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COMPETITION OF TRADING FIRMS, OIL PRICES AND EXCHANGE RATE DETERMINATION

by

Georgi Trofimov

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Postadress	Gatuadress	Telefon	Bankgiro	Postgiro
Box 5501 114 85 Stockholm	Industrihuset Storgatan 19	08-783 80 00 Telefax 08-661 79 69	446-9995	19 15 92-5

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Errata (IUI Working Paper No. 356)

P. 9, line 2: "$\lambda > p_r /p$" should read "$\lambda > p /p_r$".

P. 15, line 21: "B = (pX* - πY*)" should read "B = (p*X* - πY*)".

P. 15, line 24: "B/λ" should read "B/λ*".

P. 16, line 2: "B/λ" should read "B/λ*".

P. 16, line 3: "B/λ" should read "B/λ*".

P. 16, line 13: "B = ΔM_1*" should read "B = ΔM_1*λ*".

P. 16, line 18: "B/λ" should read "B/λ*".

the optimal exchange rate is set at the natural level, if all trading firms maximize profits in the foreign currency and no one in their own currency.

According to an optimal trade policy rule, the purchasing price of the exported good is set at half of the world market reservation price evaluated through the optimal exchange rate. This result differs from the commonly adopted practical recommendations that the domestic oil price should converge to the world market level. A simple rule of thumb for the optimal monetary intervention says that it must be bounded from above by the trade balance surplus. Another rule of thumb governs the behavior of the central bank under perfect competition among trading firms. If the world market is favourable, i.e. the reservation price of oil exceeds some threshold level, the central bank should buy the foreign currency. Otherwise it must sell it to trading firms.

Finally, as is demonstrated, under the optimal trade and monetary policy it does not matter, whether the government trades oil in domestic or in foreign currency. The optimal exchange rate, as well as the welfare of consumers and profits of trading firms, are not influenced by a pattern of oil sales.

The paper is organized as follows. The next section presents the model of a trading firm and Section 3 deals with a Cournot oligopoly situation in the foreign trade markets. Section 4 examines the optimal trade and monetary policy of the government and the central bank. Section 5 concerns the natural exchange rate with some applications to simple rules of the optimal monetary policy. The question about the foreign currency trade in the domestic market for oil is discussed in Section 6. Proofs to statements are found in the Appendix.

2. The model of a trading firm.

Consider a two-product exchange between a country and the rest of the world. The domestic economy is endowed by the stock of a good traded in the world market. Suppose it is a raw material, for example, oil. A consumer good is imported from abroad and cannot be produced inside the country. The economy is large in the sense, that the export of oil influences world prices, although the foreign market for consumer good is not sensitive to the import.

There are trading firms of two types that intermediate all the foreign trade. A firm of the first type is maximizing profit in the domestic (weak) currency and a firm of the second type is maximizing profit in the foreign (hard) currency. In what follows the weak currency is also referred to as roubles and the hard currency as dollars. Although trade occurs only during one period, trading firms differ in the sequencing of their operations. A firm of the first type buys oil in the domestic market, sells it in the foreign market, and then imports consumer goods. Firms of the second type begin the trade period purchasing consumer goods and finish it selling oil in the foreign market. Both types intermediate their trading operations by purchases in the (domestic) currency market. For the time being it is assumed, that oil is sold to both types for roubles.

In what follows the notation is used: p, p' - the purchasing and the sale price of oil; π, π' - the sale and the purchasing price of a consumer good (commodity); x_i - the volume of oil exported by a firm i; y_i - the quantity of the commodity imported by the same firm; λ - the nominal exchange rate of dollars into roubles (a price of the dollar in terms of roubles).

A trading firm of the first type solves the problem:

$$\pi y_i - px_i + \lambda m_i \to \max_{x_i, y_i, m_i} \qquad (1)$$

$$\text{subject to} \quad \pi' y_i + m_i \leq p' x_i, \qquad (2)$$

$$y_i \geq 0\,, \qquad (3)$$

where m_i is the dollar position of the firm. It is supplied to the currency exchange, if $m_i \geq 0$ and demanded, if $m_i \leq 0$.

