dot{V})\frac{R_V}{R}+\hat{V}$$

If labour supply is growing at a constant rate and the input coefficients in the front is decreasing strictly, all new labour will be used in the front provided investment is sufficiently high to absorb it. Under that assumption $v \geq \dot{V}$ and it is clear that A is a positive contribution to aggregate productivity growth. Even if that assumption does not hold, it is clear that investment must be very low indeed to make the contribution negative. That it may happen is however clear by considering the extreme case that no new investment is made and labour growth is absorbed by reinstating scrapped equipment with lower productivity. Then of course aggregate productivity would fall.

If $\hat{V}$ varies we will have a contribution to the LHS of the basic condition amounting to

$$(6.6) \qquad B=\frac{A}{F}-\dot{\hat{V}}=\frac{\dot{V}R_V}{FR^2}(2v-\dot{V})+\left[F_V-\frac{F}{V}\right]\frac{\ddot{V}}{F}+\hat{V}^2$$

Since $F_V < F/V$ the second parenthesis will always be the opposite sign of $\ddot{V}$ while the first term is positive as is the third when labour supply grows and $2v > \dot{V}$. So B is possibly negative only if $\dot{V}$ is high relative to v and/or $\ddot{V}$ is strongly positive. I.e. high and accelerating labour supply growth may decelerate aggregate productivity growth if investment cannot be kept high enough to absorb the labour supply growth.

Finally it should be noted that labour supply growth will also affect $R(V,t)$ and thus, even if $F_{tt}/F_t < 0$, influence how that translates into conditions on the capacity distribution and its relation to changes in the technique factor. It is easily verified that (4.3) still holds for a varying V, but (4.5) now reads

$$(6.7) \qquad \hat{\varphi} + \hat{\alpha} = e(R)\hat{R} + \hat{R}_t = -\, e(R)\hat{w} + \hat{R}_t$$

since $\hat{R} \neq R_t/R$ now. First we observe that (6.4) implies that

$$(6.8) \qquad -R_t = vR_V \text{ so } \dot{R} = (1 - \frac{\dot{V}}{v})R_t \equiv yR_t$$

where y simply is the flow of labour from obsolescent equipment compared to total front labour flow. In order to compare (6.6) to our previous formulation we consider the difference

$$(6.9) \qquad \hat{R}_t - \hat{\hat{R}} - \hat{R} = \frac{\dot{R}_t}{R_t} - \frac{y\dot{R}_t + \dot{y}\, R_t}{yR_t} = -\hat{y}$$

From this then follows that

$$(6.10) \qquad \hat{\varphi} + \hat{\alpha} = e(R)\hat{R} + \hat{R} + \hat{\hat{R}} - \hat{y} = -\, (1+e(R))\hat{w} + \hat{\hat{w}} - \hat{y}$$

Since

$$(6.11) \qquad \hat{y} = \frac{\dot{v} \,-\, \ddot{V}}{v \,-\, \dot{V}} - \hat{v} = \frac{\hat{v}\dot{V} \,-\, \ddot{V}}{v \,-\, \dot{V}} = \frac{\hat{v} \,-\, \hat{\hat{V}}}{v \,-\, \dot{V}}\,\dot{V}$$

where the denominator normally would be positive. We can conclude that, as long as front labour flow grows faster than total labour supply growth accelerates, labour supply growth will tend to diminish $\frac{F_{tt}}{F} - \hat{F}^2$ and thereby offsets the positive contributions from labour supply growth.

It cannot be ruled out that a labour supply growth that is high enough and accelerating strongly enough may tend to slow down productivity growth compared with the situation when labour supply is fixed. This is partly due to the possibility that not all new labour is absorbed in front production but some will actually be used for reinstatement of previously scrapped equipment. Although it cannot be disregarded without specifying the determinants of investment it seems safe to conclude that labour supply growth will in general work to accelerate labour productivity growth, but since there are also mechanisms working to the other direction the contribution to higher productivity growth ought to be relatively minor in normal circumstances when labour growth is less than growth in production.

6.2 A general ex ante production function

The assumption of unit elasticity of substitution and constant cost shares will be consistent with a wide range of measured substitution elasticities, since we have two margins here, one intensive ruled by the front production parameters and one extensive ruled by the capacity distribution characteristics. Recall that we make no assumption that the current structure is generated by a specific front production function with a form that remains stationary over time. We have only assumed the current production function to be of Cobb–Douglas type. Now we relax that assumption and allow the ex ante production function to have non–unit and even a changing elasticity of substitution. But we keep the constant returns to scale assumption.

We keep labour supply and W and D fixed like in section 4, and begin by establishing a relation between changes in the cost share and wages. By logarithmic differentiation of the ratio of factor shares in front production

(6.12) $\hat{\alpha}(1 + \frac{\alpha}{1-\alpha}) = \hat{v} - \hat{k} + \hat{w} = (1-\sigma)\hat{w}$

which implies that

(6.13) $\hat{\alpha} = (1-\sigma)\hat{w}(1-\alpha)$

Recall the following equations from the derivation of the basic condition in section 4.

(4.2) $\frac{F_{tt}}{F_t} = \hat{\varphi} - \frac{\alpha D/W}{1-\alpha D/W}\hat{\alpha}$

(4.5) $\hat{\varphi} + \hat{\alpha} = e(R)\hat{R} + \hat{\hat{R}} + \hat{R} = -(1+e(R))\hat{w} + \hat{\hat{w}}$

(4.6) $\hat{\varphi} = (1-\alpha)\hat{k} - \alpha\hat{v} + \hat{\theta} = \hat{k} - \alpha\sigma\hat{w} + \hat{\theta}$

Since wages in the model will increase monotonically with fixed labour supply the second term on the RHS of (4.2) will contribute to deceleration only if $\sigma < 1$. But the changing cost share might influence the relation between capacity flow and technical change, too.

