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Summary

Technical change in general milk processing is estimated within a homothetic frontier production function allowing neutrally variable scale elasticity.

The results show that technical progress is characterized by a rapid increase in optimal scale and a small capital saving bias, increasing the marginal productivity of labour relative to capital.

To characterize technical change, Salter's measures of bias and technical advance are utilized and interpreted within the framework of the efficiency concepts of Farrell.

1. Introduction

The purpose of this study is to analyse technical progress in Swedish general milk processing in terms of the production function. We shall try to find out how much, if any, of the change in input requirements and unit costs is attributable to each of the following three factors: (1) the shift in the production function; (2) factor substitution; (3) increasing optimal scale or elasticity of scale. This study differs from earlier studies in several ways.

The process of technical change is studied by a best-practice or frontier production function. In the literature (as far as we know) frontier functions are estimated on the basis of cross section data. See e.g. Aigner & Chu [2], Carlsson [5], Timmer [29]. Earlier time series studies are based on some sort of an average production function. See e.g. Ringstad [23] and Sato [26].

We have utilized a homothetic production function with a variable scale elasticity. Inhomogeneous production functions implying variable scale elasticity is the general rule in the production theory of Frisch [9], whereas it is the exception in empirical analysis, the bulk of which is based on homogeneous Cobb-Douglas (C-D) - or CES production functions. Homothetic functions offer the easiest possibility of specifying variable scale elasticity, because the scale elasticity is constant along an isoquant and independent of factor ratios. (See Førsund [12]). Empirical studies are found in Nerlove [21], Zellner and Revankar [30] and Ringstad [22] and [24]. As far as we know, only homogeneous best-practice functions have previously been estimated in the literature (see e.g. Aigner and Chu [2], Seitz [27],[28] and Timmer [29]). Thus we have generalized the approach of Aigner and Chu [2] to allow for variable scale elasticity. (This generalization was performed in Førsund [11] and Førsund and Jansen [16] via estimating cost functions on cross section data.)

The analysis is based on a complete set of cross section time series data for 10 years of 28 individual plants producing a homogeneous product. Estimation of production functions on the basis of time series data are usually carried out on a very high level of aggregation. Cross section data on individual plants producing a homogeneous output are rather scarce except in the field of agriculture and electricity generation. (See e.g. Christensen and Greene [6], Dhrymes and Kurz [7], Komiya [19], and Nerlove [21]. The analysis in Ringstad [23] is, however, based on pooled time series cross section data but the level of aggregation is rather high as the base unit of the industry construction is the two-digit group.

Earlier studies have almost exclusively been limited to estimating Hicks-neutral technical progress in production functions fitted as an average of the sample. Exceptions here are e.g. Ringstad [24] and Sato [26] studying non-neutral technical progress.

In this study technical progress is analysed by introducing trends in all the parameters of the frontier production function. In particular trends are introduced in both of the scale function parameters, thus making it possible to study whether optimal scale changes over time.

To further elucidate the progress of technical advance we have generalized, in a Farrell inspired way, Salter's measure of technical advance.

We will employ the following notations in this paper:

x = quantity produced milk in tonnes

L = working hours by production workers

K = user cost of capital in Swedish crowns (1964-prices)

n = number of units

T = number of years

2. Estimation of Frontier Functions

When estimating frontier functions three general approaches are found in the literature (see Johansen [18], ch. 8 for a critical evaluation of some of the approaches): i) utilizing the whole sample, but restricting the observed points in the output-input space to be on or below the frontier, ii) eliminating "inefficient" observations and estimating an "average" frontier function from the subset of efficient points, iii) allowing some observations to be above the frontier either by eliminating a certain percentage of the most efficient observations (fitting a "probabilistic" frontier a´la Timmer[29]) or putting different weights to be placed on positive and negative residuals as Aigner et al [3] or specify both an efficiency distribution proper and pure random variation of efficiency (see Aigner et al [4] and Meeusen & van der Broeck [20]).

We will here utilize approach i) and generalize the programming method in Aigner & Chu [2] to allow for neutrally variable returns to scale.

The best-practice production function is pre-specified to be a homothetic function of the general form

$$(1) \qquad G(x,t) = g(v,t)$$

where x = rate of output (single ware production), v = vector of inputs, G(x,t) a monotonically increasing function, and g(v,t) homogeneous of degree 1 in v. The returns to scale properties are given by the scale elasticity function

$$(2) \qquad \varepsilon(x,t) = \frac{G(x,t)}{x \cdot G'(x,t)}$$

As regards the generation of the actual data several schemes can be envisaged. One hypothesis is that the production structure is of the putty-clay type (Johansen [18]) with simple Leontief (limitational) ex post functions. To simulate the actual performance of plants an efficiency term with respect to the utilization of the inputs distributed in the interval (0,1) can be introduced multiplicatively on the r.h.s. of Eq (1). We will adopt this scheme and in addition assume that the plants are operated on the "efficient corners" of the isoquants. Ex post the plant managers can only choose the rate of capacity utilization. With these assumptions concern about "slack" in fulfilling marginal conditions with respect to inputs is not relevant. The frontier function can be regarded as a pessimistic estimate of the ex ante or planning production function. However, it is not possible on our level of aggregation to identify unique vintages. Technical change is characterized by successive improvements, while we assume discrete time with one year as the unit, and fixed coefficients for each year.

As regards the estimation procedure a key question is whether a specific distribution of the efficiency terms is assumed or not. If sufficient information is available (or if one is bold enough) to postulate a specific distribution the natural procedure is to derive maximum likelihood estimates as pointed out in Afriat [1]. However, in this paper we will not follow this approach. The case of specific efficiency distribution is treated in the Appendix.

A natural objective - with the information available - is that the observations should be close to the frontier in some sense. In order to keep the estimation problem as simple as possible it is here chosen to minimize the simple sum of deviations from the frontier with respect to input utilization after logarithmic transformation, subject to on or below frontier constraints.

As regards the form of the production function the following specification is employed (cf Zellner-Revankar [30]):

$$(3) \qquad G(x,t) = x^{\alpha-\gamma_4 t} e^{(\beta-\gamma_5 t)x} = g(v,t) = Ae^{\gamma_3 t} \cdot \prod_{j=1}^{2} v_j^{a_j-\gamma_j t}$$

Technical change is accounted for by specifying the possibility of changes in the constant term, A, and the kernel elasticities, a_j, for labour, L, and capital, K, and the scale function parameters α, β.

