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Comparative Keynesian Dynamics

by

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COMPARATIVE KEYNESIAN DYNAMICS[1]

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In spite of its limitations, and if for no other reason than its pedagogical value, the Hicksian version of the Keynesian macro analysis is likely to retain its interest for a long time. That pedagogical value is based on the integration of monetary and commodity sectors in a single coherent framework that illuminates with graphical simplicity how empirical properties of supply and demand determine the employment effects of monetary and fiscal policy.[2]

It is my purpose here to show that intriguing new insights can still be derived from the standard fix-price model. I replace the IS curve with an IY curve representing the dependence of investment demand on aggregate income. This is done by using the usual LM curve, which portrays the temporary equilibrium dependence of interest on aggregate income, to eliminate the interest variable in the investment function. By this means we gain an enhanced appreciation for the nonlinear effect on aggregate demand induced by the demand for money and investment goods. Then, assuming a Robertsonian lag in disposable income, one derives a difference equation that facilitates a comparative dynamic analysis involving explicit quantity adjustments.

Using this model Day and Shafer derived conditions for which the Keynesian "disequilibrium"[3] is not only unstable but for which dynamic adjustments can be cyclic or even nonperiodic, involving unpredictable, more or less random, yet bounded fluctuations in GNP.[4] My first purpose here is to explain how these results obtain in somewhat less technical terms. My second purpose is to answer the question immediately raised by these novel findings: do they rest on bizarre or implausible properties of aggregate demand and supply for money and goods, ones that would contradict assumptions explicity or implicitly underlying standard macroeconomic discussions? The answer, which may be surprising, is "no!" The underlying money and investment demand curves can have classic "textbook" forms compatible with IS-LM curves that have quite conventional qualitative profiles.

Finally, I want to use the model to illustrate how policies that influence the money supply can lead to quite different dynamic patterns of adjustment that depend on the initial conditions. By shifting the economy from one regime to another monetary policy can drastically alter qualitative behavior of GNP as well as its equilibrium position, which may or may not be attained depending on which regime has been entered. In this Keynes-Hicks-Robertson world it is literally possible for monetary authorities to trigger the onset of - or to eliminate chaos! Although based on an entirely different theoretical argument the moral of the present story would therefore appear to be somewhat like that of Simons, Friedman and Lucas.

Certainly the specification of a Robertsonian lag is not new and the use of an IY curve is nothing more than a shift in point of view.[5] However, the latter enables us to illustrate the process of adjustment with the graphical phase diagram so convenient when nonlinearties are present. And it enables us to exploit techniques of dynamic analysis that reveal a whole new realm of complicated macroeonomic behavior in the standard model that possesses a pervasive qualitative feature of observed data, that of nonperiodic, unpredictable fluctuation.

Although the analysis is explicitly dynamic, it must inspire most of the objections associated with the simplifying assumptions listed above. Consequently, it cannot in its present state help settle important contemporary issues of monetary and fiscal policy. What it can do is alert students of the subject to the possibility that dramatic, qualitative changes in macro-behavior and the existence of essentially unpredictable fluctuations in GNP can be induced by intrinsic properties of economic structure without the gratuitous help of unexplained or unexplainable exogenous shocks. This possibility will have important implications indeed if subsequent research shows that it also occurs in models in which the strong simplifying assumptions have been relaxed and in which parameter values are obtained with appropriate data and econometric techniques. All of that, however, is a task for future endeavor.

I

In order to make the discussion self-contained we review the Keynes-Hicks model. Let m, g and l be indexes denoting money, goods and labor, respectively. The aggregate system of demand-supply relationships in which prices and wages are fixed while income (GNP), Y, and the interest rate, r, are variable is

(1a) $D^m (r,Y) = S^m (r,Y) \equiv M$

(1b) $D^g (r,Y; A,\mu) = S^g (r,Y) \equiv Y \leqq Y^f$

(1c) $D^l (r,Y) \leqq S^l (r,Y) \equiv L$

The equality in a) expresses an assumed instantaneous adjusting of interest rates so that the money market clears in each period. Assuming that the supply of money, M, is exogenously determined as indicated in the right hand identity, the LM curve is derived:

(2) $r = L^m (Y;M)$.

Underlying (1b) is the assumption that supply adjusts completely to demand under conditions of less than full capacity and excess labor supply. These assumptions are usually used to derive the IS curve by setting $Y \equiv S^g (r,Y) \leqq Y^f$ where Y^f is full capacity output and by setting the supply of labor $S^1(r,Y) \equiv L$, say, so that a strict inequality in (1c) reflects the amount of involuntary unemployment at the fixed price-wage level. The values of r and Y that satisfy the three relationships of (1) then give the Keynesian disequilibria.

Here, however, I am going to use the LM curve to obtain the aggregate demand curve

(3) $D(Y; A,\mu, M) := D^g [L^m (Y; M), Y; A,\mu]$

in which μ is a parameter introduced to reflect the intensity of induced investment. If $\mu = 0$ then there is no induced but only the autonomous investment (included in A). If $\mu > 0$ then induced investment enters the picture.

In addition to the general equilibrium-disequilibrium framework (1), there are the specific functional forms for the constituent structural relationships that reflect Keynes' empirical propositions. Given the highly simplified supply-side assumptions on the right side of the relationships in (1) these functional forms involve only the demand functions for money and goods. The crucial point in this reconsideration is that the Keynesian forms imply a nonlinear bulge in aggregate demand.

To see this consider first the qualitative features usually assumed for the demand for money, namely $\partial D^m/\partial r < o$ and $\partial D^m/dY > o$. These of course imply that the LM curve is upward sloping, i.e., that $dr/dY > o$. Given a fixed money supply it is reasonable to assume also that the LM function is unbounded as income increases. Take, for example, the case in which the demand for money is comprised of separable transactions and liquidity components as shown respectively in the right hand side of

(4) $D^m (r,Y): = kY + L(r),$

where k is the reciprocal of the transactions velocity of money. L(r) is the liquidity preference function which is assumed to be downward sloping and bounded below by some minimal rate r^o so that it incorporates a liquidity trap. Also, as is usual, it is assumed that as the interest rate increases liquidity demand approaches zero. Given these assumptions L(r) is invertable (except at $Y = M/k$) and the LM curve can be written

(5) $r = L^m(Y;M): = L^{-1} (M-kY)$

which is a positively sloped function with intercept $L^{-1}(M)$ and becoming unbounded at M/k because of the liquidity trap.

Next, recall that the demand for goods is comprised of separate consumption and investment components

(6) $D^g (r,Y): = A + \alpha Y + \mu I(r,Y)$

in which A is the sum of autonomous consumption and autonomous investment demand, α is the marginal propensity to consume, I(r,Y) is the induced investment demand function, and μ is the parameter measuring the strength of induced investment as noted above. It is usually assumed that $\partial I/\partial r < 0$ and $\partial I/\partial Y > 0$ and that induced investment demand approaches or reaches zero when interest rates get high enough.