According to (1)-(3) the first-type firm maximizes gross profits from trade in the domestic market (the difference between commodity sales and oil purchases), plus the dollar cash residual exchanged into roubles (1). The budget constraint (2) relates to operations in the foreign markets: purchasing of the consumer good and the dollar cash residual cannot be in excess of oil sales. The constraint (3) prevents the export of consumer goods.

A firm of the second type maximizes dollar profits in a symmetric way:

$$p' x_j - \pi' y_j + \mu_j/\lambda \to \max_{x_j, y_j, \mu_j} \qquad (4)$$

$$px_j + \mu_j \leq \pi y_j, \qquad (5)$$

$$y_j \geq 0, \qquad (6)$$

where μ_j is a firm j demand for dollars (roubles), if $\mu_j \geq 0$ ($\mu_j \leq 0$). Its net trade profit (4) is the sum of gains from the foreign markets for goods and a rouble cash rest, exhanged into dollars. The budget constraint (5) relates to trading operations in the domestic markets: purchases of oil and the rouble cash residual must be met by sales of commodities.

In what follows we deal with competition among trading firms in the world oil market, in the domestic market for consumer goods and in the domestic currency market. The purchasing market price of the commodity is given exogenously and is not influenced by the trading firms. The purchasing price of oil is supposed to be under the control of government in the interest of the optimal foreign trade policy.

3. Competition of trading firms.

Consider a Cournot type oligopoly of n trading firms. Demand functions for oil and commodity are linear in corresponding prices. The market-clearing world price for oil is:

$$p' = p_r - a_1 X, \qquad (7)$$

where p_r is the *reservation* price of the world oil market and a_1 the parameter characterizing the elasticity of demand with respect to the amount exported from the country, $X = \Sigma_{i=1}^{n} x_i$. The domestic market-clearing price for the consumer good is a linear function of total import $Y = \Sigma_{i=1}^{n} y_i$:

$$\pi = \pi_r - a_2 Y, \qquad (8)$$

where π_r, a_2 - parameters similar to p_r, a_1. Parameter π_r is referred to as the reservation price of the consumer good.

Given the exchange rate λ and the purchasing oil price *p*, trading firms solve the problems (1)-(3) and (4)-(6) under the conditions (7)-(8), being perfectly informed about the markets and competing in the Cournot sense.

Suppose the exchange rate belongs to the interval that ensures the profitability of the foreign trade:

Assumption 1.

$$\lambda > p_r/p, \qquad (i)$$

$$\lambda < \pi_r/\pi'. \qquad (ii)$$

Proposition 1. Equilibrium export and import flows are linear in prices and identical for both types of trading firms:

$$x_i^* = (p_r - p/\lambda)/a_1(n+1) \qquad (9)$$

$$y_i^* = (\pi_r - \pi'\lambda)/a_2(n+1) \qquad (10)$$

$$i = 1,...,n.$$

Export and import flows do not differ among firms because of the perfect information about prices and the absence of price discrimination in the domestic market for oil. In what follows we omit the subscript of a firm.

The profitability assumption can be formulated in equivalently, once the trading firms act as Cournot oligopolists. Define the dollar rate for oil as a price ratio $e_1 = p/p'$, and the dollar rate for the consumer good in the similar way as $e_2 = \pi/\pi'$. The following proposition demonstrates a link between these rates and the nominnal exchange rate of the dollar.

Proposition 2. The nominal exchange rate is higher than the dollar rate for oil and lower than the dollar rate for the consumer good:

$$e_1 < \lambda < e_2.$$

Proof follows immideatly from the calculation of sale-purchasing price indeces relying on (7)-(10). The dollar rate for oil is the harmonic average of the nominal exchange rate and the price index p_r/p:

$$e_1 = (\lambda^{-1}n/(n+1) + (p_r/p)/(n+1))^{-1}. \qquad (11)$$

This together with (i) implies, that $e_1 < \lambda$. The dollar rate of the consumer good is the arithmetical average of the exchange rate and the price index π_r/π':

$$e_2 = \lambda n/(n+1) + (\pi_r/\pi')/(n+1). \qquad (12)$$

Therefore, and because of (ii) $e_2 > \lambda$.

