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TRADE AND SECURITY, I: ANARCHY

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Trade and Security, I: Anarchy

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Abstract: Market exchange is subject to an endogenously-determined level of predation which impedes specialization and gains from trade. Utility-maximizing agents opt between careers in specialized production and careers in predation. Three types of equilibria may emerge, autarky (with no predation and no defense), an insecure exchange equilibrium (with predation and defense), or a secure exchange equilibrium (in which defense completely deters predation). We analyze the influence of key parameters on the type of equilibrium which emerges. We also analyze changes in the welfare of groups of agents (the predators and specialized producers in both the richer region and the poorer region) as exogenous shocks occur in the technology of security. Since changes in security have terms of trade effects, some producers may be hurt by enhanced security. We show cases of 'immiserizing security' in which large poor countries are harmed by increased security.

Insecurity of claims to property is a prominent issue in the analysis of contemporary economic growth, whether one looks to the economic transition of Eastern Europe, the international diffusion of new technologies, the disruption of trade in parts of Africa, or the costs of increasingly evident administrative corruption. This paper presents a general equilibrium model in which the security of property is endogenously determined along with the levels of production and exchange. We assume that rational agents allocate labor across productive and predatory activities. On the one hand, an increase in predatory activity will increase the risk associated with market transactions and impede specialization and trade; on the other hand, a decrease in the volume of trade will shrink the pool of property which is subject to appropriation and diminish the incentive to engage in predation.

The general equilibrium model permits us to explore several questions: Can exchange arise at all when predatory forces are powerful and defense is uncoordinated? Under what condtory activities shift as offensive and defensive institutions and technologies change? Given that changes in production affect the terms of trade, can some producers actually lose from enhanced security? The terms of trade of a large country improve as the volume of trade falls; could such a country benefit from increased predation? If only one country supplies predators, that country's terms of trade will improve as labor is withdrawn from production; would even the productive forces of the "predating" country therefore prefer lax enforcement? We provide simulations suggesting that large poor countries may be hurt by enhanced enforcement of property rights, a case we call "immiserizing security," and we then tentatively approach a final question: What incentives might there be to move beyond the anarchy of individual action toward a world of coordinated defense--or a world of organized crime? We plan to explore this question in detail in future work.

The model can be applied as a parable about both contemporary and historical phenomena. Contemporary Russia provides dramatic examples of the evolution of private defense, organized Mafias, and new forms of state security in response to insecure exchange. A less prominent example is provided by the international diamond trade of Sierra Leone, which has recently been revived though the provision of defensive services "unsettlingly reminiscent of an older world order, . . . when private armies cleared the way for European companies to pursue commercial interests in Africa" (Rubin, 1997, p.49). In the terms of our model, the initial predation-induced collapse of specialized production and exchange in Sierra Leone appears to exemplify the case in which autarky is the only equilibrium. The exchange equilibrium subsequently emerged when a mercenary army was given a contract to defend trade in return for a share of the gains.

Historically, specialization and exchange have sometimes been supported by coordinated defense of trade and sometimes by coordination among predators concerned about killing the goose that laid the golden egg. "Piracy was business for the Barbary rulers, and the corsairs formed a guild thar defensive action was coordinated and the equilibrium was characterized by a very low level of specialization and exchange. Spanish-American trade seems to have plummeted around 1630 as the costs of defending the annual fleets increased and the number of predators rose. The long "seventeenth century recession" in the Spanish colonies might be understood as a move toward diversified local production induced by decreased security of trans-Atlantic trade. We plan to examine this case in detail in future work.

This paper is about anarchic economies, so it is awkward to introduce governments. However, "the immiserizing security" effect may provide an insight about government behavior both contemporaneously and historically. Large poor economies such as India and Indonesia have been tolerant of predation on international trade, either through regulations which permit officials to extract payment from traders or through alleged tolerance of piracy. The model of this paper shows that a benevolent government in such a country might very well serve its citizens by going easy on predation.

A possibly more convincing application of the insight of this model is to the early history of commercial relations between nations. Many less-developed nations of the world were at one time quite tolerant of predation on trade. It is sometimes claimed that one-sided commercial treaties imposed by Europeans through overwhelming naval power served to impoverish the less-developed "partners." Perhaps immiserizing security may be useful in thinking about European trade with large poor countries like China and India in the 18th and 19th centuries.

In other future work we will develop a similar formal model of *legitimate* predation through the legal attack and defense of contracts. The security of contracts is a significant issue in cross-border trade, raising the same problems of international coordination and conflict of interest which arise in the present model. Trade agreements and other forms of international economic integration can fruitfully be understood in part as security arrangements.

The present paper has five sections. We first set out the basic elements of the model, specifying production, preferences, and the parameters of the final-stage choice of the levels of production and exchange. In the second section, we explore international equilibria for given levels of insecurity and show that insecurity can keep a trading equilibrium from emerging in spite of the technological differences which ordinarily generate gains from trade. The third section of the paper endogenizes the level of insecurity, formally specifying the agent's decisions to allocate labor to defense and predation. The fourth section incorporates all three decisions, predation, defense, and production, into one grand model which shows that autarky, secure trade, and trade subject to predation are all possible general equilibrium outcomes. The fifth section uses numerical simulation to explore the impact of changes in the technologies of predation and defense, demonstrating the possibility of "immiserizing security."

I. The Elements of the Model

In general equilibrium, agents endogenously allocate labor across defense, predation, and the production of two goods. Agents differ in their relative productivity in the two lines of activity, giving rise to incentives to specialize in production and exchange goods through the market. Predation is assumed to take the form of banditry, the seizure of the traded goods. Defensive expenditure reduces the risk of banditry. Insurance is impossible due to the nonobservability of defensive effort and the possible collusion of predators and producers.[1] The degree of specialization and the gains from trade depend on the balance among predation, defense and production.

The model is related to that of Grossman and Kim (1995), who also explore a general equilibrium of predation, production and defense. Grossman and Kim have a single good technology subject to predation in the form of seizure of the agent's endowment. We assume perfect security of the 'endowment' in the sense that autarky is secure. Our focus is instead on the interesting issues which arise once specialization in production and exchange are possible.

We model exchange between two regions, called 'countries' for convenience in applying labels and techniques from international trade theory.[2] Each country is composed of agents with identical production technologies, so there is no reason to trade internally nor any domestic predation or need for defense. Countries differ in that the constant opportunity cost of good 1 in terms of good 2 is higher in the home country than in the foreign country. In the standard Ricardian trade model, the home country will specialize in good 2 and import good 1, and both the home and foreign countries will enjoy (weak) gains from trade. In our model, the closest one can come to the Ricardian equilibrium is the nonaggression equilibrium in which no resources are in fact devoted to predation. However, even in this case, some labor must be devoted to defense in each country in order to deter incipient predation. There are two other possible equilibria. One is autarky, where specialization and exchange are

[1] In our motivating examples, Sierra Leone and 17th Century Spanish America, agents could not obtain insurance. Elsewhere, partial insurance exists and a serious treatment requires a general equilibrium model of insurance, production, exchange and predation. We abstract from this complexity here.

[2] The label 'country' tends to connote a sovereign nation with an active government; an undesirable connotation in the present context. Our sequel paper addresses an active government which can overcome the free rider problem in defense (but may be predatory itself).

deterred by the expectation of banditry. The other is an equilibrium in which positive levels of banditry, defense and international trade all coexist.

A. Preferences and Production Functions

All agents share identical homothetic preferences. Each agent has a Ricardian technology for producing each of two goods. Trade can occur between two economies due to Ricardian comparative labor productivity differences, but it is inhibited by losses due to banditry. Autarky foregoes the gains from trade but avoids the losses due to banditry.

The production technology for each agent in the home economy is described by:

(1.1) $a_1 y_1 + a_2 y_2 \leq l^G$

where a is a unit labor requirement with a subscript denoting the index label of a good, y is a production level of a good and l^G is the amount of labor devoted to goods production. The aggregate level of production and productive labor will be denoted by upper case letters. An asterisk will designate the foreign economy. Subscripts with variable labels denote partial differentiation.

B. Determination of Production and Exchange

Individual levels of predatory and defensive labor are chosen before the final stage, in which productive labor is allocated across goods and the level of trade is determined. Moreover, aggregate levels of predatory and defensive labor have already determined a fixed probability of successful trade, π. The agent must now decide how much of each good to produce and to trade at price p (price of good 1 in terms of good 2).