The corresponding elasticity of scale function is:

$$(4) \qquad \varepsilon(x,t) = \frac{1}{\alpha-\gamma_4 t+(\beta-\gamma_5 t)x}$$

With this specification the estimation problem is reduced to the most simple problem of solving a standard linear programming problem. The objective function to be minimized becomes:

$$(5) \qquad \sum_{t=1}^{T} \sum_{i=1}^{n} \Big(\ln A + \gamma_3 t + (a_1-\gamma_1 t) \ln L_i(t) + (a_2-\gamma_2 t)\cdot \ln K_i(t) - (\alpha-\gamma_4^t) \ln x_i(t) - (\beta-\gamma_5 t)\cdot x_i(t)\Big)$$

Note that although the objective function is linear in all the unknown parameters, the specification yields satisfactory flexibility as regards technical change.

Concerning the constraints of the LP-model, the expression within the brackets in (5) constitutes (T+1) · n constraints securing the observed input points to be on or below the frontier:

(6) $$\ln A + \gamma_3 t + (a_1-\gamma_1 t) \cdot \ln L_i(t) + (a_2-\gamma_2 t) \cdot \ln K_i(t) - (\alpha-\gamma_4 t) \cdot$$

$$\times \ln x_i(t) - (\beta-\gamma_5 t) \cdot x_i(t) \geqq 0$$

In addition, we have the homogeneity constraint

(7) $$\sum_j a_{j,t} = \sum_j (a_j-\gamma_j \cdot t) = 1 \qquad t = 1,\ldots,T$$

Since (7) must be satisfied for all t the specification (3) implies the restriction:

(8) $$\gamma_1 + \gamma_2 = 0$$

It is not necessary to enter (7) for all T years because if it holds for one year and (8) is valid it must hold for all other values of t. For convenience we have chosen t=0 for the constraint (7). (Note that the choice of time index t=1,...,T, is not trivial. Our choice implies that the factor elasticities can never obtain extreme values for year 1 if the trends are different from zero.) In addition we want the kernel elasticities including trends to be restricted to the interval (0,1).

(9) $$0 \leqq a_{j,t} \leqq 1 \qquad t = 1,\ldots,T$$

In view of (7) and (8) these constraints reduce to

(10) $$a_j - \gamma_j T' \geqq 0 \qquad j = 1,2$$

We also want the scale parameters including trends to be non-negative

(11) $$\alpha-\gamma_4 T' \geqq 0$$

(12) $$\beta-\gamma_5 T' \geqq 0$$

We have found it reasonable to avoid the possibility of a too abrupt change in the scale function in the last year i.e. the optimal scale can exist in the next last year but might not exist in the last year, by putting T' = 2T. Thus the non-negativity conditions will hold in the future for as long a period as the observed. This seems reasonable from a prediction point of view.

Finally we have the reasonable restrictions from the economic point of view

$$\beta, \alpha, a_1, a_2, \gamma_3, \gamma_4, \gamma_5 \geqq 0$$

$\ln A$, γ_1, γ_2 unrestricted

3. The Data

In the empirical part of this study we have utilized primary data for general milk processing from 28 individual dairy plants during the period 1964-1973. We have received all data from SMR (Svenska mejeriernas riksförening), a central service organization for the dairies in Sweden.

The processing of milk in a dairy can be divided into different stages of which each one can be refered to as a production process. The data used in this study refer to the production process general milk processing. This process includes reception of milk from cans or tanks,storing, pasteurizing and separation. All milk passes this process before it goes further to different processes for consumption milk, butter, cheese or milk powder etc. Thus this stage defines the capacity of the plant. Moreover general milk processing is often treated as a separate unit in cost accountings.

A strong reason for our choice of this part of a dairy is that it makes it possible to measure output in physical or technical units (tonnes) avoiding value added or gross output. This means that our estimated production function is a true technical production function in the original sense.

Thus milk is regarded as a homogeneous product which is a very realistic assumption. Output is measured in tonnes of milk delivered to the plant each year. The amount of milk received is equal to the amount produced. There is no measurable waste of milk at this stage. According to SMR any difference is due to measurement errors. (Differences were of the magnitude of kilos.)

The labour input variable is defined as the hours worked by production workers including technical staff usually consisting of one engineer.

Capital data of buildings and machines are of user-cost type,including depreciation based on current replacement cost, cost of maintenance and rate of interest. The different items of capital are divided into five different subgroups depending on the durability of capital which varies between 6 and 25 years, so the capital measure is an aggregated sum of capital costs from these subgroups.

Capital costs,divided into building capital and machine capital,are calculated on the basis of these subgroups as a sum of the capital costs of the subgroups. The capital measure has been centrally acconted for by SMR according to the same principles for all plants and after regulary capital inventory and revaluations of engineers from SMR. Afterwards we have aggregated building capital and machine capital into one measure. Thus we have assumed that the conditions of the composite commodity theorem are fullfilled. In fact the relative prices of buildings and machine capital have developed almost proportionally during the 10-year period. The price index have moved from 100 in 1964 to 158 in 1973 for buildings and to 161 for machine capital. An alternative would be to retain the disaggregation of building and machine capital but in the case of a C-D kernel function implying a unitary elasticity of substitution. This seems to be a less realistic assumption. Note that this capital measure is proportional to the replacement value of capital, which can serve as a measure of the volume of capital. See Johansen & Sørsveen [17].

As the data is not adjusted for capacity utilization we have investigated a measure based on monthly maximum amount of milk received compared with the yearly average. This ratio is fairly stable over time, and the differences between plants are not very great. In consequence we have not corrected for capacity utilization. The increasing output over time for most of the plants support the assumption.

4. Empirical Results: Frontier Estimates

The estimates of the parameters of the frontier or best-practice production function are shown in Table I and the figures below.The different runs performed have been denoted Case 1 to Case 4. Case 1 is regarded as the main case while the other cases represents the sensitivity analysis. In Case 2, the sensitivity of trend specifications is shown because only Hicks neutral technical progress is assumed. In Case 3 and 4 another kind of sensitivity analysis is performed. In Case 3 we have excluded the largest plant from the sample and in Case 4 we have excluded the four smallest plants. The results show the sensitivity with regard to the observations.

TABLE I Estimates of the frontier production function. Combined time series cross section analysis. Estimates of the production function

$$x^{\alpha-\gamma_4 t} e^{(\beta-\gamma_5 t)x} = Ae^{\gamma_3 t} L^{(a_1-\gamma_1 t)} K^{(a_2-\gamma_2 t)} \quad (t=1 \text{ in } 1964,\ t=10 \text{ in } 1973$$

Case	Constant term ln A	Trend A $\gamma_3 \cdot 10^2$	Labour elasticity $a_1-\gamma_1 t$		Trend L $\gamma_1 \cdot 10^2 = -\gamma_2 \cdot 10^2$	Capital elasticity $a_2-\gamma_2 t$		α	Trend α $\gamma_4 \cdot 10^2$	$\beta \cdot 10^5$	Trend β $\gamma_5 \cdot 10^6$	Optimal scale x for $\varepsilon=1$	
			1964	1973		1964	1973					1964	1973
1 28×10	-6.02	0	.81	.86	-.19	.19	.14	.32	.56	1.47	.73	48 644	99 325
2 28×10	-7.58	6.22	.73	.73	—	.27	.27	.19	—	1.52	—	53 425	53 425
3 27×10	-6.81	0	.83	.91	-.91	.17	.09	.22	.62	2.14	1.07	38 158	77 815
4 24×10	-8.83	0	.72	.74	-.19	.28	.24	.05	.13	2.02	1.01	49 613	95 284

The Main Result

Technical change for Case 1 is characterized by an increasing kernel elasticity of labour and a mirror image decreasing kernel elasticity of capital. For constant factor prices this implies that the units should increase the labour-capital ratio. The technical change can in this sense be characterized as capital saving.

