

# **PRODUCTIVITY MEASUREMENT REVIEW**

**PRODUCTION EFFICIENCY  
OF THE  
INDUSTRIAL FIRM**

**SOME METHODS  
OF MEASUREMENT**

*by*  
**Erik RUIST**



Special Number December 1961

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Erik RUIST

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### EDITOR'S PREFACE

It has always been one of the aims of the Productivity Measurement Advisory Service to disseminate works of particular interest which have appeared in the less accessible languages of O.E.C.D. Member countries. This special number of *Productivity Measurement Review* is a cover-to-cover translation of a book originally published in Swedish. The author, now Director of Statistics at the "Jernkontor" (Swedish Steel Industry Association), was connected with the productivity measurement work of the European Productivity Agency from its inception, as witness his contribution to Volume I of the productivity measurement manual *Productivity, Efficiency and Wages*.

The purpose of the book is to collate, discuss and develop some of the methods suggested in various tracts, which purport to provide an assessment of productivity within the firm. The author includes detailed descriptions of conversion co-efficients for heterogenous components of output, as well as for the main input factors. The measurement of the effects of product changes is discussed, and a system is outlined for the re-allocation of indirect man-hours. We are proud to be able to present to our wide readership a scholarly work which satisfies a long-felt need by gathering together many ideas on the subject under a single cover.

The book contains a discussion on the notion and content of the terminology employed in productivity measurement, but the reader will soon notice that the author prefers to talk about "production efficiency". This is divided into two parts, "price efficiency" and "technical efficiency". The former measures the extent to which the firm has adapted itself to existing relationship between factor prices, while the latter has to do with the organization and use of the factors. It is the second aspect which most interests the author. He measures it by a ratio: unit output divided by a weighted aggregate of total input requirements for this unit output. An efficiency index yields unity in the base year and, with improving efficiency, greater than unity values for subsequent years. The component ratios for labour, raw material and capital inputs are, however, calculated as unit input requirements, where lower values indicate higher efficiency. These ratios can be combined into a total index from which the efficiency index is obtained as its reciprocal.

The author holds that only a combined measure thus constructed can express the degree of technical efficiency achieved in a firm, thereby aiding the main objective of any industrial firm, which is to obtain a given production at the lowest possible cost.

With this approach, the author is not only in line with plant-level productivity methods recently developed in other countries (e.g. at the

Aachen Institute for Research on Scientific Management) but also with a growing tendency in national productivity accounting theory to provide combined input measures, as is shown by Reddaway and Smith for the United Kingdom (*Economic Journal*, March 1960) or by the new Kendrick study on U.S. productivity trends.

If more satisfactory solutions could be found to overcome the ambiguities inherent in the weighting process necessary to combine heterogeneous inputs as well as the comparison of input structures which change under the impact of price substitution, this new approach of output per combined input might well mark a turning point in productivity measurement. It must be acknowledged that, as far as plant-level measurement is concerned, Dr. Ruist has brought us nearer to this objective. His theoretical discourse is well illustrated with examples of the practical applications of productivity measurement in real terms—applications which provide practical guidance for the Works Manager.

Our thanks go to the "Industriens Utredningsinstitut" (Industrial Institute for Economic and Social Research) who hold the copyright of the book, and to their erstwhile Head, Dr. Jan Wallander, for their kind permission to republish this stimulating treatise in the *Productivity Measurement Review* series.



*FOREWORD BY THE FORMER HEAD OF THE I.U.I.*

The concept of productivity plays a considerable part in general politico-economic discussions. At the same time, the measure of changes in productivity from the viewpoint of the individual firm is tremendously interesting, because it may be regarded as a key to an assessment of the quality of this firm's performance.

It is therefore natural that we, at the Institute, have long been interested in the problems raised in connection with the concept of productivity. In *Industriproblem 1950* the main problems were covered under the title "What is Productivity?" In the meantime, we have also considered it important to try to establish an inventory of the methods suggested to measure plant level efficiency.

The purpose of this book is to draw up such an inventory and at the same time to present a discussion on the notion of efficiency and its contents. This book, as well as the communication mentioned above, is the work of the Institute's former collaborator, Dr. Erik Ruist, now Chief of the Department of Statistics of the Jernkontor. Thanks to the kind permission of the Jernkontor, we have been able to call upon Dr. Ruist for this assignment. The Institute wishes to extend its grateful appreciation for this. Mr. Sven Fajerson, a research worker, collected the basic material for this book.

*Stockholm, May 1960.*

JAN WALLANDER.

### *AUTHOR'S INTRODUCTION*

The effort to raise technical efficiency in post-war years has been reflected in the recent tremendous interest in seeking to establish a measure of efficiency. In many countries, comparisons between different firms of one branch have been achieved according to various principles and by the use of one or several measures. These surveys were often followed by advice to the firms found to be in the worst positions. A number of such studies have been published.

In Sweden, until now, the interest in efficiency studies of this type has been quite limited. As far as is known, only two important surveys of an industry or sector have been made, one on book printing and the other on cotton spinning and weaving mills. On the other hand, several big firms have established indices aimed at following their own trend of efficiency from year to year. Such indices proved to be valuable additional assets to the financial considerations, though these must obviously predominate.

As yet, there is no literature available to explain the appropriate calculation methods of these measures. This book attempts to compensate to a certain extent for this omission. However, it is limited to the study of a measure of efficiency in the production departments and thus has nothing to do with the problem of the efficiency of the firm as a whole. Neither will there be any discussion of the possibilities for measuring the efficiency of different groups of workers or of machines. The results of the co-operation of all the production factors will be the only subject of analysis.

It must also be pointed out that the analysis of the causes of changes in efficiency should be adapted to the special structure of the firm or of the branch, and for this reason it is not possible to make any general conclusions. The discussion, therefore, is limited to the consideration of the measurement of efficiency only.

This book possesses the characteristics of a manual. Every chapter concerns a particular group of problems and may be read practically independently from the rest of the book. Chapter 1 is conceived as an introduction with a discussion of the principles on which the other chapters, whose purpose is more practical, are based. It is complete in itself and should supply a convenient basis for a decision as to which calculations should be started within the firm. The following three chapters cover the problems which arise in connection with the computation of the input figures of the different production factors, which provide the basis of the efficiency calculation. Chapter 2 covers such problems as are common to all input figures and depend on the annual changes in the composition of output. Special difficulties in the measurement of labour input are discussed in Chapter 3, and in Chapter 4 the other input figures, particularly that of capital.

The purpose of Chapter 5 is to explain how a single measure of efficiency can be obtained by combining the input figures calculated according to the methods described in Chapters 2 to 4. Chapter 5 may therefore be read immediately after Chapter 1 if desired. The final chapter presents some points of view concerning industry or sector surveys. It may also be read directly after Chapter 1. It attempts to illustrate the difference in the problems and possibilities of industry or sector surveys as opposed to internal firm indices. For a complete description of the methodology of an industry or sector comparison, the reader is referred to the comprehensive literature available.

## Chapter 1

### PRODUCTION EFFICIENCY — THE PRINCIPLES OF ITS MEASUREMENT

#### WHAT IS EFFICIENCY?

All firms try to be as efficient and as rational as possible. It is therefore natural that they should want a measure to assess in what proportion their efforts at scientific management yield any results. They also want to be able to answer the question: is the firm more scientifically managed and more efficient than previously?

Before trying to establish a standard or a measuring instrument, one must know precisely what has to be measured. Unfortunately, efficiency is a vague notion, and everyone can suit his own taste in giving meaning to the word. In fact, it is also quite natural that two people may have different opinions as to what is rational under definite conditions. No one can give an objective definition of what is meant by "rational" without establishing a relationship with what one wishes to accomplish by a definite action, in other words with an aim or a purpose. Before being able to say whether a firm is run scientifically or not, one must state the objective set to the firm and used as a basis of judgment.

After the objective has been set, the important thing is to find a criterion, a standard by which to measure how close to this objective one has come, or to what extent a definite action has brought one nearer to the aim. It so happens that such an aim may be stated in absolute figures: "Production should reach 500,000 tons within five years"—which makes it easy to establish a yardstick of the extent of accomplishment of purpose. But the type of purpose is more frequently: "as well as possible"—for example "as much profit as possible". Even an increase of profit from one year to the other is no sure token that the criterion "as much profit as possible" has been more successfully applied during the second year. A variation in the profits, whichever way it occurs, may have been caused as much by internal action as by external conditions beyond the firm's control. Fluctuations in the price of raw materials and semi-finished products and even of the firm's own productions are usually governed by external conditions.

A measure of the degree of accomplishment—a measure of efficiency—should rather bring out the difference between the effective amount of profit and the ideal amount that could have been attained under these given external conditions. This measure, however, can hardly ever be assessed practicably, and it remains very questionable whether it is at all possible to define theoretically how it should be calculated under ideal conditions.

It appears that in many cases the numerical value of an efficiency measure can be obtained by replacing the somewhat hazy maximum amount by, for example, the best achievement attained by others in the same branch, or by studying variations of efficiency rather than its absolute value. The important point under any circumstances, when the objective is of the "as well as possible" type, is to keep the measure of efficiency related to what could have been achieved in one way or another, but not to the absolute level attained. This will appear more positive in the light of what follows.

#### THE ECONOMIC ASPECT OF THE FIRM

The conception of the purpose for which a firm ought to strive depends upon the point of view from which it is considered: the national economy, the economy of the firm or the social aspect.

The objective of industry as a whole, considered from the national economic point of view, may be soundly worded as: "As much production as possible by making full use of the available production factors." This being stated, manpower stands out as the important production factor whose course is the least open to influence since, if long-term views are adopted, more capital goods can be purchased and raw materials can only constitute a bottleneck for production increase in special cases. For this reason among others, it is natural that the efficiency of industry as a whole from the national economic point of view often came to be measured in terms of volume of output per man-hour. Certainly, a subsidiary cause for this has been that consumption per individual, or long-term material standards of living, develop along parallel lines with production per individual. It is in this respect that the worker's status as a consumer, as much as his capacity to produce, has made it common to relate production results to labour input.

Many objections could be raised against this measure of efficiency. Thus, it can be pointed out that it also takes labour to build invested capital, so that a replacement of labour by machines in a firm does not necessarily involve a lessening of labour demand in the whole of the industrial cycle.

This objection gathers still more weight if one tries to measure the national economic efficiency of a branch of industry or of a firm by means of output per man-hour. Then, not only invested capital but also raw materials procured outside the branch or the firm contain transfers of manpower inputs from previous processing stages.

However, we will not discuss here the possibility of establishing a better measure. The purpose of this book is rather to investigate thoroughly the possibilities for a firm to measure its efficiency by the yardstick of its own objective.

#### THE FIRM'S SELF-DEFINED OBJECTIVE

It is obvious that the self-defined objective of a firm is never the one resulting as above from national economic considerations, because the firm cannot and need not consider the supply of any production factor as limited. However, for the majority of firms, it is not a simple matter to state an objective. Most firms lack a written policy, and the classical assumption

of economic theory that the firm strives towards the highest possible profits is relatively vague, as long as the period to be considered has not been defined. Also, in the short run, the effort to escape strong fluctuations in production and in activity, and the effort to create as good a social environment as possible, are often diametrically opposed to the effort to attain the highest possible profits.

To state an objective that can be applied to a majority of firms is therefore very difficult and will not be pursued here. This implies also that we are not seeking to build a measure of efficiency for the firm as a whole. Instead, we concentrate entirely on the production departments. One reason for this is that the definition of an objective for such departments should be very similar for most firms. Another reason is that their need for a measure of efficiency appears most important.