The production vector is **y** and the trade vector is **m**, where exports appear as negative quantities. The agent's final stage decision problem is:

(1.2)
$$\begin{aligned} &\max_{\mathbf{y},\mathbf{m}} \ \pi u(\mathbf{y}+\mathbf{m}) + (1-\pi)u[\mathbf{y}+\min(\mathbf{m},0) \\ &\text{subject to} \\ &\mathbf{a}'\mathbf{y} \leq l^G \\ &\mathbf{p}'\mathbf{m} \leq 0. \end{aligned}$$

The first constraint is the Ricardian production constraint at the individual level. The second is the exchange constraint for the states in which shipments are successful. The agent maximizes the utility of consumption; in the event of predation, consumption is equal to the production level for imported good and equal to the production level less the stolen exports for exported good.

The first order conditions of the maximization program (1.2) reveal the characteristics of the choices which the agent will make. Intuitively, exchange is risky, since shipments may be lost, and it is expensive, since shipments must be defended. Therefore, it may be best to avoid the complete specialization which would be optimal in a pure Ricardian model. Exchange may be so risky or expensive that autarky is preferable.

To analyze the levels of output and trade in detail, it is useful to denote the case where shipments are successful with a superscript G (Good state) and where they are not successful with a superscript B (Bad state).

In the Bad state, the utility function is not differentiable with respect to **m** at the autarky point, but elsewhere yields

$$u^B_{m_j} = \begin{matrix} u^B_j \text{ for } m_j < 0 \\ 0 \quad \text{for } m_j > 0 \end{matrix}$$

Since the home country imports good 1, at a bad state interior solution $u^B_{m_1} = 0$, and the first order conditions in the trade vector *(m_1, m_2)* imply:

$$(1.3) \quad \frac{\pi u^G_1}{E[u_2]} = p.$$

This can also be written as:

$$(1.4) \quad \frac{u^G_1}{u^G_2} = p + p\frac{1-\pi}{\pi}\frac{u^B_2}{u^G_2}.$$

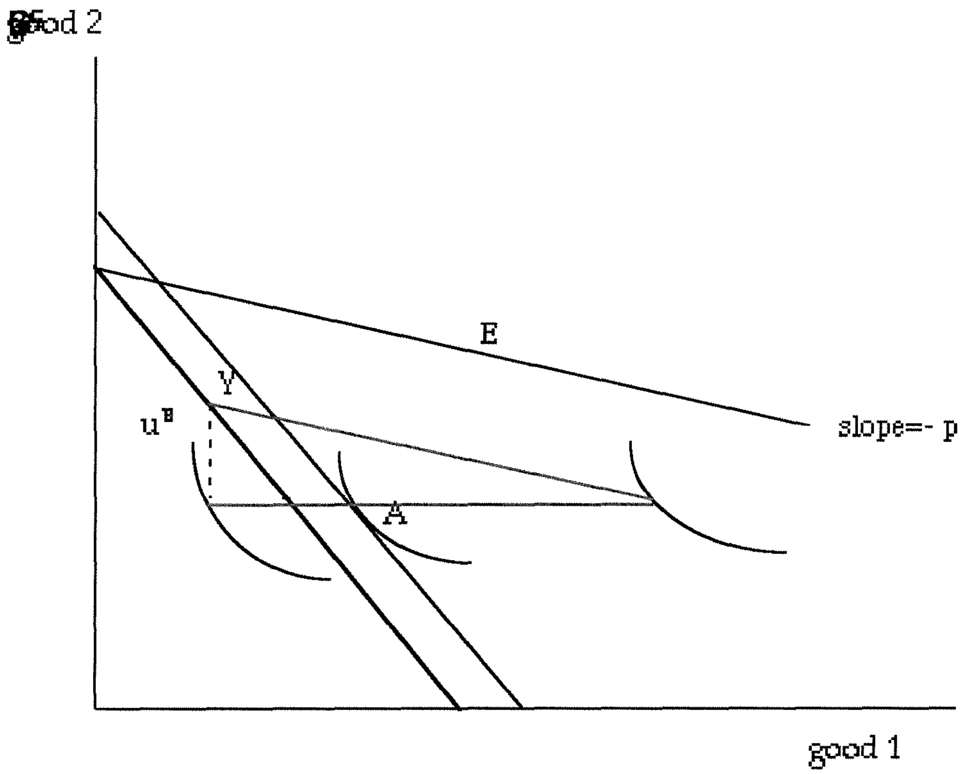
The first order conditions in the output vector (y_1, y_2) require, at an interior solution, the two conditions:

$$\pi u^G_1 + (1-\pi)u^B_1 = \lambda a_1$$
$$\pi u^G_2 + (1-\pi)u^B_2 = \lambda a_2$$

where λ is the Lagrange multiplier for the labor constraint. Taking the ratio of the first equation to the second and using (1.3)

(1.5) $$\frac{a_1}{a_2} = \frac{E[u_1]}{E[u_2]} = \frac{\pi u_1^G}{E[u_2]} + \frac{(1-\pi)u_1^B}{E[u_2]} = p + \frac{(1-\pi)u_1^B}{E[u_2]}.$$

Thus, the interior solution involves a specialization in which both the "marginal rate of expected substitution" and the marginal rate of substitution in the good state are greater than *p*. The solution may allow some of both goods to be produced. (The marginal rate of expected substitution cannot generally be ranked relative to the marginal rate of substitution in the good state.) A diagram illustrates.



The heavy line represents the Ricardian production frontier with the supply of labor to defense deducted. Let Y represent optimal production with trade and possible predation, permitting utility denoted G in the good state and utility denoted B in the Bad state. In autarky, no labor is spent on defense, and point A is the autarky consumption bundle, with utility u^A. As optimal output with risky trade, Y, approaches optimal output under autarky, A, the potential loss and the potential gain from trade are both reduced. Point E represents Eden, in which predation is completely deterred through defensive expenditures, and exchange can take place at relative price p with perfect security. Since Eden is maintained by spending resources to deter predation, the value of output is less than it would be if all labor could be devoted to specialized production.

When p is equal to the slope of the production frontier, the agent is indifferent to trade in the Eden case, but loses from trade if predation occurs. By continuity, there is an interval of values of p relative to a_2/a_1 and of π relative to $(1-\pi)$ for which autarky continues to dominate trade for the agent. Eventually, favorable enough terms of trade make it optimal to accept some risk, at an interior point like Y. As a limiting case, with right angle isoutility loci, an infinitely risk averse agent (who maximizes his minimum utility) will stay at autarky no matter how favorable the price. At the other limiting case, with straight line isoutility loci and risk neutrality, the agent will completely specialize.

The system (1.3), (1.5) and the two constraints of the maximization program give four equations to determine the four variables **y**, **m**. These variables are implicit functions of the exogenous variables p, π, $\boldsymbol{a}$. With concave utility, the solution is globally unique.

II. The Trading Equilibrium

This section sets out the determination of exchange equilibrium in terms of parameters of the model, focusing especially on boundary cases where one or another type of equilibrium obtains. For concrete results we develop a Cobb-Douglas preferences version of the model and use it as a basis for simulation.

In the nonagression equilibrium, in which defense deters all predation, the model collapses to the classic Ricardian model. The equilibrium price p is guaranteed to lie between autarkic values of the prices, which are equal to the comparative labor productivity ratios. Without loss of generality, maintain the assumption that the home economy has a comparative advantage in good 2. Let L^G and L^{*G} denote the aggregate supply of labor to production at home and in the foreign country respectively. In the classic case, with incomplete specialization p

is at one or the other limiting ratio of the unit labor requirements; otherwise p is determined by the relation:

(2.1) $$p = \frac{u_1}{u_2}\left[\frac{L^G / a_2}{L^{*G} / a_1^*}\right].$$

Here, we use the homothetic form of preferences. The terms of trade are determined by effective relative country size and by the shape of the utility functions. Larger countries tend to have worse terms of trade, and countries whose exports are inelastically demanded tend to have better terms of trade. In the present model, the secure trading equilibrium must be supported by a voluntary defense effort by the home and foreign agents. Thus it is impossible that the equilibrium price p would lie at the autarky price level for either economy, because in that case agents could do better by selecting autarky and saving the defensive resources for productive use. In all cases, the trading equilibrium without predation will be unique with the standard regularity conditions on demand.