In the case of our model the deviation of "real" dollar rates e_1 and e_2 from the nominal exchange rate λ is explained by the imperfectness of competition among trading firms and the profitability conditions (i)-(ii). If the number of trading firms increases, both dollar rates e_1 and e_2 converge to the nominal exchange rate, as it is seen from (11)-(12).

4. Foreign trade and monetary policy.

The government and a monetary authority (the central bank) can manage trade and monetary policies by implementing direct or indirect control of export and through monetary intervention. Suppose these activities are perfectly coordinated and pursued to maximise total welfare. The latter incorporates the welfare of households and the central bank preferences in the monetary sphere.

The stock of oil in the country is the domestic asset of the government. It conducts the trade policy by setting a price for oil sold to trading firms and does not influence imports directly. The revenue from the oil sales is supporting the monetary policy of the central bank. This policy is implemented in the currency market and constrained by the two conditions: a) the central bank balance and b) the currency market clearance. Influences of monetary policies in other countries and speculations against the rouble are disregarded.

Both foreign trade and monetary policies deal out an efficient allocation of currency between trading firms and the state.

A representative consumer is the utility-surplus maximizer, choosing a quantity of the imported good c as:

$$u(c) - \pi c \to \max \tag{13}$$

where the utility function $u(c) = c(\pi_r - a_2 c/2)$ is defined for $c \leq \pi_r/a_2$. For the sake of simplicity it is assumed that households do not use the hard currency.

Preferences of the central bank in the monetary sphere are indicated by an indirect utility function, depending on dollar and rouble funds denoted as M_1 and M_2. It is given in the additive form:

$$V(M_1, M_2) = v(M_1) + M_2, \tag{14}$$

where $v(M_1)$ is the utility of dollars, measured in roubles. It is a monotonously increasing, differentiable and concave function defined for $M_1 \geq 0$, satisfying Inada conditions:

Assumption 2. $v'(0) = \infty$, $v'(\infty) = 0$.

According to Assumption 2 the hard currency is unboundedly valuable for the monetary authority if there is a shortage, while it depriciates if abundance occurs. Roubles and dollars are in a sense complementary currencies for the central bank, because the marginal rate of substitution for $V(M_1, M_2)$ changes from zero to infinity on all isoquants.

The central bank balance is expressed as:

$$M_2 - M_{02} = pX^* - \lambda(M_1 - M_{01}), \tag{15}$$

where M_{01} is the initial stock of hard currency possessed by the central bank (the initial *stabilization fund* for rouble), M_{02} is the initial stock of roubles, $X^* = nx^*$ is the total export in the Cournot-Nash equilibrium. Both initial funds are given exogeneously. Equation (15) says that monetary flows $M_2 - M_{02}$ and $\lambda(M_1 - M_{01})$ are met by the sales of oil pX^*.

A trading firm of the first type supplies $m^* = p'x^* - \pi'y^*$ dollars in the currency exchange. A trading firm of the second type demands $\mu^*/\lambda = (\pi y^* - px^*)/\lambda$ dollars. The currency market is hence in equilibrium if:

$$n_1 m^* - n_2 \mu^*/\lambda = M_1 - M_{01}, \tag{16}$$

where n_1 and n_2 is the number of firms belonging to the first and the second type, i.e. $n_1 + n_2 = n$. It means, that the central bank can enhance the stabilization fund only by purchasing dollars in the currency market, where it confronts the trading firms.

The government and the monetary authority are choosing the control variables M_1, M_2, λ, p to Pareto-maximize utilities u(c) and $V(M_1, M_2)$ under the constraints (15)-(16) and the Cournot-Nash equilibrium in the foreign-trade markets for goods. Since both utilities are measured in roubles, this is equivalent to the scalar welfare-maximization problem:

$$u(Y^*) + V(M_1, M_2) \rightarrow \max_{M_1, M_2, \lambda, p \in R^+} \tag{17}$$

subject to the conditions (15)-(16),

where $Y^* = ny^*$ is the total import in the Cournot-Nash equilibrium.