An objective for the production departments of an industrial firm could be worded as follows: to produce different products of given qualities and in quantities stated by the management of the firm (or by its sales departments) for the lowest possible cost. Only such costs as can be controlled by the production departments themselves will be considered in this respect. A measure of the accomplishment of this purpose can be considered as the measure of the firm's production efficiency. The following description is designed to give guidance as to when and how such a measure can be estimated. In this respect, it should be observed that no consideration was given to the operation of the purchase and sales departments. A firm which has attained a high degree of production efficiency cannot be expected to attain other than mediocre overall results in the face of unfavourable buying and selling conditions.

A measure of the extent to which the firm fulfills the purpose which has been formulated above for its production departments will indicate in what proportion its costs are higher than those of an optimum firm with exactly the same production. Obviously, the closer it operates to minimum cost, the more efficient will the firm be. It is, however, generally impossible to estimate the costs of an hypothetical optimum firm. It is, therefore, necessary to reduce somewhat such an ambitious project and to content ourselves with a measure of efficiency which provides incomplete information. If it is not possible to deduce how far the firm falls short of the minimum cost possible today, there are two related questions of almost as much interest which it is possible to answer, to some extent at least, *viz.*:

1. How efficient is the firm under present conditions in relation to some earlier period? and
2. How efficient is the firm under prevailing conditions in relation to other firms in the same field?

The question of the development of efficiency in the firm will be discussed first. However, in Chapter 6, some points of view on comparisons with others in the same field will be given.

The advantage in introducing this basis of comparison—the same firm at another period or a similar firm—is that it is no longer necessary to measure the absolute distance to the minimum, but simply to compare two similar distances. This is often considerably easier. Thus, without having any absolute measure of efficiency, one can say it is unchanged when actual and minimum costs have undergone the same changes.

An example of this is in the price changes of raw materials, manpower and other production factors. An equal price increase of all production factors swells actual and minimum costs by the same amount, and for this reason production efficiency is unchanged. On the other hand, a changed relationship between the prices of the different factors may have a different effect and should receive special treatment.

Even should changes occur outwith the control of the technical management, for example if the composition of production is changed as a result of modified orders from the sales department, minimum costs as a rule alter as much as actual costs, and production efficiency is not affected.

If the measure of efficiency is to be largely independent of the prices of the production factors, it should generally be based on the quantitative elements in the costs, namely on the input figures of the various production factors. Thus, roughly, lower input figures mean higher efficiency. Before this is developed, it is advisable to analyse a simple example of how the measure of efficiency could possibly be rendered independent of the composition of production.

The following discussion concerns labour input per unit produced, but the same reasoning can be applied to the other input figures.

#### THE PROBLEM OF SEVERAL PRODUCTS

If the firm makes several products, the input of different production factors per unit produced cannot be calculated directly, since there is no common measure of a unit of production. Even if one can measure total production in tons for example, it is not certain that this is appropriate for a measure of the type we are now contemplating. Thus, the labour input is different for a ton of bars or a ton of heavy plate in steelworks, a kilometre of thick compared with thin thread in a spinning mill, or for a ton of milk-bottles and a ton of small drug flasks in a glass factory.

It is necessary, therefore, to resort to some new system of measuring in order to obtain a total output figure for this special purpose. A method of converting all production into units of some standard product, with the help of evaluation co-efficients for each product, is then required. Such co-efficients are calculated with the aid of special studies for a "normal" base year in the case of a comparison in time, or for a standard firm in the case of a comparison between several firms in the same industry. When a calculation of the labour input per unit produced is at issue, we first establish, as exemplified in more detail in Chapter 2, the unit labour requirement for every product during the base period (or for the standard firm). By comparing these input figures with those of the standard product, we obtain the coefficient of every other product.

It was thus observed in a shoe factory that during the base period 1 man-hour was required to make a pair of standard men's shoes with sewn welts, and 1.6 man-hours for a pair of ladies' shoes of a given type. A pair of ladies' shoes therefore corresponded to 1.6 pairs of men's shoes. The total production of 1,000 pairs of ladies' shoes and 1,000 pairs of men's shoes can therefore be said to correspond to 2,600 pairs of men's shoes, that is 2,600 "standard" pairs. If 2,080 work-hours are expended this year on this output, the labour input is  $2,080/2,600 = 0.8$  hour per standard pair as against 1 for the base period. Thus, the labour input was reduced by 20 per cent.

It should be noted that the above method of determining an overall measure for the volume of output does not assume an annual allocation of man-hours for different products; merely for the base period. Thus the allocation of work is a once-for-all affair. Afterwards, quantitative data on the output of every product and the total number of production man-hours are sufficient. However, difficulties arise with the introduction of new products, or if the quality of the former products is changed substantially. Apart from this difficulty, which is discussed in Chapter 2, the multi-product firm can apply the same method as the firm producing a single product, irrespective of changes in the composition of output.

#### PRICE EFFICIENCY AND TECHNICAL EFFICIENCY

We can now return to the main question; how to calculate production efficiency? In order not to complicate the description unnecessarily, it may be appropriate to consider firstly, a production process with only two co-operating production factors, let us say labour and raw material. The expenditure of raw material can change with higher or lower labour loading, that is to say in proportion to the amount of labour put into production. Thus, raw material input per unit increases on account of a higher rate of wastage or spoilage, through breakage, etc., when labour input per unit is reduced. Then, evidently, it is a question of price to decide how much labour should be expended in order to save raw material. If the material is cheap, this can be an inducement to cutting down further the labour input and permitting a larger percentage of wastage, while on the other hand it pays to process more carefully and slowly with a more costly material. Thus, what is rational and therefore economical for such a firm depends upon the relationship between the price of labour and that of raw material. As this relationship changes from time to time, the optimum relationship between the labour input and raw material input is also modified. The expression "price efficiency"<sup>1</sup> has been coined for the measure indicating the extent to which a firm has adapted itself to the optimum position in every situation. There is no doubt that this plays a part in production efficiency.

Another way to increase efficiency, given a certain price relationship between raw materials and labour, is to arrange some organisational changes, so that either the raw material input or the labour input is reduced without changing the other, or even so that both diminish. This "intrinsic" method is, according to Farrell, a higher "technical efficiency". It is the complement of "price efficiency".

In order to distinguish one component from the other in a measure, it is necessary to be aware of the substitution possibilities between input of raw materials and labour input, according to the prevailing technique.

The difference between the two types of efficiency changes, which is important to the development of the analysis, is best emphasized by Diagram 1. The two input figures concerned have been chosen as co-ordinates. The continuous line curve constitutes a boundary which shows what com-

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1. M. S. Farrell, "The Measurement of Productive Efficiency", *Journal of the Royal Statistical Society*, Ser. A, Vol. 120 (1957), page 253.



binations of labour and raw material input are technically possible at the outset. The firm can consequently choose between the positions which lie on the curve and above the curve to the right. But it is only necessary to consider the positions on the curve, since there always is at least one point *on* the curve which is better (i.e. has a lower input of one of the production factors, without having a higher input of the other) than a given point above the curve. A curve of this type is hereafter called a *technical curve* for it describes the state of technology under definite conditions.

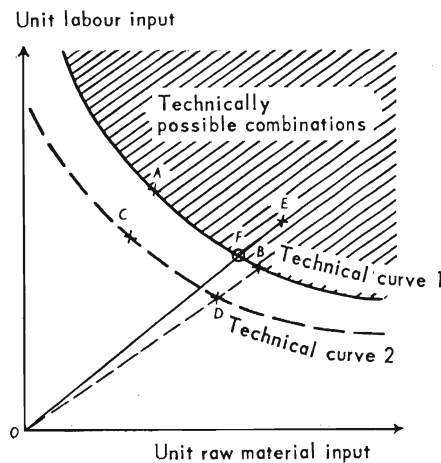


Diagram 1

We now suppose that *A* is the optimum point on the technical curve at the prevailing relationship between prices of labour and raw material, and that the firm chooses the combination of raw material and labour input per unit of its production which corresponds to this point. Now, if the price relation is altered in such a way that labour becomes more expensive compared to raw material, the optimum relationship between both input figures is modified and the minimum cost moves over to point *B*.<sup>1</sup> In order to preserve its high price efficiency, the firm must then undertake a change of disposition and advance along the curve to point *B*. Technical efficiency is not modified by such movements along the technical curve. On the contrary, an increased technical efficiency is obtained if both output figures are reduced by way of organisational changes, better deployment of the labour force, etc., which move the entire technical curve to position 2 on the diagram. A move from *A* to *C* brings about a higher degree of technical efficiency without a simultaneous change in price efficiency, while a move from *A* to *D* also results in a modified price efficiency.

1. Albeit provided that the firm can consider prices of labour, raw materials and other production factors as given, so that the firm itself has no influence whatsoever over them.

The technical curve's outlook must be known very accurately to whoever wants to estimate the price efficiency of the firm. In practice, one can seldom draw technical curves of such an accuracy as to permit this estimation.

Instead, when a comparison over time for a single firm is involved, one has to start from the supposition that the price efficiency of the firm is the optimum at all times, that is to say that the firm always found itself at the point of its technical curve which provided the minimum cost—or at least never was very far from it. Generally, this supposition is necessary in order to estimate technical efficiency. If this assumption is correct, a change in technical efficiency will bring about an equal change in production efficiency, which is exactly what we are looking for.

Because of the practical difficulties involved in measuring price efficiency, we shall not attempt to make such a measurement for the isolated firm. In Chapter 6, possible evaluations of price efficiency in an inter-firm comparison will be discussed in brief. The rest of this Chapter is concerned with the possibility of measuring technical efficiency.

#### A MEANS OF DETERMINING THE POSITION OF THE TECHNICAL CURVE

From the foregoing, it is obvious that technical efficiency is constant along a technical curve. The degree of efficiency to which a definite technical curve corresponds in a given state of things, according to the preliminary principles of reasoning, depends solely on how far it falls short of the prevailing "optimum technical curve" under the same conditions. This optimum technical curve describes the productive capacity of a firm which applies all known techniques and experience to economize on its input. Consequently, the optimum curve changes position as new technical experience develops. As already said, it is generally impossible to determine periodically the position of such a curve. We are, therefore, forced to establish, more or less arbitrarily, an optimum position as a basis of comparison. In this respect, the only practicable thing would be to compare in every case the actual figures with the absolute minimum point, the point where all input figures are zero. This corresponds to the origin of the diagram. This choice means that any move of the firm's technical curve toward the origin is regarded as enhanced technical efficiency, even if technical improvements in the rest of the industry are further ahead and if the firm has thus lost ground with regard to the best technical curve actually attained by others. Thus, at this point it has been necessary to depart from the principle that the comparison should be effected against a moving minimum.

Thus, technical efficiency comes to depend solely on input figures. It is natural, therefore, to associate with every curve a number which can be regarded as a measure of its position. A comparison of technical efficiency between two points on different technical curves can then be achieved by dividing this measure on one curve by that of the other.

However, the practical application of this principle raises two difficulties. We know only one point on each curve, so that knowledge of the shape of the curves is generally lacking. Moreover, there is no obvious way to indicate the position of a curve by a single measure. The first difficulty can be overcome with the help of different kinds of approximations, which are described in Chapter 5. The second difficulty involves that, even after

the shape of the curves has been set, different methods can give different measures of their relative positions. Thus, no univocal measure of changes in the technical efficiency of a firm can be obtained. However, the disparity of the results given by the different methods is seldom large enough to play an important part in the interpretation of the figures.