Substituting for r we get what I shall here call the IY function

(7) $H(Y;M) := I[L^m(Y;M),Y]$

whose "shape" is crucial to the point at issue. Consider

(8) $$\frac{dI}{dY} = \frac{\partial I}{\partial r}\frac{dr}{dY} + \frac{\partial I}{\partial Y}.$$

The second term on the right is positive or zero. The first term is negative, a consequence of the assumed forms of the demand for money and investment goods. Since dr/dY becomes unbounded as income increases and because induced investment is chocked off when interest rates rise enough, the IY function must reach zero at some income, say Y^o. Because of the possible stimulating influence of the income level, investment demand might rise when income increases from relatively low levels. The crucial fact for our argument is that it must fall to zero because of the "crowding out" effect of the transactions demand for cash. The implication is that for $\mu > 0$ aggregate demand has a nonlinear profile first lying above the line given by $A + \alpha Y$ but eventually declining to that line at Y^o.

II

The Robertsonian Lag specifies that aggregate demand in a given period depends on aggregate income of the preceding period so that

(9) $I_{t+1} = \mu I(r_t,Y_t) = \mu H(Y_t;M)$

and

(10) $C_{t+1} = \alpha Y_t$

so

(11) $Y_{t+1} = \theta(Y_t) := A + \alpha Y_t + \mu H(Y_t;M)$.

What we want to do is to see how the qualitative behavior of GNP over time, as it is generated by this equation, changes when the intensity of induced investment represented by parameter μ is modified.

If $\mu = 0$ then of course we have the usual convergent Kahn-Keynes multiplier. If μ is very small then induced investment cannot change this picture very much. As μ increases, however, the dynamics of GNP can change a very great deal. Exactly how the dynamics change depends on the profile of the IY curve.

As we have seen this profile is one in which autonomous expenditure A and induced consumption Y are augmented by induced investment, possibly with a growing income effect for low income levels but necessarily falling to zero as income approaches the value Y^o. The latter limit is not influenced by μ so that an increasing importance of induced investment raises aggregate demand to the left of Y^o but leaves it unchanged to the right of Y^o. This kinked, twisting effect of increasing μ leads to growing complications of behavior.

To see what happens as μ is increased consider Figure 1. When μ is small as shown in (1a), a sequence of output adjustments converges monotonically to an income level somewhat bigger than $A/(1-\alpha)$. For $\mu = \mu_2$ cycles emerge but converge to a stable Keynesian disequilibrium. As μ is increased to μ_3, the Keynesian disequilibrium becomes unstable and a stable two period cycle emerges as shown in (1c).

As μ increases still more this cycle becomes unstable and a stable four period cycle emerges. Smaller and smaller changes in μ lead to a succession of period doubling stable cycles; as each stable cycle emerges its predecessor becomes unstable. This sequence converges to a value, say μ^c, such that for values of μ at and above μ^c unstable

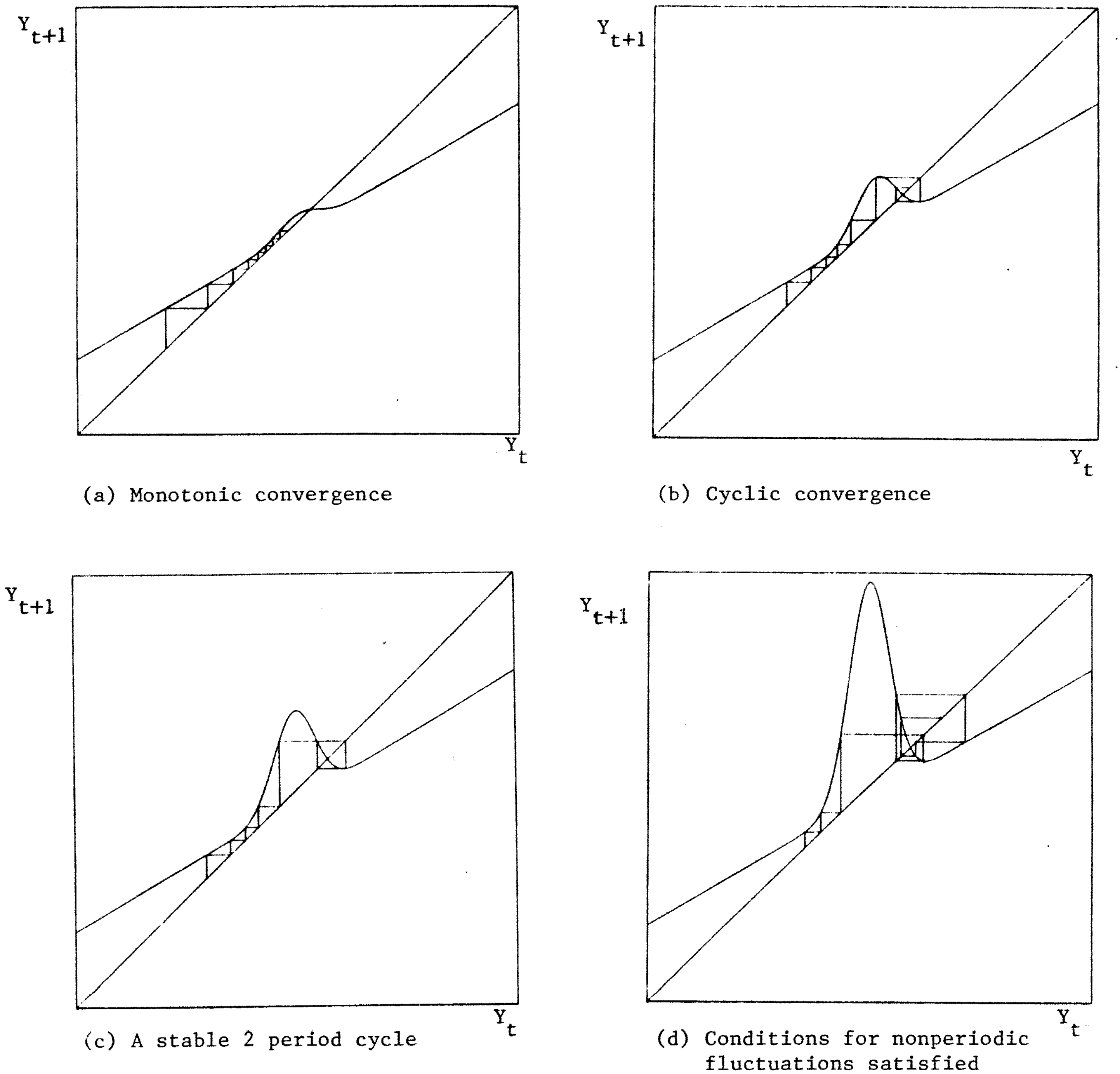
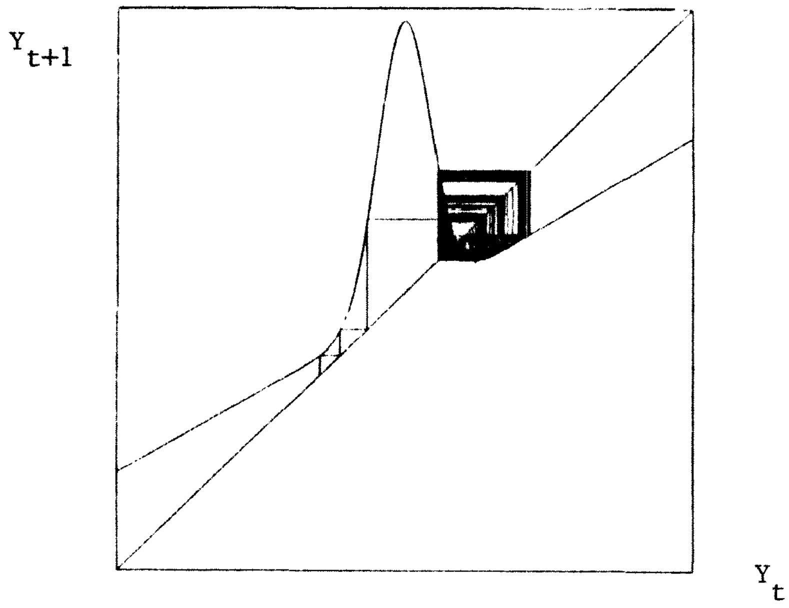
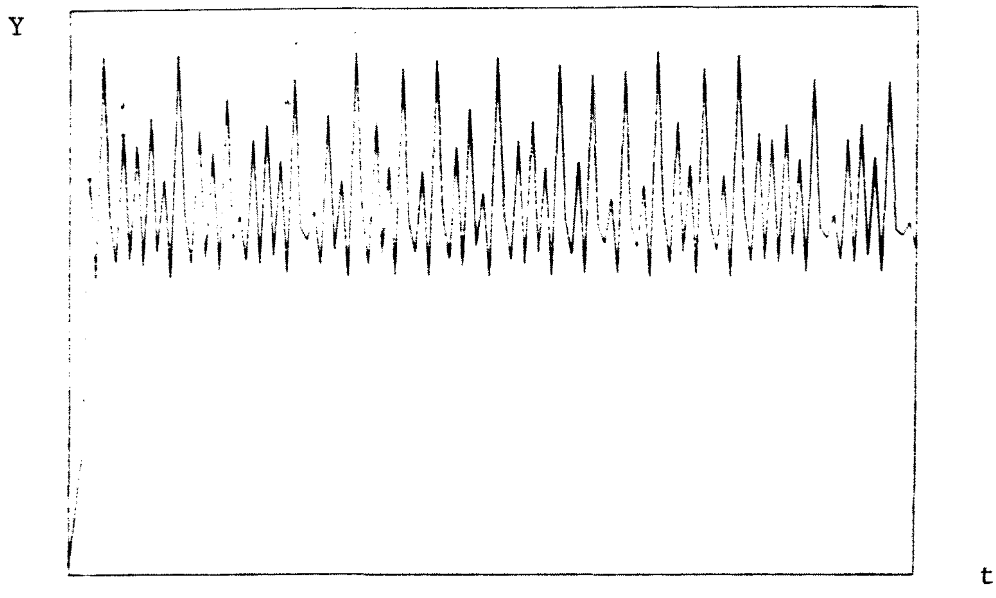