Using the definition of the capital cost share, $k/\varphi D = 1 - \alpha$, we have

(6.14) $\hat{\varphi} - \hat{k} = \frac{\hat{\alpha}\alpha}{1-\alpha} = \alpha(1-\sigma)\hat{w}$

implying that (4.6) can be rewritten as

(6.15) $\hat{w}(\alpha\sigma+(1-\sigma)\alpha) = \alpha\hat{w} = \hat{\theta}$

and we see that the relation (4.8) remains valid, so the basic condition in the form (4.10) will now translate to

(6.16) $-(1+e(R))\frac{\hat{\theta}}{\alpha} + \hat{\theta} - \hat{\alpha}(1 + \frac{\alpha D/W}{1-\alpha D/W}) < (1-\alpha D/W)\frac{\varphi}{F}$

since we now have a non–zero $\hat{\alpha}$ on the LHS of (4.5) or in still more fundamental terms

(6.17) $-(1+e(R))\frac{\hat{\theta}}{\alpha} + \hat{\theta} - \frac{(1-\alpha)(1-\sigma)}{\alpha(1-\alpha D/W)}\hat{\theta} < (1-\alpha D/W)\frac{\varphi}{F}$

If $\sigma < 1$ we will then have a negative contribution to the LHS compared to the unit elasticity case. We can then conclude that $\sigma < 1$ will contribute to deceleration and vice versa.

6.3 Total factor productivity

The production function, $F(V,t)$, has no simple relation to the standard neoclassical one where production is a function of a capital stock and a labour flow. Although it is possible to define a measure of the vintage capital stock either in physical or value terms this measure will in general not be independent of the aggregate labour input, and therefore does not allow functional separation according to the Leontief(1947) aggregation theorems[7]. In general an aggregate production function hypothesis constructed on the basis of data from a vintage structure will

[7] Cf. Nadiri(1970) for a short summary of other aggregation problems in connection to production functions.

tend to underestimate long run production possibilities in the economy that could be realized through investment since all aggregate observations will be well within the ex ante production possibility frontier. It will in general also distort the actual substitution and scale properties of the ex ante function, see Johansen(1972) for a more detailed discussion. Comparisons could easily be misleading and it is important to appreciate that the explicit aggregation in the above structural production functions makes the concept of technical change in the neo–classical production function very different from the corresponding concept in the front production function.

Disregarding this aggregation problem we use the common growth accounting approach and write $\Phi(K,V,t) = F(V,t)$ as a linearly homogeneous function of some index of the capitalstock and labour supply at a given time t. Assuming neutral technical change and profit maximization we can then calculate an accounting measure of total factor productivity growth, $\hat{\Theta}$. With λ as labours cost share we get

(6.18) $\hat{\Theta} = \hat{\Phi} - \lambda\hat{V} - (1-\lambda)\hat{K}$

Again assuming zero growth in labour supply and add the assumption of a constant capital/output ratio, thus implying that a decrease in labour productivity growth here must be matched by an equal decrease in the rate of capital accumulation. Then (6.18) simplifies to

(6.19) $\hat{\Theta} = \lambda\hat{K}$

implying that if rate of capital accumulation is decreasing at a faster rate than increases in λ, total factor productivity will also slow down. Using the definition of $\lambda = \frac{wV}{F}$ we get

(6.20) $\hat{\lambda} = \hat{w} - \hat{F}$

which is positive only if aggregate marginal labour productivity increases faster than average labour productivity. We can also write

$$(6.21) \qquad \hat{\hat{\Theta}} = \hat{\lambda} + \hat{\hat{K}} = \hat{w} - \hat{F} + \hat{\hat{F}} = \hat{\hat{w}} - e(R)\hat{w} - 2\hat{F}$$

by (3.2) and (4.5) and the definition of λ. From this we can conclude that since $\hat{F} > 0$ and $\hat{w} > 0$ a positive elasticity of the capacity distribution will guarantee that total factor productivity growth will move in the same direction as labour productivity for some positive acceleration of technical change. Obviously this is only a sufficient and not necessary condition. Moreover, adding and subtracting $e(R)\hat{F}$ in the last member of (6.21) we easily see that $e(R) > 0$ actually will make growth in the labour share contribute to decleration in Θ!

Note that a rising capital/output ratio[8], $\hat{\Phi} - \hat{K} < 0$ would work in (6.18) to decrease the level of total factor productivity growth. If we assume that the capital ratio has been constant initially a rising capital/output ratio also must slow down capital accumulation less than the slowdown in production for some period of time. This gives a negative contribution to total factor productivity growth.

These calculations are only intended to show two things. First that deceleration in standard measures of total factor productivity growth may be explained by the same kind of mechanism as labour productivity deceleration in the model used here. Second that labour productivity in the context of heterogeneous capital structures is a natural productivity concept to use.

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8 Boskin and Lau(1991) for example finds a rising capital/output ratio when estimating a meta–production function over US, UK, West Germany, France and Japan.

Section 7.

CONCLUSIONS

The results here show that in a simple model with heterogeneous capital acceleration in the rate of technical change can take place at the same time as a deceleration in the growth rate of aggregate productivity. An elasticity of the capacity density that is not too negative, or equivalently a supply schedule that is not too convex will guarantee this when expectations are fixed. Moreover, the larger the gap between the highest and the lowest input coefficient or equivalently the higher the capital elasticity of the front production function, the more convex the supply schedule may be for given rates of change in technique, and still aggregate productivity growth would slow down. Changes in expectations may work in either direction, and results so far do not support any definite conclusion, although it is not too far–fetched to conjecture that shortening of expected life times will tend to reinforce deceleration tendencies.

Of course, the capital structure may also be such as to enhance productivity acceleration and compensate for retardation in technical change. This paper has concentrated on the slowdown aspect here since mechanisms like this may have contributed to the productivity slowdown in the 70's at least in the initial stages. If that is so, the other side of the coin would lead us to believe that the echoes from high investment activity in the 60's may reach us in the 90's. Until empirically verified this is of course purely speculative, but the possibility seems to justify some effort to be spent on empirical work about irregularities in capital structures.

The main importance of this theoretical exercise is, however, not these specific results, but the demonstration that the historically given capital structure of an economy may crucially determine the transmission from enlarged production possibilities to actual economic productivity growth measures. Expectations of future price movements can both reinforce and attenuate these echoes from the past. To the extent that such expectations are based on the recent past they will probably tend to reinforce slowdowns and speedups at least in the initial stages.

The growth depressing effect overtaking accelerated technical change reveals important economic effects from investment decisions taken one or two generations ago. It seems worth pointing out two things.