Two different methods of constructing such a measure are shown in Chapter 5. The one which seems the most attractive from various stand-points can be described briefly as follows, with the help of Diagram 1. The firm is assumed to have found itself at point *A* during the base period, and the technical curve through *A* was traced one way or another. The position of the firm during the comparison period is represented by point *D*. If one now draws a line from the origin *O* through *D*, the technical curve of the base period is cut at *B*. As *A* and *B* are considered to be on the same level, we can now compare *D* with *B* instead of with *A*. By a move from *B* to *D*, the inputs (in this case labour and raw material input) are reduced by the same percentage. It is, therefore, natural to take this general change as a measure of the reduction in input of the production factors over a movement from *A* to *D*.<sup>1</sup>

It may be added that, if the price of the factors has not changed during this period, this general change in input figures usually agrees very closely with the cost changes.

If the desired ultimate result of the calculations is an efficiency index that increases with increasing efficiency (as opposed to the input figure, which decreases), it may prove convenient to use the changes in the reciprocals of the input figures. Thus, if the reduction of the input figure from point *B* to point *D* was 20 per cent, that is to say from 1.0 to 0.8, this can be expressed as an increase of technical efficiency from 1.0 to  $1/0.8 = 1.25$ , or of 25 per cent.

An efficiency index computed in this way is very readily understood. It brings out the relationship between the production volume obtained, during the year of comparison, and the production volume which would have been obtained from the base year technique with the same factor input.

It must be noted that, with the method described above, it was necessary to know the shape of only one technical curve. In the example, the curve of the base period was used, as it is the simplest when one is concerned with a comparison over time, but in principle any other curve can serve as a basis of comparison. Thus, a comparison between points *E* and *D* on Diagram 1 can be made in such a way that each point is compared with the curve drawn through *A*, after which the quotient of the index figures thus obtained gives the desired relationship. If for example the input figure at point *E* is 1.20 times that of point *F* and the relationship between the input figures at *D* and *B* is 0.80, the measure of the position of *D* in relation to *F* will be  $0.80/1.20 = 0.67$ . If another technical curve had been used as a basis of comparison, the result expressed in figures would probably have

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1. The calculation problem, finding a measure of technical efficiency as explained here, is formally identical with the general problem of indices. The type of solution referred to here has also been applied to the theory of indices. See S. Malmqvist, "Index Numbers and Indifference Surfaces", *Trabajos de Estadística*, Vol. IV (1953), page 209.

been different. However, the possibility of using a technical curve, as a basis of comparison, that does not pass through any of the points compared, has a definite practical significance, as will be shown in Chapter 6.

The method of calculating an efficiency index, as described above, where only two production factors are involved, can easily be extended to the more realistic case with several production factors, although this cannot be so easily illustrated in diagrammatic form. In considering the question: "How do the production costs of the firm change in relation to the minimum cost?" it is obviously necessary to include all the important types of costs in the calculation of efficiency. In this case, capital constitutes a special difficulty.

#### INPUT OF CAPITAL

Since part of the technical change recorded in industry has involved an increase in capital input per unit produced to obtain a decrease in labour input, it is important to consider capital input too, when calculating technical efficiency. Analogically to the case discussed above, with labour and one raw material as the only production factors involved, one can say that the reduction in labour input displayed the characteristics of a move along the curve of the diagram. The industry has thus borne the consequences of the successively heightened relation between the price of labour and of capital. Only a part of the reduction in labour input has, therefore, presented the characteristics of improved technical efficiency, that is to say of a shifting of the curve. It is this move which has, in Sweden, sometimes been called "Horndalseffekt"<sup>1</sup>. For example, it can be due to the fact that labour, capital or some other production factor has become more efficient, or that there has been more efficient co-operation between the input factors.

Because of the special characteristics of capital, there may be a definite divergence according to whether one considers the measure of efficiency from a short or a long-term point of view. From a short-term point of view, it is of immediate importance that the firm should use its existing machinery as efficiently as possible. Efficiency can then either be better or worse when compared with other firms in the industry, depending on the age of machinery. However, from a long-term point of view, the firm should also change its machinery, so that it succeeds in achieving high efficiency throughout the firm. It is principally the latter aspect which will be treated in this book. No consideration will, therefore, be given to the fact that obsolete invested capital cannot be readily exchanged against more efficient equipment.

Measurement is much more of a problem for capital than for labour. Thus it is extremely rare to be able to determine off-hand the capital input for every individual product over a single period (base period), as for labour input. It is, however, possible to establish an approximate measure with the help of a capital value and price index, as will be further discussed in Chapter 4.

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1. See for example Erik Lundberg, *Kapital och teknik*, TVF 30, 1959, page 301 (issued by the Swedish Academy of Engineering Sciences). At the Horndal steelworks in Central Sweden the labour requirement per unit of output showed a steady decrease at an annual rate of about 2 per cent over a considerable period of time (1933-1950) in spite of the fact that investment expenditure was virtually nil during this period.

The mutual replacement of capital and labour can sometimes be of the same relatively simple character as the substitution between raw material and labour. This is, for example, the case with an automated machine unit, which only needs to be serviced when there is a mechanical stoppage or when it requires more raw material. The problem of the most economic harmony between the number of machines and the number of workers has been studied, particularly in respect of automatic weaving looms where a break in the thread is the most usual cause of mechanical stoppage. If a worker services many looms, it occurs relatively often that one or several of the looms have stopped and remain idle pending intervention, while the worker is busy on another weaving loom. On the other hand, if he has fewer looms to service, he will find himself empty-handed during a greater part of the working hours. The different combinations of the number of looms (of the exact type used by the firm) and the number of workers (or rather of working hours) which can be realised in this respect are grouped on the technical curve of the firm. Where there is a modified relation between wages and capital costs, the firm may benefit by moving along the curve, for example, by letting every worker service more machines and thereby increasing the idle moments of the machines but reducing that of the workers. This is also a manifestation of modified price efficiency. A higher technical efficiency is only obtained if the frequency of thread breaks is lowered, or if the repair time for each mechanical stoppage is reduced by better deployment or installation of better machines.

In this particular case, at least in principle, no difficulty is involved in determining the shape of the technical curve. But most often the conditions of substitution are not of this simple type. Generally, an increased use of capital involves the replacement of an existing machine by another, or the purchase of new machines to perform certain operations formerly done by hand. Thereby, obviously, technical efficiency can be increased at the same time. Whether the change in input figures, involved in such a substitution is considered as a mere adaptation of price, which leaves the new position on the same technical curve as the old one, depends upon the way in which the amount of capital is measured. This problem will be further discussed in Chapter 4.

Thus, as in the former example, it is usually necessary to construct the technical curve with the help of certain schematical suppositions, as in Chapter 5.

Sometimes the possible technical curve of a new firm may serve as a basis of comparison in evaluating the efficiency development of the established firm. A new firm can be fully endowed with the latest technical developments, though different technical solutions are, of course, possible on some points. The possibilities involved in this case—even though they are not all economic—correspond to various points on the technical curve representing up-to-date technique.

Finally, it is useful to point out that the shape of the technical curve depends on the volume of production. In fact, a production increase that does not entail an equivalent increase of the number of workers is quite usual. A consequent decrease of labour input per unit ensues. If basic installations are required, not directly connected with production volume, capital input per unit also becomes dependent on the rate of production. Thus, the technical curve possesses different shapes for different sizes of

firms. By drawing such curves for different sizes of firms, it is possible to construct a curve that constitutes a limit for all this group of curves, towards the origin and within the axes—the “ hull of the group of curves ” in mathematical language. This limit curve has been called “ the line of technical limitation ” or *T*-line<sup>1</sup>. There are certain methods of estimating the approximate shape of the *T*-line for an industry which can constitute a basis of comparison between firms (see Chapter 6).

#### WHY NOT “ PRODUCTIVITY ” ?

Many readers have certainly wondered why, in this chapter, they have not yet found the word “ productivity ”. Yet this notion, most often in terms of output per man-hour, has played an important part in the discussion concerning the methods of measuring the efficiency of an industrial firm. The discussion has often been characterised by a search for a way of expressing the problem to which the measure of productivity provides an answer. In the present book, we have gone the opposite way and tried to construct a measure that constitutes an answer to our initial question: how does the production efficiency of an industrial firm develop when the objective of the producing departments is to reduce costs as much as possible ?

It is, therefore, not surprising that the measure attained is not one of productivity. Neither is there any inducement to discuss here the question to which the measure of productivity constitutes an answer.

Meanwhile, it must be pointed out that the measure of productivity as it is generally defined—output per man-hour—is the reciprocal of the concept used in this book—labour input per unit produced. The reason for the use here of the latter concept is that on the one hand it does not evoke misinterpretations to the same extent as productivity, and on the other hand it is easier to manipulate with computing techniques. Thus, for example, the labour input in different processing stages can be aggregated to a total labour input, which is not possible with productivity expressed in such terms as output per man-hour.

#### THE DEVELOPMENT OF INDUSTRIAL EFFICIENCY — 1920 TO 1957

As an example of how useful the foregoing method of efficiency measurement can be, even though it is not extended to the calculation of a univocal measure, the development of labour and capital input per unit produced, for the whole of Swedish industry from 1920 to 1957, has been established in Diagram 2. The installed energy in HP for the direct power requirements of machines and equipment has been chosen as a measure of capital input. This is, of course, a very incomplete picture of the amount of capital, because among other things it only concerns machines, but it has been selected because it was recorded in the annual industrial statistics. The production index used as a measure of output was not intended for comparisons with input figures and can give doubtful results over such a long period.

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1. R. Bentzel & Ö. Johansson, “ Om homogenitet i produktionsfunktioner ”, *Ekonomisk Tidskrift* LXI (1959), page 159.

Nevertheless, the diagram is of some interest. Even without the establishment of a technical curve as a basis for comparative purposes, the development in different periods can be analysed in the terms which have been discussed above. During periods of generally progressive business activities (1921-29, 1932-37, 1945-49 and 1954-57) the development was characteristic of increased technical efficiency with a simultaneous reduction of both labour and capital inputs. When the volume of output diminished (1920-21, 1930-32, 1939-41 and 1944-45) the exploitation of capacity declined, that is to say the capital cost per unit increased, and the change was more or less characteristic of a move along a technical curve.

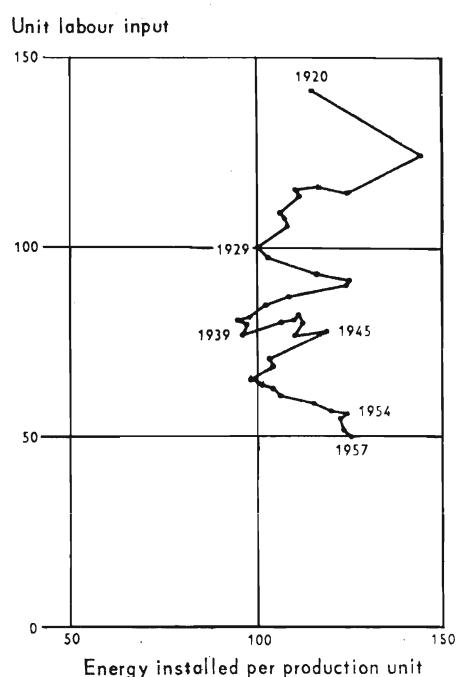


Diagram 2

It is doubtful whether such periods can appropriately be included in an analysis of this kind. However the most interesting development was during the period 1948-54, when capital input per unit increased significantly in spite of a general increase in the volume of production. During this period, therefore, it seems that, for industry as a whole, the development of technical efficiency has been insignificant, assuming that the figures are reliable. Instead, the changes which took place seem to have meant an adjustment to the changed relationship between wages and capital costs.