The estimated trends in the scale elasticity function implies a considerable increase in optimal scale; about a doubling during the period. The Hicks neutral term turned out to be on its zero lower boundary. The impact on the production surface of these changes is shown in Fig.1. Cutting the production function with a vertical plane through the origin along the average factor ray, a ray corresponding to the average factor ratio, one obtains the classical text-book S-shaped graph of the production function. For this average factor ratio the development through time gives the impression of a rapid technical progress due to the increase in optimal scale.

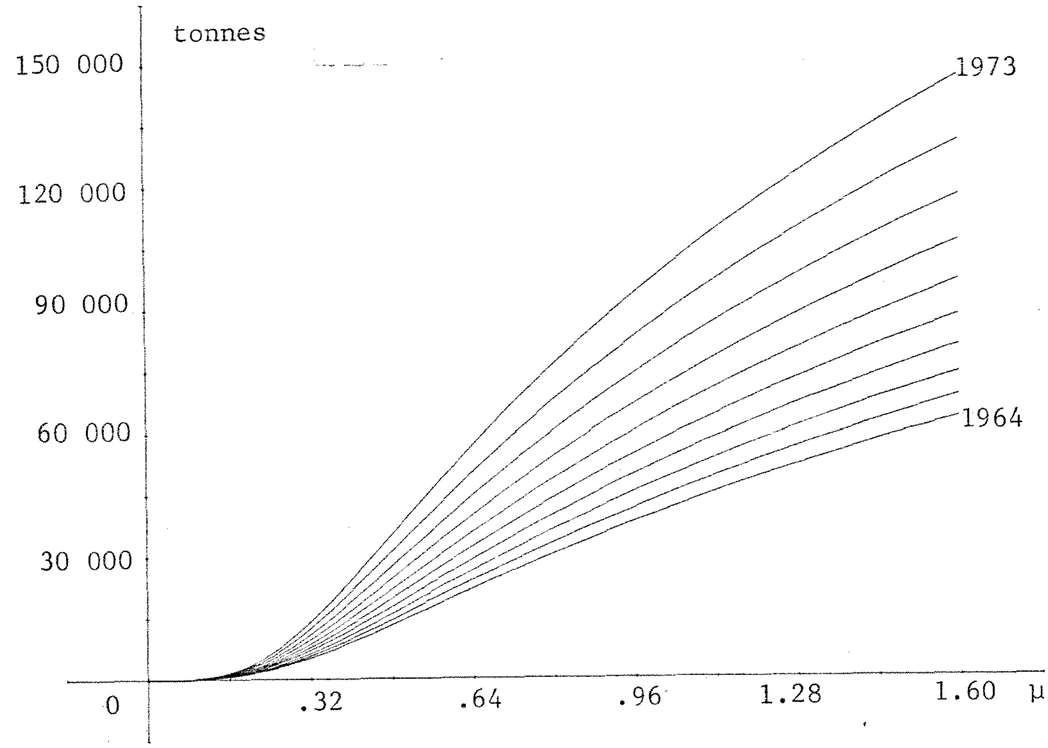

FIGURE 1 The change in the frontier production function through time. Combined time series cross section analysis. The production function cut with a vertical plane through the origin along a ray, $(\mu L^o, \mu K^o)$, $L^o = 13\ 000$ and $K^o = 200\ 000$

$$x^{\alpha-\gamma_4 t} e^{(\beta-\gamma_5 t)x} = Ae^{\gamma_3 t} (\mu L^o)^{(a_1-\gamma_1 t)} (\mu K^o)^{a_2-\gamma_2 t)}$$

The shift in the elasticity of scale function can be studied in Fig.2 where the function is plotted for different years. The level of $\varepsilon=1$, i. e. optimal scale is indicated. The scale elasticity shift through time in a such a way that optimal scale increases at an accelerating rate; from 6% at the start to 10% at the end of the period.

The output of the largest plant has been in the interval 111 000 - 141 000 tonnes in the period 1964-73, while the average output has increased from 29 000 tonnes to 39 000 tonnes. Thus the largest unit has had a scale elasticity less than one during the period while the average output corresponds to scale elasticities considerably greater than one.

It is obvious from Fig.1 that the production function is not concave over its entire domain. In Førsund [11] it is shown that the production function with the functional specification utilized in this paper, is concave for the values of output corresponding to $\varepsilon < \sqrt{1/\alpha}$. In Case 1 here the estimate of α

is .32 in 1964 and .27 in 1973, yielding that the production function is concave for $\varepsilon < 1.77$ in 1964 and $\varepsilon < 1.92$ in 1973, which corresponds to an output of 17 583 and 33 961 respectively.