FIGURE 1 COMPARATIVE KEYNESIAN DYNAMICS:

Qualitative Changes Caused by Shifts in the Intensity of Induced Investment periodic cycles of all orders exist and, also, there exists an uncountable set of GNP levels, called a scrambled set, any one of which leads to unstable nonperiodic fluctuations. Within that set GNP does not converge to a cycle of any order but wanders in an erratic, more or less random pattern.[6] Figure 1d portrays aggregate demand for a value of μ which can be shown to exceed the critical value μ^c. Figure 2a shows time series data for GNP implied by the model for the set of specific functional forms underlying the aggregate demand function shown in Figure 1d and which is described mathematically in section IV below. One notes that during the course of this "simulation" the economy varies the pattern of apparent cycles so that it appears to switch from cycles of one periodicity to cycles of another periodicity, but in a more or less erratic manner. Figure 2b shows these same data as they appear on the phase diagram. One observes here how the cycles do not converge on a fixed "orbit" but spread themselves more or less continuously throughout an interval. The principal result is that the Keynes-Hicks-Robertson adjustment model can display limitless variety of behavior including stable and unstable cycles and more or less random motion all depending on the strength of induced investment and the amount of the money supply.[7]



(a) Wandering cycles on the "phase diagram"



(b) The corresponding time series data

FIGURE 2
APPARENTLY NONPERIODIC FLUCTUATION
(Time series data gathered by the example shown in Figure 1d and described in section IV. For parameter values see Note 10.)

III

I cannot think of a place in the literature where the aggregate demand function including induced investment is represented graphically so I would not be surprised if some readers were startled at the shape which it exhibits when μ is positive, especially when it has a large enough value to allow for "chaos." The question that might then very naturally arise is, "what about the shape of the LM curve and of the IS curve that could be derived in the usual way assuming equilibrium (instead of adjustment) on the goods markets? Wouldn't these have to have bizarre shapes in order to be consistent with instability, especially in the extreme case of chaos?

A start toward an answer can be made by considering the condition for local instability of a stationary state, say Y^s, of the adjustment process (11). In order to get locally expanding oscillations around such a Keynesian disequilibrium point we have to have $\theta'(Y^s) < -1$.[8] This implies that $\alpha + \mu dH/dY < -1$ or

$$(12) \qquad \mu\frac{dH}{dY} < -(1+\alpha).$$

We know from the analysis in Section II that the slope of the IY curve H(Y;M) becomes negative for sufficiently high income. Consequently, there is always some intensity of induced investment (μ) big enough so that (12) is satisfied. Consequently, locally expanding cycles are always theoretically possible,[9] a result we have already obtained by means of the graphical argument in the discussion of Figure 1.

How is such a situation related to IS and LM? Using (7) we find that (12) implies

(13) $$\left.\frac{dr}{dY}\right|_{LM} = \frac{dL^m(Y)}{dY} > \frac{\frac{(1+\alpha)}{\mu} + \frac{\partial I}{\partial Y}}{-\frac{\partial I}{\partial r}} .$$

Now the slope of the IS curve is

(14) $$\left.\frac{dr}{dY}\right|_{IS} = \frac{\frac{1-\alpha}{\mu} - \frac{\partial I}{\partial Y}}{\frac{\partial I}{\partial r}} .$$

Using this together with the right side of (13) we see that the instability condition (12) can be expressed as

(15) $$\left.\frac{dr}{dY}\right|_{LM} > \left.\frac{dr}{dY}\right|_{IS} - \frac{2}{\mu\frac{\partial I}{\partial r}} = \frac{\frac{1+\alpha}{\mu} + \frac{\partial I}{\partial Y}}{-\frac{\partial I}{\partial r}}$$

The slope of the LM curve is always positive. The denominator on the right is positive because $\partial I/\partial r < o$. From (14) it is evident that the slope of the IS curve can be positive at a given point if $\partial I/\partial Y$ and μ are large enough there. Obviously from (15) this increases the slope of the LM curve required to get instability. On the other hand if the IS curve is downward sloping a sufficiently steep LM curve can always cause instability. Clearly, the larger μ, the less steep the LM curve need be to produce instability, regardless of whether the IS curve is upward or downward sloping. On the other hand, the chance that the IS curve is upward sloping increases with μ. But note that if $\partial I/\partial Y \equiv 0$ as is often assumed in the Keynesian literature then the IS curve is always downward sloping though as μ increases it becomes progressively flatter. To summarize, <u>locally expanding but bounded cycles are quite compatible with conventional looking IS and LM curves</u>.

Can these cycles also be erratic or chaotic within a conventional context? The answer does not involve the local instability condition (15) but rests rather on a nonlinearity of aggregate demand which leads to sufficient "overshoot". Evidently, however, such "global" conditions are quite compatible with (15) which merely guarantees that expanding cycles must occur for awhile no matter how close an income level comes to the Keynesian disequilibrium.