## SUMMARY

Many of the difficulties encountered by a firm wishing to obtain a measure of its production efficiency have been mentioned in this chapter. The next four chapters indicate a number of methods which can be used to solve or overcome many of these problems.

It may be appropriate here to sum up the basic assumptions made for the discussion of the method:

*Objective.* The technical departments of the firm will process as cheaply as possible the assortment of goods in the quantity and quality determined by the management of the firm.

*Measure of efficiency.* A measure of production efficiency will express the extent of fulfilment of this objective. It appears that production efficiency should be divided into two components, price efficiency and technical efficiency. The former, which shows how well the firm has adapted itself to prevailing price relationships between the input factors, is difficult to measure in an isolated firm. The methods of measurement described hereafter are therefore solely concerned with technical efficiency. It depends entirely upon the input figures of the different factors of production. All the relevant input figures should be accounted for in the calculations.



## Chapter 2

### THE PROBLEM OF MEASUREMENT — PRODUCTION VOLUME

The problem of measurement which can arise when a firm attempts to calculate a series of input figures according to the principles set out in Chapter 1 is discussed in this and in the next two chapters. The problem in question here refers partly to the calculation of production volume and is then roughly the same whatever input figure is used, and partly to the measurement of factors of production, in which case the problem can vary according to the different input figures.

Thus, in this chapter, the general problems related to the measurement of the volume of production will be discussed. The problems specially related to the measurement of labour input are dealt with in Chapter 3; those of energy and raw material input and capital input are dealt with in Chapter 4. Of these last two chapters, the one concerning labour input is the more complete. It even covers certain methods which can also be applied to other input factors after minor changes.

#### SYMBOLS

In order to simplify the analysis, it is convenient to introduce symbols for the different input figures per unit produced. In connection with labour input, the abbreviation HOK (Hours worked by Operatives per 100 Kg; Heures Ouvrier par 100 Kilos) in an EPA study on cotton weaving mills, and OHP (Operative Hours per unit of Production) in an English research, have been coined. None of these abbreviations obtained general approval, and there is therefore no necessity to use either of them here. It seems more convenient to use a symbol which can even be interpreted by those who are not capable of remembering the meaning of a certain combination of letters. Since labour input per unit is a quotient, the fraction bar should appear in the formula as in the definition of speed (m/sec). Since  $h$  is an internationally used symbol for time and  $q$  for quantity,  $h/q$  would appear to be a convenient formula for labour input per unit of output. Correspondingly, the raw material input per unit produced will be represented by  $r/q$  and capital input per unit by  $c/q$ .

In calculations of input figures within a firm, one is generally interested mostly in the changes which occur over a stated period of time. These changes are conveniently expressed as indices with their starting point in a definite base period. This period is called "year 0" and the period for which the actual calculations are performed is called "year 1". The

calculations, of course, may be in respect of quarters or months, but the method is the same.

In the examples, the annual relatives have not been multiplied by 100, and accordingly the index of the base period is 1 and not 100, as is customary. When computing the indices of an actual firm, it is more convenient to multiply all numbers by 100. Corresponding to the symbols presented above, an index of the development of labour input per unit will be called an  $h/q$  index, and so on.

#### THE PROBLEM WITH SEVERAL PRODUCTS

Should the process, department or firm studied produce goods of different kinds, some means of aggregating the quantities produced must be made. It is not certain that a simple addition is appropriate to the calculation of an input figure, even if there exists a common unit of measurement, e.g. tons. A series expressed in man-hours per ton is misleading in this respect, if the products require different amounts of labour and if the relation between them changes from one period to the other. If, for example, product A required 10 hours per ton and product B 1 hour per ton during a year, and if 1,000 tons of each product have been turned out, the average labour input has evidently been 5.5 hours per ton. Now, if the production of B increases to 2,000 tons while the production of A remains unchanged at 1,000 tons, the average labour input falls to 4 hours per ton, without any gain in efficiency.

On the other hand, the importance of dissimilarities should not be over-emphasised. In many firms the proportion of the products in a group does not change to the extent of affecting the average input of the input factors. Thus, even if different sizes of suits require different amounts of fabric, this does not cause the consideration of every size as a special product when calculating the raw material requirement, since one can rely on a reasonably constant proportion of the different sizes. This type of relatively unchanged harmony within a line of goods is very common in reality. Before going through any more complicated calculations, one should therefore examine whether the work can be simplified by a convenient grouping of goods produced by the same kind of output process. In a rolling mill of a Swedish steel works, where it was desired to study the changes in labour input per unit produced, there were some 400 combinations of qualities and dimensions for which the labour input was considerably divergent. However, after studying overall output for a period, the variety of dimensions proved to be so stable that it was only necessary to consider quality. Thus, instead of reckoning with 400 products, only 20 qualities needed to be brought out in the calculations. Subsequent sub-division by size proved that there were no adjustments important enough to warrant a revision of the calculations.

In some cases such an unchanged pattern of output can concern the whole firm, in which case it is only necessary to reckon with one product. This is generally the case when the entire output stems from a single raw material and when the processing is based on its composition. Thus, in an oil refinery, the quantity of crude oil used should be a good indication of the scope of production when it comes to measuring the input of factors other than raw material. Should it prove impossible to consider, in this way, the whole manufacturing process as that for one product, the problem

remains of expressing the output of the firm or of the department by a single figure in a manner convenient for the calculation of the input figure. This can be done either by converting all the goods into a *standard product* or by calculating a *production index*. In fact the second method stems from the first, and conversion into a standard product is therefore dealt with first.

Conversion into a standard product is made by assigning to every product a coefficient with which to multiply its output. The purpose of this coefficient is to act as a balance for the dissimilarity between the respective goods and the standard product. In a study concerning labour input per pair of shoes produced, a specific type of men's shoes with sewn welts was chosen as the standard, and the equivalent output of more elaborate qualities has been calculated by taking into consideration the longer time of work they required. If it took one man-hour to produce a pair of standard shoes and two for a pair of finer shoes, it may be said that as far as labour is concerned the latter corresponds to two pairs of standard shoes. Consequently, its coefficient is 2 for a study of labour input. If a series in time is at issue, coefficients may refer to the conditions of a base period and remain unchanged until the base year requires revision. For inter-firm comparisons, coefficients can either be calculated in a selected representative firm or for the average of all the firms involved. In both cases the coefficient will be applied equally to all the firms.

Corresponding calculations can be made when input figures other than those of labour are concerned. Thus, in a study of fuel input in blast furnaces, the type of pig-iron produced had to be taken into account. Choosing pig-iron for steelmaking as the standard and finding the coke consumption for that quality to be 600 kg/t. the conversion figure for a certain type of foundry iron will be 1.5 if the coke outlay for this type at the same time was 900 kg/t. For a comparison with other blast furnaces or with another period, the output can be converted into tons of standard pig-iron, and the difference in coke consumption obtained, independently of whether the composition of output is the same in both cases.

Thus, conversion into a standard product aims at eliminating the effect of changes in the composition of total output on the input figure studied, in comparisons between two periods or two firms. Moreover, conversion into a standard can even eliminate other differences, for example in machine equipment, if the effect of these differences on the input figure is known. This possibility will be further discussed in Chapter 5, page 74.

An example of the calculation of the input figure by means of conversion to a standard product is given below. It concerns labour input in a paper mill which produces three kinds of paper. The number of man-hours per ton for every kind of paper during the base year has been calculated after special research. Kraft paper is used as the standard product, and the conversion figure for the other qualities is obtained by dividing the labour input per ton of each of the others by 10, the labour input for Kraft paper:

CATEGORY OF PAPER	MAN-HOURS PER TON	CONVERSION FACTOR
Kraft paper .....	10	1.0
Sulphite wrapping paper .....	24	2.4
Grease-proof .....	41	4.1

The calculation of output volume and labour input is illustrated in the following synoptic table:

	YEAR 0		YEAR 1	
	TONS	STAND- ARD TONS	TONS	STAND- ARD TONS
Production:				
Kraft paper .....	4,000	4,000	6,000	6,000
Sulphite wrapping paper .....	3,000	7,200	2,500	6,000
Grease-proofs .....	400	1,640	500	2,050
<i>Total (q) .....</i>	—	<i>12,840</i>	—	<i>14,050</i>
<i>Total work-hours (h) .....</i>		<i>128,400</i>		<i>130,600</i>
<i>h/q .....</i>		<i>10.0</i>		<i>9.3</i>

Thus, average labour input per ton was reduced from 10.0 hours to 9.3 hours, or by 7 per cent.

It should be noted that it is generally possible to express the production of a firm in units of a standard product, even if the quantity produced of various goods is expressed in different units. Thus in a mechanical workshop, if the production of an ordinary man's bicycle requires 15 hours' work and that of 1 ton of casting of a certain type 12 hours' work, the latter product can be chosen as the standard, so that the conversion figure for a bicycle becomes  $15/12 = 1.25$ .

This last example may seem a little unrealistic, but in fact it illustrates what one is compelled to do in many cases. In such cases, however, it is usually advisable to express the result in the form of a production index.

#### RULES FOR THE CALCULATION OF THE PRODUCTION AND LABOUR INPUT INDICES

A production index is obtained by dividing the production of every period, expressed in units of the standard product, by the production of a base period calculated in the same manner. The same conversion figures should be used for each period.

When production changes are expressed in the form of indices, absolute values of the various inputs per unit can no longer be obtained. However, it is generally quite sufficient to obtain an index of the development. This can be calculated as the quotient of two other indices. Thus, for example, the index of labour input per unit (*h/q* index) will be:

$$h/q \text{ index} = \frac{\text{labour index}}{\text{production index weighted with man-hours}}$$

the labour index shows the development of the whole number of man-hours and the production index is calculated as described above. The result in figures will be exactly the same as if the labour input per unit of standard product in different periods was divided by its equivalent for the basic period.

The use of symbols can simplify the calculations. However, it is not necessary to read these sections printed in small type, where the method is presented in mathematical symbols, to understand it. The calculation methods will be clear from the numerical examples and from the written rules given in addition and printed in italics.

The following symbols are introduced:

$q_j$  = the production of product  $j$ ;

$h_j$  = the total number of man-hours in the production of product  $j$ ;

$H = \sum_j h_j$  = the whole number of man-hours for all products;

$v_j$  = a conversion figure of product  $j$  in the production index.

The figures (0) or (1) set in superior indicate to which period of time the information refers.

If the standard product receives the figure  $j = 1$ , the conversion figure of product  $j$  is:

$$\frac{h_j^{(0)} \cdot q_1^{(0)}}{q_j^{(0)} \cdot h_1^{(0)}}$$

and the average man-hours per ton of standard product is:

$$\begin{array}{ccc} \text{for year 0:} & & \text{for year 1:} \\ \frac{H^{(0)}}{\sum_j q_j^{(0)} \frac{h_j^{(0)}}{q_j^{(0)}} \cdot \frac{q_1^{(0)}}{h_1^{(0)}}} = \frac{h_1^{(0)}}{q_1^{(0)}} & = & \frac{H^{(1)}}{\sum_j q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}} \cdot \frac{q_1^{(0)}}{h_1^{(0)}}} \end{array}$$

Thus, the total  $h/q$  for the base period is automatically equal to that of the standard product. Dividing the  $h/q$  of year 1 by the  $h/q$  of year 0 gives the  $h/q$  index:

$$\frac{H^{(1)}}{\sum_j q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}} \tag{a}$$

This is the simplest way to calculate the  $h/q$  index.

