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Nr 45, 1981

GROWTH, EXIT, AND ENTRY OF FIRMS

by

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June, 1981

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1. INTRODUCTION

According to the Marshallian theory of supply for a competitive industry, all firms are assumed to have identical U-shaped long-run average cost curves. There are no barriers to entry and positive (negative) excess profits cause new firms to enter (existing firms to leave) the industry. In the long run changes in total demand only change the number of firms in the industry. Although this is the standard textbook story it is unsatisfactory because it is inconsistent with certain pervasive empirical facts, namely, that industry growth comes mainly from existing firms, that most firms grow at rates which do not systematically depend on the scale of their operations and that there is simultaneous entry into and exit out of an industry.[1]

Subsequent theoretical work has further developed the static Marshallian theory.[2] Still, there exists as yet no model of industry supply analyzing the determinants of the gross flows of firms into and out of an industry, which is also consistent with the above-mentioned empirical facts. The purpose of this paper is to fill this vacuum by presenting a model which under traditional assumptions both (1) explains the simultaneous entry and exit of firms based on maximizing behavior and (2) shows the existence of an industry equilibrium for growth, entry and exit of firms when the firms are continuously growing.

The model presented here assumes (like the static theories) perfect competition and constant long-run average costs and (like the dynamic theories) that firms maximize the present value of net earnings and that there are growth costs internal to the firm. In addition it assumes that

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1 Gibrat (1931), Simon and Bonini (1958), Hart (1962) and Marris (1964) among others.

2 See for instance Lucas (1967), Howrey and Quandt (1968), Gaskin (1970), Myers and Weintraub (1971) and Brock (1972).

there are short-run variations in labor productivity in existing firms and in the costs of potential entrants of overcoming barriers to entry. The latter assumptions allow us to explain the simultaneous occurrence of exit and entry in an industry.

This paper is organized as follows: Section 2 derives the optimal investment policy and growth for a firm already in the industry. Section 3 derives the functions determining the rates of entry and exit. Section 4 presents the industry equilibrium solution. We note that this solution is characterized by a series of short-run equilibria each of which implies equality between short-run demand and supply at a time-constant output price, given a constant growth rate of industry demand. Using comparative dynamic analysis we then show how the industry equilibrium is affected by changes in the growth of industry demand and the costs of entry and exit. Finally, we note how the model compares with other theories of supply and with empirical findings.

2. VALUATION AND OPTIMAL BEHAVIOR OF A SINGLE FIRM

2.1 The valuation

Consider an industry consisting of a large number of firms which produce a homogeneous product. The typical firm is owner-controlled and assumed to maximize the present value of all future net cash flows. The firm uses two homogeneous inputs, labor and capital, the quantities of which at time t are represented by L_t and K_t respectively. Labor but not capital is a perfectly variable input. No substitutability exists between these inputs in production[1] and there are constant returns to scale. There are also internal adjustment costs such that the higher the rate of capital accumulation at every point in time, v_t, the lower is the current output, Q_t, given K_t.

$$Q_t = \alpha_{Kt} K_t (1-c_{vt}) \tag{1}$$

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[1] This assumption rules out an analysis of the choice of technique, but the introduction of substitutability would not add anything new of interest for our purposes.

[2] Besides mathematical convenience the quadratic form of the adjustment cost function seems rather well to describe the behavior of many types of similar costs such as costs for hiring and training new workers, inventory costs, machine setup costs, etc, (Gould [1968], p 49.)

The reason for using v_t as an explanatory variable in this function is the above mentioned observation that the rate of growth, except of the smallest firms, is roughly independent of the size of the firm.

$$L_t = \alpha_{Lt} K_t \tag{2}$$

$$c_{vt} = \alpha_{vt} v_t^2 \tag{3}$$

$$v_t = \dot{K}_t / K_t \tag{4}$$

where α_{Kt} α_{Lt} and α_{vt} are positive coefficients and $\dot{K}_t = dK_t/dt$.

Let us now introduce a very simple form of uncertainty in our model. We assume unexpected short-run variations in how well the firm's managers succeed in adapting it to changing external conditions, which cause stochastic changes in the productivity of labor, i.e. in the amount of labor required per unit of capital. All such changes, ε_t, are assumed to be normally distributed with zero expectation and constant variance σ. The ε_t are also independent with zero covariance between all pairs of time intervals.

Since the decisions taken by the managers normally should have impacts of certain duration there is reason to expect that each random change in labor productivity will persist for some period of time. For simplicity we assume that these periods are of the same constant length T and that there are no other random impacts than those working through variations in the labor-capital ratio. This means

$$\left.\begin{aligned} \alpha_{Lt} &= \alpha_L + \int_{i=0}^{T} \varepsilon_{t-i}\,di \\ &= \alpha_L + \varepsilon_{Lt} \\ \alpha_{Kt} &= \alpha_K \text{ and } \alpha_{vt} = \alpha_v \end{aligned}\right\} \tag{5}$$

where α_L can be seen as the long-run time invariant value on the labor/capital ratio and ε_{Lt} as the sum of all short-run changes in this ratio which have occured during the historical period t-T to t.[1]

In addition we assume that depreciation, δ, of the capital stock, product price , p , price of capital goods, p_K, and wage rate, p_L, are constant. Defining the rate of return on capital, r_t, as the ratio between profit, V_t, and value of the capital stock, $p_K K_t$, we obtain

$$r_t = V_t/p_K K_t = \frac{p\alpha_K}{p_K}(1-\alpha_v v_t^2) - \frac{p_L}{p_K}(\alpha_L+\varepsilon_{Lt}) - \delta \qquad (6)$$

We imagine a world without information costs, which, in our case, means a perfect market in which the firm's shares are traded. On this market all existing and potential shareholders of the firm have access to equal and cost-less information about the actual price of its shares and all other relevant facts about the firm. The market is cleared in each time interval when the dividend yield plus the capital gains per share equals the shareholders' discount rate, κ. The solution to this differential equation will then imply a value of the shares equal to the present value of the expected future dividends. This value at t is

$$Z_t = \int_{j=t}^{\infty} p_K K_j \ (r_j - v_j) \ e^{-\kappa(j-t)} \ dj \qquad (7)$$

where K_j, r_j and v_j now express the mean expected future values of the capital stock, rate of return and rate of growth respectively conditional on the presence of earlier random impulses lasting at most till the future date t+T.