Assumption 2 rules out corner solutions. Insertion of (15), (16) into (17) brings the latter into an unconstrained problem with control variables p and λ. Economically it means that the government and the central bank manage only the oil price and the exchange rate (through the monetary intervention). Suppose the latter is optimal $\lambda = \lambda^*$. Then the optimal oil price is calculated in line with a simple rule.

Proposition 3. The optimal purchasing price of the exported oil is equal to: $p^* = p_r\lambda^*/2$.

According to this rule the government maximizes total sales of oil pX^* (this is so because the total welfare does not depend on the stock of the real asset which is supposed to be unlimited). The optimal oil price is the half of the world market reservation price denominated in the domestic currency through the optimal exchange rate. For example, if the reservation oil price is 150 $/ton and the optimal dollar rate is 80 roubles/$, the optimal oil price should be set at the level 6000 roubles/ton. This is approximately 2-2,5 times higher than the equilibrium oil price in the domestic market of Russia in the first quarter of 1992. The difference between these two prices can be interpreted as a licence price.

Notice that Proposition 3 ensures the fulfilment of the profitability Assumption 1(i) for the optimal purchasing oil price and an optimal exchange rate. The following Theorem deals with the optimal monetary policy of the central bank.

Theorem 1. There exists a unique optimal exchange rate. It provides the increase of the stabilization fund $\Delta M_1^* = M_1^* - M_{01} =$

$$= (p_r^2/4a_1 - \pi\pi'/a_2)n/(n+1).$$

The optimal monetary intervention can be viewed as the derivative of the foreign trade turnover with respect to the exchange rate evaluated in domestic prices as: $\Delta M_1^* = p(X^*)_\lambda' + \pi(Y^*)_\lambda'$. One can see it from (A7) in the proof of Theorem 1. Note that the optimal intervention depends only on exogeneous parameters (because $\pi = (\pi' n + \pi_r)/(n+1)$) and can be estimated empirically.

5. The "natural" exchange rate.

Up to this point we considered the competition of a finite number of trading firms. It is now reasonable to extend the analysis to the case of the perfect competition. When the total number of trading firms goes to infinity, an optimal exchange rate tends to a limit value λ_n, which is called a "natural" exchange rate (in the sense that there are no barriers to entry into the sphere of foreign trade).

Proposition 4. The natural exchange rate is the half of the reservation-purchasing price index for the imported good: $\lambda_n = \pi_r/2\pi'$.

Note that the natural rate λ_n does not depend on both the reservation oil price and the limit proportion of types of firms. It is determined by the preferences of domestic consumers on the one hand, and by the competitiveness of the foreign market for consumer goods, on the other hand. The higher price domestic consumers are ready to pay for the imported good and the cheaper it is abroad, the higher is the natural rate of dollar.

Under the optimal foreign trade and monetary policy, and in the case of the perfect competition of trading firms, the "law of one price" is fulfilled in a modified form: $\lambda_n = 2p^*/p_r = \pi_r/2\pi'$. In other words, the exchange rate of the dollar is half of the reservation-purchasing price index for the commodity and the doubled reciprocal of the same index for oil.

As it turns out, an optimal exchange rate is bounded from above by the natural exchange rate. Under imperfect competition of trading firms the optimal exchange rate is determined (irrespective of other parameters) by the number of firms belonging to each type n_1 and n_2. It is maximal, when none of the trading firms belongs to the first type.

Theorem 2. Let the competition of trading firms be imperfect ($n < \infty$). Then for $n_1 = 0$ the optimal exchange rate is set at the natural level: $\lambda^* = \lambda_n$, while for $n_1 > 0$ it is lower: $\lambda^* < \lambda_n$.

Thus, if there is a finite number of trading firms each earning the foreign currency, the optimal exchange rate will be the same as under perfect competition. One can show that it is increasing as the number of the second type of firms grows. In other words a higher number of dollar profit-making trading firms implies a lower rouble market value.