If, instead, an arbitrary series of weights  $v_j$  is used for the different products, the conversion coefficients to the standard product are calculated on the basis of these weights, and finally the production index is obtained by dividing the production of year 1 converted into standard by its equivalent for year 0, one obtains:

$$\frac{\sum_j q_j^{(1)} \frac{v_j}{v_1}}{\sum_j q_j^{(0)} \frac{v_j}{v_1}}$$

If both numerator and denominator are multiplied by  $v_1$ , the production index is obtained:

$$\frac{\sum_j q_j^{(1)} v_j}{\sum_j q_j^{(0)} v_j} \tag{1}$$

Thus, to calculate the production index, one need never divide the conversion figure by that of any standard product. The calculation of the production index according to the formula can be expressed verbally:

#### Rule 1 — PRODUCTION INDEX

*To calculate the production index, the production of every product for year 1 is multiplied by its conversion coefficient. The total of the numbers thus obtained is divided by the equivalent total for year 0.*

An example of such a calculation will be found on page 30.

We now choose the labour input per unit of every product for the base period as a conversion figure, i.e.:

$$v_j = \frac{h_j^{(0)}}{q_j^{(0)}}$$

Thus the production index will be:

$$\frac{\sum_j q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}}{\sum_j q_j^{(0)} \frac{h_j^{(0)}}{q_j^{(0)}}} = \frac{\sum_j q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}}{H^{(0)}}$$

Dividing the labour index  $H^{(1)}/H^{(0)}$  by a production index thus established, one obtains

$$\frac{H^{(1)}}{H^{(0)}} \cdot \frac{H^{(0)}}{\sum_j q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}}$$

which can be abbreviated, so that one has the  $h/q$  index:

$$\frac{H^{(1)}}{\sum_j q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}} \quad (2)$$

viz., the same expression as that obtained in formula (a). This method of calculation will be used as much as possible in what follows.

This calculation of the  $h/q$  index by means of formulae has obviously equivalents for the other input figures. The formula can be expressed more commonly in words:

#### Rule 2 — INPUT INDEX

*The calculation is made by dividing the total number of man-hours (the raw material amount, the capital amount) of year 1 by a calculated number of hours (calculated raw material, capital amount), which corresponds to what would have been required by the present production, had the unit input figures of the base period still prevailed.*

According to this method, the calculation of the  $h/q$  index for the above-mentioned paper mill should be as follows:

	YEAR 0		YEAR 1	
	MAN-HOURS PER TON	PRODUCTION IN TONS	CALCULATED NUMBER MAN-HOURS	
	$\frac{h_j^{(0)}}{q_j^{(0)}}$	$q_j^{(1)}$	$q_j^{(1)}$	$\frac{h_j^{(0)}}{q_j^{(0)}}$
Kraft paper .....	10	6,000		60,000
Sulphite wrapping paper .....	24	2,500		60,000
Grease-proof .....	41	500		20,500
<i>Total</i> .....				<i>140,500</i>

The  $h/q$  index is then obtained by dividing the real number of man-hours of year 1, 130,600, by the calculated 140,500:

$$\frac{130,600}{140,500} = 0.93.$$

The information required for such a calculation is:

1. The labour (raw material, capital) input per unit for each product during the base period.
2. The output of each product during the present period.
3. The total number of man-hours for the present period.

The first of these is usually the most difficult to draw up. However, the conversion coefficients which apply to a certain base period can be used for a longer period ahead. Their computation is, therefore, in some ways a once and for all affair, and there is no need to compute them again as the available information is not always easily obtained from routine accounting. The calculation of the conversion figure depends, to some extent, on the type of production and on the form of the accounts. Some of the difficulties which can arise in this respect are discussed in the following section.

The above example concerned a calculation of labour input. However, when other input figures are involved, the conversion coefficient to be used in the conversion of production into a standard product, and consequently in the calculation of a production index, should correspond to the difference between the goods, in input terms, to which the comparison refers. If one constructs an index series of labour input per unit as above, the conversion figure must reflect the difference in labour input in a base period, but if one studies raw material input, the conversion figure should be based on the difference between the goods in consideration of this, and so on. In this manner, indices of input carry an easily understood value. Thus, the index of labour input per unit during a certain period depends on the relation between the actual number of man-hours during that period and the number that would be needed for the same production if the technique (inputs per unit) of the base period still prevailed.

To this end, different production indices should be used in studies of different input figures. Thus a production index, weighted with man-hours per unit, will be used in the calculation of the  $h/q$  index; another weighted with raw material consumption per unit used to calculate the  $r/q$  index and so on. In many cases this will not be applicable in practice. It is preferable to work with only one production index, to which all the desired indices of production factors can relate. But such proceedings can only give the desired information in some special cases, viz., if the composition of production is not changed or if the different input figures are proportional to each other, so that a product requiring twice as much labour input as the standard also requires twice as much capital input.

The latter case is usually rare. On the other hand the composition of production can be so stable, at least for a while, that no important part is played by whatever input factor is used for the weighting of the production index, so that the same production index can apply to the calculation of the unit input of any production factor.

Where the composition of production was not stable and it is still desired to calculate the index of, e.g., labour input and capital input according to the same production index, it must be remembered that, for this very reason, the result can be misleading, or at least difficult to judge. In this case it is appropriate to use such weights in the computation of the production index so that at least one series of inputs per unit is right, and to use the others only as a rough indication of development. An alternative solution is to eliminate from a series of inputs the influence of alterations in the other ones. (See Chapter 5, page 74.)

An example of the effect of using such weights in the calculation of the production index is shown in the following recapitulation of the Swedish production of hot rolled steel from 1949 to 1958, calculated in three different ways: viz., by weighting the different products by the man-hours required per unit, by raw material (crude steel) input per unit, and by the weight of the finished product in tons. The data on labour input concern the number of man-hours in the rolling mill and have been taken from an American survey of the '30s. For raw material input, we used the coefficient usually applied by the U.N. Economic Commission for Europe. Thus, none of the series of weights is directly adapted to Swedish conditions, but they nevertheless seem to produce results very close to what would have been obtained had Swedish figures been available.

The following input figures were used:

	LABOUR INPUT MAN- HOURS/TON	CRUDE STEEL INPUT
Semi-finished products for sale .....	5.1	1.12
Railway track materials .....	9.2	1.20
Beams and girders .....	7.2	
Reinforcement bars .....	9.0	
Other bars .....	9.7	
Wire rods .....	9.1	
Strips .....	12.1	1.35
Heavy plates .....	6.9	
Sheets .....	21.9	
Seamless tubes .....	25.5	1.20

The production index can now be calculated according to Rule 1. Output is multiplied first by the respective inputs per unit (see the next table). The production index can then be calculated by dividing the totals for the year 1958 by those of 1949:

$$\text{According to man-hour weights: } \frac{18,752}{11,751} = 1.60;$$

$$\text{According to crude steel weights: } \frac{1,988.0}{1,114.7} = 1.78;$$

$$\text{Directly in finished product weight: } \frac{1,607.7}{894.6} = 1.80.$$



	PRODUCTION '000 TONS		CALCULATED MAN-HOUR INPUT, '000 HOURS		CALCULATED CRUDE STEEL INPUT, '000 TONS	
	1949	1958	1949	1958	1949	1958
Semi-finished products						
for sale .....	10.0	103.8	51	529	11.2	116.3
Railway track materials	30.1	70.7	277	650		
Beams and girders ...	10.6	37.7	76	271		
Reinforcement bars ..	67.4	235.0	607	2,115	599.5	1,103.5
Other bars .....	269.5	394.1	2,614	3,823		
Wire rods .....	122.0	182.1	1,110	1,657		
Strips .....	59.5	86.1	720	1,042		
Heavy plates .....	80.7	182.7	557	1,261	378.1	603.4
Sheets .....	139.9	178.2	3,064	3,903		
Seamless tubes .....	104.9	137.3	2,675	3,501	125.9	164.8
<i>Total</i> .....	<i>894.6</i>	<i>1,607.7</i>	<i>11,751</i>	<i>18,752</i>	<i>1,114.7</i>	<i>1,988.0</i>

Apparently, a relative shift occurred in production requiring a lower labour input per ton. According to the "objective of measurements" outlined in Chapter 1, viz., that the index shall only reflect changes occurring in the manufacturing departments, such a shift should not influence the index, as it is the result of the activity of the sales departments. It would, therefore, be misleading to calculate an *h/q* index from a production index weighted by crude steel requirements or by weight of finished product, as this would give too favourable a picture of the reduction in labour input. It would be equally misleading to calculate an index of raw material input by means of a production index based on man-hours. This would probably show an increased raw material consumption, even if it had in reality diminished.

Differences on this scale thus mean that the same production index should preferably not be used for the calculation of several input figures. If for some reason it is necessary to do so, one should make sure in every case that the differences in weights do not have any appreciable influence on the result.

#### CHAIN INDEX

Such input figures valid for a definite base year, and used as weights in different kinds of production and input indices, sooner or later become so outdated that they no longer reflect the actual relationships between the factor requirements of the different products. Even a comparison of two consecutive years may be misleading if it rests on input figures which differ too much from the actual ones. This makes it advisable to change the base period at certain intervals. There is no general rule as to how often this should happen; the greater the difference in the progress of different products, the more often the base year should be changed. However, as a rule, an interval of five to ten years should prove about right.

A drawback to the change of base year is that the series is completely broken and that all comparisons retrospective from the new base year are made more difficult. However, it is possible to connect the indices, com-

puted for different periods, with a *chain index*, whereby a coherent single series is obtained.

A chain index is obtained by multiplying every index figure of the later series by the index figure which the later base year had in the earlier series. An example will demonstrate this.

	FIRST SERIES BASE YEAR 0	SECOND SERIES BASE YEAR 5	CHAIN INDEX
Year 0 .....	1.00		1.00
1 .....	0.96		0.96
2 .....	0.93		0.93
3 .....	0.88		0.88
4 .....	0.87		0.87
5 .....	0.84	1.00	$0.84 \cdot 1.00 = 0.84$
6 .....		0.97	$0.84 \cdot 0.97 = 0.81$
7 .....		0.92	$0.84 \cdot 0.92 = 0.77$
8 .....		0.85	$0.84 \cdot 0.85 = 0.71$

The series of chain indices in the example has its *basis of comparison* in year 0, that is to say the index of that year is 1.00. The *weight base*, that is to say the year to which the conversion coefficients refer, changes. Year 0 serves as a weight base for years 0 to 5, year 5 for years 6 to 8. Merely on account of the change of weight base, the meaning of the long series is somewhat vague. In fact, one can no longer speak of a direct comparison with the cost of today's production in terms of man-hours, capital or raw materials, if the technique of the base year has been applied. However, if one keeps in mind the character of the series of chain indices with several "links", a continuous series is to be preferred, from a practical point of view, to a number of shorter series.