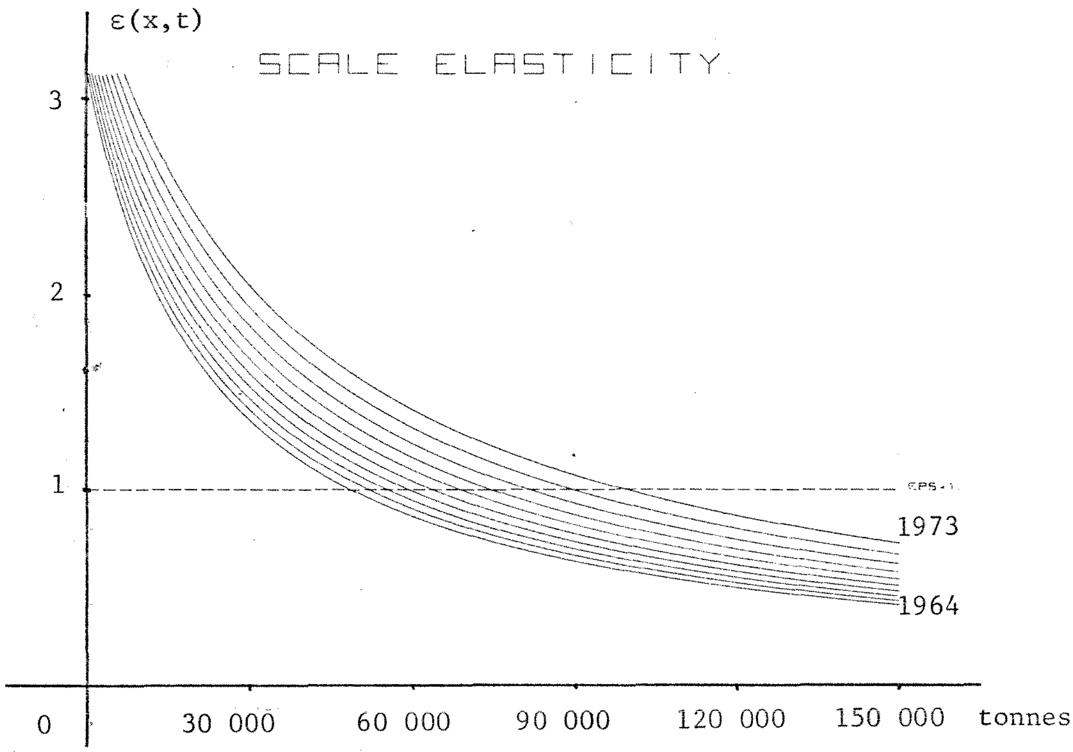

FIGURE 2 The plotting of the elasticity of scale function for all 10 years

$$\varepsilon(x,t) = \frac{1}{\alpha - \gamma_4 t + (\beta - \gamma_5 t)x}$$

The characteristics of technical advance can also be illustrated in the input coefficient space (cf. Salter [25] ch. 3) by the development of the technically optimal scale curve (see Frisch [9], ch. 8) which we here will call the efficiency frontier. See Førsund and Hjalmarsson [13]. The efficiency frontier is the locus of all points where the elasticity of scale equals one, i.e. it is a technical relationship between inputs per unit of output for production units of optimal scale. Thus the efficiency frontier represents the optimal scale of the frontier production function. In the input coefficient space the frontier or ex ante production function defines the feasible set of production possibilities while the efficiency frontier is a limit towards the origin of this set. (This consideration has been elaborated in detail in Førsund [10].) The development of the efficiency frontier and the observed input coefficients for 1964 [▪] and 1973 [x] are shown in Fig. 3. Note that for homothetic functions the shape of the efficiency frontier is identical with the shape of the isoquants

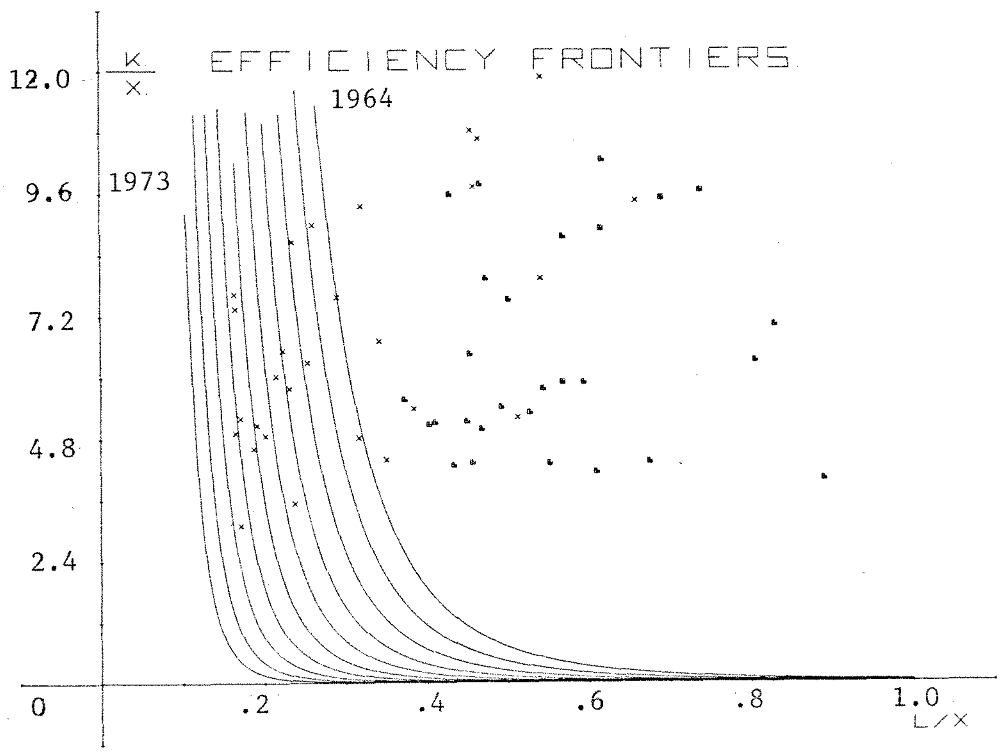

FIGURE 3 The changes in the efficiency frontier through time combined time series cross section analysis. Estimates of the production function

$$x^{\alpha-\gamma_4 t} e^{(\beta-\gamma_5 t)x} = Ae^{\gamma_3 t} L^{(a_1-\gamma_1 t)} \cdot K^{(a_2-\gamma_2 t)}$$

with the efficiency frontier

$$\left(\frac{L}{x}\right)^{a_1-\gamma_1 t}\left(\frac{K}{x}\right)^{a_2-\gamma_2 t} \cdot Ae^{\gamma_3 t} \cdot \left(\frac{e(\beta-\gamma_5 t)}{1-(\alpha-\gamma_4 t)}\right)^{\alpha-\gamma_4 t-1} = 1$$

The speed with which the efficiency frontier moves towards the origin is clearly exhibited. For instance,along the ray of the average factor ratio, the input coefficients of the 1973 frontier are about 40% of the input coefficients on the 1964 efficiency frontier. It is also interesting to note that 17 of 28 units in 1973 have passed the 1964 efficiency frontier.

The increasing slope of the efficiency frontier illustrates the capital saving bias even if the trends in the kernel elasticities of labour and capital are rather small. The estimated capital saving technical progress is contrary to what one would guess a priori. Examples of labour saving techniques which have been introduced in the dairies are easy to find: Changes of milk reception from cans to tanks,self-cleaning separators and one storey buildings. The observed capital-labour ratio has increased substantially for all the production units over the ten year period. Fig. 3 reveals that all the units have reduced their input coefficients of labour while about half of the input coefficients of capital have increased. But the relative price increase of labour has been considerably higher than for capital, the price indexes for the last year being 2.45 and 1.60 for labour and capital respectively (1 for the base year). The results are therefore not in conflict with the observations. Capital saving progress means in our context that the marginal productivity of labour is increasing over time. Put this way it may seem as reasonable as the other way round.[1)]

Sensitivity Analysis

In Timmer [29] a kind of sensitivity analysis was performed by estimating the "probabilistic" frontier, by discarding efficient units on the frontier from the first run and then reestimating a new frontier without the most efficient units. The purpose was to investigate the effect of the most " extreme" observations. The result was that the new frontier without the " extreme" observations differed a lot from the original frontier but was more similar (except for the constant term) the traditional average production function for the same data set. When assessing frontier estimation, however, one must keep in mind that the raison d'être of frontier function estimation is that the most efficient units should count unproportionally.