Moreover, the exchange price p will never be at either of the autarkic limits in an equilibrium with positive levels of predation, because in that case agents in the incompletely specialized economy can do better by moving to autarky. In an equilibrium with predation, it will not generally pay to completely specialize, so the solution of (2.1) does not apply, and the analysis of the diagram must be applied to both economies simultaneously.

Generally, there are two linear constraints (the balance of trade constraint and the full employment constraint) and two marginal efficiency conditions (for trade and for production) to determine the optimal choice of the four variables: per capita production of goods 1 and 2, and excess demand for goods 1 and 2. These per capita functions will be implicit functions of p,π, and the technological parameters. To aid in the analysis of the production and exchange equilibrium system, the Appendix develops the special Cobb-Douglas case, for which closed form solutions obtain for trade and production.

A. International Exchange Equilibrium

The international equilibrium of the two country version of the model is determined by using the market clearing condition for the home country's imported good:

(2.2) $N^G m_1(p,\pi,\alpha,a_2,l^G) + N^{*G} m_1^*(p,\pi,\alpha^*,a_1^*,l^{*G}) = 0$.

Here, m denotes per capita excess demand, * denotes the foreign values, $\mathbf{N}^G$ equals the number of identical agents in production and defense (equal to L^G/l^G, the aggregate labor devoted to production divided by the fraction of the individual's labor endowment which is devoted to production), and we write the excess demand for good 1 as an implicit function of the variables which the agents take as exogenous at this stage. We assume a complete separation between legitimate trade and the thieves' market in which captured goods are exchanged. In proceeding with the analysis, relative country size alone matters, so we divide through by $\mathbf{N}^G$. Also, it is convenient to work with import demand functions for both countries, taking advantage of the budget constraint to write

$$m_1^* = -\frac{1}{p} m_2^*.$$

Assuming that tastes are identical and Cobb-Douglas with the parametric expenditure share for good 1 (the home country import) denoted by γ, the international equilibrium condition (2.2) reduces to:

$$(2.3) \quad \frac{\gamma f(p,\pi,\alpha)}{pf(p,\pi,\alpha)+\alpha} l^G / a_2 - \frac{1}{p} \frac{\gamma f^*(1/p,\pi,\alpha^*)}{f^*(1/p,\pi,\alpha^*)/p+\alpha^*} \frac{N^{*G}}{N^G} l^{*G} / a_1^* = 0.$$

Here, f is the import penetration ratio m_1/y_1, as defined in the Appendix. We use the relation $-m_2^*/p = m_1^*$, and we note that the foreign import penetration ratio f* gives the ratio of foreign imports to foreign output of importables.

The existence, uniqueness and stability of equilibrium in this model are standard, provided there is a range of mutually advantageous prices. This follows since both import demand functions are downward sloping everywhere in the relevant range of prices. The sole question of existence arises from the effect of lower π in reducing and eventually eliminating the range of potential equilibrium prices.

Ricardian theory implies that the trading equilibrium price lies between the two autarky price ratios: $1/\alpha^* \leq p \leq \alpha$. The interior equilibrium price in the standard Ricardian Cobb-Douglas case is:

$$(2.4) \quad p = \frac{\gamma}{1-\gamma}\frac{N^G l^G / a_2}{N^{*G} l^{*G} / a_1^*}.$$

In contrast, in our model (2.3) does not yield a closed form solution for p, and when π equals one the equation is not even defined. However, even in our model, given the comparative cost ratios α and α^*, there are limits to the differences in effective country size ratios and consumption share ratios which are consistent with a trading equilibrium.

Risky trade restricts possible international prices to a narrower range than $1/\alpha^* \leq p \leq \alpha$. As the probability of successful shipment falls, the range of prices consistent with trade may actually vanish. To see how this arises, consider the incipient autarky price at which the home country is just barely unwilling to trade. The import penetration ratio would equal zero which, drawing on the Appendix, implies:

$$\frac{m_1}{y_1} \equiv f(\cdot) = \left[\frac{(1-\pi)p}{\pi(a_1/a_2 - p)}\right]^{-1/(1-\gamma)} - 1 = 0, \text{ implying}$$

$$\text{p} = \pi\alpha.$$

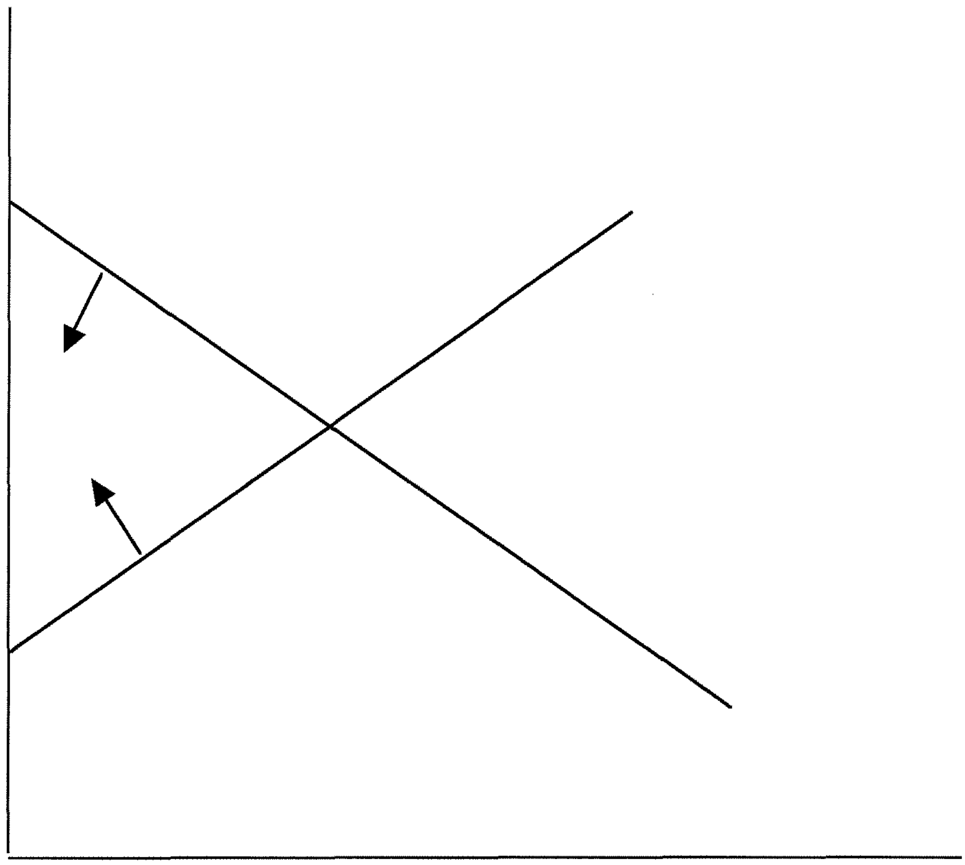
For the foreign economy, the incipient autarky price is similarly defined by:

$$1/p = \pi\alpha^*, \text{ or}$$

$$\text{p} = 1/\pi\alpha^*.$$

Solving the two equations simultaneously, the critical value of π which defines the range of trading equilibria is: $\tilde{\pi} = (\alpha\alpha^*)^{-1/2}$. When π falls to that level, both countries will cease trading. For values of π below the critical value, a trading equilibrium does not exist, since the upward sloping export supply function has a vertical intercept above the vertical intercept of the downward sloping import demand function. Trade can occur only at a mutual loss.

Trade Equilibrium and Security



The diagram above illustrates. The equilibrium at *E* permits trade according to comparative advantage, and mutual benefit despite some predation. With lower values of π, the relative positions of the import demand and export supply schedules will eventually reverse, resulting in equilibria which will not be chosen by rational agents who compare their real income in autarky with their real income in the loss-making equilibrium. Incorporating the need to cover the fixed costs of trade in order for there to be net gains, there are still tighter limits on the admissible range of prices.

<u>Proposition 1. Nonexistence of risky trade equilibrium</u>

For π sufficiently small in the special Cobb-Douglas case, no trading equilibrium exists.

Proposition 1 is useful because it shows that even leaving aside the fixed cost of trading, when traders are faced with exogenous predation, the market cannot always find a price at which voluntary exchange will occur. Larger fixed costs will shift π upward but will also shrink the interval of welfare-improving relative prices.