Assuming that the shareholders are indifferent between certainty and uncertainty we now take the neoclassical standpoint that the firm chooses a growth path in the future, which maximizes Z_t, given that the initial

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[1] Note that all changes before t-T do not longer exist at t due to the restricted duration T of each change.

capital stock, K_t, is predetermined from the history of the firm. [1)] As will be seen in the next section, this planned growth path must be continuously revised with the passage of time due to new stochastic disturbances and disappearance of old ones.

2.2 Optimal behavior of the firm

The optimality condition

On the growth path which maximizes the share value, the following Euler differential equation must be satisfied[1]

$$r_t + (\kappa - v_t) \frac{\partial r_t}{\partial v_t} - \kappa = \dot{v}_t \frac{\partial^2 r_t}{\partial v_t^2} \tag{8}$$

where $\dot{v}_t = dv_t/dt$. On the assumptions that the discount rate is greater than the growth rate and that the adjustment cost function increases at an accelerating rate, the present value of investment projected at any time approaches zero as the date of the investment recedes into the distant future. Thus, fulfillment of this optimality condition represents a true interior maximum.

Consider now a steady state situation in the case where no random impulses prevail. In such a situation the rate of return and the growth rate are constant over time, i.e., $r_t = r^*$, $v_t = v^*$ and $\dot{v}_t = 0$. This means that (8) can be written

1) Lintner (1964) has shown that uncertainty could be compatible with the presupposition that the present value of expected future dividends, given a constant discount rate, is a maximizing variable even when the shareholders have risk aversion. This is done by the use of a parabolic utility function with the property of a constant proportional risk aversion equal to all shareholders. However, the result requires a more simple form of uncertainty than we have assumed namely that the stochastic variations in the firm's rate of return, ε_{Lt} are independent over time.

$$r^* + (\kappa - v^*) \left(\frac{\partial r}{\partial v}\right)^* = \kappa \tag{9}$$

What (9) tells us is that when the dynamic restraints are only in the form of growth costs, the firm's investment rule is that the marginal rate of current dividends from growth should equal the discount rate. Optimality conditions for investment with the same meaning as (9) have been derived earlier, given the same basic assumptions as value maximization, growth costs, etc.[1]

Other interesting findings are apparent from (9). First, the firm should invest to obtain an average rate of return which is higher than the discount rate. This result is at variance with traditional investment theory which states equality in optimum between these two rates under perfectly competitive conditions, but it agrees with the conclusion arrived at by Gordon and Lintner (op cit). Second, only in the special case of a stationary state, when the size of the firm is constant, will the rate of return equal the discount rate.[2]

The growth rate

We are now in a position to derive the firm's rate of growth as a function of the product price and average cost. The first step in this derivation is to linearise the left side of the optimum condition (8) around *the steady growth rate*, v^*, and insert (6). Then we obtain[3]

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[1] Gordon (1962), chapt 4, Lintner (1964), Bergström and Södersten (1976) and Eriksson (1978), chapt 4.

[2] These results follow from our assumptions regarding the adjustment cost function, which mean that $(\partial r/\partial v)^* < 0$ for $v^* = 0$ and $(\partial r/\partial v)^* = 0$ for $v^* = 0$. Note also that if the adjustment cost function were only linearly increasing instead of increasing at an accelerating rate with the growth rate the marginal rate of present value $MR(v^*) = r^* + (\kappa - v^*)(\frac{\partial r}{\partial v})^*$ should be constant due to $(\frac{\partial^2 r}{\partial v^2})^* = 0$. Obviously no interior optimum could then exist. Depending on $MR \gtreqless \kappa$ the optimal growth rate should become either infinitely great, indeterminate or zero.

[3] Appendix 1.

a first-order differential equation for *the transitory growth rate*, v_t, whose general solution is

$$v_t = v^* - \varepsilon_{vt} \tag{10}$$

where[1]

$$\varepsilon_{vt} = a \int_{j=t}^{t+T} \varepsilon_{Lj}\, e^{-(\kappa - v^*)(j-t)}\, dj \tag{11}$$

and $a = p_L/p)/\ 2\ \alpha_K \alpha_v$ (12)

ε_{vt} expresses the discounted sum, ε_{vt}, of all random impacts still prevailing during the nearest period t to t+T. From (10) - (12) it is then clear that a lower real wage rate, p_L/p, or higher capital productivity, α_K, will reduce the size of adaptation in the firm's rate of growth in response to the random shocks. Moreover, a more steeply rising adjustment cost function (higher α_v) has a similar effect on its dynamic reaction pattern. This result is intuitively clear; high costs associated with the installment of capital make the firm unwilling to quickly change the rate of capital accumulation in response to temporary changes in the rate of return.

The next step is to determine what explains the growth rate v*. Inserting the rate of return function in (9) we find that v* is a function which increases at a declining rate when the sales margin increases and that v* equals zero when the sales margin is zero.

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1 Note that $\varepsilon_{Lj} = \left\{ \int_{i=j-t}^{T} \varepsilon_{t-i}\, di \right\}$ for $j > t$ and that ε_{Lj} diminishes to zero when j increases to t+T due to the expected values on all new random impacts are zero.