IV

Nonlinear examples of the kind illustrated in Figure 1 are often used in standard texts such as Branson [1979], or the much earlier Hanson [1949] or Bailey [1962]. Define the liquidity demand for money to be

(16) $L(r) := \lambda/r, \; r > 0,$

where λ is a parameter. If the "transactions" demand is kY where k is a parameter then the LM curve is

(17) $r = \lambda/(M-kY).$

For the investment function use

(18) $I(r,Y) := b[Y/(\theta Y^f)]^{\beta} \, (\rho/r)^{\gamma} ,$

where b, θ, ρ, β, and γ are parameters. The first multiplicative term in brackets represents the influence of the GNP level on investment. If $\beta = 0$ then it has no influence and we have a "pure" Keynesian model. If $\beta > 1$ the "Kaldorian" effect matters. Y^f is full capacity output and θ a proper fraction. θY^f might be thought of as the "optimal" or "desired" level of capacity utilization. If income is less than θY^f then the investment demand associated with the Keynesian term is reduced. Otherwise it is increased. If $\beta > 1$ then low levels of output have a strongly depressing effect but levels greater than θY^f have a strongly stimulating influence.

The following term, $(\rho/r)^\gamma$, is the Keynesian investment term which reduces investment demand as the interest rate increases. If $r = \rho$ the interest rate effect on investment is "neutral." If r is less than ρ then the effect is stimulating; if r is greater than ρ then the effect is depressing. The higher γ is the more pronounced are these influences.

Substituting (17) into (18) we obtain the IY function

$$(19) \qquad I = H(Y;M) := B\, Y^\beta (M-kY)^\gamma$$

where $B = b(\rho/\lambda)^\gamma (\theta Y^f)^{-\beta}$ is a constant. If β and γ are both larger than unity then this function has a "cocked-hat" shape, first rising gradually, but at an increasing rate, then at a declining rate until a maximum investment is reached at

$$(20) \qquad Y = (\beta/(\beta + \gamma)) \cdot (M/k).$$

Beyond this level of GNP investment declines, first at an increasing, then at a decreasing rate approaching zero as income approaches M/k. The GNP adjustment equation (11) becomes

$$(21) \qquad Y_{t+1} = A + \alpha Y_t + \mu B Y_t^\beta (M-kY_t)^\gamma.$$

An analysis of this equation would show the possible presence (depending on the parameters) of multiple Keynesian disequilibria and stationary states. It would also be discovered that obtaining general conditions for the emergence of viable, erratic cycles is not such an easy task. Intuitively, what has to be done is to increase β and γ so that induced investment is "peaky," that is, its initial response to income is negligible but then increases sharply with a quite abrupt onset of "effect reversal." This makes the largest of the Keynesian disequilibria strongly unstable when μ is increased enough, so that the overshoot condition to get the increasingly high ordered cycles culminating in nonperiodic fluctuations can occur with values bounded above by M/k. This is what is exhibited in Figures 1 and 2 which are based on the equations (16) - (21).

In this case the IS curve is given by

$$(22) \qquad r = K\, Y^{\beta/\gamma}\, [(1-\alpha)Y-A]^{1/\gamma}, \quad Y \geqq Y^k = A/(1-\alpha)$$

where $K = \rho b^{1/\gamma} \,/\, (\theta Y^f)^{\beta/\gamma}$. It is undefined for incomes below Y^k for no interest rate can clear the excess demand for goods in this range. It is downward sloping just above Y^k, reaches a minimum at $Y^* = [\beta A-(1-\alpha)Y']/[(\beta-1)(1-\alpha)]$, beyond which point it slopes upward. If this value exceeds M/k, the IS curve is downward sloping throughout its feasible range, otherwise it contains a hook. In Figure 3, T.Y. Lin has plotted the IS-LM curves corresponding to the aggregate demand situation shown in Figure 1d. We see that they have a "normal" configuration with upward sloping LM curve and downward sloping IS curve!

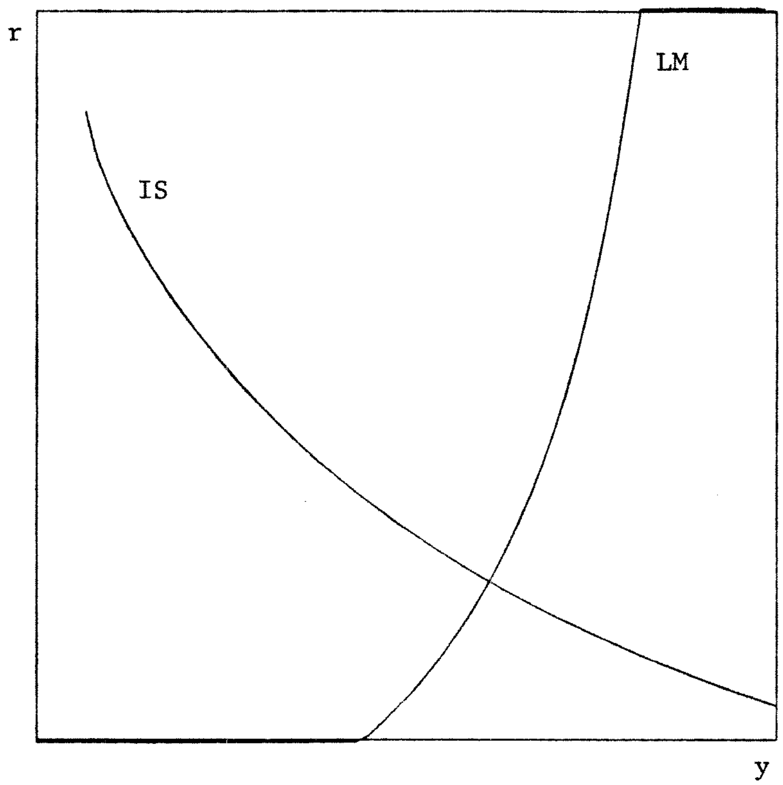


FİGURE 3

THE IS AND LM CURVES

V

We now consider the case in which each of the basic functions is linear except that we introduce nonnegativity restrictions on the demand for money, interest rates and the induced demand for investment goods. Linear functions underly many standard textbooks such as Gordon [1978], quantitative policy analyses such as Hall [1977], and theoretical exegesis such as Smyth and Peacock [1974]. Here the nonnegativity restrictions usually ignored in such treatments are made explicit. They imply "kinked" or piecewise linear shapes for the IS, LM, IY and aggregate demand functions. Because of these kinks a locally unstable Keynesian multiplier process produces bounded oscillations which, as we shall see, may be nonperiodic for a family of parameter values.

First, let the liquidity preference schedule be

$$(23) \qquad L(r) := \begin{cases} L^o - \lambda r, & 0 < r \leqq L^o/\lambda \\ 0, & L^o/\lambda \leqq r. \end{cases}$$

Here L^o and λ are parameters. In this example (so as to eliminate an unnecessary parameter) the liquidity trap interest rate is 0. If kY is the transactions demand the implied LM curve is

$$(24) \qquad r = L^m(Y) := \begin{cases} 0 & , 0 \leqq Y \leqq Y'' \\ (k/\lambda)(Y - Y'') & , Y'' \leqq Y < M/k, \end{cases}$$

where the income given by

$$(25) \qquad Y'' := (M - L^o)/k$$

is the level at which the interest rate emerges from the liquidity trap.