The diagram also shows that the analysis leading to Proposition 1 is actually quite general. The Cobb-Douglas case serves to pin down the exact shape of the import demand and excess supply functions and the value of the vertical intercepts. In more general cases, the vertical intercepts should move with lower π just as in the diagram. Existence will always depend on a π large enough so that the relative positions of the two autarky prices are not reversed.

B. Predation and the Terms of Trade

It is very useful to characterize the response of the terms of trade, *p*, to changes in parametric levels of predation. Based on the standard comparative static methods applied to (2.2), *dp/dπ* has the sign of $m_{1\pi} - \frac{1}{p} m^*_{2\pi} \frac{N^{*G}}{N^G}$. Both partial derivatives are positive, so when they are equal (a natural benchmark case), *lowering π has the effect of improving the terms of trade of the larger country.* In Figure 1, the excess demand function of the larger country shifts to the left by more. This benchmark result cannot be proved to hold generally even in the Cobb-Douglas case (see Appendix). For the symmetric case ($\alpha=\alpha*$ and $\gamma=1/2$), country size alone matters and the large country's terms of trade improve with decreases in π. For asymmetric parameters, for sufficiently different country size it appears likely that the larger country will experience terms of trade improvements as π falls, and simulation results confirm this. In general we have a presumption:

Presumption 1:

There is a presumption that higher predation improves the terms of trade of the larger country.

Given the terms of trade effect, the impact of decreased security on the welfare of the larger country is ambiguous. In contrast, the smaller country loses both from increased insecurity directly and from a terms of trade deterioration. The analysis suggests that the two nations may have opposing interests in security arrangements when terms of trade effects are powerful. Under the presumption, the large nation may prefer less secure trade. To finish the welfare analysis of this case requires simulation, as the welfare of the country enjoying a terms of trade improvement changes according to the magnitude of $dp/d\pi$, which is a deeply nonlinear function of the parameters. Moreover, π is determined by the endogenous choice of entry into predation and defense, and implicitly is thus a nonlinear function of the deep parameters of the model. The full model must be developed before returning to the welfare analysis.

III. Equilibrium Predation and Defense Decisions

At the earlier stage of the game, agents make "occupational" choices about whether to enter specialization with defense or predation. We characterize this decision and then consider a rational expectations trading equilibrium and its possible collapse to an autarky equilibrium.

A. Labor in Predation and Defense

Predation occurs on the flow of trade as opposed to the endowment as in Grossman and Kim (1995). One can defend against predation, and the technologies of predation and defense are represented in a relative effectiveness parameter. There is an exogenous proportionate 'spoilage' of the stolen goods, with the remainder of the stolen goods being resold on spot markets (which we assume for expositional simplicity to be separated from legal markets).

The technology of predation and defense is captured in the probability of successful shipment π, which is a function of the aggregate amounts of labor devoted to predation and to defense. At the aggregate level the probability is equal to the proportion of the value of shipments which escapes seizure. There is no aggregate uncertainty. The probability or proportion of successful shipment is

(3.1) $$\pi = \frac{1}{1+\theta L^B / L^D}.$$

Here, the superscript B denotes the labor devoted to (B)anditry and the superscript D denotes the labor devoted to (D)efense. The upper case L denotes the aggregate supply of labor to each activity. The parameter θ is meant to capture the relative efficiency of offensive and defensive activity. Obviously, π is defined on the unit interval for all nonnegative levels of labor in each activity.

The individual productive agent treats the probability of successful shipment as a parameter in making the agent's own commitment to defense.[3] However, the agent still faces a choice between devoting at least a minimal amount of labor to defense and shipping successfully with probability π, or devoting no labor defense and losing any shipment. Formally, the individual probability of successful shipment is equal to:

(3.2) $$\pi^i = d(l^{Di})\pi,$$

where d(.) is a dummy variable operator which returns the value 1 for $l^{Di} \geq \bar{l}$ and returns the value zero elsewhere. With this specification, we introduce a realistic element of fixed cost to the trading and defense decision while avoiding,

[3] With a small enough number of productive agents, the agent may realize that π is endogenous, but still avoid any more than the fixed cost of defense due to the free rider problem.

for simplicity, further modeling of choice of defensive effort.[4] Fixed costs are important in all sorts of exchange, especially at low levels of trade.[5]

This specification of the probability function can be rationalized as follows. Successful trade requires both a successful trip to market with the exports and a successful trip home with imports. The probability of success is independent on each portion of the round trip, with the joint probability being π. Shipments of number S flow to A possible meeting points. Both bandits and shippers pick meeting points to use and to attack according to some random process, possibly based on optimal strategies. Routes to and from meeting points are so various (and only one way shipments are available to attack) than bandits only attack meeting points. The determination of outcomes is that greater force wins (or wins more often) and in the event of a tie the probability of successful exchange is 1/2. We do not specify the random process by which forces are allocated and shipments are sent to meeting points, nor the process which determines outcomes, but claim that our simple specification has reasonable qualitative properties.[6] The expected number of bandits per meeting point is L^B/A. The goods shipments may be defended with expected defensive intensity L^D/S. The term in the probability function $\theta L^B/L^D$ gives the odds of successful banditry.[7] Then in this case $\theta \frac{L^B}{L^D} = \frac{L^B/A}{L^D/S}$ and the parameter θ is interpreted as the number of shipments per meeting point, S/A.[8] Moving away

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[4] The model with endogenous choice of defense levels adds little so long as there are constant returns to defense. Let the private probability of success be equal to $\frac{l^{Di}}{\bar{l}}\pi$ for $0 \le l^{Di} \le \bar{l}$. The optimal choice of defense will be either 0 or $\bar{l}$ save by chance.

[5] The descriptive literature on early long distance trade reveals that trading companies maintained agents in foreign cities who operated secure storage facilities, exchanged goods between ship arrivals, and gathered commercial intelligence.

[6] Taking the model literally, reasonable specifications of the allocation process produce more complex odds functions which all share the property of being increasing in defensive labor and decreasing in offensive labor.

[7] Strictly speaking, the odds of successful banditry are $(1-\pi^{1/2})/\pi^{1/2}$.

[8] The same formal model can be interpreted as a single market of circumference A, where exchange is safe inside the market but exposed to predation as it passes the circumference. Such concentration in a 'port town' suggests coordination but could be anarchistic as in the present paper.

from the literal interpretation, θ is a parameter reflecting the natural relative advantage of offensive forces.

The offense attacks the trade flow. Let M_i denote the aggregate trade flow of good i (positive for imports and negative for exports) and let p denote the relative price of good 1. The total value of trade at risk is $p|M_1|+|M_2|$. Pirates can resell stolen goods on the thieves market at price p^B for the relative price of good 1. There is an exogenous proportionate loss of stolen goods equal to β. The expected total gain to banditry is

(3.3) $$(1-\pi)\left[p^B|M_1|+|M_2|\right](1-\beta)=(1-\pi)\mathbf{p}^{B\prime}|\mathbf{M}|(1-\beta),$$

where $\mathbf{p^B}'$ is the vector $(p^B, 1)$ and $\mathbf{M}'$ is the vector (M_1, M_2).[9] In equilibrium, the returns from banditry to the individual (expression (3.3) divided by L^B) must equal the returns from production and defended exchange, if both banditry and exchange are to occur.[10] Complementary slackness controls entry; dominated activities will not be used at all.

The timing of decisions is unimportant in the anarchistic model of this paper, in which agents are small and disorganized. We find it convenient to imagine the sequence where production and exchange are chosen last, following the defense decision, which follows the offense decision. We think of individuals as making a life choice to enter the productive or predatory sectors on the basis of equal expected utility in each. This occupational specialization comes first.[11] Given a productive career choice, the defensive expenditures come before the allocation of productive labor, reflecting investments in monitoring and storage systems. The analysis of individual behavior is done by

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[9] If the thieves market and the legitimate market are integrated, then $p^B=p$. Due to the Cobb-Douglas structure there is a tight connection between the two prices anyway, with the separated markets assumption simplifying the accounting.