First. It may very well take a long time — on the order of centuries — before the final effects of technical changes are reached, presumably long after the equipment implementing it was scrapped, because price responses to imbalances in the capacity distribution will result in further imbalances in the structure. Thus any anomalies in the history of the economy will tend to be more or less reproduced later on when equipment installed during an anomaly is scrapped. Furthermore, by the time final effects are approached further changes have occurred imposing their own adjustment paths on the original one. The comparison of steady state paths should therefore be expected to yield very limited information about the short and medium term effects of any sizable change in the rate of technical efficiency growth.

Second. It bears stressing that the slowdown in the model is not due to any economic inefficiency. On the contrary, given price expectations, the representative firm acts optimally within the given framework and a social planner would face essentially the same mechanisms. If a faster transmission of technical change into productivity change is desirable for some reason, any policy aiming to increase the speed of that transmission must take account of the dependency on the existing structure and should not be expected to admit generalization to rules only dependent on current macro variables. E.g. if aggregate productivity is depressed because of a transition from backflat to backsteep regions of the supply schedule and for some reason a planner would like to speed it up, subsidizing capital costs would perhaps be improductive in the short run, since it would work against the transfer of labour. On the other hand it could prove highly efficient in order to boost productivity growth in the neighbourhood of the converse transition. Of course, subsidies affect the economy in many other ways so any policy recommendations would have to take many more mechanisms into account than this very imprecise sketch.

Further research would probably be well spent on disaggregation, at least into a capital and a consumption goods sector, since the relation between the prices of these two sectors may play a crucial role in determining aggregate productivity. Suppose e.g. that capital equipment prices go down relatively to the consumption prices. That should increase capital intensity in both sectors and lead to transfers of labour to the less capital intensive production. The rate of aggregate productivity growth would then depend on which sector it is that have the fastest change in technical efficiency.

The results here only answer the question how, at a specific moment, the capital structure transmits technical change into productivity growth. It does not say anything specific about the duration of such a relation or the long run path of the economy. This is a very essential question, which may gain some illumination by simulation studies even if the dynamic model, as can be suspected, turns out to be analytically intractable.

Of course, since the productivity slowdown is the obvious inspiration for this work, empirical testing of the degree to which the hypothesis can explain real data must be high on the research agenda. Since the essential feature of the vintage model is its covariation in variables separated by long but varying periods of time, standard econometric techniques seem ill suited to the task of analyzing changes in this time structure. Spectral analysis and similar techniques may be a better choice.

APPENDIX

1. List of variables used

For ease of reference the notation for economic variables and functions that are used in the main text is listed here.

Latin letters

D	present value of one investment unit over expected life time
e	elasticity of the capacity density in the input coefficient domain
f	*ex ante* production function
F	aggregate output as a function of aggregate labour
k	real investment
K	capital stock measure
ℓ	life time of capital equipment
P	present value of expected profit
r	interest rate
R	maximal labour input coefficient
t	current time
v	labour flow in new investment
V	total labour supply
w	real wage
W	present value of expected wage changes
X	minimal labour input coefficient
y	ratio of labour released by scrapping and labour absorbed in front production.

Greek letters

α	labour elasticity of front production function
θ	technique factor
Θ	total factor productivity
λ	labours income share in the aggregate
ν	rate of wage change
ξ	labour input coefficient
Π	average labour productivity in total production
σ	elasticity of substitution
τ	time index of vintage
φ	capacity flow in new investment
Φ	aggregate production as a function of capital and labour
ψ	production capacity density over input coefficients
Ψ	accumulated capacity distribution

Top notation:

$\dot{x}$	time derivative of x
$\ddot{x}$	second time derivative of x
$\hat{x}$	logarithmic time derivative of x
$\hat{\hat{x}}$	second order logarithmic time derivative of x
$\tilde{x}$	expected value of x

2. The basic condition in terms of the minimal input coefficients.

Equation (4.5) will hold under the assumptions in section 4 if we interchange the maximal labour/output ratio for the minimal one. This is obvious since the Cobb–Douglas assumption fixes the ratio X/R. But it may be instructive to derive this independently.

From (3.7) it follows that the front rate of change in labour flow

$$\text{(A2.1)} \qquad \hat{v} = \frac{\psi'(X)}{\psi(X)}\dot{X} + \hat{X} + \frac{\ddot{X}}{\dot{X}} = (e(X)+1)\hat{X} + \frac{\ddot{X}}{\dot{X}} \quad \text{when } \dot{X} < 0$$

By noting that $\hat{\hat{X}} = \ddot{X}/\dot{X} - \hat{X}$ this can be rewritten as

$$\text{(A2.2)} \qquad \hat{v} = (2 + e(X))\hat{X} + \hat{\hat{X}}$$

and since $\hat{X} = \hat{v} - \hat{\varphi}$

$$\text{(A2.3)} \qquad \hat{\varphi} = (1 + e(X))\hat{X} + \hat{\hat{X}}$$

so the changes in flow variables will be dependent on elasticities of the density function as well as the second order proportional changes in productivity and it follows, when $\hat{X} = \hat{R}$, that the elasticities in both ends of the utilized part of the capacity distribution must be equal. Note that, this holds even if V is variable while (4.5) holds only for V fixed.

3. Convexity of present value function

It is convenient here to take advantage of the constant returns to scale we have assumed in the front production function and work with the average labour productivity π as a function of capital intensity κ and define present value per labour unit p by

$$\text{(A3.1)} \qquad p(\kappa)v = P(k,v) \ \text{ i.e. } \ p(\kappa) = \pi(\kappa)D - wW - \kappa$$

and maximize this instead with the first order condition

$$\text{(A3.2)} \qquad p'(\kappa) = \pi'(\kappa)D - 1 + \pi D_\kappa - wW_\kappa$$

where D and W depends on κ via $\tilde{\ell}$. By the scrapping condition we have

(A3.3) $\quad w\tilde{\nu}(t+\tilde{\ell}) = \pi(\kappa)$

It is now easy to verify by differentiation that $W_\kappa = \tilde{\nu}(t+\tilde{\ell})D_\kappa$ so the last two terms will always cancel out. That could also have been inferred by the envelope theorem. But the problem with dependence on capital intensity in the expectations will recur in the second order derivative and it is this that causes the present value function to possibly fail concavity even if the ex ante production function is concave by definition.