The investment demand curve is also linear except for a non-negativity kink.

$$(26) \qquad I(r,Y) := \max\{0, \beta(Y-Y')-\gamma r\}.$$

If Y' is positive then according to this assumption induced investment is zero until the threshold Y' is reached. After this point income stimulates investment. If we suppose $Y' < Y''$ then the interest depressing effect will not take place until income reaches the level Y'' when the liquidity trap is escaped. Above Y'' the interest effect depresses investment. If Y' is negative then $-\beta Y'$ is equivalent to an autonomous investment term and the stimulating effect of income on investment holds throughout its full range. In the remainder of this section I shall assume $Y' > 0$.

The net effect of these separate influences is obtained from the IY function which is derived by substituting (24) into (26). It has four separate branches. The first branch occurs in the range $0 \leqq Y \leqq Y'$ when investment is zero. The second occurs for incomes between Y' and Y'' when interest is still in the liquidity trap but the stimulating effect of income on induced investment is operating. The slope here is β.

The third branch occurs in the interval (Y'', Y^o) where the interest depressing effect of interest comes into play as the transactions demand pushes the interest rate up out of the liquidity trap. After some rearranging of terms we see that in this branch investment equals $(\gamma k/\lambda)Y'' - \beta Y' - \sigma Y$ where $\sigma = \gamma k/\lambda - \beta$. Therefore, if

$$(27) \qquad \gamma/\lambda > \beta/k$$

the slope of the IY curve is negative which means that in this range the interest depressing effect (caused by the rise of interest rates to clear the money markets) overpowers the stimulating effect of income increases. This "crowding out", which we shall assume in what follows, is enhanced by increases in k or γ or by decreases in λ or β, or if the ratio of interest rate effects γ/λ is greater than the ratio of the income effects. It implies that eventually investment is driven to zero when income reaches the point

$$(28) \qquad Y^o := [(\gamma k/\lambda)Y'' - \beta Y']/\sigma,$$

where we recall, $Y'' = (M-L^o)/k$. To summarize, the IY curve is

$$(29) \qquad H(Y;M) := \begin{cases} 0 & , 0 \leqq Y \leqq Y' \\ \beta(Y-Y') & , Y' \leqq Y \leqq Y'' \\ \sigma(Y^o-Y) & , Y'' \leqq Y \leqq Y^o \\ 0 & , Y^o \leqq Y. \end{cases}$$

Aggregate demand of course has four branches that correspond to the branches of the investment function so that the adjustment equation is found to be

$$(30) \qquad Y_{t+1} = \Theta(Y_t) := \begin{cases} A+\alpha Y_t & , 0 \leqq Y_t \leqq Y' \\ A-\mu\beta Y'+(\alpha+\mu\beta)Y_t & , Y' \leqq Y_t \leqq Y'' \\ A+\mu\sigma Y^o+(\alpha-\mu\sigma)Y_t & , Y'' \leqq Y_t \leqq Y^o \\ A+\alpha Y_t & , Y^o \leqq Y_t \leqq M/k. \end{cases}$$

Four examples are shown in Figure 4. The parameters of this model were chosen so that the shapes of the aggregate demand curves are roughly similar to those of the smooth nonlinear model described in section III and shown in Figure 2.

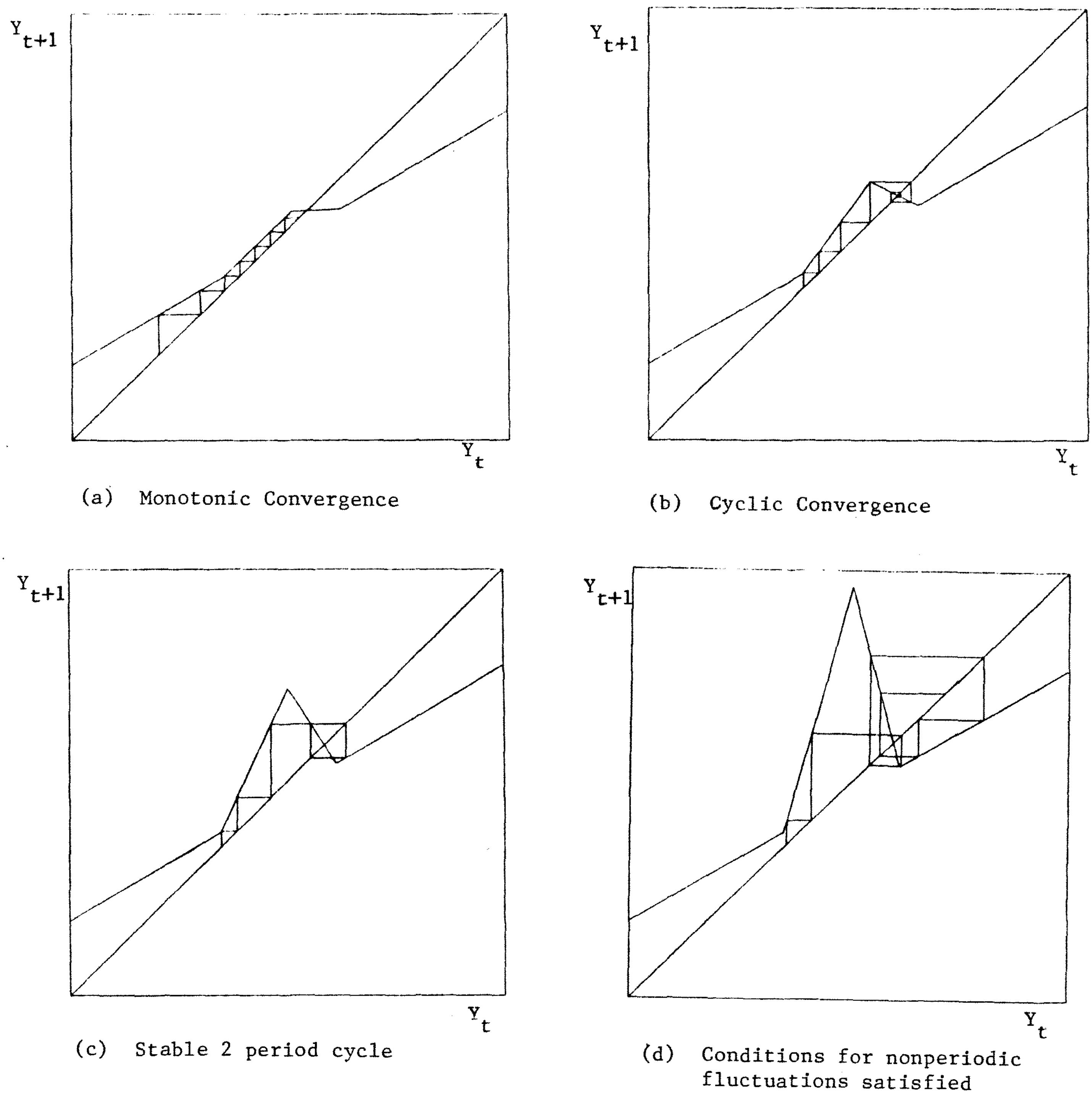


FIGURE 4

COMPARATIVE KEYNESIAN DYNAMICS FOR THE PIECE-WISE LINEAR MODEL

Obviously, when $\mu = 0$ the model reduces to the usual Kahn-Keynes multiplier process that converges to $Y^k = A/(1-\alpha)$ if $0 < \alpha < 1$ is assumed. If μ is positive, however, various possible time paths and stationary states are possible. Evidently up to three of the latter can occur depending on the relative magnitudes of Y', Y^k and Y''. For simplicity and to reduce our task to the most interesting issues, let us confine attention here to the situation illustrated in Figure 4 in which only one stationary state exists and it occurs in the third "monetary crowding out" segment of the aggregate demand curve. Clearly, the usual stable multiplier process is possible in which GNP converges monotonically or cyclically to the stationary value given by