Using a logarithmic form of this relationship we get:[1]

$$v^* = \beta_0 \, m^{\beta_1} \tag{13}$$

$$m = (p-c)/c \tag{14}$$

$$c = [p_L \alpha_L/\alpha_K + p_K(\kappa+\delta)/\alpha_K] \tag{15}$$

where $\beta_0 > 0$, $0 < \beta_1 < 1$. The sales margin m is given by (14). It is easily verified that c is the minimum average cost when the firm does not grow. $p_L\alpha_L/\alpha_K$ and $p_K(\kappa+\delta)/\alpha_K$ are the cost parts accruing to labor and capital respectively and $p_K(\kappa+\delta)$ is the well-known user cost of capital. Now, one can see that the behavior implied in (13) agrees with the common view that not until the exogenously given product price exceeds the average cost can the competitive firm permanently profit from expansion.[2]

The valuation ratio

The valuation ratio of the firm, z_t, is the ratio between the value of its shares, Z_t, and value of its assets, $p_K K_t$. The equilibrium condition for clearing the market for the shares - see page 5 above - can be translated into changes in this ratio: The percentage change in the valuation during every time interval plus the dividend yield equals the net discount rate. After some algebraic simplifications the solution of this first-order differential equation gives [3]

$$z_t = Z_t/p_K K_t = z^*(1 + \varepsilon_{Dt}) \tag{16}$$

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[1] Appendix 2. The logarithmic specification of the function for the firm's growth rate, as well as of the functions for the rates of exit and entry below, is done in order to make it easier to trace the directional effects in section 4. Note that this does not change the sign of the effects.

[2] Compare Englund (1979), chapt 2.

[3] Appendix 2.

steady-state valuation ratio and

$$\varepsilon_{Dt} = \int_{j=t}^{t+T} \left[\frac{[(r_j - v_j) - (r^* - v^*)]}{z^*} \right] e^{-(\kappa - v_j)(j-t)} dj \qquad (17)$$

ε_{Dt} can be seen as the present value of the future differences between the transitory and steady-state values of the dividend yield during the space of time t to t+T. Since ε_{Dt} is built up by the same random elements as in the stochastic term, to the growth rate, v_t, the short-run changes in z_t are similar to those of v_t. Therefore also all exogenous influences, which increase the speed at which the firm's adjustment costs rise with the growth rate will dampen the swings in z_t.

z* is given by[1]

$$z^* = (r^* - v^*)/(\kappa - v^*) \qquad (18)$$

This equation says that the steady-state valuation ratio equals the ratio between the steady-state paid-out dividens per unit value of assets (r-v*) and the net discount rate (κ-v*), Using (18) it can be shown that z* increases at an accelerating rate with incrased sales margin, m, and that m = 0 involves z* = 1.[2] In logarithmic forms this relationship can be expressed as

$$z^* = \gamma_0 \, m^{\gamma_1} + 1 \qquad (19)$$

where $\gamma_0 > 0$ and $\gamma_1 > 1$.

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1 Appendix 2.

2 Appendix 2.

3. DETERMINANTS OF EXIT AND ENTRY

3.1 The exit function

We have discussed how the firm's optimal growth rate and valuation ratio are determined. In this section we will deal with quite another type of decision facing the firm, namely whether or not it should stop producing altogether.

Let us first present some basic assumptions in our analysis. Besides a competitive market for the firm's shares, a competitive market for its existing real assets is also assumed. On this market the firm's owners can sell its assets at an exogenously given unit price, $\hat{p}_K$. We conceive of all buyers as being outside the industry in which the firm operates so that selling means that there is exit from the industry. Since existing equipment in reality probably has a lower value in alternative uses we assume that $\hat{p}_K < p_K$ which is the price on new capital goods. The ratio $\pi_D = (p_K - \hat{p}_K)/p_K$ (the percentage loss brought down upon the owners due to the selling) can then be seen as a measure of the relative cost of exit.

Wealth maximization means that the owners will cease the production and sell the assets if they find that the value of share falls short of the value of assets less the liquadation costs. Expressed in relative terms this rule for exit means that the transitory valuation ratio, z_t, is less than one minus π_D, i e

$$z_t = z^*(1 + \varepsilon_{Dt}) < (1 - \pi_D) \qquad (20)$$

We define the probability of exit, b_{Dt}, as the probability that the exit criterion is met at t, i e

$$b_{Dt} = \text{Prob}\ [z^*(1+\varepsilon_{Dt}) < (1-\pi_D)] \tag{21}$$

It is obvious that b_{Dt} is a decreasing function of z^* or π_D. Applying a logarithmic specification of this relationship we get

$$b_{Dt} = \lambda_0 (z^*)^{\lambda_1} (1-\pi_D)^{\lambda_2} (1+\varepsilon_{Dt})^{\lambda_3} \tag{22}$$

where $\lambda_0 > 0$, $\lambda_1 < 0$ and $\lambda_2 > 0$. (22) is the individual exit function showing the determination of the *transitory rate of exit* at t for each single firm. On the basis of (22) we also define the *permanent rate of exit*, b_D^*, as determined by

$$b_D^* = \lambda_0 (z^*)^{\lambda_1} (1-\pi_D)^{\lambda_2} \tag{23}$$

3.2 The entry function

Let us think of a pool of potential entrants consisting of individuals which are about to start production in the industry. These entrepreneurs are assumed to have the same time preference and the same managerial ability as existing producers once they have aquired the industry-specific know how as the existing producers have. New firms are thus presumed to display a long-run behavior which is similar to the existing ones.

Since our analysis concerns entry into a competitive industry it is reasonable to presume that only the costs of acquiring this knowledge constitute the costs of barriers to entry. Now, we introduce a time-invariant relative entry cost variable common to all potential entrants, π_E, defined as these costs per unit value of assets in a new firm. We also assume (in analogy to the above postulated variations in the transitory valuation ratio of an already established firm) that the transitory relative entry cost, π_{Et}, for a typical entrant is randomly determined by

$$\pi_{Et} = \pi_E(1+\varepsilon_{Et}) \tag{24}$$

where ε_{Et} is the random component which is specific for this entrant.