(A3.4) $\quad p'' = \pi''D + \pi'D_\kappa$

The second term here may be positive and hence outweigh the first term, thus indicating a convex present value function. Obviously this depends crucially on how expectations react to changes in κ. Within our unspecified expectation framework it is not possible to be sure about the effect. But as Bliss(1968) have shown it is possible to say somewhat more on a steady state path with perfect foresight. Then $\tilde{\nu}(t+\tilde{\ell}) = e^{\beta\ell}$ where we skip the tilde since expected life time equals actual life time, which is fixed, as is the technical change parameter β. Using the scrapping condition (A3.3), and differentiating both sides w.r.t. κ it follows

(A3.5) $\quad \ell_\kappa = \frac{\pi'}{\pi}$

and differentiating D then yields

(A3.6) $\quad p'' = \pi''D + \pi' e^{-r\ell}\frac{\pi'}{\pi} < 0$

as a condition of strict concavity. Without specification of the production function this still does not guarantee concavity. There are examples of concave production functions that do not fulfill this requirement. But with our Cobb–Douglas assumption, $\pi = \kappa^{1-\alpha}$ it is possible to be more specific.

(A3.7) $$p'' = -D\alpha(1-\alpha)\pi/\kappa^2 + e^{-r\ell}\pi(1-\alpha)^2/\kappa^2 =$$
$$= (1-\alpha)\frac{\pi}{\kappa^2}(-\alpha D + e^{-r\ell}(1-\alpha))$$

so if

(A3.8) $\quad e^{-r\ell} < \frac{\alpha D}{1-\alpha}$

p will be concave in κ. $D = (1-e^{-r\ell})/r$ on the steady state path which in general will be substantially higher than 1. But given only that $D > 1$ it will be enough if $\alpha > 0.5$ and for any reasonable values of r and ℓ it can be substantially less.

Although this does not prove the concavity of p on a general path it at least seems reasonable to guess that the problem with convexity is less likely to occur when we use a Cobb–Douglas ex ante production function and restrict the labour elasticity to empirically reasonable values.

4. Conditions for a falling ratio W/D

Recall the definition of W and D from (3.13)

$$D = \int_t^{t+\ell} e^{-\int_t^z r(x)dx} dz \quad \text{and} \quad W = \int_t^{t+\ell} \nu(z) e^{-\int_t^z r(x)dx} dz$$

where $\nu(z)w(t) = w(z)$, where we to simplify notation skips the tilde on expectational variables.

We parametrized the expectations functions, by assuming that a constant average rate of interest is used as well as a constant expected exponential rate of wage increases. I.e. we wrote in (5.11)

$$D = \int_0^{\ell} e^{-rz} dz = \frac{1-e^{-r\ell}}{r} \quad \text{and} \quad W = \int_0^{\ell} e^{(\beta-r)z} dz = \frac{1-e^{(\beta-r)\ell}}{r-\beta}$$

Differentiating W and D w.r.t. time and dividing through we got (5.14)

$$\hat{W} - \hat{D} = \left[\frac{W_\ell}{W} - \frac{D_\ell}{D}\right]\dot{\ell} + \frac{W_\beta}{W}(\dot{\beta} - \dot{r}) - \frac{D_r}{D}\dot{r}$$

and also (5.15)

$$\beta\dot{\ell} + \dot{\beta}\ell = \hat{W} - \hat{D}$$

and assumed (5.16):

$$\hat{r} = \delta\hat{\beta} \;\; \text{so} \;\; \dot{r} = \delta\frac{r}{\beta}\dot{\beta}$$

deriving then (5.17)

$$\hat{W}-\hat{D}=\underbrace{\left[\frac{W_\ell}{W}-\frac{D_\ell}{D}\right]}_{a}\dot{\ell}+\underbrace{\left[\frac{W_\beta}{W}\left(\frac{\beta-\delta r}{\beta}\right)-\frac{\delta r}{\beta}\frac{D_r}{D}\right]}_{b}\dot{\beta}$$

and (5.18)

$$\hat{W}-\hat{D}=\frac{a\ell-b\beta}{a-\beta}\dot{\beta}\ \text{if}\ a\neq\beta$$

1. We will now prove that the denominator is strictly negative and the sign of the numerator will be the opposite of the sign of $1-\delta$.

(A4.1) $$a-\beta=\frac{e^{(\beta-r)\ell}D-e^{-r\ell}W-\beta DW}{DW}$$

where the sign is determined by the numerator, which when expanded reduces to

(A4.2) $$r(e^{(\beta-r)\ell}-e^{-r\ell})-\beta(1-e^{-r\ell})<0\ \text{iff}$$

$$\frac{r}{\beta}<\frac{e^{r\ell}-1}{e^{\beta\ell}-1}$$

which will hold true as long as $\beta<r$ since then $\int_0^\ell e^{\beta z}dz<\int_0^\ell e^{rz}dz$.

2. We then proceed to the second assertion and prove $a\ell-\beta b$ to have the opposite sign of $(1-\delta)$

(A4.3) $$a\ell-\beta b=\ell e^{-r\ell}\left[\frac{e^{\beta\ell}}{W}-\frac{1}{D}\right]-\left[\frac{\beta-\delta r}{W}\left[\frac{W}{r-\beta}-\frac{\ell e^{(\beta-r)\ell}}{r-\beta}\right]-\right.$$
$$\left.-\frac{\delta r}{D}\left[\frac{\ell e^{-r\ell}}{r}-\frac{D}{r}\right]\right]=$$
$$=\ell e^{-r\ell}\left[\frac{e^{\beta\ell}}{W}\left[\frac{r(1-\delta)}{r-\beta}\right]-\frac{1-\delta}{D}\right]-\frac{(1-\delta)\beta}{r-\beta}=$$
$$=(1-\delta)\left[\frac{r\ell e^{-r\ell}(e^{\beta\ell}-1)}{(1-e^{(\beta-r)\ell})(1-e^{-r\ell})}-\frac{\beta}{r-\beta}\right]$$

Taking the expression in brackets and multiplying with $1-e^{(\beta-r)\ell}$ will not change its sign, if $\beta<r$, and we get

(A4.4) $$r\ell\frac{e^{\beta\ell}-1}{e^{r\ell}-1}-\beta W=\beta\left[\ell\frac{\int_0^\ell e^{\beta z}dz}{\int_0^\ell e^{rz}dz}-W\right]$$

Obviously, when $\beta=r$ the bracket above vanishes, since W then equal ℓ

and the ratio is unity. Otherwise we know the integrands to be strictly positive and can rewrite the bracket in (A4.4) once again, using B for the integral involving β and P for the one involving r

(A4.5) $\quad \ell B - PW = H$

The sign of H will determine the sign of $a\ell - \beta b$ when δ is given. We will prove that H is negative in the interior of the set $\Omega = \{\ (r,\beta) : 0 \leq \beta \leq r\ \}$. It is easy to verify that $H = 0$ when $\beta = 0$ or $\beta = r$. We will proceed by examining how H changes when we increase r for any fixed positive β. Note that the first term in (A4.5) depends only on β and the second only on r. Hence only PW changes as we increase r, keeping β fixed. We shall show that PW increases monotonically with r, and so H must be negative in the interior of Ω.