Theorem 2 implies that for any optimal exchange rate the Assumption 1(ii) holds, since $\lambda^* \leq \pi_r /2\pi'$. The suggestive economic interpretation is that optimal trade and monetary policy significantly decreases the upper bound of the dollar rate. The import profitability requirement (ii) bounds the exchange rate by the reservation-purchasing price index π_r /π', while the optimal policy determines the upper limit for it as half of this ratio.

Theorem 2 provides some practical implications for optimal dollar interventions. If the central bank is not perfectly informed about some exogenous parameters, say demand coefficients a_1 and a_2, it cannot calculate ΔM_1^* precisely, but it can estimate bounds for the monetary intervention using the trade balance surplus: $B = (pX^* - \pi Y^*)$.

Corollary. Let the competition of trading firms be imperfect. Then the optimal monetary intervention is bounded from above by the trade balance surplus: $-\Delta M_1^* \leq -B/\lambda$. (For $n_1 = 0$ $\Delta M_1^* = B/\lambda$).

The Corollary says, that in the case of the negative trade balance surplus the central bank cannot sell more than $-B/\lambda$ dollars, while in the opposite case it must buy at least B/λ dollars.

Considering how perfectly competitive trading firms operate in the currency market, it turns out that both types of firms are simultaneously purchasing or selling dollars, subject to a level of the reservation oil price.

Proposition 5. Let $n \to \infty$. Both types are selling dollars, if and only if

$$p_r \geq p_{r0} = \pi_r(a_1/a_2\lambda_n)^{1/2}.$$

Note, that in the limit case of perfect competition the initial stabilization fund does not change ($\Delta M_1^* = 0$) if the reservation oil price is just at the threshold level $p_r = p_{r0}$. The Proposition 5 implies, that the trade balance surplus B is positive, if $p_r > p_{r0}$, and negative, if $p_r < p_{r0}$. In both cases it coincides with the monetary intervention $B = \Delta M_1^*$.

Proposition 5 demonstrates the influence of the world oil market on the distribution of "roles" in the domestic market for currency. If the former is favourable, i.e. p_r is higher than the threshold level p_{r0}, then the latter is satiated by dollars, since perfectly competitive trading firms of both types sell them. In this situation the central bank is buying dollars and adding B/λ to the initial stabilization fund M_{01}. The central bank becomes a pure seller of hard currency under the unfavourable world market for oil, i.e. if p_0 is low.

6. Should the government sell oil for dollars?

The answer to this question is not obvious. Economic intuition suggests alternative ways of reasoning. On the one hand, the trade of oil in the domestic market for the foreign currency could be justified by fiscal purposes.

Possibly the government and the central bank could gain more hard currency by directly selling oil for dollars, rather than for roubles. On the other hand, artificial narrowing of the currency market can negatively influence the value of rouble. To support it the government should sell oil for roubles enforcing trading firms to buy them in the currency market. Although both views are reasonable, the formal analysis of the problem provides another result.

Consider the above model in a slightly modified form. Suppose the government sells oil to trading firms, setting the price p'' \$/ton. Both types of trading firms maximize net profits in a similar way to (1)-(3) and (4)-(6):

$$(p' - p'')\lambda x + (\pi - \pi'\lambda)y \rightarrow \max$$

for the first type and

$$(p' - p'')x + (\pi/\lambda - \pi')y \rightarrow \max$$

for the second (the subscript of a firm is omitted).

Let the profitability conditions similar to those in Assumption 1 hold with (i′) stating, that $p'' < p_r$, instead of (i). It means, that the purchasing dollar price of oil must be less than the reservation world market price. Equilibrium trade flows are the same as in the above model:

$$x^* = (p_r - p'')/a_1(n+1)$$

$$y^* = (\pi_r - \lambda\pi')/a_2(n+1).$$

However, the behaviour of different types of trading firms in the currency market does not contain the symmetry peculiar for that model and expressed by the budget constraints (2) and (5). Consider how a firm of the first type supplies dollars. First, it sets the trade margin $p'x^* - \pi'y^*$ which is the difference between oil sales and purchases of the consumer good in the foreign markets. Second, it exchanges roubles yielded in the domestic commodity market for $(\pi/\lambda)y^*$ dollars. Third, the firm sells a remainder of dollars $(\pi/\lambda)y^* - p''x^*$ once oil has been purchased. Hence, its net supply is $(p' - p'')x^* - \pi' y^*$.