Given these assumptions wealth maximization should imply that he decides to enter as soon as he finds that the present value of his new firm net of his initial investment expenditures exceeds the entry costs. Expressed in relative terms the criterion of entry is[1])

$$(z^*-1) > \pi_{Et} \tag{25}$$

We define the probability of entry as the probability that the entry criterion is met, i e

$$b_{Et} = \text{Prob}\ [(z^*-1) > \pi_E(1+\varepsilon_{Et})] \tag{26}$$

Apparently higher z^* or lower π_E implies higher value of b_{Et}. Applying again a logarithmic specification, we get

$$b_{Et} = \theta_0(z^*-1)^{\theta_1}\ \pi_E^{\theta_2}(1+\varepsilon_{Et})^{\theta_3} \tag{27}$$

where $\theta_0 > 0$, $\theta_1 > 1$ and $\theta_2 < 0$. (27) is the individual entry function showing the determination of the *transitory rate of entry* at t for each single entrant. Then we get the corresponding function for the *permanent rate of entry* given by

$$b^*_E = \theta_0(z^*-1)^{\theta_1}\ \pi_E^{\theta_2} \tag{28}$$

[1] Because the entry costs are the only disadvantage faced by the potential producer he expects, after overcoming of this disadvantage, that the valuation ratio of his new firm will equal the long-run expected valuation ratio z* of existing firms.

4. MARKET EQUILIBRIUM

This section deals with the establishment of a dynamic equilibrium for the industry, given that the total demand for the industry's output expands at a constant rate over time. The first important step in modelling such an equilibrium solution is to specify the growth rate of the industry's output as a function of the product price. The second step is to state the total rate of growth of output to be equal to the total rate of growth in demand and examine the consequences of that.

4.1 The function for the total growth rate

Let us start the specification of this function by defining the industry output at time t as

$$Q_{Tt} = N_t \bar{Q}_t \tag{29}$$

where N_t is the number of firms in the industry and $\bar{Q}_t$ is the mean output from these firms. Differentiation of (29) with respect to time, then, gives

$$v_T = v_N + \bar{v} \tag{30}$$

according to which *the rate of growth in total production*, v_T, equals *the percentage change in the number of firms*, v_N, plus *the rate of growth in the mean production of existing firms*, $\bar{v}$. [1]

By assuming also a fixed relation, ρ, over time between the number of potential and existing firms, we find, due to the large number of both potential and existing firms that[2]

1) This interpretation of (30) presupposed in fact that there are no systematic association between size and probability of exit for each existing firm and between size and probability of entry for each potential firm.

2) See appendix 3

$$v_N = \bar{v}_E - \bar{v}_D = \rho\, b^*_E - b^*_D \qquad (31)$$

and

$$\bar{v} = v^* \qquad (32)$$

where $\bar{v}_E$ is the proportion (mean rate) of inflow of new firms and $\bar{v}_D$ is the proportion (mean rate) of outflow of old firms during each period. b^*_E and b^*_D are the individual permanent rates of entry and exit respectively given by equations (28) and (23). v^* is the steady-state rate of growth of each firm given by equation (13).

What remains to do is to express b^*_E, b^*_D and v^* as functions of p. This gives us[1]

$$v_T = \underbrace{\rho\theta_0\left[\gamma_0\left(\frac{p-c}{c}\right)^{\gamma_1}\right]^{\theta_1} \pi_E^{\theta_2}}_{\bar{v}_E} - \underbrace{\lambda_0\left[\gamma_0\left(\frac{p-c}{c}\right)^{\gamma_1} + 1\right]^{\lambda_1} (1-\pi_D)^{\lambda_2}}_{\bar{v}_D} + \underbrace{\beta_0\left(\frac{p-c}{c}\right)^{\beta_1}}_{\bar{v}} \qquad (33)$$

(33) is the function for the total rate of growth. The first, second and third terms on the right-hand side of this equation show the influence on that growth from entering, leaving and existing firms respectively. From this function it is clear that a higher output price, p, causes industry output to increase at more rapid rate.[2] It is also clear from (33): when p is equal to the minimum average cost, c, there is no entry of new firms nor any growth of existing firms (the first and the third terms become zero).

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[1] First, we insert (14) and (19) in (23) and (28). After that we insert (23) and (28) in (31) as well as (13) and (14) in (32). Then we get (33) from (30).

[2] Note that $\theta_0 > 0$, $\theta_1 > 0$, $\theta_2 < 0$, $\lambda_0 > 0$, $\lambda_1 < 0$, $\lambda_2 > 0$, $\gamma_0 > 0$, $\gamma_1 > 0$, $\beta_0 > 0$ och $\beta_1 > 0$.

4.2 The equilibrium solution

At any instant of time the supply of the industry's output is fixed. But over time output can increase. This might be visualized as rightward shifts in a vertical short-run supply curve of the industry. Recall that the rate at which this curve is moving equals the total growth rate, v_T. Let us now introduce a negatively sloped industry demand curve and assume that it moves rightwards at an exogenously given rate, η.[1] In the long run equilibrium for the industry implies a series of short-run equilibria, each of which is distinguished by the intersection of the short-run supply and demand curves at a time-constant output price. The necessary condition for such an equilibrium is [2]

$$v_T = \eta \qquad (34)$$

Equation (34) completes our model. We are now in a position to show how the growth of existing firms and the rate of entry and of exit of new and old firms are interrelated. We can do that in a summary way by describing some key relationships in the model graphically.