2a) First, noting that the partial derivative must be taken *before* evalutating the expression

(A4.6)
$$\frac{\partial}{\partial r}(PW)\Big|_{\beta=r} = \ell \int_0^\ell z e^{rz} dz - \frac{\ell^2}{2}\int_0^\ell e^{rz} dz = \frac{\ell}{r}(\ell e^{r\ell} - P - \frac{r\ell}{2}P) =$$
$$= \frac{\ell}{2r^2}\,(r\ell e^{r\ell} - 2e^{r\ell} + 2 + r\ell) =$$
$$= \frac{\ell}{2r^2}\sum_{i=2}^{\infty}\frac{(r\ell)^i}{(i-1)!}(1 - \frac{2}{i}) > 0$$

since all terms of the Taylor–expansion is non–negative. So we then have established that PW increases as we cross the zero line of H. By continuity H will then be negative in an open set where $\beta < r$.

2b) To demonstrate that this open set in fact can be identified with the interior of Ω we shall show that the partial derivative of PW w.r.t. r remains positive throughout the interior.

(A4.6)
$$\frac{\partial}{\partial r}(PW) = P_r W + W_r P > 0 \Leftrightarrow -W_r < \frac{W}{P}P_r$$

To determine this some lengthy algebraic exercises is needed. We start by

$$(A4.7)\qquad W_r = \frac{\ell e^{(\beta-r)\ell} - W}{r-\beta} = \frac{1}{(r-\beta)^2}[(r-\beta)\ell e^{(\beta-r)\ell} - 1 + e^{(\beta-r)\ell}] =$$

$$= \frac{e^{(\beta-r)\ell}}{(r-\beta)^2}\,[(r-\beta)\ell + 1 - e^{(r-\beta)\ell}] =$$

$$= -\frac{e^{(\beta-r)\ell}}{(r-\beta)^2}\sum_{i=2}^{\infty}\frac{((r-\beta)\ell)^i}{i!} =$$

$$= -e^{(\beta-r)\ell}\,\ell^2\sum_{i=0}^{\infty}\frac{((r-\beta)\ell)^i}{(i+2)!} = -e^{(\beta-r)\ell}\,\ell^2\,S_1$$

which clearly is negative. Note the shorthand for the sum! In an analogue manner we have

$$(A4.8)\qquad P_r = \frac{\ell e^{r\ell} - P}{r} = \frac{1}{r^2}\,(r\ell e^{r\ell} - e^{r\ell} + 1) =$$

$$= \frac{1}{r^2}\sum_{i=2}^{\infty}\frac{(r\ell)^i}{(i-1)!}(1-\frac{1}{i}) = \ell^2\sum_{i=0}^{\infty}\frac{(r\ell)^i}{(i+2)!}(i+1) = \ell^2\,S_2$$

obviously positive. Next step is

$$(A4.9)\qquad e^{(r-\beta)\ell}\,W = \frac{e^{(r-\beta)\ell} - 1}{r-\beta} = \ell\sum_{i=0}^{\infty}\frac{((r-\beta)\ell)^i}{(i+1)!} = \ell\,S_3$$

and

$$(A4.10)\qquad P = \frac{e^{r\ell} - 1}{r} = \ell\sum_{i=0}^{\infty}\frac{(r\ell)^i}{(i+1)!} = \ell\,S_4$$

It is easy to verify that condition (A4.6) is equivalent to

$$(A4.11)\qquad S_1 < \frac{S_3}{S_4}\,S_2$$

This is still hard to determine so some further work is needed

$$(A4.12)\qquad S_1S_4 - S_2S_3 = S_1(S_4 - S_2) - S_2(S_3 - S_1)$$

Now

$$(A4.13)\qquad S_4 - S_2 = \sum_{i=0}^{\infty}\frac{(r\ell)^i}{(i+1)!}(1-\frac{i+1}{i+2}) = \sum_{i=0}^{\infty}\frac{(r\ell)^i}{(i+2)!} > 0$$

and so have the same form as S_1. We also have

$$(A4.14)\qquad S_3 - S_1 = \sum_{i=0}^{\infty}\frac{((r-\beta)\ell)^i}{(i+1)!}(1-\frac{1}{i+2}) = \sum_{i=0}^{\infty}\frac{((r-\beta)\ell)^i}{(i+2)!}(i+1) > 0$$

turning out to have the form of S_2. By comparison term by term we easily verify that

$$(A4.15)\qquad S_3 - S_1 > S_1 \text{ and } S_2 > S_4 - S_2$$
$$\text{or } \frac{S_1}{S_3 - S_1} < 1 \text{ and } \frac{S_4 - S_2}{S_2} < 1$$

Dividing through in (A4.12) by $S_2(S_3 - S_1)$ we do not change sign so

$$(A4.16)\qquad \frac{S_1}{(S_3 - S_1)} \frac{(S_4 - S_2)}{S_2} - 1 < 0 \Rightarrow S_1 < \frac{S_3}{S_4} S_2$$

We have then proved that PW increases monotonically with r in the set Ω for each fixed β. It follows that $\ell B - PW < 0$ in the whole interior of Ω.