A firm belonging to the second type behaves more simply. Since it is interested in hard currency profits, it has only to "get rid" from roubles received in the domestic market for consumer good and to buy $(\pi/\lambda)y^*$ dollars.

The central bank also somewhat changes its actions in the currency market. This is so, because the budget constraint (15) modifies to

$$M_2 - M_{02} = p''X^* - \lambda(M_1 - M_{01}) \qquad (18)$$

and the market demand of the monetary authority for dollars D is less than the expansion of the stabilization fund:

$$D = M_1 - M_{01} - p''X^*. \qquad (19)$$

The last condition means, that a part of dollars is bought for roubles, while another part is exchanged for oil. The central bank and the government solve the modified problem (17) with (15) substituted for (18) and the new condition (19).

Proposition 6. If the government sells oil for the hard currency, the central bank supplies $S = -D = -\pi(Y^*)_\lambda' = \pi\pi' n/a_2(n+1)$ *dollars to the monetary market.*

Thus in the modified model the central bank is always a pure seller of hard currency. The description of the currency market is summarized by an equilibrium equation:

$$n_1(p' - p'')x^* - n_1\pi' y^* + S = n_2(\pi/\lambda)y^* \qquad (20)$$

Now we are able to formulate the main result of this Section.

Theorem 3. If the government sells oil for the foreign currency, the optimal exchange rate will be the same as when the oil is traded for the home currency.

Formally the Theorem follows from the identity of the market-clearing conditions (20) and (16), that takes place under the optimal policy of economic authorities. It implies the optimal intervention prescribed by Proposition 6 and the oil price setting at the half of the reservation price level $p'' = p_r/2$, that is equivalent to the optimal pricing rule, obtained above (see Proposition 4 and the proof of Theorem 3 in the Appendix).

As a result welfare will not be influenced, if the government sells oil for the foreign currency and (of coarse) the central bank is precommitted to the optimal monetary policy. Trading firms of both types will neither loose, nor gain from this.

7. Concluding comments.

The paper examined the optimal trade and monetary policy in a market characterized by imperfect competition among trading firms. The highly-stylized model of foreign trade proposed here deals with a situation peculiar for a large incompetetive economy, endowed with a large stock of physical assets traded in world markets. The imperfections of competition among trading firms is explained by institutional and informational barriers that make entry into the sphere of the foreign trade difficult.

As is demonstrated, optimal trade and monetary policy can support the domestic currency. The optimal exchange rate (a price of the foreign currency) is bounded from above by a natural level which is twice as small as the upper bound imposed on exchange rates by the import profitability requirement. The optimal exchange rate is set at the natural level, if all trading firms are maximizing profits in the foreign currency or in the limit case of the perfect competition.

The optimal domestic price of the traded good (oil) is set by the government as half of the world market reservation price. Thus the government is maximizing the total amount of exports. This is a reasonable inference under the assumption that the stock of the real assets is large and its reduction is not counted in the welfare function. Otherwise the optimal domestic oil price must be higher than the sales-maximizing level.

As it turns out, the optimal exchange rate does not depend on currency which the oil is sold for to trading firms. As a consequence, the total welfare of the state and profits of trading firms do not change, if the government trades the real asset against the foreign currency instead of the domestic currency.

Appendix.

1. Proof of Proposition 1.

Budget constraints (2) and (5) turn into equities in equilibrium. Inserting $m_i = p'x_i - \pi' y_i$ into (1) and $\mu_j = \pi y_j - px_j$ into (4), we have a problem identical for both types:

$$(p' - p)x_i + (\pi - \pi')y_i \to \max \qquad \text{(A1)}$$

subject to market-clearing conditions (7),(8) and the constraints (3),(6). The first-order conditions immedeatly imply (9) and (10).