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1) In Førsund and Hjalmarsson [15] the technical progress was estimated to be labour saving. However, the data set for two dairies for one year each have since been corrected. The measurement errors made one of the dairies considerably more labour intensive.

In our case we are more interested in another kind of sensitivity analysis. As it is one dominating large firm we are interested in its influence on the scale properties of the production function.Incidentally it is once on the frontier. The influence on the results of the smallest plants, of which one is once on the frontier, are also of interest because one can suspect that if these plants were to be built today new and more efficient techniques might be available for the same scale of output. The Hicks neutral case is, of course also of interest because most earlier studies have been limited to this case.

In Case 2 with only neutral technical progress the elasticity of scale function is constant and optimal scale obtains a moderate value, somewhat higher in 1964, than for Case 1, but considerably lower in 1973. On the other hand the trend in the constant term is now rather high so neutral technical progress amounts to about 6% which is a rather high value. (Cf. Ringstad [23].) Labour elasticity is also lower and capital elasticity higher in this case. Thus with this specification a 60% higher capital-labour ratio is optimal for the same relative factor prices,than for Case 1 in 1964,and 130% in 1973.

The objective function (5), the sum of slacks, increases with 3.6% from Case 1 to Case 2, and is thus not negligible. In Case 1, 6 units were on the frontier, while in Case 2, 5 units were on the frontier. Moreover, in Case 1, one unit is on the frontier in 1973, the unit with the lowest input coefficient of labour, but in Case 2 no unit is on the frontier after 1971. With the flexible specification in Case 1 it pays in terms of reduced objective function to shift the ratio between the kernel elasticities in favour of labour, such that this highly labour efficient unit appears on the frontier.

The exclusion of individual observations in Case 3 and 4 has some influence on the results. The exclusion of the largest plant in Case 3 reduces optimal scale and increases capital saving bias. An inspection of data shows that the input coefficients of labour and capital have been very stable for this plant which has tended to reduce the capital saving bias. The opposite is true for the four smallest plants whose input coefficients for labour, which are among the highest in the sample, have decreased relatively more than for most other plants. This explains the large reduction in capital saving bias in Case 4 where all these small plants are excluded. In this case, however, the level and development of optimal scale is very similar to Case 1.

If small obsolete plants are included the frontier may give a pessimistic bias over the relevant range. However, removing these units has created a much stronger bias. The small units are not replaced by observations of technologically new plants of the same scale,so really we have no control over what happens with the frontier. It turns out that the four smallest plants now in the sample are very close to the frontier,and one small unit being on the frontier at the start and another at the end of the period.

5. The Characterization of Technical Change

In order to assess the importance of the various parameter changes reported in Table 1 we will here follow Salter's [25] proposals for characterizing technical advance:

i) Relative change in total unit cost assuming cost minimization and constant factor prices.

ii) Relative change in factor ratioes for constant factor prices (bias measure).

iii) Relative change in the elasticity of substitution. (This is introduced by Salter in order to sort out the various influences on productivity change).

Since we work with production functions with constant substitution elasticities (and equal to 1) it is the two first measures that are of interest here.

Salter considered only two factors. We will first state the measures for the case of n factors and then introduce the specific homothetic function employed here.

The relative change in unit cost for discrete time is, in general:

$$(13) \qquad T = \bar{c}_{t+1}(X_{t+1},q_1,..q_n)/\bar{c}_t(x_t,q_1,..,q_n),$$

where $\bar{c}(\cdot)$ is the average cost function and q_i, $i=1,..,n$, are the factor prices, equal for both periods. Salter compares unit costs for the same output level, i.e. $x_t = x_{t+1}$. He notes the lack of reference to economies of scale in the measures, and suggests ways of measuring the impact of scale change on unit cost and factor bias. However, it might be preferable to make use of the relationship;

(14) $\bar{c} = \varepsilon \partial c/\partial x = \varepsilon c'_x$

where ε is the scale elasticity (Frisch [9]). Insertion in (13) yields

(15) $T = \{\varepsilon_{t+1}(x,q_1,..,q_n)\ c'_{x,t+1}(x,q_1,..,q_n)\}/\{\varepsilon_t(x,q_1,..,q_n)\cdot$

$\cdot c'_{x,t}(x,q_1,..,q_n)\}$

The change in unit cost is split up in the change due to change in the elasticity of scale and the change in marginal cost, for constant output and input prices.

When working with inhomogeneous production functions it is natural to concentrate on the change in the minimum unit cost, i.e. when $\varepsilon = 1$. This corresponds to the unit cost along the efficiency frontier in the input coefficient space. From (15) we then have:

(16) $T = c'_{x,t+1}\ (x^*_{t+1},\ q_1,..,q_n)/c'_{x,t}(x^*_t,\ q_1,..,q_n)$

where x^*_{t+1}, x^*_t are the output levels that corresponds to $\varepsilon_{t+1} = \varepsilon_t = 1$

It might be of interest to note the similarity between this measure of technical advance and Farrell's [8] concept of overall efficiency. (See Førsund [11], Førsund and Hjalmarsson, [13] for interpretations of the Farrell measures in a setting of inhomogeneous functions.) This can be illustrated in the two factor case. Let P in Fig. 4 be the point of reference on the efficiency frontier for the base period. Q' is the point on the efficiency frontier for a later period where the factor prices are the same. A measure analogous to the Salter measure i) above, assuming cost minimization, is then the relative change in unit cost from P to Q', i.e. the unit cost reduction possible when choosing techniques from two different ex ante functions for constant factor prices and realizing optimal scale. This change is equal to OR/OP in Fig. 4 which is also the Farrell overall efficiency measure for a production unit with observed input coefficients given by P relative to next periods efficiency frontier.