We can then finally conclude that in Ω

$$(A4.17)\qquad \hat{W} - \hat{D} < (>)\, 0 \text{ iff } \hat{r} > (<)\, \hat{\beta}$$

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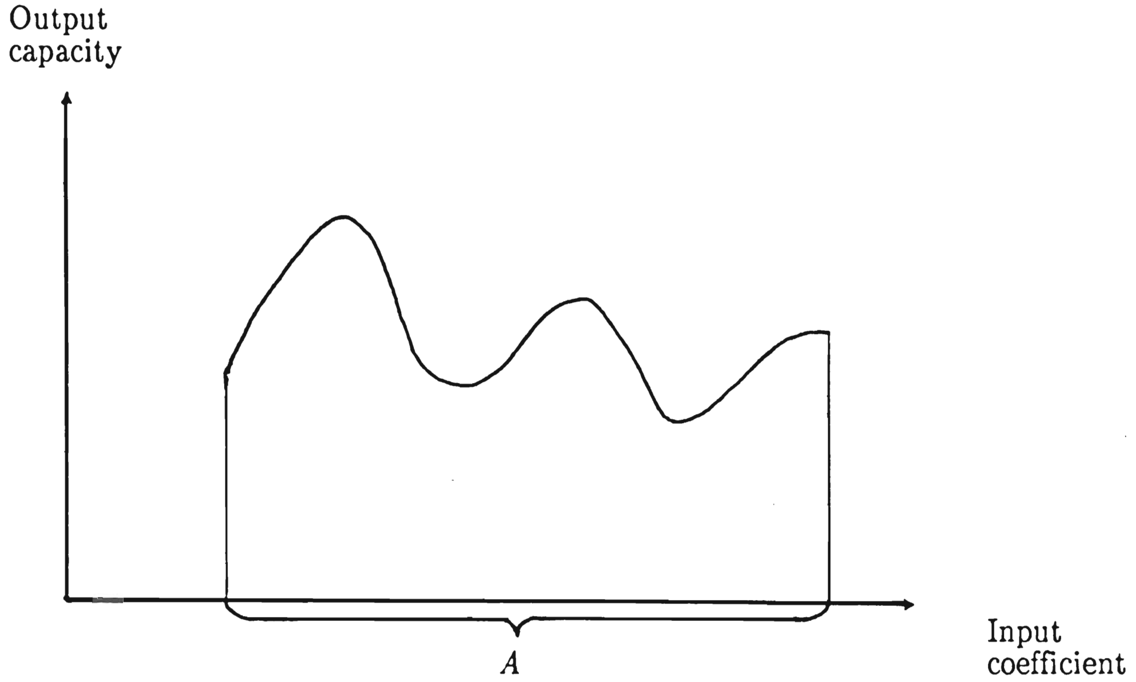
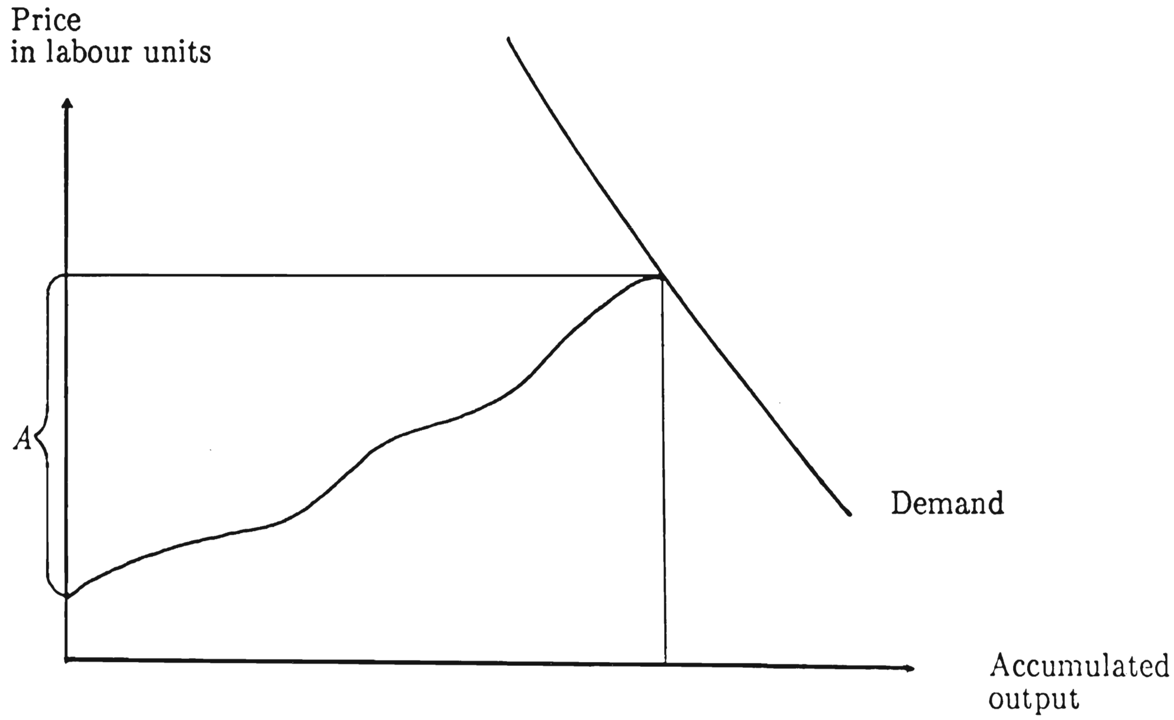
Fig. 1

a)