(31) $$Y^s = (A+\mu\sigma Y^o)/[1-\alpha+\mu\sigma].$$

The stability condition is that the slope of the aggregate demand curve evaluated at Y^s is less than unity in absolute value. If, however,

(32) $$\Theta'(Y^s) = \alpha-\mu\sigma < -1$$

then divergent cycles emerge in the neighborhood of Y^s. Rearranging the above expression to get $\mu\sigma > 1+\alpha$, we see that so long as the "monetary crowding out" effect is present ($\sigma > 0$), <u>oscillations emerge for any intensity of induced investment satisfying</u>

(33) $$\mu \geqq (1+\alpha)/\sigma.$$

From the discussion in section II we know that, as μ is increased beyond the level given in (33), successive bifurcation points are reached in μ in which cycles of higher and higher order emerge until a critical value μ^c is reached at which periodic cycles of every order and an uncountable set of nonperiodic fluctuations exist. Which one from this infinitely varied set of possibilities occurs depends only on the initial condition. I am not going to derive this result here. The interested reader can find a discussion of the procedure in Day and Shafer. What one finds is that this "chaotic" regime exists for a continuum of parameter values (α, β, γ, λ, k, L^o, Y', M). In Figure 5a the time series of Y_t is shown for parameter values satisfying the chaos condition. Figure 5b shows the same data as they appear on the "phase diagram." Again we see both the erratic nature of the time series and how the cycles spread themselves over the phase space.

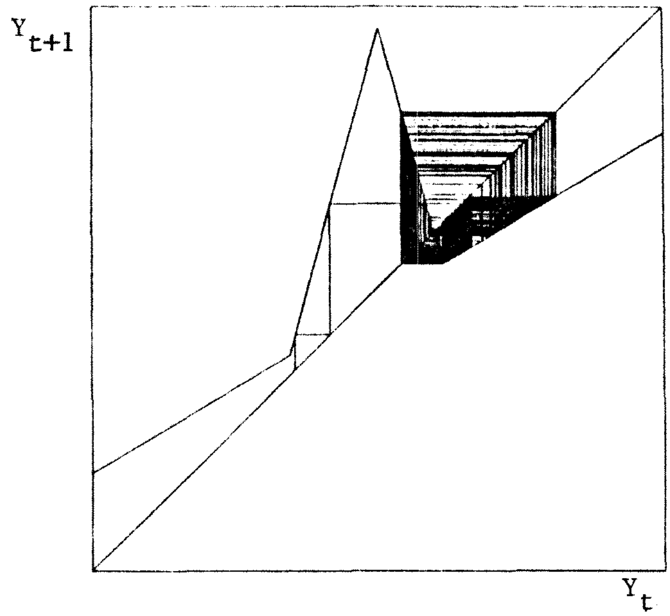
Following the analysis of section 3 we can derive the IS curve and see what the cyclical instability condition implies for the static IS-LM framework. Below Y' investment is insensitive to the interest rate so the IS curve cannot be defined in the interval [0,Y']: no positive interest rate can clear the commodity markets in that range. Above Y' we have the non zero segment of the investment function (26). If $\alpha + \mu\beta > 1$, the marginal propensity to spend from income is upward sloping, the market clearing interest rate could become negative if $A - \mu\beta Y'$ is negative. Or if $\alpha + \mu\beta < 1$ the IS curve is downward sloping and the market clearing interest could go negative above $A - \mu\beta Y'$. In any case we have for the IS curve

$$(36) \qquad r = \frac{A - \mu\beta Y' + (\alpha + \mu\beta - 1)Y}{\mu\gamma}, \quad Y > Y'.$$

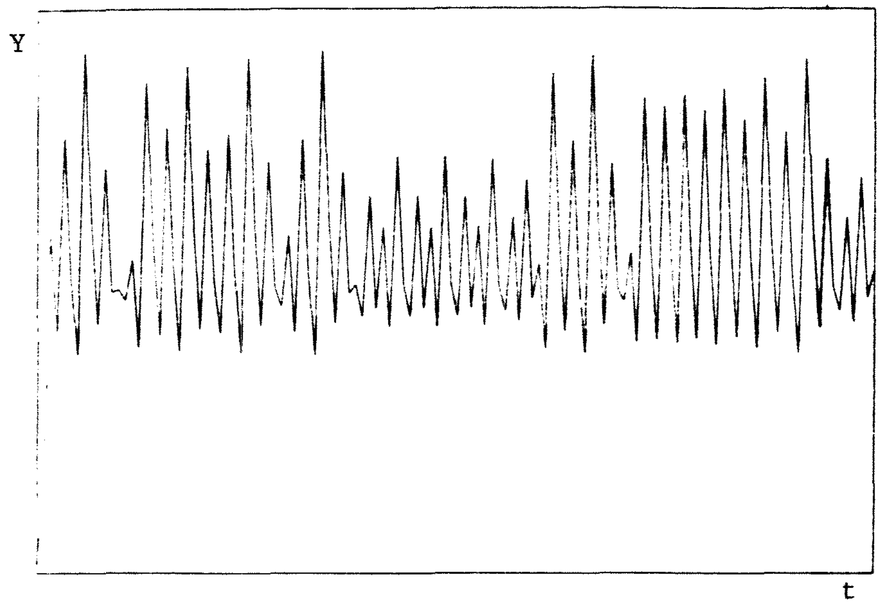
Evidently, when μ is small enough the IS curve is downward sloping. As μ increases the IS curve becomes flatter and then becomes positive.

Now the slope of the LM curve (24) is k/λ so from (15) we find that the cyclical instability condition (32) is satisfied when

$$(37) \qquad \frac{k}{\lambda} \geqq \frac{\alpha + \mu\beta - 1}{\mu\gamma} + \frac{2}{\mu\gamma} = \frac{\frac{1+\alpha}{\mu} + \beta}{\gamma}$$



(a) Wandering cycles on the "phase diagram"



(b) The corresponding time series data

FIGURE 5

APPARENTLY NONPERIODIC FLUCTUATIONS
FOR THE PIECE-WISE LINEAR MODEL

(Time series data generated by example shown in Figure 4d and described in this section. For parameter values see Note 11.)

Thus, if the IS curve is positively sloped the LM curve must be very steep, while if the IS curve is negatively sloped the LM curve need not be so steep for locally expanding cycles to occur. Given one or the other of these two cases increases in μ, which reduce the term on the right, reduces the slope of the LM curve sufficient to bring about instability.

Figure 6 shows the IS-LM curve corresponding to the chaotic example of Figure 4d. Even though the present model looks and behaves like its smooth predecessor, the IS curve is positively sloped! In Figure 7 T.Y. Lin and I found a set of parameters for the piece-wise linear model that leads to chaos but for which the IS curve is negatively sloped. The aggregate demand profile is now no longer much like the smooth case. These perhaps surprising results are due to the fact that the piece-wise model is an extremely crude approximation of the smooth one (or vice-versa) so that the stability properties of curves drawn so as to "look something alike" may be quite different. None-the-less, our central point is well established: the new kind of behavior is quite compatible with conventional pictures of macroeconomic structure.