[10] This specification is equivalent to pooled shares in banditry, where the aggregate proportion of goods stolen is certain and all individual risk is removed. With income-risk neutral bandits, as assumed in the Cobb-Douglas utility function, such pooling is irrelevant as the agent is indifferent between the expected per capita income with certainty and the uncertain stream with the same expected value. We prefer the individual uncertain return interpretation, as risk pooling presumes coordination.

[11] Allowing for some members of the household to enter banditry complicates the notation but adds nothing essential to the analysis.

backward induction. The timing assumption will be important when analyzing coordinated offense and defense in our sequel.

B. Allocating Labor to Production and Defense

For productive agents, the choice between defense and production is made anticipating the outcome of the last stage. We may substitute the equilibrium values of the Appendix functions (7.2)-(7.5) into the Cobb-Douglas utility function to obtain:

$$(3.4)\quad v^G(p,\pi,\alpha,l^G)\equiv\left\{\pi\left[\frac{1+f}{pf+\alpha}\right]^{\gamma}+(1-\pi)\left[\frac{1}{pf+\alpha}\right]^{\gamma}\right\}\frac{l^G}{a_2}.$$

We assume the agent has 1 unit of labor in total, so $l^G=1-l^D$. For those choosing production, the next choice is between autarky and trade. In autarky, the agent earns utility equal to $v^A=\frac{1}{a_2}\alpha^{-\gamma}$, which exceeds trade utility at f equal to zero. In specialization, the minimum level of defense is the fraction $\bar{l}$. The agent selects the level of defensive labor (and implicitly productive labor) to solve:

$$(3.5)\quad \max_{l^D} d(l^D)v^G(p,\pi,\alpha,1-l^D)+[1-d(l^D)]v^A.$$

The agent will either choose autarky with no defensive labor, or the minimum defensive labor and specialization. In the latter case, the agent enjoys utility v^G evaluated at $l^G=1-\bar{l}$. The agent undersupplies defense from a social point of view because he takes the aggregate probability as given. Even a 'large' agent will undersupply defense due to the externality involved, so our setup simply makes the coordination problem more severe.

We allow both foreign and domestic agents to select defensive expenditures, so an analogous program to (3.5) characterizes defensive expenditures by the foreign agent.

$$(3.6)\quad v^{*G}(p,\pi,\alpha^*,1-l^{*D})=\left\{\pi\left[\frac{1+f^*}{f^*/p+\alpha^*}\right]^{1-\gamma}+(1-\pi)\left[\frac{1}{f^*/p+\alpha^*}\right]^{1-\gamma}\right\}\frac{1-l^{*D}}{a_1^*}.$$

(3.7) $\max_{l^{*D}} d(l^{*D})v^{*G}(p,\pi,\alpha^*,1-l^{*D})+[1-d(l^{*D})]v^{*A}$.

Here, the foreign agent's autarky utility is defined as $v^{*A} = \frac{1}{a_1}\alpha^{*-(1-\gamma)}$. We assume for simplicity that the fixed cost of defense is identical for the foreign and home agents.

C. Allocating Labor to Predation

Banditry pays by seizing shipments. In the aggregate, the prize vector is $[(1-\beta)(1-\pi)M_1, -(1-\beta)(1-\pi)M_2$. We have two possible ways to treat the exchange of the stolen goods. We could assume that they find their way into legitimate commerce again, and this is the appropriate setup when the household is treated as an integrated producing and predating agent. Alternatively, we may assume a separation of legal and illegal exchange. This is appropriate for the case developed here. The details are not essential; we need only some simple and plausible way to model the utility of the representative bandit in order to model entry.

We assume that agents have identical tastes, so the stolen goods are exchanged on a thieves' market with equilibrium price

$$p^B = \frac{\gamma}{1-\gamma}\frac{-M_2}{M_1} = \frac{\gamma}{1-\gamma}p.$$

The assumption of no arbitrage between legitimate markets and the thieves' market is restrictive, as the thieves' market price is not generally equal to the legal market price.

The representative bandit realizes expected utility from his activity equal to:

$$\begin{aligned} v^B(p^B,\pi,\beta,M_1,M_2,L^B) &\equiv (p^B)^{-\gamma}(1-\pi)(1-\beta)\frac{p^B M_1 - M_2}{L^B} \\ &= \left(\frac{\gamma}{1-\gamma}p\right)^{-\gamma}(1-\pi)(1-\beta)\frac{pM_1/(1-\gamma)}{L^B} \end{aligned} \quad (3.8)$$

In the second line we replace p^B using the Cobb-Douglas special case and use the international exchange equilibrium condition to replace the ratio of M's. The agent chooses Banditry if his utility exceeds that available from autarky or from specialization and production.

Foreign labor is also allowed to enter into banditry, so L^B is equal to the sum of home (N^B) and foreign (N^{*B}) bandits, and $L^B = N^B + N^{*B}$. (Note that L^D, which affects π, likewise includes both foreign and domestic defense.)

IV. Equilibrium Predation and Exchange

For rational expectations equilibrium, the entry decisions must be consistent with the information actually revealed in the trading stage of the model.

First, the equilibrium with exchange must satisfy the entry condition into banditry. Domestic entry into banditry requires that

(4.1) $v^B = v^G > v^A$

where v^G is given by (3.5). For the foreign entry into banditry,

(4.2) $v^{*B} = v^{*G} \geq v^{*A}$,

where v^{*G} is given by (3.6). For given π,p,M, (4.1)-(4.2) determines the level of Banditry resources, L^B. Only by chance will both equalities hold, implying entry into Banditry by both countries. Generally, all the Bandits will be supplied by the poorer country.[12]

Second, the supplies of labor to defense and to production must be consistent with equilibrium. The aggregate supply of home labor N is split into N^B (the number of bandits supplied) on the one hand and the supply of labor to production and defense $(N-N^B)$ on the other hand. The technology of defense gives the defense requirement relative to specialized production as:

(4.3) $N^D = \bar{l}(N - N^B)$.

[12] The case where utilities are equal between the two countries is consistent with entry to banditry in both. This is necessarily a knife edge type of equilibrium so we do not bother to analyze it.

For the foreign economy similarly,[13]

(4.4) $N^{*D} = \bar{l}(N^* - N^{*B})$.

Productive labor is thus $L^G = (1-\bar{l})(N - N^B)$ and $L^{*G} = (1-\bar{l})(N^* - N^{*B})$ for the domestic and foreign economies respectively. In rational expectations equilibrium, there will be no supply of defense (and hence autarky results) if, with trade, utility for either type of agent lies below the autarky level of utility enjoyed by that agent. The world supply of defensive and offensive labor is the sum of the supplies of the two countries,

$$\text{(4.5)} \quad \begin{aligned} L^D &= N^D + N^{*D} = \bar{l}(N + N^* - L^B) \\ L^B &= N^B + N^{*B}. \end{aligned}$$

We may now characterize a rational expectations trade equilibrium making use of previous steps. The aggregate trade volume M_1 is obtained using the knowledge that good 1 is the home country import, the special reduced form import demand function at the aggregate level (see Appendix), and the supply of productive labor of the home economy:

$$\text{(4.6)} \quad M_1 = \frac{\gamma f(p,\pi,\alpha)}{pf(p,\pi,\alpha)+\alpha}(1-\bar{l})(N - N^B)/a_2.$$

Based on the previous considerations concerning the supply of defense and inverting (3.8) and using (4.6), if a trading equilibrium exists then:

$$\text{(4.7)} \quad \begin{aligned} L^B &= \frac{1}{v^{*G}}\phi(p,\pi,\alpha,N,\beta,\gamma) \text{ for } v^G > v^{*G}, \\ L^B &= \frac{\phi / v^G}{1+\phi / Nv^G} \text{ for } v^G \le v^{*G}, \text{ where} \\ \phi(p,\pi,\alpha,N,\beta,\gamma) &\equiv \left(\frac{\gamma}{1-\gamma}p\right)^{1-\gamma}(1-\pi)(1-\beta)\frac{f(1-\bar{l})}{pf+\alpha}\frac{N}{a_2}. \end{aligned}$$

Here, $N^B=0$ if $v^G>v^{*G}$ and $N^B=L^B$ if $v^G<v^{*G}$. Equation system (4.7) and the slackness conditions for banditry entry determine defensive and offensive labor L^B, L^D.

Plugging the offensive and defensive labor quantities into the probability of successful shipment function produces a value of π:

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[13] We now impose the same technology of defense in each country for simplicity.