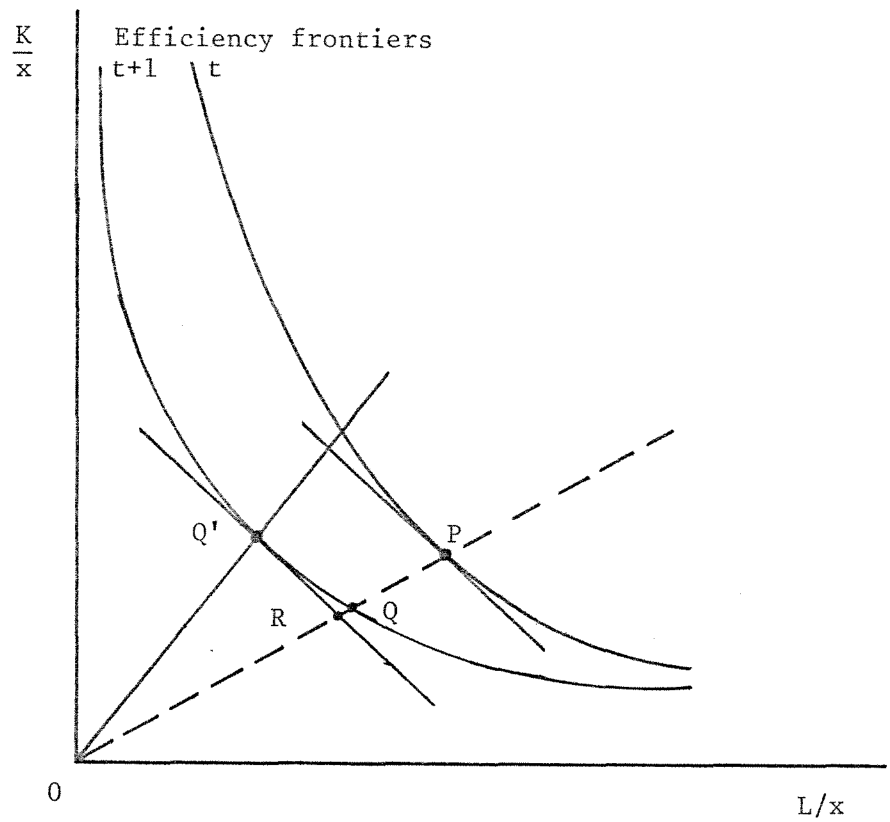

FIGURE 4 The generalized Salter measure of technical advance and its components.

The Farrell overall measure, and correspondingly the Salter technical advance measure, can be split multiplicatively into technical efficiency, OQ/OP, and price efficiency, OR/OQ. In our context this splitting shows the relative reduction in unit cost due to the movement along a factor ray and the movement along the next period efficiency frontier generated by biased technical change.

The general version of the Salter bias measure is:

$$D_{ij} = (v_{i,t+1}/v_{j,t+1})/(v_{i,t}/v_{j,t})$$

(17)
$$= (h_{i,t+1}(x_{t+1},q_1,\ldots,q_n)/h_{j,t+1}(x_{t+1},q_1,\ldots,q_n))/$$
$$(h_{i,t}(x_t,q_1,\ldots,q_n)/h_{j,t}(x_t,q_1,\ldots,q_n))$$

where the h(.)'s are the factor demand functions. It seems that Salter assume $x_{t+1} = x_t$. Relating this measure to the efficiency frontier means that the optimal scale outputs x^*_{t+1}, x^*_t should be inserted in (17). It is obvious that this bias measure must be related in some way to the price or allocative measure of Farrell since the latter measure shows the reduction in unit cost by adjusting to the optimal factor ratio (while keeping technical efficiency constant), i.e. the unit cost reduction due to changing from the optimal factor ratio on the old technology to the optimal factor ratio on the new one while keeping factor prices constant.

For the homothetic function the cost function is $c = G(x)\Lambda(q_1,\ldots,q_n)$, (see e.g. Førsund [11] and [12]), and the technical advance measure (16) becomes:

(18) $$T = G'_{x,t+1}(x^*_{t+1})\Lambda_{t+1}(q_1,\ldots,q_n)/G'_{x,t}(x^*_t)\Lambda_t(q_1,\ldots,q_n)$$

With the functional form (3) chosen here optimal scale, x^*, is:

(19) $$x^*_t = (1-\alpha_t)/\beta_t$$

The factor demand functions corresponding to the homothetic production function are in general:

(20) $$v_i = \partial c/\partial q_i = G(x)\Lambda'_i(q_1,\ldots,q_n)$$

With a C-D kernel function, which we will employ, the calcualation of the bias measure (17) becomes especially simple.

(21) $$\Lambda_t(q) = A_t^{-1}\prod_j(a_{j,t})^{-a_{j,t}}(q_j)^{a_{j,t}}$$

which yields:

(22) $$D_{ij} = \frac{\Lambda'_{i,t+1}(q_1,\ldots,q_n)/\Lambda'_{j,t+1}(q_1,\ldots,q_n)}{\Lambda'_{i,t}(q_1,\ldots,q_n)/\Lambda'_{j,t}(q_1,\ldots,q_n)} = \frac{a_{j,t}}{a_{j,t+1}}\cdot\frac{a_{i,t+1}}{a_{i,t}}$$

In order to show the Farrell splitting up of the unit cost reduction in a part due to proportional shift towards the origin and a part due to the change in the optimal factor ratio, the factor ratioes must be introduced in (18) with (21) inserted. Consider the n-1 factor ratioes

(23) $$b_{ij} = v_i/v_j, \quad j=1,\ldots,n$$

When these are given, all the other ratioes follow. The prices generating these ratioes must then be:

(24) $$q_j/q_i = a_j b_{ij}/a_i, \quad j=1,\ldots,n$$

Substituting the price ratioes in (18) with (21) inserted yields:

$$(25) \qquad T = \frac{G'_{x,t+1}(x^*_{t+1})}{G'_{x,t}(x^*_t)} \cdot \frac{A^{-1}_{t+1}}{A^{-1}_t} \cdot \prod_j (D_{ji})^{-a_{j,t+1}} \cdot \frac{a_{i,t}}{a_{i,t+1}} \cdot \prod_j (b_{ij})^{a_{j,t+1}-a_{j,t}}$$

To find the proportional cost reduction part, T_1, we may calculate:

$$(26) \qquad (v_{i,t+1}/x^*_{t+1})/(v_{i,t}/x^*_t)$$

We get $v_{i,t}$ and $v_{i,t+1}$ from (20) utilizing (21) by inserting the factor ratioes (23) constant for t and t+1. From (2) we obtain when $\varepsilon_t(x^*_t)=1$, $G'_t(x^*_t)=G_t(x^*_t)/x^*_t$. The result with a C-D kernel function is:

$$(27) \qquad T_1 = \frac{G_{t+1}(x^*_{t+1})/x^*_{t+1}}{G_t(x^*_t)/x^*_t} \cdot \frac{A^{-1}_{t+1}}{A^{-1}_t} \cdot \prod_j (b_{ij})^{a_{j,t+1}-a_{j,t}}$$

The first ratio, OS, shows the reduction in unit cost due to change in optimal scale, the second term, H, shows the cost reduction due to Hicks neutral technical change and the third term, B, shows the cost reduction due to factor bias technical change for constant factor ratio.

In view of (12) the bias cost reduction part, T_2, must then be:

$$(28) \qquad T_2 = \prod_j (D_{ij})^{-a_{j,t+1}} \cdot \frac{a_{i,t}}{a_{i,t+1}}$$

The factor neutral (Hicks) term, H, and the change in the scale function, OS, only affect the labelling of the isoquants, so they naturally belong to the proportional change term, T_1. Note that this term depends on the factor prices (factor ratioes), but that the bias cost reduction term, T_2, is independent of the factor prices. The latter term is, naturally, made up of a combination of the trends in the kernel elasticities.