2. Proof of Proposition 3.

Replacing the budget constraint (15) into (17) we have a problem with three control variables: p, λ and M_1. Suppose $\lambda = \lambda^*$ is optimal. Let $z = z(p, \lambda^*) = n_1 m^* - n_2 \mu^*/\lambda^*$ stands for the net supply of dollars by trading firms under the optimal exchange rate. Taking into account the market-clearing condition (16) and inserting $z = \Delta M_1$ into (17) we obtain an unconstrained problem with two control variables: p and z. Differentiating (17) with respect to the latter variable yields the efficiency condition:

$$v'(M_{01}+z) = \lambda^* \tag{A2}$$

and the same with respect to p implies the first-order equation:

$$v' z_p' + (pX^*)_p' - \lambda^* z_p' = 0, \tag{A3}$$

which is equivalent due to (A2) to

$$(pX^*)_p' = 0 \text{ or } (p(p_r - p/\lambda^*))_p' = 0.$$

Thus the optimal oil price provides the maximization of sales and is equal to $p^* = p_r \lambda^*/2$.

3. Proof of Theorem 1.

One can show, that under the Nash-Cournot equilibrium the first-order condition

$$u'(Y^*) = \pi \tag{A4}$$

holds as identity.

Suppose the oil price is optimal $p = p^*$. Let $z_1 = z(p^*, \lambda)$ denotes the net supply of dollars by both types under the optimal oil price. Inserting (15) into (17) and taking into account (16) we have an unconstrained problem with two control variables z_1 and λ. The former variable gives the first-order condition similar to (A2):

$$v'(M_{01}+z_1) = \lambda, \tag{A5}$$

while differentiating (17) with respect to the latter yields:

$$u'(Y^*)(Y^*)_\lambda' + [v'(M_{01}+z_1) - \lambda](z_1)_\lambda' + p^*(X^*)_\lambda' - z_1 = 0 \tag{A6}$$

or because of (16), (A4), (A5)

$$z_1 = \pi(Y^*)_\lambda' + p(X^*)_\lambda' = \Delta M_1^*. \tag{A7}$$

Taking the derivatives $(X^*)_\lambda'$ and $(Y^*)_\lambda'$ we receive:

$$\Delta M_1^* = ((p^*/\lambda)^2/a_1 - \pi\pi'/a_2)n/(n+1)$$

or using Proposition 4

$$\Delta M_1^* = (p_r^2/4a_1 - \pi\pi'/a_2)n/(n+1). \tag{A8}$$

We have proved the second part of the Theorem. To prove the first, consider the equilibrium condition (A7) as an equation on λ:

$$z(\lambda, p^*) = \Delta M_1^*. \tag{A9}$$

Since

$$m^* = p'x^* - \pi'y^* = (p_r + np/\lambda)x^*/(n+1) - \pi'y^*, \tag{A10}$$

$$\mu^* = \pi y^* - px^* = (\pi_r + n\pi'\lambda)y^*/(n+1) - px^*, \tag{A11}$$

then one can see from (A8), that (A9) is a square equation:

$$z(\lambda, p^*) - \Delta M_1^* \equiv \alpha\lambda^2 + \beta\lambda - \gamma = 0, \tag{A12}$$

with coefficients

$$\alpha = (2n^2 + n_1)(\pi')^2/a_2,$$

$$\beta = n_1 p_r^2/4a_1 - (n^2 - 2n_2)\pi_r\pi'/a_2,$$

$$\gamma = n_2\pi_r^2/a_2.$$

Since $\alpha > 0$ and $\gamma > 0$, there exists a unique positive solution λ^* to (A9).

4. Proof of Proposition 4.

Dividing (A12) by n^2 and taking the limit with respect to $n \to \infty$ implies:

$$2(\pi')^2\lambda - \pi_r\pi' = 0,$$

that is $\lambda_n = \pi_r/2\pi'$.