(4.8) $$\pi = \frac{1}{1+\theta L^{B} / \bar{l}(N+N^{*}-L^{B})}.$$

The rational expectations interior equilibrium is the value of p, L^{B} which satisfies (4.7) and the exchange equilibrium condition (2.3) when (4.8) is substituted for π in all expressions.

Interior equilibrium need not exist. Unfortunately, analytic methods are not able to reveal much about when it does. We therefore describe a few special cases and then turn to simulation.

A. Autarky

Suppose that β is large enough that $L^{D} = \bar{l}(N + N^{*})$ is sufficient to completely deter entry into banditry. Then π is equal to 1. However, it is possible that a large enough country would still not gain from trade because the equilibrium price will lie too close to the autarky price. Since some fixed cost must be absorbed to trade at all, the agents in the large country, foreseeing a loss, would not commit the necessary defense resources, and hence there is no trading equilibrium --- autarky is the only solution.

By extension, we may expect a range of values of large β for which entry into banditry is small, and π is large although less than one, but the large country still refuses to enter trade. Specifically, suppose that at π=1, the equilibrium price is equal to p_{max}. For secure trade, then, there is a trading equilibrium with all gains going to the foreign economy. However, with entry into banditry, π begins to fall. Since bandits are all drawn from the foreign labor force, the equilibrium p for any given π must rise (by downward sloping world excess demand). This implies that p rises incipiently and π falls incipiently, both of which mean that utility for the home country incipiently falls below its autarky value, and the home country refuses to enter trade. It might be possible that p would fall sufficiently due to the decrease in π to restore a trading equilibrium. However, note that both import trade volumes respond positively to increases in π at constant p, by (7.8). Unless the home response is larger than the foreign response by a sufficient margin, p cannot fall enough to allow for gains from trade for the home economy.[14] There always exist parameter values where this cannot be guaranteed and for which autarky will be the only equilibrium.

Finally, even if β is large enough to deter entry into banditry and both countries would gain enough through trade to pay the fixed cost of trading, autarky may still prevail because of a coordination failure --- the export market exists only if other agents commit to their fixed cost of trading. This is similar to the coordination failures of Murphy, Shleifer, and Vishny (1989). Unraveling this story requires a more detailed account of the nature of trade than we have previously set out. The coordination issue could be avoided if the "trading agent" can pay the fixed cost, travel to a passive agent and offer exchange at the other agent's autarky price, with all risks and all gains to be absorbed by the trading agent. This suffices to get trade going and in equilibrium all agents will be "trading agents" sharing the gains. If, however, the initial exchange requires the fixed cost commitment from both agents, there is a coordination issue. Such coordination failure equilibria need not be stable.

B. Secure Equilibria

Secure equilibria result when β is large enough that $L^D = \bar{l}(N + N^*)$ is sufficient to deter entry into banditry. There must simultaneously be a Ricardian complete specialization solution with mutual gains from trade sufficient to pay for the fixed cost of defense.

More interestingly, there is always a secure equilibrium in the model which is a self-fulfilling prophecy. If no bandits enter, then no shipments are captured and the expectation of a zero success rate is confirmed. If trade occurs, it will in this case always be perfectly secure. (Autarky may still be the only equilibrium if fixed costs of trade are too high.) Here, there is a coordination failure on the side of the bandits.

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[14] The ranking of these derivatives can be set in either direction with combinations of relative country size (N/N*) and relative country efficiency $(1-\bar{l})a_2 / (1-\bar{l})a_1^*$.

C. *Trade and Predation*

The most interesting and complex class of equilibria are the interior solutions -- allocations to both defense and predation are made and there is specialization and exchange. These give rise to a rich set of comparative statics which cannot be derived as special cases of previously known results. Unfortunately, the complexity of the model yields the comparative statics only from simulations. The simulations also serve to flesh out the discussion of existence of the three classes of equilibria.

V. Simulated Equilibria and Comparative Statics

We have argued loosely about the effects of parameters such as β on the existence of one or another class of equilibrium and on the comparative statics of the system. Simulation of the model provides a description of both which is unavailable analytically.

A. *Existence and Uniqueness of Autarkic, Secure and Interior Equilibria*

We began our simulation analysis uncertain about whether all three types of equilibrium could be found with the Cobb-Douglas model. We suspected that it would be difficult to find secure equilibria because entry into banditry is continuous, and at low levels of predation the volume of trade per bandit is high. Reversing the reasoning, we suspected that near autarky, the low volume of trade per bandit might so deter banditry that autarky would not emerge. The simulation analysis shows that it is quite easy to find both secure equilibria and autarky equilibrium. If banditry is made uneconomical enough through high values of β, it is always possible to force a secure equilibrium. If at the same time the fixed cost of entry is forced low enough, this equilibrium will dominate autarky. Conversely:

> *Proposition 2*
> *For low enough β in combination with high enough fixed cost, autarky is the only equilibrium.*

The fixed cost of defense as a proportion of the labor force need not be very large (well under 10%) to force autarky. In contrast, the interior equilibrium is rather fragile with respect to variation of key parameters acting on predation: the fixed cost of protecting trade, the loss ratio β and the relative effectiveness of predation θ. Nevertheless, there are broad ranges of parameter values for which interior equilibria exist.

One or another sort of equilibrium always exists. Conditional on equilibrium being either autarkic or perfectly secure, the equilibrium exists and is unique, by standard methods. For parameter values such that interior equilibrium is possible, if it does not exist then an equilibrium is to be found at one or the other of the limiting cases.

Uniqueness of equilibrium conditional on being in the interior seems very difficult to prove analytically, because the reduced form equilibrium conditions are so very highly nonlinear and complicated. Nevertheless, all interior equilibria we have found appear to be unique because grid searches with varying starting values failed to turn up any other equilibria. Global uniqueness is of course ruled out because of the self-fulfilling prophecy which can always deliver a secure equilibrium when starting with banditry levels sufficiently low.

B. Simulated Comparative Statics

A key question is the effect of changes in the level of security on the terms of trade and on the welfare of the two groups of producers and of the predators. This section investigates the effect of exogenous changes in two key parameters on the terms of trade and on welfare. We vary the proportion of predatory gains which are lost or spoiled, β, and the proportion of the nonpredatory labor force which is (required to be) employed in defense, $\bar{l}$. The simulations reveal cases of immiserizing security, in which one country would prefer not to have improvements in security. This "perverse" case has predator welfare behaving "normally": increases in security will decrease predator welfare. Conversely, the "normal" range for the productive agents in the poorer country has predator welfare behaving perversely: improvements in security will raise the welfare of predators. The intuition is that the supply response in predation is more powerful than the impact of changes in security for given supply of predators.

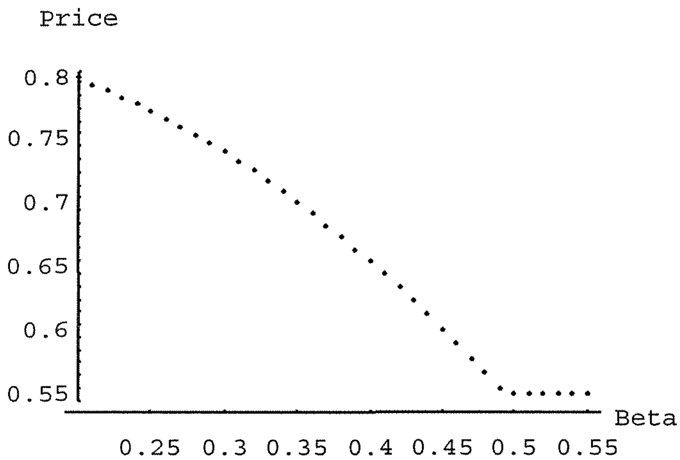
The probability of successful shipment is an endogenous variable. It turns out that π, the probability of successful shipment, is always monotonically related to β and $\bar{l}$. All our simulations trace the effect of variation in β and $\bar{l}$, for various values of the other parameters.

Simulations of interior solutions show that a fall in security improves the terms of trade of the larger "country" for most parameter values. This result is not general, but only a presumption. The simulations below all have this property, however.