In Figure 1 the 45^o-line $v_T = \eta$ and the curve $v_T = f_T(p)$ depicts the equilibrium condition (34) and the total growth function (33) respectively. The curves $\bar{v} = f_v(p)$, $\bar{z} = f_z(p)$ and $\bar{v}_D = f_D(\bar{z})$ depict the functions for the firm growth rate (13), valuation ratio (19) and mean rate of exit (23) after that we have replaced m with $(\bar{p}-c)/c$. By multiplying (28) with ρ we obtain the func-

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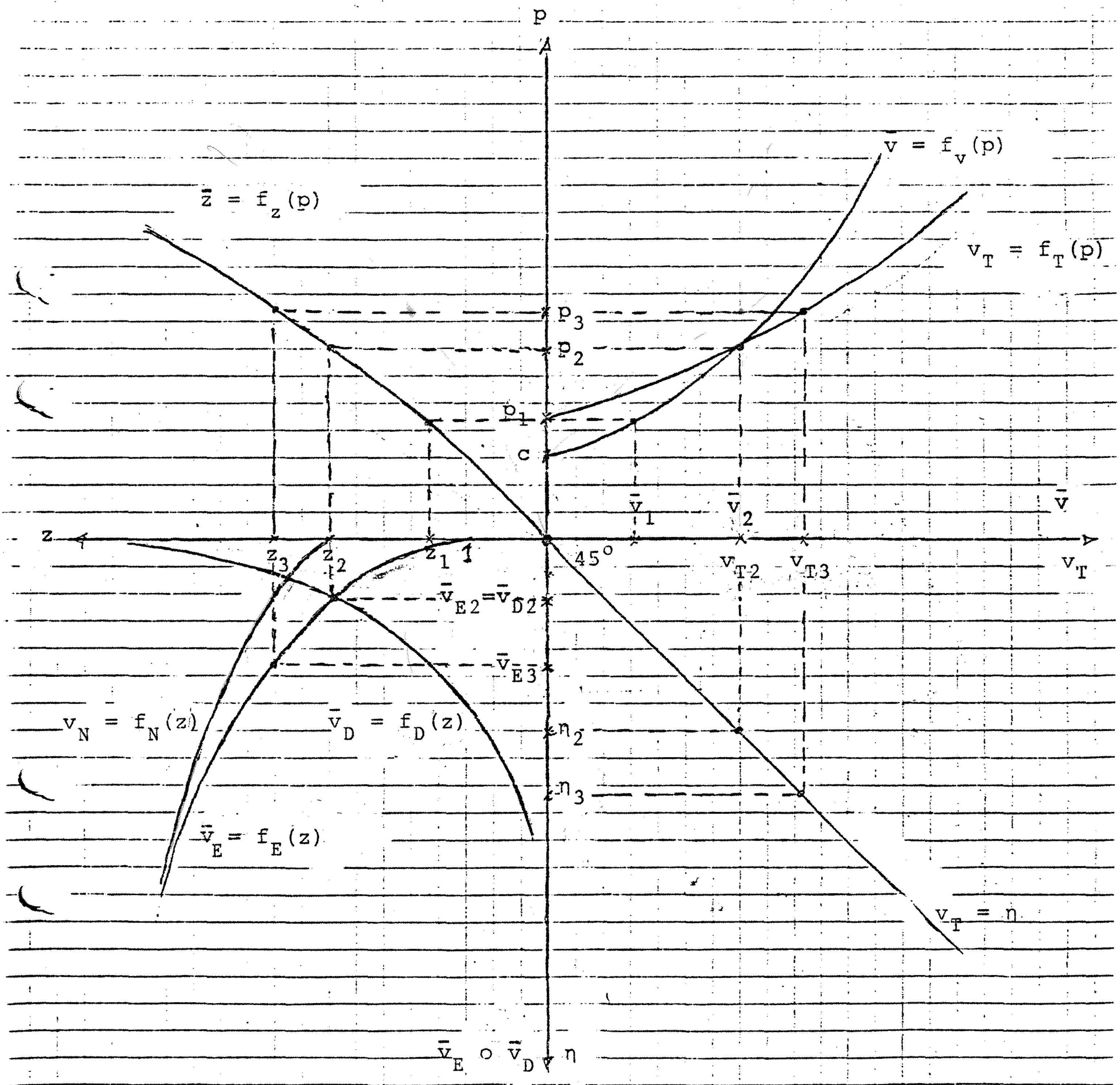
[1] That $\eta > 0$ can e.g. be a result of an exponential rise in the society's per capita income.

[2] (34) represents a stable equilibrium so far that any external disturbance which disrupts this dynamic equilibrium sets in motion forces producing the restoration of itself. Consider for instance an unexpected rise in η to a new higher permanent level. This makes the price increase at once to secure the short-run equilibrium. But the price increase increases the total growth rate during the next period. Thereby the price will fall again, which in turn retards the additional increase of the total growth rate during the period thereafter, etc; thus implying a course of the price and growth rate towards a new long-run equilibrium with higher values on these variables compared to those that existed before the rise in η.

Figure 1. The dynamic equilibrium for the industry



tion for the mean rate of entry described by the curve $\bar{v}_E = f_E(\bar{z})$. Finally the vertical distance between these two last mentioned curves gives the curve $v_N = f_N(\bar{z})$ for the percentage change in the number of firms.[1]

One important exogenous factor is the rate of change in demand. Let us first look at the case when demand decreases at a rate equal to the mean rate of exit. $(\eta < 0)$. That gives a situation in which the product price is just equal to the minimum average cost $(p = c)$. It implies that no firm grows $(\bar{v} = 0)$ and the valuation ratio for each equals one $(\bar{z} = 1)$, which in turn means that no firms will enter $(\bar{v}_E = 0)$.