One caveat, however. The choice of functional form has a crucial influence on the global dynamics of a model. Approximations used for econometric convenience may either prevent the inference of relevant possibilities, could introduce dynamic properties where they don't exist in reality, or could seriously distort the representation of behavior except in the vicinity of equilibria.

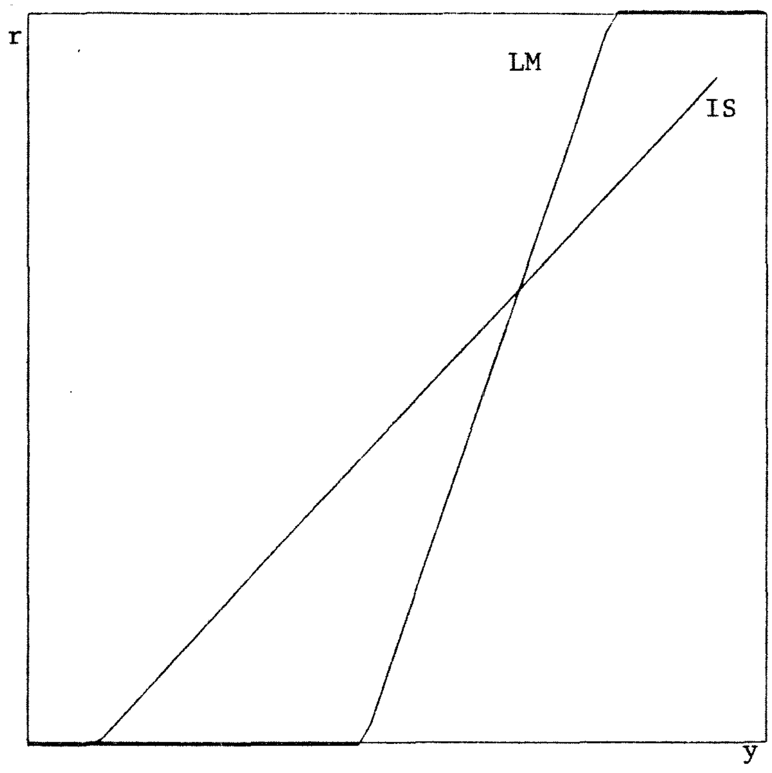


FIGURE 6

THE IS AND LM CURVES FOR THE PIECE-WISE LINEAR CASE

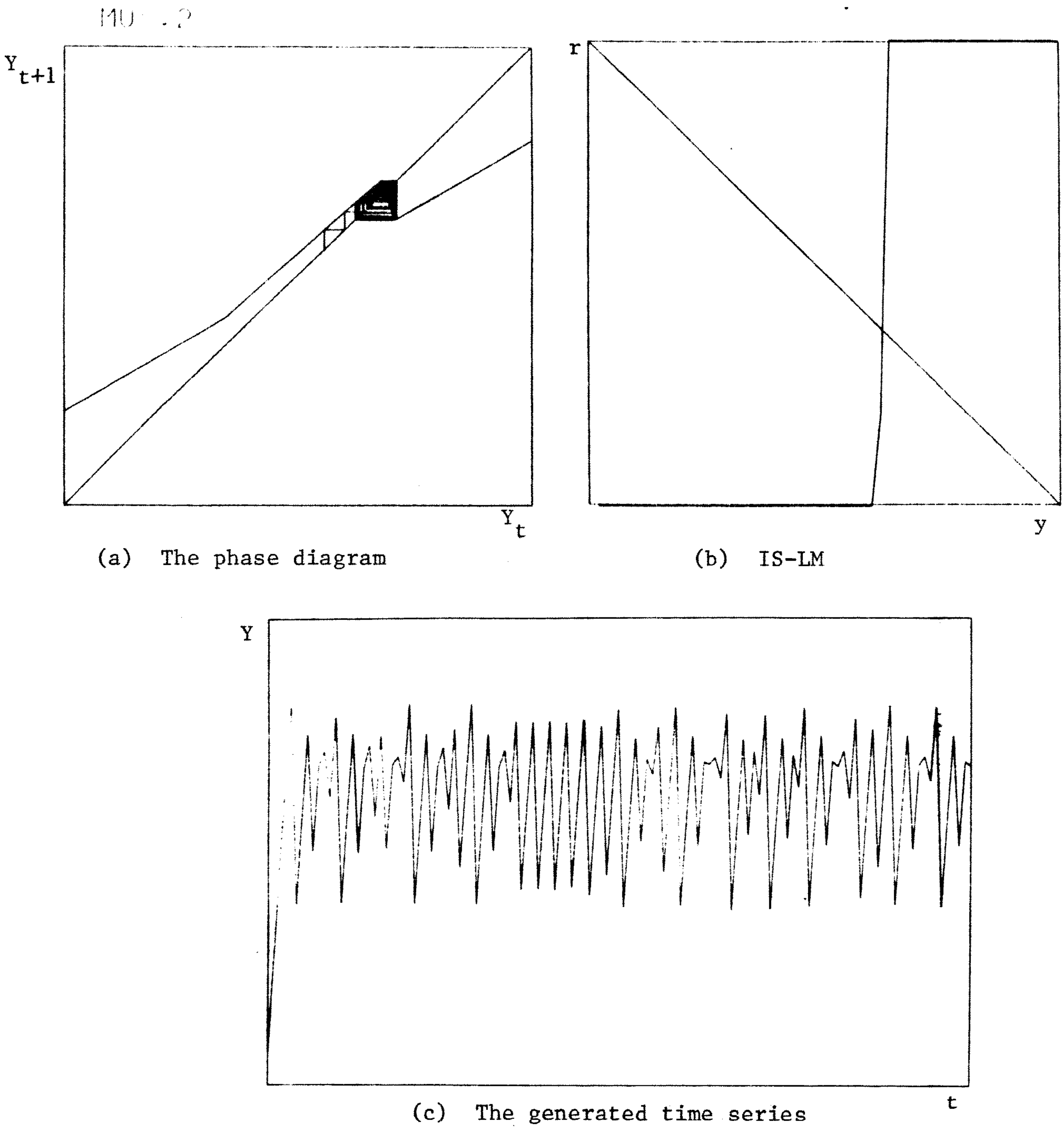


FIGURE 7

A CHAOTIC EXAMPLE FOR THE PIECE-WISE MODEL
WITH DOWNWARD SLOPING IS CURVE

VI

To summarize: so far in addition to the familiar stable and explosively unstable multiplier dynamics, we can get (1) locally unstable but bounded fluctuations in output, employment, interest, and investment, etc.; (2) these fluctuations may be periodic with the periodicity increasing as the intensity of induced investment increases; (3) at some level of induced investment intensity completely erratic fluctuations exit [in Day and Shafer, (1983) it is shown that these fluctuations can be ergodic, that is, can converge to stationary distribution functions so that time series of the macro variables are very similar to stationary stochastic processes]; (4) such complicated behavior is compatible with conventional hypotheses about the demand for money and for investment goods; and (5) it can be generated using the simplest linear and log-linear functional forms.

A comparative dynamics policy analysis within the present framework must be concerned not just with the effects of monetary and fiscal parameters on the level of output and employment and their possible influence on stable or unstable multiplier adjustments but also with their potential ability to shift output adjustments into cyclical or even to non-periodic fluctuation, or contrastingly to stabilize such instabilities when they emerge. A complete examination of such policy issues is beyond the scope of the present paper, but the new kinds of results that can be obtained can be readily illustrated.