The time functions used here are:

$$(29) \qquad \begin{array}{l} a_1(t) = a_1 - \gamma_1 t, \quad a_2(t) = a_2 - \gamma_2 t, \quad \gamma_1 = \gamma_2, \quad A(t) = Ae^{\gamma_3 t} \\ \alpha(t) = \alpha - \gamma_4 t, \quad \beta(t) = \beta - \gamma_5 t \end{array}$$

With the two inputs utilized here the technical advance measure (25) becomes:

$$(30)\quad T = \frac{\left(\frac{e(\beta-\gamma_5(t+1))}{1-(\alpha-\gamma_4(t+1))}\right)^{1-(\alpha-\gamma_4(t+1))}}{\left(\frac{e(\beta-\gamma_5 t)}{1-(\alpha-\gamma_4 t)}\right)^{1-(\alpha-\gamma_4 t)}} \cdot e^{-\gamma_3} \cdot (b_{21})^{-\gamma_2} \cdot (D_{21})^{-a_2-\gamma_2(t+1)}$$

$$\times \frac{a_1-\gamma_1 t}{a_1-\gamma_1(t+1)}$$

The other measures follow from inserting the time functions (29) in (22), (27) and (28).

6. Empirical Results: Technical Progress Measures

The estimated technical advance measures are set out in Table II for the observed average factor ratio.

TABLE II The Salter measure of technical advance and its components. K/L = 15.4 (the average factor ratio).

Type of relative unit cost reduction measures at optimal scale	28 units		27 units		24 units	
	1964/65	1972/73	1964/65	1972/73	1964/65	1972/73
T : Overall technical advance	.9207	.8882	.9186	.8816	.9415	.9038
T_1 : Proportional technical advance	.9208	.8883	.9188	.8820	.9415	.9038
OS : Change in optimal scale	.9070	.8750	.8963	.8603	.9367	.8992
B : Proportional change due to bias	1.0152	1.0152	1.0252	1.0252	1.0051	1.0051
H : Hicks-neutral advance	1	1	1	1	1	1
T_2 : Factor bias advance	.9999	.9999	.9997	.9995	1.0000	1.0000
D_{LK} : Relative change in optimal labour capital ratio	1.0377	1.0474	1.0672	1.1111	1.0094	1.0097

For the first two years the overall technical advance measure is T=.92 i.e. the average cost at the optimal scale in the second year is 92% of the average cost at optimal scale in the first year, representing a decrease in the average cost of about 9%. Between the last two years technical advance is somewhat more rapid, about 13% decrease in average costs. Overall technical advance, T, is the product of proportional technical advance, T_1, and factor bias advance, T_2. In our case technical advance is due to the movement of the efficiency frontier towards the origin, the factor bias advance, T_2, representing only .01% of the reduction in average cost. The splitting up of the proportional advance measure, T_1, reveals that the cost saving is due to the change in the optimal scale: OS increases with about 10% at the start of the period and with 14% at the end. The factor bias puts a brake on the cost saving along the factor ray chosen. The estimated factor bias, D_{LK} implies that, for constant prices or a constant factor ratio, it is optimal to increase the labour- capital ratio with 4% at the start and 5% at the end of the observed period. As already pointed out this change yields practically no returns in terms of cost saving.

Since we have found increasing optimal scale as the driving force behind cost saving it is of special interest to investigate the sensitivity of the overall technical advance measure when the specification of the production function is changed, as regards the time development of the parameters. Allowing a time trend in the constant term only, i.e. Case 2, the overall advance measure, T, becomes .94, or an average cost reduction (independent of time) of about 6 %. This is a somewhat lower cost reduction than obtained with the flexible specification, Case 1, but still a substantial figure for a sector characterized by small day to day improvements.

The dairy industry in Sweden has been characterized as relatively inefficient (Carlsson, [5]). As pointed out in a comment on that result (Førsund and Hjalmarsson, [14]) the more rapid the technical change the less efficient the industry appears based on cross section data as in Carlsson [5]. Our estimate of technical change over a period covering that year fits well into this explanation of his result.

The sensitivity of the results with respect to the units included in the estimation is also shown in Table II. When the biggest production unit is removed the results for the overall advance measure, T, is about the same, and when the smallest units are removed the measure is somewhat smaller. If the small units are "obsolete" as regards relevant ex ante designs the inclusion of these units when estimating the frontier function leads to a positive bias in the estimated technical advance. The proportional technical advance measure, T_1, follows the same pattern as the overall measure, T. But the impact of the change in optimal scale, OS, is somewhat greater when the largest unit is removed, and less than for all units when the smallest units are removed. Again, if these units are obsolete in the ex ante sense the inclusion of them gives a positive bias to the increase in optimal scale. The removal of the largest unit adds to this bias. Although the difference between the scale elasticity functions in Case 1 and Case 4 revealed in Table I is small it leads to a marked slower increase in the OS term in Case 4, 7 % and 11 % respectively, at the start and end of the period.

In Case 3 the capital saving bias increases markedly, the optimal labour-capital ratio increases with 7 % and 11 % at the start and end of the period respectively. As already mentioned the removed unit is quite stable as regards its input coefficients. However, this increased bias has still a minimal impact on the cost reduction, .03 % and .05 %. If the units are changed over time in accordance with the relevant ex ante function it does not matter much in cost terms if the factor ratio is not the optimal.

For Case 4 the change with respect to the bias is the opposite. The bias has now no impact on the cost reduction, and the increase in the optimal labour-capital ratio is .9-1.0 %. It is the change within the smallest units that gives rise to the capital saving bias, as pointed out in the previous section. If, therefore, the smallest units are technically obsolete, the technical progress has been almost neutral, but with an increasing optimal scale as the driving force.

7. Conclusions

When allowing variable returns to scale the driving force behind technical progress turned out to be a fairly rapid shift in the returns to scale function (Fig.2). The upward shift of the production frontier (Fig. 1) tended to be non neutral, increasing the kernel elasticity of labour and decreasing the kernel elasticity of capital somewhat.

The splitting up of the generalized Salter measure shows that it is the movement of the efficiency frontier (Fig. 3) along a ray towards the origin that results in the significant reductions in the average costs at optimal scale of 9-13 per cent per year. Optimal adjustment to the capital saving bias results in quite insignificant cost reductions.

The sensitivity analysis showed that the production function parameters were influenced by discarding a priori chosen units,some of which turned out to be on the frontier of the complete sample. However, the form and shift of the elasticity of scale function were fairly stable, leading to quite small variations in the cost reduction measures.