Next, assume a stationary case with unchanged total demand $(\eta = 0)$. The loss of output from leaving firms now provides room for expansion by each remaining firm as well as entry by new firms and at the same time implies that the price exceeds the average cost. This means positive net profits. They also make the mean valuation ratio greater than one creating incentive for existing firms to expand and new firms to enter the industry. The dynamic equilibrium solution associated with $\eta = 0$ is seen from the figure as $\eta_1 = v_{T1} = 0$ implying $p_1 > c$, $\bar{v}_1 > 0$, $\bar{z}_1 > 1$ and $\bar{v}_{E1} > 0$.

Third, look at the expansion case when the total demand increases $(\eta > 0)$. Equilibrium requires that price must be higher than before. This strengthens the incentives for both existing producers to expand for entrepreneurs outside the industry to become producers. At the same time, the higher valuation ratio means a lower probability for each existing firm to want to leave the industry. This solution is pictured by $v_{T2} > 0$, $p_2 > 0$, $\bar{v}_2 > 0$, $\bar{z}_2 > 1$ and $\bar{v}_{D2} = \bar{v}_{E2} > 0$.

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[1] Both the slopes and positions of the curves can be verified easily by inspection of equations (13), (19), (23), (28), (33) and (34).

The model allows us to state the effects from some of the other exogenous factors as well. Consider for example that it becomes more costly for the firms to expand (expressed by a higher α_V). However, to ascertain the changes of direction of all endogenous variables in this case is very difficult. Only one thing is clear. The output price must rise. Taking into account the facts that η is exogenously given and the equilibrium condition $\eta = \bar{v} + \bar{v}_E - \bar{v}_D$ must hold there are only two possible outcomes of directional changes for the rest. These are: a lower (higher) $\bar{v}$ is associated with higher (lower) $\bar{z}$ and $\bar{v}_E$ and lower (higher) $\bar{v}_D$.

Nothing has been said about the effects following from changes in the costs of entry and of exit. This is a subject that has received considerable attention in most models of market behavior. Both casual observation and common sense tell the same story, namely, that decreased costs of entry and of exit speed up the process of structural change through creation and destruction of firms. This conclusion also follows from our model. But the effects on the other market variables are not so clear by intuition. Therefore, let us see if we can ascertain these effects.

To begin with, recall that π_E and π_D, stand for the relative costs of starting a new firm and liquidating an old one respectively. Consider the case when π_E falls. The rise in the rate of inflow of firms will now take a greater part of the exogenously given demand expansion from the already established firms thereby depressing the output price and valuation ratio. This, in turn, will drive relatively more firms out of the industry. Then, consider the case when π_D falls. The implied rise in the rate of outflow of firms leaves greater room for expansion by remaining and new firms. The price and valuation ratio will rise which will induce more outsiders to enter the industry. [1]

The above effects of changes in the exogenous variables are summarized in Table 1.

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1 The consequences of decreased π_E or π_D can be verified from (33) and (34) by total differentiation of these equations with respect to a change in π_E and π_D respectively.

Table 1. The long-run equilibrium effects of some exogenous factors

Exogenous factors	Variables: Total growth rate (v_T)	Product price (p)	Firm growth rate ($\bar{v}$)	Valuation ratio ($\bar{z}$)	Rate of entry ($\bar{v}_E$)	Rate of exit ($\bar{v}_D$)	Rate of change in number of firms (v_N)
Growth of total demand (η)	+	+	+	+	+	-	+
Growth costs (α_v)	0	+	-	+	+	-	+
			or +	-	-	+	-
Costs of entry (π_E)	0	+	+	+	-	-	?
Costs of exit (π_D)	0	-	-	-	-	-	?

Remark: +, 0 and - mean an increase, no change and decrease in the variables.

4.3 Earlier models

In this section we will point out certain interesting similarities between our results and those from earlier well-known microeconomic models of market structure and performance.

A large number of theoretical studies have dealt with the problem of ascertaining an optimal pricing strategy for established firms, under imperfect competition, taking into account the probability of entry.[1] Depending on different choices of assumptions, the authors of these studies conclude that the optimal price set by firms should be either above, equal to, or below the highest price precluding entry (the limit price). Common to all of them is, however, the conclusion that the optimal price is higher than the competitive level, implying that the established firms get the benefit of positive excess profits. This result is a consequence of their assumption of imperfect competition. We have obtained the same result even when total demand does not change. Our result follows from the assumption that there is an ongoing exit by firms[2] and dynamic constraints restricting the increase in production from existing and entering firms.

Our model also contains the notion of a limit price. The output price, which makes the net rate of entry equal to zero in our case (p_2 in the figure) can be regarded as a limit price, since this price implies a constant number of firms in the industry (the rate of entry equals the rate of exit). Because higher barriers to entry raise p_2, our analysis is consistent with the limit pricing theories insofar as the extent to which established firms can maintain a price above minimum average cost depends positively on the height of barriers to entry.

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[1] Harrod (1951), Hicks (1954), Modigliani (1958), Osborne (1964), Baron (1971) Gaskin (1971), etc.

[2] The possibility that firms disappear is often disregarded by earlier writers or treated in a very rough manner. The rate of exit is simply defined as a negative rate of entry, which means that both entry and exit cannot exist simultaneously.

In addition, it is interesting to compare our model with the Marshallian theory of supply and with Walrasian general equilibrium theory. According to Marshallian theory the long-run equilibrium price equals minimum average cost and increases in demand only increase the number of firms. On the assumption that there are no entry costs but that there are adjustment costs, it is immediately clear that our model produces the same result. The unlimited inflow of new firms depresses the equilibrium price to the cost level c and firms do not grow. Then, increases in industry demand are met solely through entry of new firms.

In general equilibrium models of a Walras-type there is no entry or exit of firms. If we, in this tradition, were to assume prohibitive entry and exit costs, precluding any turnover of firms, we would also find that the increase in aggregate production from existing firms always equals the increase in total demand. When there is no change in demand we get the classical competitive equilibrium with constant size and zero profits of each firm. Thus, with respect to the determinants of industry supply, Marshallian and Walrasian theories can be formulated as special cases in our model, depending on the assumptions made regarding entry, exit and growth costs.