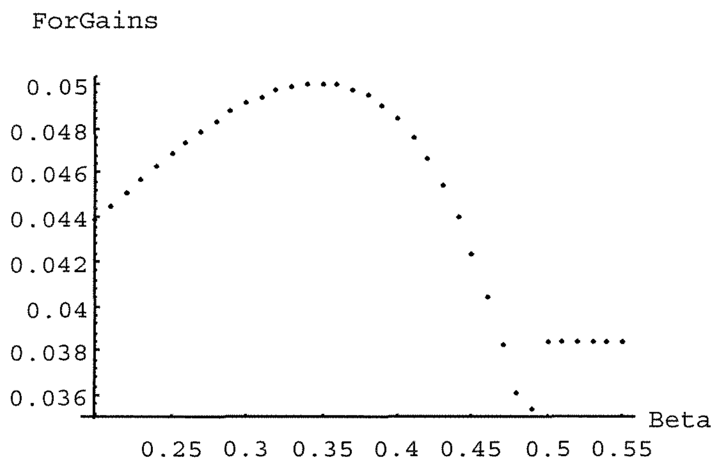
The possibility of immiserizing security arises as follows. The poorer country supplies all the bandits (since migration to the richer country is assumed impossible, but banditry is a free entry career), and thus an increase in predation will raise incipiently real incomes in the poorer country by reducing the size of its productive sector, consequently improving its terms of trade. When the poorer country is also the larger country, a decrease in security improves its terms of trade for two reasons, the "emigration" effect and the direct effect discussed in Section II. The terms of trade effect for a large poor country can dominate the negative impact on welfare of reduced trade volumes, at least when initial trade volumes are large. This phenomenon is illustrated below in simulations matching a large poor country with a small rich country.

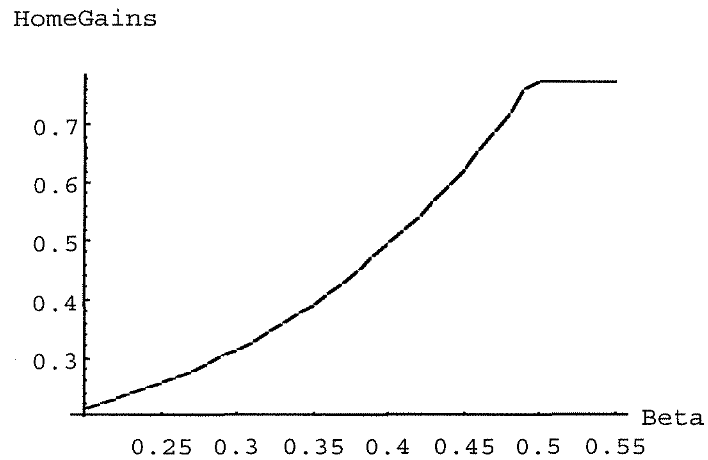
The simulations are based on the Cobb-Douglas case outlined in the Appendix, with α=2 and α^*=2, so the home country has a comparative advantage in good 2. With γ=.45, N=1000 and N*=1500, the foreign country is both larger and poorer; the home country will get more of the gains from trade both because it is smaller and because its import is less in demand. This intuition is based on reasoning from the secure equilibrium price (2.4). The values of θ and $\bar{l}$ must be set within rather narrow bounds to reach an interior equilibrium. Given these, the diagrams show the effect of varying β. In the first panel of Figure 3, the home terms of trade, the relative price of good 1 (the export of the large poor country), deteriorate monotonically as β rises (and the simulations confirm that security thereby improves). The flat region reflects reaching a secure equilibrium. The second panel shows the foreign country's gains from trade as a percentage of autarky utility. Increases in β decrease banditry and raise security (π) endogenously, lower p (the price of the foreign country's export), and yet still raise welfare on balance in both countries for the lower range of β, when trade is highly insecure and hence the volume is low. However, at higher ranges of β, implying higher π and hence larger trade volumes, the terms of trade effect and the migration effect dominate the volume effect of greater security and the large poor country is hurt by improvements in security.[15] In contrast, the smaller, richer "home" country gains from improvements in security throughout, as seen in the third panel.

Figure 3. Immiserizing Security Improvement



[15] The apparent discontinuity in the behavior of foreign gains as a function of β in the 0.45-0.5 interval is a consequence of the relative coarseness of the grid used in this diagram. A finer grid reveals a smooth convex function.)





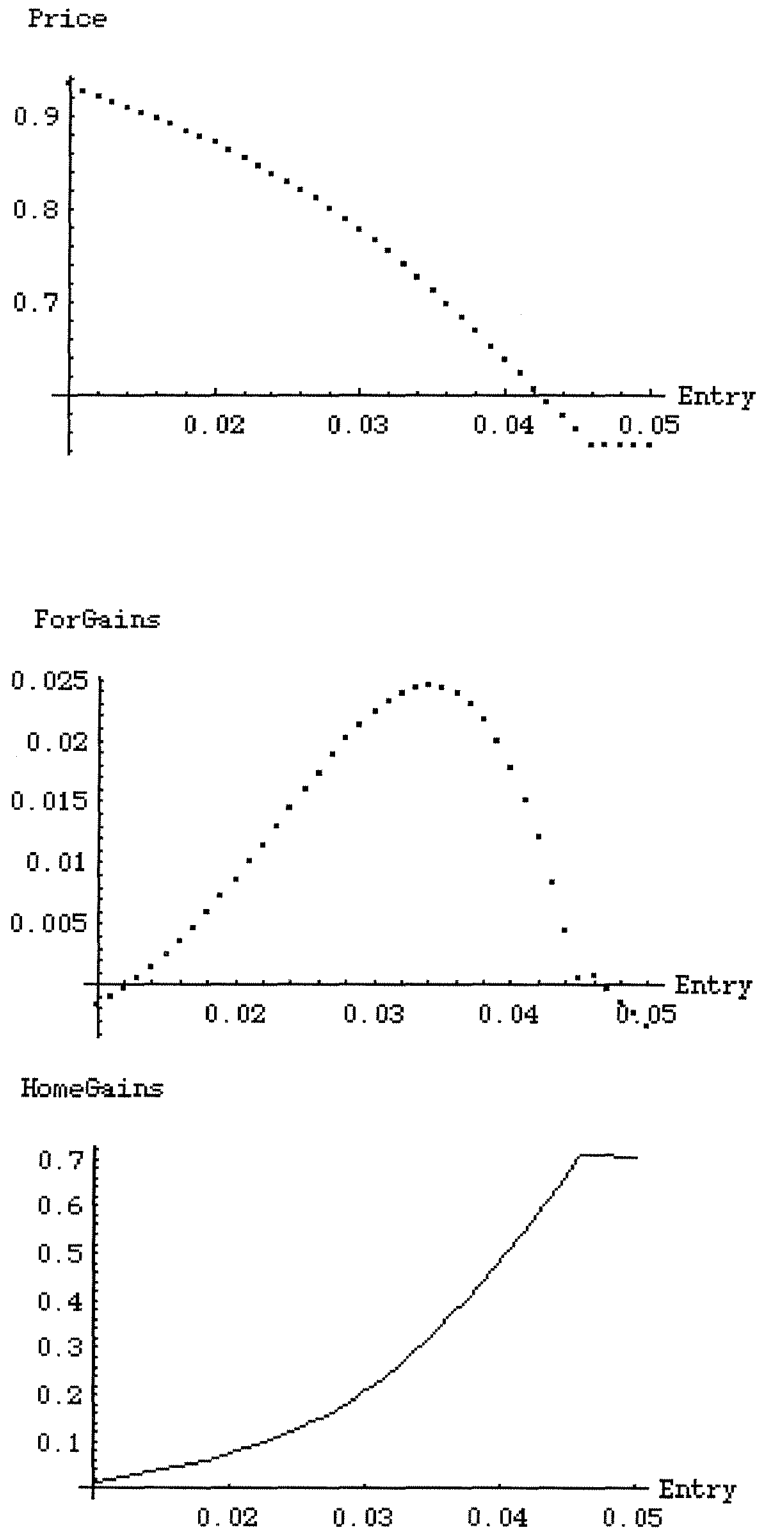
It is worth emphasizing that immiserization is here not connected with inferiority or special conditions on backward bending export supply; it comes through the direct effect of security on the terms of trade and the indirect effect through changing relative country size through entry or exit from banditry.