#### CHANGING EXTENT OF PROCESSING

Even if the production yield of the firm, measured in amounts of finished products, remains unchanged over a long period, the performance of the firm may still have changed. Thus, if the firm, instead of using bought out parts, starts to perform the whole transformation from the raw material on its own, the requirements of labour, capital, etc. naturally increase. In order to retain some degree of comparison, it is necessary either to adjust the measure of production volume or that of consumption of production factors, or else perform comparisons at departmental level. Since we are striving to establish a measure for the whole production of the firm, the latter method alone is unsatisfactory. Some type of adjustment is therefore necessary.

In many cases, input figures can be kept comparable from year to year by completely excluding certain departments. Thus, departments whose services are sometimes partly replaced by those of outside suppliers are considered separate to the firm. If the change in the extent of processing occurs once and for all, separate series of input indices can be computed before and after the change and include all the working departments for each period. By letting both calculations overlap each other during a year it

will be possible to link up the two series of indices in the manner described in the foregoing section.

However, if recourse to sub-contractors and service workshops outside the firm is to be had more often, the best method will be to have the measure of production volume reflect the changes of performance within the firm. This can be achieved in the calculation of the output volume by regarding the products of every department as finished products, regardless of whether or not they need further processing within the firm. The conversion coefficients for these products will then be the inputs of the relevant input factor in the respective departments. Thus, if a shoe manufacturing concern starts to buy ready-stamped soles instead of stamping them, this is recorded, along with an otherwise unchanged production, as a diminished production volume, which can be directly compared with the total consumption of different input factors. An example of this type of calculation will be found in Chapter 3, page 52.

Meanwhile it should be noted that this method of calculating production volume has one weakness, especially in connection with efficiency measurements—namely if, owing to less spoilage in the finishing departments, the amount of required semi-finished product diminishes in relation to the amount of finished product. Of course, this implies higher efficiency, but it will simply be recorded as a reduction of production volume and a corresponding drop in the use of production factors. Some suggestions of how to overcome this weakness are presented in Chapter 5, page 72.

#### CHANGES IN QUALITY AND NEW PRODUCTS

In the foregoing section, we assumed that the same goods were produced during the base period as during the comparison period. This is seldom the case in reality, because new goods come up continuously and old ones are modified. This makes it more difficult to calculate the production index, and thereby the input per unit. One may say that there is no fully satisfactory method of accounting for changes in quality and new products. However, there are different ways of correcting the results, and we will go over some of them here. Several typical cases may be distinguished:

1. Construction change without importance to the user.
2. Change in quality with measurable effect.
3. Change in quality with non-measurable effect.
4. Completely new product.
5. Fabrication by request with constantly changing qualities.

##### *Construction change*

If the construction of a product is modified mainly to make processing easier, while from the user's viewpoint it is the same as before, then of course the modification should influence the input figure. Thus, if the production is reckoned in pieces, no change whatsoever will be required in the calculations, and a unit of the new model will be counted as an old one (e.g. telephones). On the other hand, if production is measured by weight, some conversions may be necessary if a unit, unchanged from the user's point of view, weighs less (or more) in the new fabrication than in the old one.

#### *Change in quality with measurable effect*

A change in the quality of a product may be appreciated from two different angles, that of the consumer and that of the producer. It is not certain that both parties will agree. The consumer may be interested in the length of life of the product, its solidity or its performance. By means of a measure of any of these properties, a change in quality can immediately be expressed in figures. On the other hand, from the producer's point of view, the change in quality can probably be expressed in terms of a reduced amount of the input factors required to process a unit of the product. In social economic research, the consumer's view usually dominates, but when efficiency measurements for a specific firm are at issue, the producer's view may have to come first. Usually, however, it is difficult to find a measure of either one or the other aspect, and for this reason it is advisable to use the method which is easiest to apply to the concrete case.

As an example of a product whose quality it seems possible to measure from the consumer's angle, we have car tyres. Their longevity can be expressed objectively enough in the number of miles under well-defined conditions. Thus, one can consider tyre miles instead of just tyres as the relevant product. In a corresponding way, one can reckon military uniforms in terms of days of wear instead of in pieces. In such practical cases, a conversion factor should be calculated for the new product in relation to the old one. For example, if the new quality lasts 10 per cent longer than the old one, every unit of the new quality should count as 1.1 unit of the old one. After such a conversion of the production volume, the input index can be calculated directly.

#### *Change in quality with non-measurable effect*

Generally, it is not possible to measure the effect of a change in quality (for example a new model of motor car). In such cases, the difference between both qualities must be assessed from the processing viewpoint, and a conversion figure, to be used in the calculation of the production volume, must be obtained accordingly. As a rule, this conversion figure will consist of the quotient of the input figures (labour, raw material, etc.) of the old and new qualities under the same technique. It can be calculated in several ways—which one to choose will be a matter of exigency rather than preference.

A conversion figure of man-hours per unit is relatively simple to calculate if the new quality undergoes the same processes as the old one, perhaps even concurrently. The difference in labour requirement can then be directly measured in order to supply a conversion figure. An example will illustrate the proceedings.

Let us consider a firm with only two products, viz., perambulators (a single type) and children's tricycles (a single type). The prams remain unchanged during the whole period. On the other hand, a process improvement in quality is realised with the tricycles. These require a slightly increased labour input per unit, among other things because the number of weldings has increased. As the newly incurred work times are of the same nature as the former ones, it is possible to calculate directly the amount of increase in man-hours required. At the time the change took place, 2.0 man-hours were dedicated to the old model, and 2.4 to the new one. Thus, the

conversion figure from old to new quality is  $2.4/2.0 = 1.2$ . The number of man-hours which would have been required for the production of year 1 (after the change) according to the inputs per unit of the base year is obtained as shown in the following table. In this example, the calculated labour time was 6,540 hours. The observed total number of man-hours for year 1,  $H^{(1)}$ , was 6,050. Thus, the  $h/q$  index is  $6,050/6,540 = 0.93$ .

	YEAR 0	YEAR 1		
	MAN-HOURS PER UNIT $\frac{h_j^{(0)}}{q_j^{(0)}}$	PRODUCED UNITS	SAME, CONVERTED $q_j^{(1)}$	CALCULATED MAN-HOURS $q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}$
Perambulators . . . . .	6.0	250	250	1,500
Tricycles, old . . . . .	2.1	—	2,400	5,040
Tricycles, new . . . . .	—	2,000	—	—
<i>Total</i> . . . . .				6,540

Calculations will be simpler if we multiply the conversion figure 1.2 by the labour input per unit of the older model of tricycle for year 0, *viz.* 2.1. The result 2.52 can thereupon be used as the labour input per unit of the new model for year 0 and multiplied by the production of later periods to obtain the denominator of the  $h/q$  index corresponding to tricycles. The result will be identical to the one above.

It is sometimes possible to evaluate directly the input figure of the new quality for the base period. This will especially suit cases where the qualities are expressed rather in terms of size, for example thread counts in a weaving mill or thickness of wire in a wire-mill. In these cases, the input figure for the base period of a size that was not produced at that time is obtained by interpolation of the value of sizes which were then produced. Even in other cases, where the production of a new quality would have been technically possible during the base period, one can calculate an input figure by means of retrospective cost calculations.

The most difficult problem arises when the new quality is introduced with, or subsequent to, new production processes. An example is the case of a new machine which makes it possible to process the parts of a product at closer tolerances, and thereby with better adaptation. It is meaningless in this case to speak of the various inputs per unit for the new quality during the base period or of the input figures of the old quality according to the new technique. The way to overcome this difficulty is more or less arbitrary:

1. If the product is subject to free pricing on the market, the price relation between the new and the old product can be said to express the consumer's estimation of the difference in quality, and this quotient can be used as a conversion coefficient. This may be considered as an approximation of the case of quality changes with a measurable effect.

2. If there is no possibility whatsoever of calculating a conversion coefficient, the new quality may be considered as
- a) a completely new product to which the methods discussed below will be applied, or
  - b) a product of the same value as the old one, whereby calculations can be made directly as if nothing had happened. If an improvement in quality has really taken place, the indices of the different inputs per unit will show an upward bias. If the quality has deteriorated, the bias will be downwards.

*Completely new product*

When a new product is initiated, one should first and foremost look into the possibility of calculating what its factor requirements per unit would have been at the base period, had it been produced then, by means of comparisons with other products. If this proves to be impossible, there are two ways out: excluding the product from the calculations, or “linking up”.

*Excluding* new products from the calculations is only possible when man-hours, raw materials, etc., involved in their processing can be completely set apart from the rest. Thus, it takes nearly as thorough an analysis for every comparison period as for the base period.

It is somewhat simpler to calculate a chain index. The principle in this case is to compare firstly year 0 and year 1 *without* the new product, and then year 1 and year 2 *with* the new product. The two links are then coupled up in a chain index. The proceedings will be illustrated simply by an example. A brickyard is supposed to have produced types of bricks A and B during year 0. In the beginning of year 1, an additional new type C is introduced. To begin with, the *h/q* index of year 1 can be calculated on the basis of year 0, whereby C is excluded from the calculations, the first part of which is reproduced on the next table. The whole number of man-hours during year 1, exclusive of such hours as could be attributed to brick type C, was according to the firm's calculations 19,500. The *h/q* index is:

$$\frac{19,500}{25,000} = 0.78.$$

	YEAR 0		YEAR 1	
	MAN-HOURS PER '000 UNITS		PRODUCTION IN '000 UNITS	CALCULATED NO. OF MAN-HOURS
	$\frac{h_j^{(0)}}{q_j^{(0)}}$		$q_j^{(1)}$	$q_j^{(1)} \frac{h_j^{(0)}}{q_j^{(0)}}$
Brick type A .....	4		4,000	16,000
Brick type B .....	6		1,500	9,000
<i>Total</i> .....				25,000

In a corresponding manner, the index for year 2 is calculated on the basis of year 1, whereby brick type C is included as well.

	YEAR 1	YEAR 2	
	MAN-HOURS PER '000 UNITS	PRODUCTION IN '000 UNITS	CALCULATED NO. OF MAN-HOURS
	$\frac{h_j^{(1)}}{q_j^{(1)}}$	$q_j^{(2)}$	$q_j^{(2)} \frac{h_j^{(1)}}{q_j^{(1)}}$
Brick type A .....	3	3,500	10,500
Brick type B .....	5	2,000	10,000
Brick type C .....	8	1,000	8,000
<i>Total</i> .....			28,500

If the whole number of man-hours of year 2 was 24,500, the  $h/q$  index is:

$$\frac{24,500}{28,500} = 0.86.$$

The index links can now be coupled up, if one wishes to compare year 2 to year 0. This is done by multiplying the index numbers by each other. Thus an  $h/q$  index for year 2 on the basis of comparison of year 0 becomes:  $0.78 \cdot 0.86 = 0.67$ .

A relatively long series of linked-up indices may be obtained in this manner, in spite of the fact that, in the calculations, comparisons are limited to two consecutive periods.

However, a problem remains if one chooses to calculate a chain index, namely: when will the series be linked up? The production of a new product often requires a much larger input of production factors during the introduction period than later on, when the worst teething troubles have been overcome. Now, if the new product is included in the index too soon after its introduction, when the methods are not yet conclusively evolved, the index will later prove to show too big a drop, which can hardly be said to express what one wants to measure. This has a special effect if the new product comprises a large part of the production of the firm.