Our results concerning the effects of external factors on the industry equilibrium can be compared to those of empirical studies which formulate the testing equations conforming to the notion that most firms grow with short-run variations around a long-run trend. Almost all of these studies give convincing support to our conclusion that a higher rate of demand, by raising the profitability of existing firms, induces more individuals to start new firms and, at the same time, increases the chance of survival of each existing firm. In addition, they support the statements made above that higher barriers to entry decrease not only the rate of entry but also the rate of exit.[1]

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[1] See Hause (1962), Mansfield (1962), Orr (1974) and Du Rietz (1980).

4. CONCLUDING REMARKS

In this paper a model of industry dynamics has been presented showing how growth of existing firms, rate of entry and rate of exit are interrelated. Besides that the model has given some valuable insights into the process of structural change and the existence of an equilibrium solution in a competitive industry.

It has been possible for us to explain the theoretical riddle of why firms simultaneously enter into and leave an industry. We have also shown that positive excess profits (profit rates above the rate of return required by the shareholders on their financial investments) can persist even in perfect competition and with constant average costs in the long run, a result which is due to the presence of dynamic restraints (growth costs and costs of entry).

In addition our model is consistent with Gibrat's law, which states that the growth of each firm is determined in a stochastic way and the rate of growth is independent of firm size. Thereby, our comparative dynamic analysis has produced predictions of strong intuitive appeal, which, furthermore, are of such nature that they should be easily tested on empirical data. In the cases where it has been possible to make comparisons with earlier econometric studies, our results conform to these.

APPENDIX 1. The optimality condition, the functions for transitory growth rate and for steady-state growth rate

According to (6) and (7) the criterion functional can be written as[1]

$$Z = \int_0^\infty \left\{\left[p\alpha_K(1-\alpha_v v^2) - p_L\alpha_L - p_K\delta - p_L\varepsilon_L\right]K - p_K\dot{K}\right\}e^{-\kappa j}dj$$

$$= \int_0^\infty H\{K, \dot{K}, j\}dj \qquad (1)'$$

where $\dot{K} = dK/dj$, $v = \dot{K}/K$ and ε_L is the discounted sum of random impulses occurred before t = 0 which will at most last to the future date t+T.

The Euler differential equation that must be satisfied for maximization of this functional is[2]

$$\frac{\partial H}{\partial K} - \frac{d(\partial H/\partial \dot{K})}{dj} = e^{-\kappa j}\Big\{p\alpha_K(1-\alpha_v v^2) - p_L\alpha_L - p_K\delta - p_L\varepsilon_L - 2p\alpha_K(\kappa-v)\alpha_v v - p_K\kappa + 2p\alpha_K\alpha_v\dot{v}\Big\} = 0 \qquad (2)'$$

After division with p_K, we find that the sum of the first four terms, the fifth term and the seventh term within the bracket { } is r, $(\kappa-v)\partial r/\partial v$ $-\dot{v}\partial^2 r/\partial v^2$ respectively. Thus (2)' can be expressed as

$$r + (\kappa-v)\frac{\partial r}{\partial v} - \kappa = \dot{v}\frac{\partial^2 r}{\partial v^2} \qquad (3)'$$

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[1] To simplify the presentation the time subscripts are deleted. The definitions of the variables are found on pp. 4-7.

[2] Note that $\partial(\alpha_v v^2)/\partial K = -2\alpha_v v \cdot \dot{K}/K^2 = -2\alpha_v v^2/K$ and $\partial(\alpha_v v^2)/\partial\dot{K} = 2\alpha_v v/K$ and $\dot{v} = dv/dj$.

Reintroducing the time subscripts we find that (3)' is the optimality condition (8). Q.E.D.

We perform a Taylor expansion of the left side of (3)' around the steady state growth rate v* and use the fact that $\partial^2 r/\partial v^2 = \partial^2 r^*/\partial v^{*2} = -2p\alpha_K\alpha_v/p_K$. This gives

$$\left[r^* + (\kappa - v^*)\frac{\partial r^*}{\partial v^*} - \kappa\right] - \frac{p_L}{p_K}\varepsilon_L$$

$$(v-v^*)(\kappa-v^*)\frac{\partial^2 r^*}{\partial v^{*2}} = \dot{v}\frac{\partial^2 r^*}{\partial v^{*2}} \qquad (4)'$$

Since the sum of the terms in the bracket [ ] is zero we get[1]

$$\dot{v} = (\kappa - v^*)(v - v^*) + (p_L/2p\alpha_K\alpha_v)\varepsilon_L \qquad (5)'$$

The solution of differential equation (5)' gives the function (10) for transitory growth rate. Q.E.D.

Now to the determinants behind v*. From

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[1] This is clear from (3)' because $\dot{v} = v$ when $r = r^*$ and $v = v^*$.

$$r^* + (\kappa - v^*)\frac{\partial r^*}{\partial v^*} = \kappa \qquad (6)'$$

follows that

$$\frac{p\alpha_K}{p_K}(1-\alpha_v v^{*2}) - \frac{p_L\alpha_L}{p_K} - \delta - (\kappa - v^*)$$

$$2\,\frac{p\alpha_K}{p_K}\,\alpha_v v^* = \kappa \qquad (7)'$$

After some algebraic manipulations we get

$$v^{*2} - 2\kappa v^* = \left\{\frac{p_L\alpha_L + p_K(\kappa+\delta)}{p\alpha_K} - 1\right\}$$

$$/\alpha_v = \left(\frac{c}{p} - 1\right)/\alpha_v \qquad (8)'$$

The solution to (8)' is

$$v^* = \kappa \pm \sqrt{\kappa^2 + \left(\frac{c-p}{p}\right) / \alpha_v} \qquad (9)'$$

Because $\kappa > v^*$ the positive root can be rejected implying

$$v^* = \kappa - \left[\kappa^2 + \left(\frac{c-p}{p}\right) / \alpha_v\right]^{1/2} \qquad (10)'$$

It is seen from (10)' that $p = c \rightarrow v^* = 0$, $p > c \rightarrow v^* > 0$ and $\partial v^*/\partial[(p-c)/c] > 0$ for $p > c$. Approximating (10)' with logarithmic form we get

$$v^* = \beta_0\left(\frac{p-c}{c}\right)^{\beta_1} = \beta_0\, m^{\beta_1} \qquad (11)'$$

(11)' is the function (13) for steady-state rate of growth. Q.E.D.