For that purpose consider the "general" model of section 2 but introduce a pure "Keynesian" investment demand function in which there is no direct income effect so that $\partial I/\partial Y \equiv 0$. Denote the investment demand function by $I(r)$ in this special case and assume it is downward sloping and approaches zero as γ becomes large enough. Also let $D^m(r,Y) = kY+L(r)$. The interest rate is bounded below by $L^{-1}(M)$ so the maximum possible induced investment is $I[L^{-1}(M)]$. If investment is initially insensitive to increases in the interest rate then aggregate demand will be approximately $A + \alpha Y + \mu I[L^{-1}(M)]$ at low income levels. Eventually, however, as income increases and with it the transactions demand for cash, interest rates must rise sharply thereby reducing the demand for investment goods. As income approaches M/k the interest rate becomes indefinitely high and investment falls to zero at some value of Y, say Y^o. Aggregate demand therefore runs approximately along a straight line $A + \alpha Y + \mu I[L^{-1}(0)]$, then falls to a second, parallel straight line $A + \alpha Y$ as GNP grows.

If the supply of money is very small the "crowding out" effect just described will occur at low income levels and the Keynesian disequilibrium, Y^k, will be the usual multiplier amount, $Y^k = A/(1-\alpha)$ (provided that $Y^o < Y^k \leqq M/k$). If, however, the money supply is very large so that the Keynesian disequilibrium occurs while interest is still quite low then approximately $Y^k = (A + I[L^{-1}(M)])/(1-\alpha)$ (provided that $Y^k < Y^o \leqq M/k$). Below the amount $M^\ell = kA/(1-\alpha)$ and above the amount $M^\mu = k(A+I[L^{-1}(M)]>/(1-\alpha)$, changes in the money supply will have little influence. In between them, however, lies a range of money supplies that gives a "window of monetary effectiveness." Suppose the chaos condition is satisfied for some values of M within this window. We then have the following intriguing possibilities.

Suppose the money supply is initially compatible with the chaos conditions and erratic investment, interest and income cycles emerge.

Either a sufficient reduction or, contrastingly, a sufficient expansion in the money supply would push the economy into zones of stabilizing macro behavior, in the first case to a high unemployment disequilibrium, in the second case to a low unemployment disequilibrium. Monetary policy would have two alternative routes to stabilization and two unstable regimes.

VII

It would be apocryphal to claim on the basis of this analysis that the "Fed" can cause or present "chaos" in the economy. All we have shown is that among the possible impacts of policy are stabilizing and destabilizing ones including the inducement or suppression of erratic economic oscillations. Note that it is not erratic behavior of the money supply that causes the problems. Erratic oscillations are intrinsic under the conditions discussed for fixed money supplies within a specific range. Thus, if policy is to rest on stable rules rather than on capricious discretion, then those rules must lead to instruments set at efficacious levels. Of course these complicated possibilities depend on whether or not empirical conditions favor them. The need for accurate econometric estimates of structure would therefore appear to be extremely important even if forecasting is virtually hopeless.

Notes

1. The present paper was begun at the Industrial Institute of Economic and Social Research, Stockholm and was completed at the Modelling Research Group, Department of Economics, USC, Los Angeles. The simulations and computer graphics were prepared by T.Y. Lin.

2. All this was achieved, of course, at the cost of extreme, over-simplifying assumptions. Namely, (1) money markets clear with instantaneously adjusting interest rates; (2) the quantity of commodity supply adjusts instantaneously to demand on the commodity markets with fixed prices, (3) the supply of labor and wage rates remain fixed in the face of deficient demand; and (4) capital stock remains fixed. Of course, some of the deficiencies due to these assumptions have been remedied in subsequent literature, but at the cost of substituting other oversimplifications or of adding considerable complication, or by demanding increased sophistication of interpretation.

3. I follow the Clower-Leijonhufvud practice of referring to the Keynesian static solution as a "disequilibrium." See Benassy [1982, p. 3].

4. Thus the problem of unbounded change does not arise and the search for floors and ceilings by the early business cycle theorists was not really necessary. The necessary bounding nonlinearities are present in the Keynes-Hicks-Robertson theory right under the theoretical noses of the cycle theorists of that time.

5. Since Modigliani's [1944], Samuelson's [1948, pp. 281-283] and Hansen's [1953] analyses, various authors have investigated dynamic

versions of the Keynesian system, for example Peacock [1962] and Smyth [1974]. Some of these linearize the consumption, investment and demand for money functions with the effect that the complicated dynamics under consideration here are precluded. Other authors, like Modigliani and Samuelson, are concerned exclusively with local stability. They exploit linearizations of the structural relationships in the neighborhood of equilibria, again, with the effect of precluding an analysis of nonperiodic fluctuations. Pohjohla [1982] introduces a progressive linear tax function into the linear Peacock and Smyth models. This creates a nonlinearity in the consumption function leading to a chaos result. Here we are concerned, however, with global results in which the phenomenon of interest is shown to depend not on some kind of government of interference but on precisely the nonlinear properties Keynes thought were intrinsic to the demand for money and investment goods. For a recent discussion of the dynamic multiplier see Hicks, pp. 1388.

6. For these complicated dynamics to be interesting they have to unfold in the interval [0,M/k] because income levels above M/k do not have enough money to sustain them. Thus for cycles to persist the system must not be "too unstable." Notice that the aggregate demand function develops a local maximum and minimum when cycles emerge. These critical levels depend on μ. Let Y_{μ}^{min} and Y_{μ}^{max} denote this dependence. Suppose the interval $(Y_{\mu}^{min}, Y_{\mu}^{max})$ contains a Keynesian disequilibrium. If $Y_{\mu}^{max}{}_{c} < M/k$ then the output adjustments can never carry GNP above the monetary bound so that the complicated dynamics for all μ such that $Y_{\mu c}^{max} \leqq Y_{\mu}^{max} \leqq M/k$ are feasible.

7. Since the possibility of erratic oscillations in a deterministic model is the most novel finding in the present context I shall focus attention on it in the subsequent discussion. Sufficient conditions for their presence are the existence of income levels, say Y^a, Y^b, Y^c, Y^d in which $Y^b = \theta(Y^a)$, $Y^c = \theta(Y^b)$ and $Y^d = \theta(Y^c)$ and such that either

$$Y^d \geqq Y^a > Y^b > Y^c$$

or

$$Y^d \leqq Y^a < Y^b < Y^c$$

These are the sufficient overshoot conditions of Li and York [1975] which various investigators have discussed in an economic context, for example Benhabib and Day [1980a,b; 1982a,b], Day [1982][1983]. These references contain citations to the relevant mathematical literature. What we can show, and it is done rigorously in Day and Shafer [1983] is that by increasing μ, one or the other of these inequalities can be brought about. In the present Keynes-Hicks-Robertson model either one can occur. The first set of inequalities occurs in the example of Figure 1, the second set occurs in the example given by Day and Shafer.

8. The following convention for the derivative of the function $\theta(\cdot)$ evaluated at a point Y^s is used,

$$\theta'(Y^s) := \left.\frac{d\theta(Y)}{dY}\right|_{Y = Y^s} .$$

9. We have also to make sure that they are feasible. See note 6.

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