As an illustration to the characteristics of the introduction period, two American studies may be quoted. The man-hours required for dissimilar units in a long series is in question. When Liberty Ships were built during the war, it appeared, by and large, that in the shipyards which took part in the production, the number of man-hours per ship diminished by about 18 per cent for each doubling of the serial number. Thus, the fortieth Liberty Ship built in the shipyard required 18 per cent less man-hours than the twentieth, which had required 18 per cent less than the tenth. Corresponding observations were made on the production of machines in longer series. With a surprising stability, the diminution in man-hours is about 20 per cent when the number of produced units is double. Thus, during the introduction period, man-hour requirement diminishes relatively fast, and later on it lessens at a significantly slower rate.

Due to this "learning factor", the choice of the linking point can affect the development of the  $h/q$  index considerably, as is apparent from the example below. Product B has been introduced during year 1.

	YEAR			
	0	1	2	10
Production A, units .....	800	1,000	1,100	800
Production B, units .....	—	50	400	2,000
Man-hours/unit, A .....	4	3	3	..
Man-hours/unit, B .....	—	10	5	..
Total No. man-hours .....				7,600

If an  $h/q$  series of indices is calculated from year 0 to year 10, one can envisage linking up either in year 1 or in year 2. To link up in year 1, an index figure for year 1 based on year 0 is first required. As this only includes a single good, it can be obtained by a direct comparison of the man-hours per unit in both instances. Thus the index will be  $3/4 = 0.75$ . Thereafter, year 10 is compared in the usual manner to year 1, at which point both goods are included in the calculations. The result is:

$$\frac{7,600}{800 \cdot 3 + 2,000 \cdot 10} = 0.34.$$

The index for year 10, with year 0 as a comparison basis, is obtained by multiplying both of these index figures:

$$0.75 \cdot 0.34 = 0.26.$$

On the other hand, if one chooses to link up in year 2, the index for that period, in relation to year 0, must be first calculated with the development of product A as its starting point. The result is  $3/4 = 0.75$ . The index of year 10 in relation to year 2 will be:

$$\frac{7,600}{800 \cdot 3 + 2,000 \cdot 5} = 0.61.$$

Then the chain index for year 10 in relation to year 0 is:

$$0.75 \cdot 0.61 = 0.46.$$

Thus in one case 0.26 is obtained, and in the other 0.46, as a measure of the same thing.

This example serves as a warning against considering a chain index as a precision tool. However, the discrepancy between results obtained by linking up at different points of time is of lesser import when the new product is not responsible for such a large portion of the total production as in the example.

As a practical rule for cases where a new product must be included in the index, one can say that this should not be done as long as production has the characteristics of an innovation, but must take place as soon as the new product has been in full production for the whole year.

#### *Fabrication by request*

An extreme case of continuous changes in quality occurs in the firm which manufactures individually important units upon order, for example ships. Series seldom occur, but a year's production can be composed of some ten units, all more or less different. The quantity of material under process is large and can vary within wide limits.



In such a firm, none of the above methods is convenient, and it is difficult to specify any general way of measuring the production volume or the input figure for each year.

However it is much easier if one gives up endeavouring to aggregate the units produced over a period and calculates instead an index figure for each finished unit. Of course, such calculations are *ex post*. The input figure may be calculated for the whole unit, or more conveniently for every part, or for every group of operations (*vide* section on "Measurement of Operations", page 49).

According to this method, the initially-recorded figures concern the total input of any production factor in a certain process or for a certain part of the production unit, for example, the ship. The figures recorded for the last ship built may not be at all comparable to those of the one immediately preceding it. However, if a ship built earlier is of about the same type, a comparison can eventually be made after a certain conversion. In particular, it is often possible to make a summary adjustment concerning the ship's size. The important thing is then to express this size in a way that suits the context. It may be any usual measure, like the gross registered tonnage, but could just as conceivably be the total weight in steel of the ship or the number of linear meters of the welded joints.

In this way, several series of input figures are obtained, each of them concerning a certain type of ship, for example tankers and dry cargo ships. They can be grouped in a diagram whose abscissa is a time axis. The indication, concerning a certain ship, is placed in line with her day of launching or delivery, or at another convenient moment. Thus, the horizontal spacing between the points on the curve will be uneven. However, the curves should be able to convey a certain conception of the development of input figures for the firm as a whole. If the different curves lie at widely different levels, it may prove convenient to use a logarithmic scale on the vertical axis.

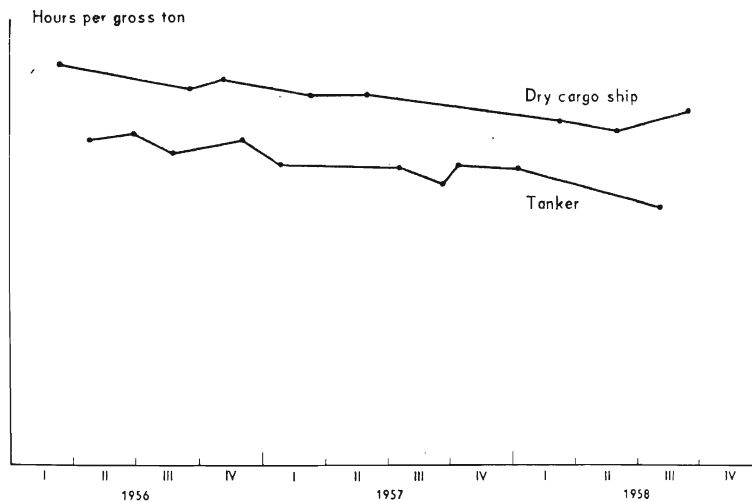


Diagram 3

Diagram 3 shows an hypothetical example of this kind of representation. If it is not possible to express directly how a certain input figure is apt to change, for example with modifications of size, one can apply the technique of indirect calculation of the conversion figure as further described on page 46. This implies the tracing of a diagram for every type of ship, where the vertical axis concerns the total input of the production factor studied, and the horizontal axis, the measure used for the ship's size. Every ship is considered as a point on the diagram, after which the points are connected to each other in chronological order. When the diagram contains a sufficient number of points, it should be possible to differentiate between the changes in input which depend upon the different sizes of ships and those which reveal a trend caused by measures designed to increase efficiency, etc.

### Chapter 3

## THE PROBLEM OF MEASUREMENT — UNIT LABOUR INPUT

### MEASUREMENT OF THE AMOUNT OF LABOUR USED

How to measure labour input in production largely depends upon the way the question is put. According to the general reasoning followed in Chapter 1, the important thing is to extricate, as much as possible, the influence on the  $h/q$  of such factors as can be controlled by the technical management. In this light, the data on labour time must correspond as closely as possible to the amount of work which has been put into production. They should include time wasted on the premises but not, for example, vacations. It is therefore more appropriate to express this in number of man-hours than in number of workers.

Still, from the production standpoint, all man-hours do not have an equal value. In rate of pay as well as in performance, there is for example a difference between apprentices, unskilled labour and specialised workers. Corresponding to what was done for production, it should also be possible to achieve the aggregation of different groups of man-hours into a standard hour, for example by the token of payment per hour. However, such a calculation is often difficult because a convenient distinction between workers from this standpoint is lacking in the accountant's files. If the relation between the number of workers in different groups does not show too great a variation, it is usually quite sufficient to consider all man-hours as of equal value and simply to aggregate them.

However, it should also be observed that the performance which corresponds to a man-hour can also vary with the difference in the workers' personal efficiency. But when a time series within a firm is at issue, personal efficiency may usually be considered as roughly constant. Modifications in the labour input per unit produced will thus show, in a general way, how well the available labour has been utilised. However, in a comparison between several firms, one cannot entirely overlook the possibility of discrepancies in personal efficiency. But there is no possibility of using a measure other than man-hours for labour input.

A long-established tendency in industry is that, when the input in actual labour per unit produced diminishes, the number of monthly paid staff increases. For this reason it may be interesting to include monthly paid staff as well in the  $h/q$  calculations. Generally, however, they cannot be included directly, because there is a lack of data on man-hours expended. Moreover, the contents of such an employee's time are so different from those of a worker's time that a straight aggregation would be of little interest.

Thus, if the working time of monthly paid staff is to be treated in a manner different from that of workers, it is necessary to delineate between both categories. As long as the purpose is not to compare different firms with each other, but simply to establish a time series for the  $h/q$  of a firm, the exact position of this delineation is not too important, provided it does not change from year to year. In most firms it should prove practicable to count all hourly paid wages in the  $h/q$  measure and exclude all monthly paid salaries. Methods for the inclusion of input from monthly paid staff are described in Chapter 5.

#### CALCULATING THE $h/q$ INDEX

As shown on page 29, there are two different kinds of information concerning labour input and which are required for the calculation of an  $h/q$  index—on the one hand, the total man-hours for each period, and on the other, the labour input per unit for every individual line of goods during the base period. The first should not be difficult to calculate, once it has been decided which hours refer to which category of workers, and whether all man-hours should be considered as of equal value or not. Labour used per unit for various goods can generally be deduced *ex post* or by summing job cards or similar documents. It is, however, an exception to be able to allocate, in this way, all the man-hours of the base period among goods, required for the calculation of the  $h/q$  index in accordance with the simple Rule 2 (page 28).

Difficulties can be of different kinds. Roughly, they should relate to one of the following groups:

1. Too much work would be involved in establishing how many man-hours corresponded to every product during the base period, even for a single department, because many products underwent the same operations, and it is difficult to record anything but the total time spent in these operations.

This difficulty can be overcome by means of one of the following methods:

- a) Standard and other calculations used to determine the conversion co-efficients. See page 43.
- b) Indirect assessment of conversion co-efficients. See page 46.
- c) Measurement of operations rather than products. See page 49.

All these methods can be used in combination with those described below to avoid other difficulties. With minor changes they can also apply to other input figures besides labour input.

2. Certain man-hours cannot be directly related to any particular product (for example man-hours in auxiliary departments such as the boiler room, the maintenance workshop or transport).

A method of allocation (see page 50) can be used in certain cases to overcome this difficulty. It involves a sharing out of the man-hours of the auxiliary departments among the finished products. If this is not practicable, for example because of the difficulties indicated under (3), the overhead method (see page 50) should be applied.

3. It is not possible to distinguish the different finished products in the earlier stages of production, because they originate partly from the same raw materials, and severe spoilage or breakage occurs in the course of processing. Thus, in integrated steelworks it is practically impossible to share among the finished products in the rolling mill the hours devoted to the processing of pig-iron. Pig-iron is used together with other raw materials in steelmaking, but the proportions of these different raw materials can vary. Moreover, the quantity of crude steel required per ton of approved rolled product changes according to the type and quality of product, as the proportion of scrap produced in the rolling process also varies between products.

Where this difficulty is important, the overhead method is the most convenient. It implies that every department of the firm be considered in itself and that the man-hours in the auxiliary departments be re-distributed as an overhead to be added to the man-hours in the producing departments. The overhead need not be as important everywhere. This method, as described on page 50, is applicable to a majority of firms. If difficulties arise in measuring the production of every department, it can for example be combined with "sample of products" or with "measurement of operations" (see pages 55 and 59).

4. For firms with a great number of products it may entail a lot of work in the calculation of the  $h/q$  index on the basis of data for every product. Moreover, the lines of products grouped for internal production statistics can bring together goods which require a very dissimilar amount of labour per unit. The counting of such groups as individual goods in the  $h/q$  calculations might, therefore, involve some risk of distortion of the results, as a move within a production group, including goods of different labour requirement is recorded as a modified  $h/q$ .