APPENDIX 2. The functions for transitory valuation ratio and steady-state valuation ratio

Differentiation of the valuation ratio

$$z = Z/p_K K \qquad (12)'$$

with respect to time gives

$$\dot{z} = [p_K K\dot{Z} - Z\dot{p}_K K]/(p_K K)^2 = (\dot{Z} - vZ)/p_K K \qquad (13)'$$

Since $\dot{Z} = \kappa Z - p_K K(r-v)$ according to the stock-market equilibrium (13)' can be expressed as

$$\dot{z} = (\kappa - v)z - (r - v) \qquad (15)'$$

Due to $\dot{z} = 0$ when $r = r^*$, $v = v^*$ and $z = z^*$

$$z^* = (r^* - v^*)/(\kappa - v^*) \qquad (16)'$$

Adding $(\kappa - v^*)z^*$ and $(r^* - v^*)$ to the right side of (15)' we obtain

$$\dot{z} = (\kappa - v)z - (\kappa - v^*)z^* - [(r - r^*) - (v - v^*)] \qquad (17)'$$

the solution of which is

$$z = \int_0^\infty \{(\kappa - v^*)z^* + (r - r^*) - (v - v^*)\} e^{-(\kappa - v)j}\, dj \qquad (18)'$$

Approximate $\int_0^\infty (\kappa - v^*)\, z^* e^{-(\kappa - v)j}\, dj$ with z^*. Then we get from (18)' the function (16) for the transitory valuation ratio. Q.E.D.

Now to the determinants behind z*. From (10.)' we get

$$(\kappa - v^*) = \left[\kappa^2 + \left(\frac{c-p}{p}\right) / \alpha_v\right]^{1/2} \qquad (19)'$$

By the definitions of r^* and c it follows

$$(r^* - \kappa) = \alpha_K (p-c)/p_K - p\alpha_K \alpha_v v^{*2}/p_K \qquad (20)'$$

Now insertion of (19)' and (20)' in

$$z^* = (r^* - \kappa)/(\kappa - v^*) + 1 \qquad (21)'$$

gives

$$z^* = 1 + \left\{\frac{\alpha_K}{p_K}(p-c) - \frac{p}{p_K}\alpha_K\alpha_v \left[\kappa - \left(\kappa^2 + \left(\frac{c-p}{p}\right) / \alpha_v\right)^{1/2}\right]^2\right\} / \left[\kappa^2 + \left(\frac{c-p}{p}\right) / \alpha_v\right]^{1/2} \qquad (22)'$$

It is seen from (22)' that $p = c \rightarrow z^* = 1$, $p > c \rightarrow z^* > 1$ and $\partial z^*/\partial[(p-c)/c] > 0$. Logarithmic approximation of (22)' gives

$$z^* = \gamma_0\left(\frac{p-c}{c}\right)^{\gamma_1} = \gamma_0 m^{\gamma_1} \qquad (23)'$$

(23)' is the function (19) for the steady-state valuation ratio. Q.E.D.

APPENDIX 3. The functions for the percentage change in number of firms and mean rate of firm growth

If N_D, N, N_E and N_P denote the number of leaving, existing, entering and potential firms respectively at time t we get from (22) and (27) in the text

$$N_D/N = \sum_{i}^{N} b_D^i/N = b^*_E \sum_{i}^{N} (1 + \varepsilon_D^i)^{\lambda_3} /N \qquad (24)'$$

$$N_E/N_P = \sum_{j}^{N_P} b_E^j/N_P = b^*_E \sum_{j}^{N_P} (1 + \varepsilon_E^j)^{\theta_3} /N_P \qquad (25)'$$

where b_D^i and b_E^j are the individual probability of exit for existing firm, i, and probability of entry for potential firm, j, at t. ε_D^i and ε_E^j are the random impacts on b_D^i and b_E^j respectively.

Since ε_D^i and ε_E^j can be regarded as symmetrically distributed around 0 over all existing and potential firms N_D/N approaches b_D^* when N becomes large and N_E/N_P approaches b_E^* when N_P becomes large, i.e.

$$N_D/N = b_D^* \qquad (26)'$$

$$N_E/N_P = b_E^* \qquad (27)'$$

v_N is defined as

$$v_N = (N_P/N)(N_E/N_P) - (N_D/N) \qquad (28)'$$

Inserting (26)' and (27)' in (29)' give

$$v_N = \rho\, b^*_E - b^*_D \qquad (29)'$$

where $\rho = N_D/N$. (29)' is (31). Q.E.D.

It follows from (10) that

$$\bar{v} = \sum_{i}^{N} w^i v^i = \sum_{i}^{N} w^i (v^* - \varepsilon_v^i) \qquad (30)'$$

where $w^i = Q^i / \sum^N Q^i$ and Q^i is the output from existing form i. v^i is ^{i} the transitory growth rate of this firm and ε_v^i is the sum of random impact on v^i. Since ε_v^i can be regarded as symmetrically distributed around 0 over all existing firms $\bar{v}$ approaches v* when N becomes large, i e

$$\bar{v} = v^* \qquad (31)'$$

(31)' is (32). Q.E.D.

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