Appendix

As stated in Section 2 introducing a stochastic variable in the production function to simulate differences in technology between units, one may then proceed to derive maximum likelihood (ML) estimators. To investigate this approach consider the following specification of the production relation (1) (where the time dimension is dropped for notational ease):

$$\text{(A1)} \qquad G(x) = g(v)u, \; u \in (0,1],$$

where u is the stochastic variable implying input-neutral differences between units with respect to what they get out of their inputs. (We assume that each unit has perfect knowledge of its own production function; u is the econometrician's own device of simulating differences.) If the inputs are assumed to be exogeneous and u is assumed to be identically and independently distributed, writing (A1) on logarithmic form the simaultaneous probability distribution for the sample is

$$\text{(A2)} \qquad f(x_1,\ldots,x_n) = \prod_i h\{\ln G(x_i) - \ln g(v_i)\} \cdot |J|$$

where

$$\text{(A3)} \qquad |J| = |\partial \ln u_i / \partial \ln x_j| = \prod_i |\partial \ln G(x_i)/\partial \ln x_i|,$$

and h(.) is the distribution function for lnu. On logarithmic form becomes:

$$\text{(A4)} \quad \ln f(x_1,\ldots,x_n) = \sum_{i=1}^{n} \ln h\{\ln G(x_i) - \ln g(v_i)\} + \sum_{i=1}^{n} \ln|\partial \ln G(x_i)/\partial x_i|$$

Specific functional forms must now be inserted enabling us to derive ML-estimators. Introducing the one-parameter distribution

$$\text{(A5)} \qquad h(\ln u) = (1+a)e^{(1+a)\ln u}, \; a > -1,$$

insertion in (A4) yields

$$\text{(A6)}\quad \begin{aligned} \ln f(x_1,\ldots,x_n) &= \sum_{i=1}^{n} \{\ln(1+a) + (1+a)(\ln G(x_i) - \ln g(v_i))\} \\ &+ \sum_{i=1}^{n} \ln|\partial \ln G(x_i)/\partial x_i| \end{aligned}$$

If ML-estimates for the production function parameters were available an ML-estimator for a is:

$$\text{(A7)}\quad \begin{aligned} &\frac{\partial \ln f}{\partial a} = \frac{n}{1+a} + \sum_{i=1}^{n} \{\ln G(x_i) - \ln g(v_i)\} = 0 \\ &\Rightarrow a = \frac{n + \sum_{i=1}^{n} \{\ln G(x_i) - \ln g(v_i)\}}{-\sum_{i=1}^{n} \{\ln G(x_i) - \ln g(v_i)\}} = \frac{n}{-\sum_{i=1}^{n} \{\ln G(x_i) - \ln g(v_i)\}} - 1, \end{aligned}$$

where ML-estimates are inserted for the G(.) and g(.)-function parameters. Note that $E[\ln G(x_i) - \ln g(v_i)] = -1/(1+a)$, i.e. the estimator for a is derived from using the estimated average of lnu as estimator of the expected value $(-1/(1+a))$ of lnu.

Inserting (A7) in (A6) to obtain the concentrated log likelihood function, it seems to be very difficult to avoid solving a non linear programming problem to obtain ML-estimates when specific functional forms for the functions G(.) and g(.) are introduced. Comparing (A6) with the objective function (5) in Section 2, we see that it is the last term on the r.h.s. of (A6) that creates problems in this respect. Insertion of the functional forms given in (3), Section 2, yields:

$$\text{(A8)}\quad \begin{aligned} \ln f &= \sum_{i=1}^{n} \{\ln(1+a) + (1+a)(\alpha \ln x_i + \beta x_i - \ln A - \sum_j \ln v_{ij}\, a_j)\} \\ &+ \sum_{i=1}^{n} \ln|\beta + \alpha/x_i| \end{aligned}$$

The concentrated log likelihood function is:

$$\ln f^* = n\ln n - n - n\ln\left(\sum_{i=1}^{n} \ln g(v_i) - \ln G(x_i)\right) + \sum_{i=1}^{n} \ln\left|\partial \ln G(x_i)/\partial x_i\right| \tag{A9}$$

$$= n\ln n - n - n\ln\left(\sum_{i=1}^{n} \left(\ln A + \sum_{j=1}^{m} \ln v_{ij}a_j - \alpha \ln x_i - \beta x_i\right)\right) + \sum_{i=1}^{n} \ln\left|\beta + \alpha/x_i\right|$$

One could now proceed by using (A9) as the objective function and derive the estimates of the G and g functions by maximizing (A9) subject to the on or below the frontier contraint (6) in Section 2 and the homogenity contraint, (7) , on g(.), and then use (A7) to estimate a.

However, if one has access to a LP program only, it may seem worth while to try the following iteration procedure:

1. Start with the following objective function:

$$\sum_{i=1}^{n} (\ln G(x_i) - \ln g(v_i)) = \sum_{i=1}^{n} \left(\alpha \ln x_i + \beta x_i - \ln A - \sum_{j=1}^{m} \ln v_{ij}a_j\right) \tag{A10}$$

Maximize this subject to our constraints (6), $\ln G(x_i) - \ln g(v_i) \leq 0$, $i = 1,\ldots,n$, and (7), $g(v_i)$ homogenous of degree 1 in Section 2:

$$\alpha \ln x_i + \beta x_i - \ln A - \sum_{j=1}^{m} \ln v_{ij}a_j \leq 0 \tag{A11}$$

$$\sum_{j=1}^{m} a_j = 1 \tag{A12}$$

2. Estimate a according to (A7) by using these estimates. (Actually, in this first round the value of the objective function is the denominator in the first expression on the r.h.s. of (A7). This denominator is in general the sum of slacks of the constraints (A11).

3. The step 1 and 2 estimates of a, α, β, A, a_j are inserted in (A8) yielding the value of the objective function. (In this calculation it should be utilized that the sum of slacks appear in the expression.)

4. The coefficients of α,β in the objective function (A10) are changed according to the partial derivatives with respect to α,β of the objective function (A8):

(A13) $$\partial \ln f/\partial\alpha = (1+a) \sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{n} 1/(\beta + \alpha/x_i)x_i$$

(A14) $$\partial \ln f/\partial\beta = (1+b) \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} 1/(\beta + \alpha/x_i)$$

The new coefficients in the objective function (A10) for α and β become:

(A15) For α: $$\sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{n} 1/[(\beta+\alpha/x_i)x_i(1+a)].$$

(A16) For β: $$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} 1/[(\beta+\alpha/x_i)(1+a)].$$

The step 1 and 2 estimates are used.

5. The new problem (A10 - A12) is solved and new estimates obtained. Step 2 and step 3 are repeated. The last value of the objective function (A8) is compared with the value previously obtained. If the last value is greater by a δ factor or more, the procedure continues with step 4. If the last value is less by a δ factor or more, the iterations are stopped.

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