5. Proof of Theorem 2.

The equilibrium equation (A12) can be transformed in a following way:

$$n^2(2\pi'\lambda-\pi_r)\pi'\lambda/a_2 + n_2(2\pi'\lambda-\pi_r)\pi_r/a_2 + n_1[(\pi')^2\lambda + p_r^2/4a_1]\lambda = 0. \quad \text{(A13)}$$

For $n_1 = 0$ (A13) is:

$$n^2(2\pi'\lambda - \pi_r)\pi'\lambda + n_2(2\pi'\lambda - \pi_r)\pi_r = 0, \quad \text{(A14)}$$

that gives $\lambda^* = \pi_r/2\pi' = \lambda_n$.

For $n_1 > 0$ the left-hand side of (A14) becomes negative and hence $\lambda^* < \lambda_n$.

6. Proof of Corollary

Under the optimal policy the trade balance surplace is:

$$B = (p_r/2)X^* - \pi Y^* = [(p_r^2/4)/a_1 - (\pi/\lambda^*)(\pi_r - \lambda^*\pi')/a_2]n/(n+1).$$

The optimal intervention is: $\Delta M_1^* = [(p_r^2/4)/a_1 - \pi\pi'/a_2]n/(n+1)$. Therefore $B = \Delta M_1^*$, if $\pi_r - \lambda^*\pi' = \lambda^*\pi'$ or $\lambda^* = \lambda_n$. This implies, that $\Delta M_1^* > B$ if and only if $\lambda^* < \lambda_n$. According to Theorem 5 $\Delta M_1^* = B$ if $n_1 = 0$ and $\Delta M_1^* > B$ if $n_1 > 0$.

7. Proof of Proposition 5.

If the total number of trading firms tends to infinity, then individual trade flows (9)-(10) as well as surpluses of cash (A10)-(A11) degenerate to zero. Therefore we are to show, that for $n \to \infty$ inequalities

$$\lim_{n1\to\infty} n_1 m^* \geq 0 \text{ and } \lim_{n2\to\infty} n_2 \mu^* \leq 0$$

are fulfilled, if and only if $p_r \geq p_{r0}$.

Inserting $p = p^*$ and $\lambda = \lambda_n$ into (A10) implies:

$$m^* = p'x^* - \pi' y^* = \{[(p_r + (p^*/\lambda_n))n/(n+1)]p_r/2a_1 -$$

$$\pi'(\pi_r - \lambda_n{}^*\pi')/a_2\}/(n+1) = [(n+2)p_r{}^2/4a_1(n+1) - \pi_r\pi'/2a_2]/(n+1).$$

Consequently $\lim_{n1\to\infty} n_1 m^* \geq 0$ if and only if $p_r \geq (2\pi_r\pi' a_1/a_2)^{1/2} = p_{r0}$.

Using (A11) it is shown in the same way that $\lim_{n2\to\infty} n_2\mu^* \leq 0$, if and only if $p_r \geq p_{r0}$.

8. Proof of Proposition 6. replicates the proofs of Proposition 3 and Theorem 1. Taking into account condition (18) one can show, that the demand of the central bank for dollars is:

$$D = \Delta M_1{}^* - p^*X^* = (p_r{}^2/4a_1 - \pi\pi'/a_2 - p_r{}^2/4a_1)n/(n+1) =$$

$$- \pi\pi' n/a_2(n+1). \qquad \text{(A15)}$$

9. Proof of Theorem 3.

According to: a) the market-clearing equation (20), b) the optimal intervention rule (A15) and c) the optimal oil pricing rule: $p'' = p_r/2$ an excess demand for dollars is:

$$(n_1\pi' + n_2(\pi/\lambda))y^* - n_1(p' - p'')x^* + D = [n_1\pi' + n_2(\pi_r/\lambda + n\pi')/(n+1)]\times$$

$$(\pi_r - \pi'\lambda)/a_2(n+1) - n_1(p_r + np'' - (n+1)p'')p_r/2a_1(n+1)^2 - \pi\pi' n/a_2(n+1) =$$

$[(n^2 - 2n_2)\pi_r\pi' - (2n^2 + n_1)(\pi')^2\lambda + n_2\pi_r^2/\lambda]/a_2(n+1)^2 - n_1p_r^2/4a_1(n+1)^2.$

Equating this to 0, we obtain (A12).

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