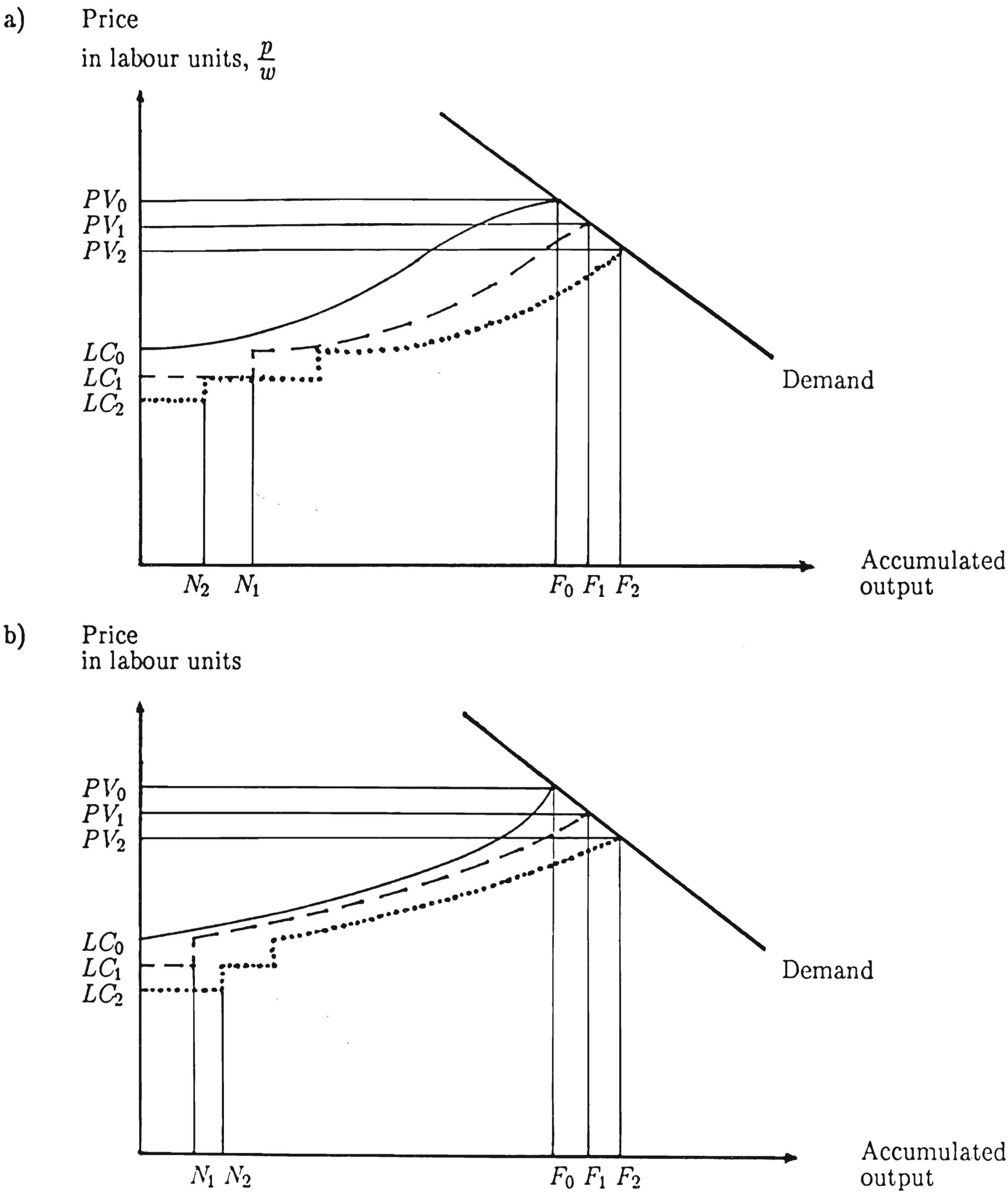
The distribution of capacity ranked by input coefficients

b)



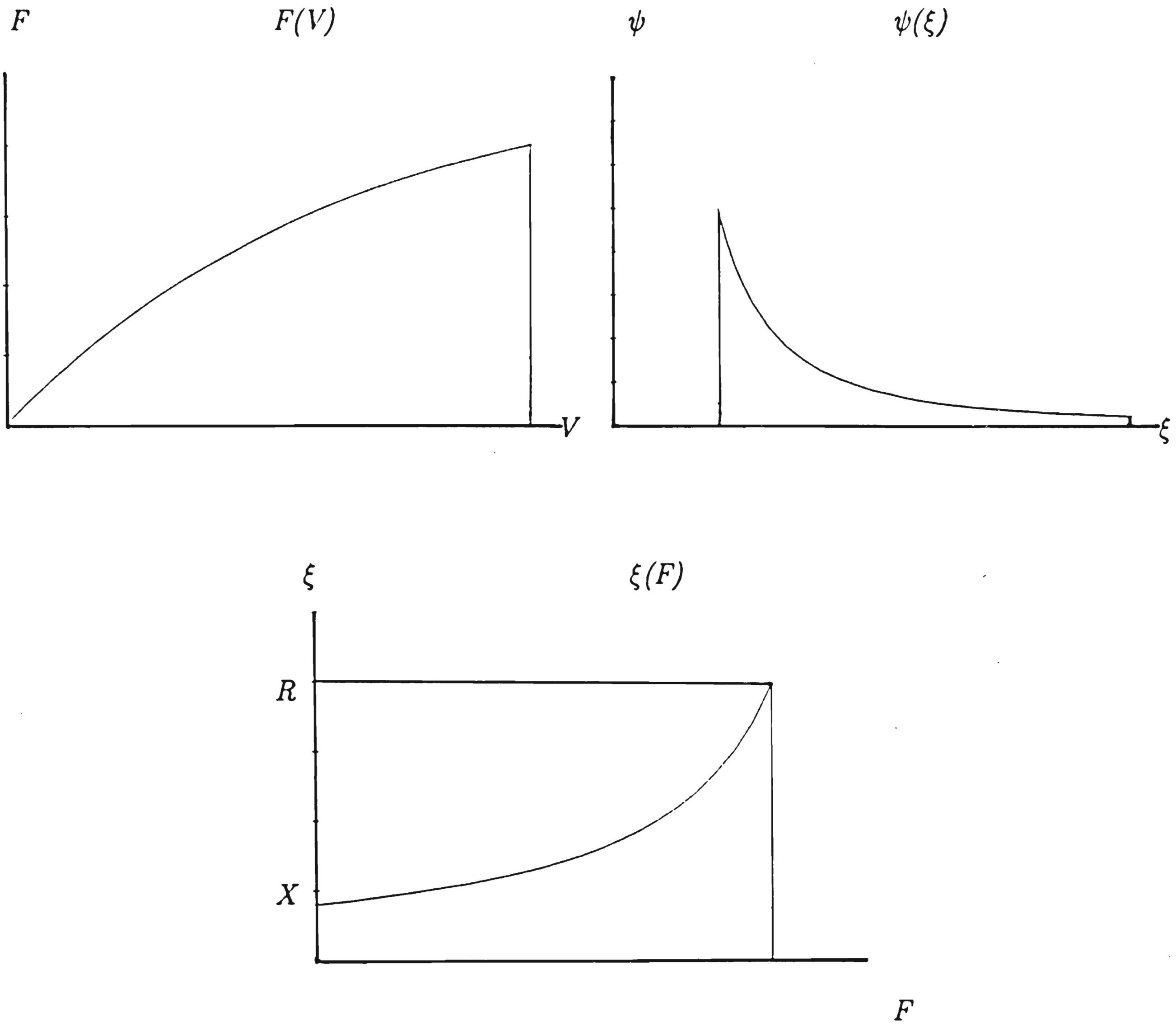
Supply and demand schedule. Supply given by the inverse of the accumulated capacity distribution. I.e. the labour cost of production in each unit. *A* represents the same span of input coefficients in both diagrams.