In the second set of simulations, we examine the effect of a rise in $\bar{l}$, the minimum defense level, on welfare in the two countries. This can also be regarded as the payoff to coordinated levels of defense effort, assuming costless coordination.[16] A common value of $\bar{l}$ is altered in both countries simultaneously. The defense effort is labeled 'Entry' in Figure 4 below, standing for the Entry cost of the specialized production and exchange activity. It is expressed as a proportion of the labor force not employed in banditry. The terms of trade of the

[16] In practice, governments and various other institutions organize collective action for defense, in a costly manner to be examined in future work. Defense is a classic public good. In this case the free rider problem is international, presenting some extra complications of organization.

large poor country monotonically deteriorate as defense effort rises (and security improves as a result). The first panel of Figure 4 shows this. As before, the flat region shows secure equilibrium. The next two panels depict welfare effects of changes in defense effort. For very low levels of defense (hence trade volume), both nations gain from increased defensive effort. At higher trade volume the terms of trade effect dominates and the large poor country prefers less defense. This happens through the immiserizing security mechanism explained above.

Figure 4. Coordinated Defense and Welfare



Notice that the foreign country loses from trade at both low and high levels of defense, so for these ranges of the security requirement, autarky will be the actual equilibrium.

Summarizing the results of the two cases, we have

Proposition 3

There exist parameter values for which improvements in security via greater defense or greater punishment of bandits are immiserizing for large poor countries.

Taken together, the results of Figure 4 suggest a potential for conflict due to divergent interests in greater security. It is natural, based on the foregoing, to investigate noncooperative approaches to the provision of international security. We plan to do so in our sequel paper. It is easy to imagine equilibria in which one (small, rich) partner gains and the other loses from trade; yet the loser's welfare is locally increasing in the provision of security. Conversely, it is also possible that the small rich country may pay too much for security and thereby lose from trade.

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APPENDIX: AGENTS' DECISIONS IN THE COBB-DOUGLAS CASE

A closed form solution for production and trade obtains if we assume that utility is a Cobb-Douglas function of the consumption bundle:

$$u = x_1^{\gamma} x_2^{1-\gamma}.$$

Here, x denotes consumption. With some judicious substitution, we obtain a closed form solution for the quantities in four steps.

First, we obtain a solution for the import penetration ratio. The combination of the efficiency conditions (1.4) for imports and (1.5) for output implies

$$\frac{\pi u_1^G}{(1-\pi)u_1^B} = \frac{p}{a_1/a_2 - p}.$$

For the Cobb-Douglas case this implies

$$\frac{\pi\left(x_2^G/x_1^G\right)^{1-\gamma}}{(1-\pi)\left(x_2^B/x_1^B\right)^{1-\gamma}} = \frac{\pi}{1-\pi}\left(x_1^B/x_1^G\right)^{1-\gamma} = \frac{p}{a_1/a_2 - p}.$$

Here, we have used the fact that $x_2 = y_2 + m_2$ in each state. Now note that

$$x_1^B / x_1^G = y_1/(y_1 + m_1).$$

Solving this expression for the import penetration ratio m_1/y_1 we obtain

$$\text{(7.1)} \quad \frac{m_1}{y_1} = \left[\frac{(1-\pi)p}{\pi(a_1/a_2 - p)}\right]^{-1/(1-\gamma)} - 1 \equiv f(p,\pi,\alpha),$$

where $\alpha = a_1/a_2$. The import penetration ratio is undefined at $\pi=1$, as is appropriate since in that case the classic Ricardian model obtains and production will either be equal to zero or indeterminate.

Second, we obtain the consumption ratio in the two states in terms of the import penetration ratio and the production ratio. We substitute into the ratio of consumption in the two states using $m_2 = -pm_1$ to solve in terms of m_1/y_1 and y_2/y_1.

$$\frac{x_1^B}{x_2^B} = \frac{y_1}{y_2 + m_2} = \frac{1}{y_2/y_1 - pm_1/y_1} \quad \text{and}$$

$$\frac{x_1^G}{x_2^G} = \frac{y_1 + m_1}{y_2 + m_2} = \frac{1 + m_1/y_1}{y_2/y_1 - pm_1/y_1}.$$

Third, we solve for the production ratio. Substituting the preceding expressions for the consumption ratios into the efficiency condition for imports and using $f(p)$ for the import penetration ratio m_1/y_1 we obtain:

$$\frac{\pi u_1^G}{\pi u_2^G + (1-\pi)u_2^B} = p = \frac{\pi\gamma\left[\frac{1+f(\cdot)}{y_2/y_1 - pf(\cdot)}\right]^{\gamma-1}}{\pi(1-\gamma)\left[\frac{1+f(\cdot)}{y_2/y_1 - pf(\cdot)}\right]^{\gamma} + (1-\pi)(1-\gamma)\left[\frac{1}{y_2/y_1 - pf(\cdot)}\right]^{\gamma}}.$$

This expression may be solved for y_2/y_1 to yield:

$$\frac{y_2}{y_1}=\frac{p\left\{\pi(1-\gamma)[1+f(\cdot)]^{\gamma}+(1-\pi)(1-\gamma)\right\}}{\pi\gamma[1+f(\cdot)]^{\gamma-1}}+pf(\cdot)$$
$$=p\frac{1-\gamma}{\gamma}+pf\frac{1}{\gamma}+p\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}(1+f)^{1-\gamma}$$
$$=p\frac{f}{\gamma}+\alpha\frac{1-\gamma}{\gamma}=\frac{pf+\alpha}{\gamma}-\alpha.$$

Finally, in combination with the full employment constraint $a_1y_1+a_2y_2=l^G$ the production ratio yields the closed form solution for y_1, m_1, y_2, m_2 as functions of the exogenous variables *p* and π and the technology parameter α.

(7.2) $$y_1=\frac{\gamma l^G/a_2}{pf(p,\pi,\alpha)+\alpha}.$$

Then in turn:

(7.3) $$m_1=\frac{\gamma f(p,\pi,\alpha)}{pf(p,\pi,\alpha)+\alpha}l^G/a_2$$

(7.4) $$y_2=l^G/a_2-\alpha\gamma\frac{l^G/a_2}{pf(p,\pi,\alpha)+\alpha}$$

(7.5) $$m_2=-\gamma pf(p,\pi,\alpha)\frac{l^G/a_2}{f(p,\pi,\alpha)+\alpha}.$$

Now we are in a position to consider the partial equilibrium comparative statics of system (7.2)-(7.5). It is immediate that a rise in 'effective size' l^G/a_2 will raise trade volume, as is intuitive. We anticipate that a rise in π will raise the level of trade m_1 and the degree of specialization measured by y_2. A rise in α should also raise trade as it increases the gap between the autarky price ratio and the price available through trade.

To develop these ideas it is necessary as a preliminary step to differentiate the import penetration ratio function $f(p,\pi,\alpha)$.

$$f(p,\pi,\alpha)\equiv\left[\frac{p(1-\pi)}{(\alpha-p)\pi}\right]^{-1/(1-\gamma)}-1,\text{ hence}$$

(7.6)
$$f_p=-\frac{1+f}{1-\gamma}\left[1/p+1/(\alpha-p)\right]<0$$
$$f_\pi=\frac{1+f}{1-\gamma}\left[1/\pi+1/(1-\pi)\right]>0$$
$$f_\alpha=\frac{1+f}{1-\gamma}\left[1/(\alpha-p)\right]>0.$$

Now we are in a position to analyze the properties of the per capita import demand function $m_1(p,\pi,a)$. Differentiating (7.3) with respect to p:

(7.7) $$m_{1_p}=m_1\left[\frac{f_p}{f}\left(1-\frac{pf}{pf+\alpha}\right)-\frac{1}{pf+\alpha}\right]<0.$$

The negative sign follows from noting that the square bracket term is negative for positive imports.

As for the response of m_1 to a rise in π, we can show that this is positive and approaches zero as complete specialization is approached:

(7.8) $$m_{1_\pi} = m_1\left(1-\frac{p}{pf+\alpha}\right)\frac{f_\pi}{f}>0.$$

Deriving the foreign economy's excess demand functions in the Cobb-Douglas case simply replicates the steps above, recognizing that the role of goods 1 and 2 is switched, and recognizing that the relative price of imports for the foreigner is *1/p* and that the marginal rate of transformation relevant to the steps above is that for the import good in terms of the export good , so $\alpha^* \equiv a_2^*/a_1^*$. All properties are the same, *mutatis mutandis*.