Fig. 2



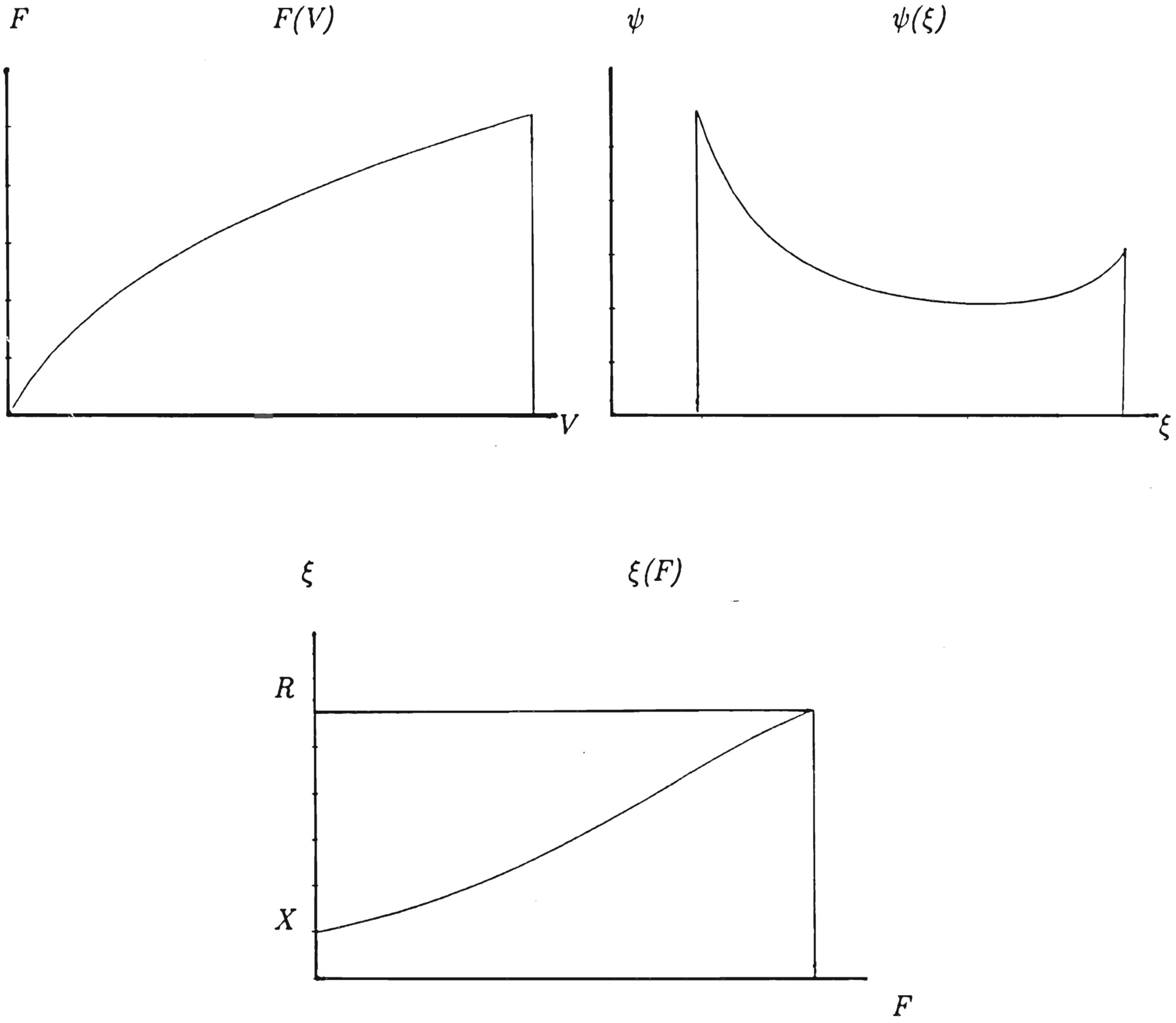
In a) a backflat and in b) a backsteep supply schedule. PV is present value cost per produced unit, LC is labour cost per produced unit, N is added new capacity. Subindex 0 denotes initial values, subindex 1 values after adaptation to the first change, subindex 2 values after the second change in best practice input coefficient. Note how the added capacity diminishes in a) in step 2 while it increases in b).

Fig. 3



The steady state production function $F(V) = \frac{\varphi}{\gamma}(1 - e^{-(\gamma/v_0)V})$, and the corresponding capacity distribution, $\psi(\xi) = \frac{v_0}{\xi^2\gamma}$, and supply schedule, $\xi(F) = \frac{v_0}{\varphi - \gamma F}$. The graphs are generated with Mathematica, using the value 32 for total labour supply, V, $v_0 = 1$, $\gamma = 0.05$, $\varphi = 0.25$

Fig. 4



Production function $F(V) = \frac{1}{c}[\ln(V+\frac{a}{c}) - \ln(a-b(V+\frac{a}{c})) + \ln(c-b)]$, and the corresponding capacity distribution density, $\psi(\xi) = \frac{a}{2bc\xi[(a/2b)^2 - (a/bc)\xi]^{0.5}}$, and supply schedule, $\xi(F) = \left[\frac{c}{b + (c-b)e^{-cF}}\right]\left[a - \frac{ab}{b + (c-b)e^{-cF}}\right]$.

The graphs are generated with Mathematica, using the value 32 for total labour supply, V, $a = 2$, $b = 0.02$, $c = 0.5$ This production function is arbitrarily chosen for being reasonably simple to calculate and yet provide an example of backflat supply. F in fact is the lower half of an inverted logistic curve and no economic significance should be attached to the parameters. Note that $\xi(F)$ is weakly concave to the right.

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