In order to be able to obtain some grasp of the development of the  $h/q$  within the firm, the calculations may be based on a sample of products (see page 55). However a progressive evolution toward a measurement based on the entire product range must be attempted.

The next sections describe methods which can be applied to different situations in which standard Rule 2, page 28, cannot be applied directly. Their common characteristic is to give a result which is an approximation to what could have been obtained with Rule 2.

The descriptions of the different methods are independent from each other, and it is therefore unnecessary to read them all in order to use one particular method.

#### APPLICATION OF STANDARD CALCULATIONS

Should it prove difficult to observe the actual amount of labour used per unit for every product, and if the firm works with standard calculations, one can use instead the standard man-hours required as a conversion figure for the calculation of the index. The calculated number of hours used in assessing the  $h/q$  index according to Rule 2 is thus based on the standard man-hour figure instead of the observed man-hours of the base period. In this way, the importance of a change in the  $h/q$  index, between the base period when the standard hours were assessed and a later period, will be

somewhat vague. But of course, on the contrary, two other periods may be compared with each other, exactly as in the simplest case. Thus, a comparison between years 1 and 2 is not made via the real year 0 but via standard times, a difference which is insignificant. It is convenient to calculate an index figure for year 0 as well and with it to divide the general index figures, whereby a new series is obtained in which the actual year 0 is again equal to 100 (or 1 as in the examples and formulae reproduced here). However, in this case, on top of the other data necessary elsewhere, the total number of man-hours and the output of every individual product for year 0 are also required.

$s_j$  stands for standard man-hours for product  $j$ . Calculated according to Formula (2), with the observed unit man-hours figure for the base period replaced by  $s_j$  the  $h/q$  index will then be for year 1:

$$\frac{H^{(1)}}{\sum_j q_j^{(1)} s_j} \quad (b)$$

Dividing this formula by its equivalent for year 0, we get:

$$\frac{H^{(1)} \sum_j q_j^{(0)} s_j}{H^{(0)} \sum_j q_j^{(1)} s_j} \quad (3)$$

As can be seen, this result can also be obtained by dividing the employment index  $H^{(1)}/H^{(0)}$  by a production index, where the conversion figure of every product is its standard man-hours.

Thus, the  $h/q$  index described above may be simply calculated according to

### Rule 3 — $h/q$ INDEX WITH STANDARD MAN-HOURS

*Divide the employment index by a production index, where the conversion coefficient for every product is its standard man-hours.*

In a Swedish firm manufacturing soap, detergents, etc., this method is applied to the entire production process, including labour time for internal as well as external transport and for storage.

It should be noted that if new standard calculations are made at equal intervals, they need not be used immediately to renew the conversion system. If the structure of production is not further altered, it may prove profitable to maintain the conversion figures unchanged for at least a period of five years. However, renewed standard calculations make possible more elaborate comparisons. More especially, if standard calculations are accurately made, modifications in standard man-hours depend mainly on technical and organisational changes. An  $h/q$  index, in which all man-hour figures for year 0 as well as for year 1 concern standard man-hours during the respective years, should thus be considered as giving a measure of the total effect of these changes on labour requirement. A series of indices on the relationship between real labour time and the time required for the same year's production according to recent standard time calculations can equally be considered to measure changes in work intensity. The product of both these indices is the  $h/q$  index calculated according to Rule 3. Thus the calculations described here separate this index into two parts. However, when interpreting such a separation, one should be very careful to allocate

the changes in the different parts to particular causes. The separation depends entirely on the way the calculation of standard man-hours has been made.

If standard man-hours are calculated for year 0 as well as for year 1, the comparison between them is made as follows:

$$\frac{\sum_j q_j^{(1)} s_j^{(1)}}{\sum_j q_j^{(1)} s_j^{(0)}} \quad (c)$$

The comparison between real labour and time allowed, from standard calculations, is made by the following formula (compare (b) above):

$$\begin{array}{cc} \text{for year 0:} & \text{for year 1:} \\ \frac{H^{(0)}}{\sum_j q_j^{(0)} s_j^{(0)}} & \frac{H^{(1)}}{\sum_j q_j^{(1)} s_j^{(1)}} \end{array} \quad (d)$$

Dividing the ratio for year 1 with that for year 0 to get an index of development, we obtain:

$$\frac{H^{(1)} \sum_j q_j^{(0)} s_j^{(0)}}{H^{(0)} \sum_j q_j^{(1)} s_j^{(1)}}$$

This, multiplied by (c) above, gives:

$$\frac{H^{(1)} \sum_j q_j^{(0)} s_j^{(0)}}{H^{(0)} \sum_j q_j^{(1)} s_j^{(1)}} \cdot \frac{\sum_j q_j^{(1)} s_j^{(1)}}{\sum_j q_j^{(1)} s_j^{(0)}} = \frac{H^{(1)} \sum_j q_j^{(0)} s_j^{(0)}}{H^{(0)} \sum_j q_j^{(1)} s_j^{(0)}}$$

which is the same as formula (3), if standard times refer to year 0.

For every operation, a certain English firm calculates the relationship between the actual number of man-hours and the number required according to the prevailing standards (according to formula (d) above). No total index for the firm is calculated. On the other hand, a study of the whole family of index figures for all operations is studied as a statistical distribution whose mean and variance are followed monthly, by random sampling of some work shifts within every group of operations and recording the results. The trend is followed on a control chart.

Even if standard calculations do not contain direct information on man-hours but only on labour costs, this can still be used for the calculation of the  $h/q$  index. Cost data are converted into man-hours by dividing them by the hourly paid wages of the groups involved for every type of product. However, cost data can also be used directly. It is assumed that the relation between labour costs and man-hours, that is to say hourly wages, is about the same for all the products. Then, the quotient of labour costs for two products is about the same as that of their man-hour requirements and can be used to convert output into a standard product. The subsequent calculation of the  $h/q$  index is then by means of the production and employment indices. With this method it will be possible to use standard or other cost calculations, even though they give no data whatsoever on man-hours but only on labour cost. However, to ensure that labour costs be roughly

proportional to man-hours, the calculations for all products must refer to the same period, so that changes in wage levels are counteracted.

In a survey of the Danish shoe industry in 1952, where every operation was considered separately, the job cost of every type of product was used as the conversion figure. This usually presents a good illustration of the differences in labour input for every process, at least if all job prices were established at the same time.

If  $k_j$  represents the cost of labour, calculations are made according to the formula:

$$\frac{H^{(1)} \sum_j q_j^{(0)} k_j^{(0)}}{H^{(0)} \sum_j q_j^{(1)} k_j^{(0)}} \quad (4)$$

The following example shows how the calculations are carried out. They refer to a firm with only two products.

	YEAR 0			YEAR 1	
	LABOUR COST PER UNIT	PRODUCTION IN UNITS	TOTAL LABOUR COST	PRODUCTION IN UNITS	CALCULATED TOTAL LABOUR COST
	$k_j^{(0)}$	$q_j^{(0)}$	$q_j^{(0)} k_j^{(0)}$	$q_j^{(1)}$	$q_j^{(1)} k_j^{(0)}$
Product 1 .....	3.25	10,000	32,500	12,000	39,000
Product 2 .....	4.50	5,000	22,500	4,000	18,000
<i>Total</i> .....			55,000		57,000

This gives a production index of:  $57,000/55,000 = 1.04$ .

The employment statistics of the firm show that the entire number of man-hours was 9,420 for year 0 and 9,250 for year 1. Thus the occupation index will be:  $9,250/9,420 = 0.98$  and the  $h/q$  index:  $0.98/1.04 = 0.94$ .

#### INDIRECT CALCULATION OF CONVERSION FIGURES

This method assumes that the conversion figures between different products are first established after watching for some time how the product mix influences a summarily computed  $h/q$  index. It is easy to apply this method when the difference between several products can be measured (for example thread counts) or when only two products or groups of products are manufactured. The  $h/q$  value calculated on the basis of total quantity (for example tons) may then be related to the measure of the average product (for example a thread count) or to the proportion of one of the two products in the output of the period. The value obtained for different periods may be illustrated, for example, on a graph scaled on the y-axis in  $h/q$  and on the x-axis in percentages or in the average measure. At first, not much information can be obtained on the real changes in  $h/q$ , but after several observations have been made, a basis for assessing the influence of the factors studied and for estimating trends,



will be obtained. The method can also be profitably applied to a jobbing firm producing important units on order, where the  $h/q$  is measured for each unit instead of each period of time.

This method has mostly been used in inter-firm comparison. Every firm is then represented by a point on the graph, which gives an immediate basis for assessment. In Diagram 4, this method has been applied to the data from a study of Swedish cotton spinning mills in 1956<sup>1</sup>. It shows the  $h/q$  in the spooling process as a function of the proportion of thread spooled automatically. It appears that a negative correlation exists between that proportion and the  $h/q$ , so that, on the average, the  $h/q$  is lower when the proportion of thread spooled automatically is higher. By drawing a regression line, either freely or with the aid of some statistical technique, an estimating equation can be read or calculated for an average  $h/q$  in automatic spooling and separately for the other types of spooling.

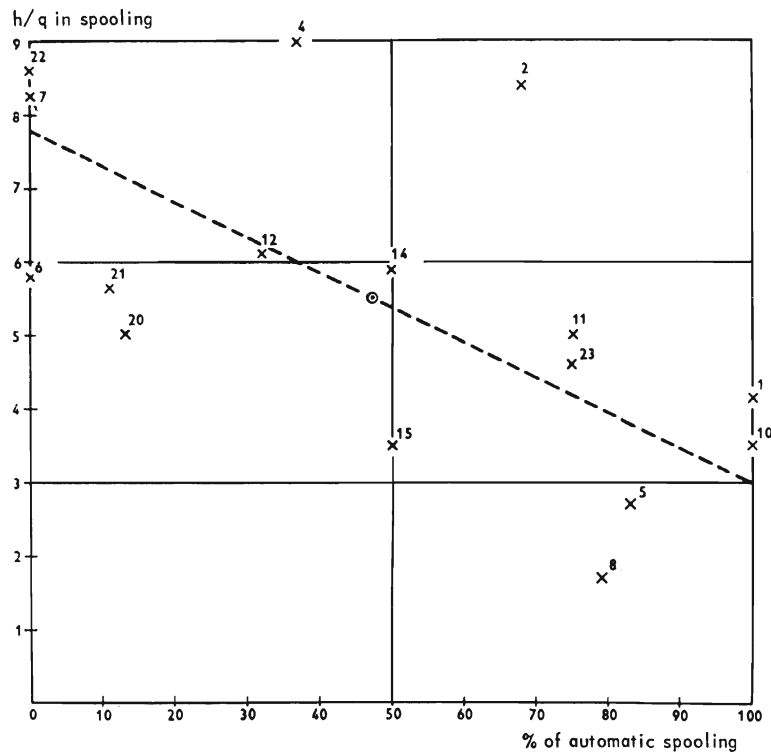


Diagram 4

1. *Produktivitet och kapacitetsutnyttjande i svensk bomullindustri* (Productivity and capacity utilisation in the Swedish cotton industry). Partsutredningen för textilindustrin. Stockholm, 1958, page 54.

The line in the diagram was traced freely, but it runs through the point which represents the average of both variables. By reading off on the y-axis for 0 and 100 per cent automatic spooling, we obtain an  $h/q$  estimate of about 3 for automatic spooling and up to 8 for other types. These unit man-hour figures can then be used as conversion factors to eliminate the dissimilarities in the proportion of automatic spooling and to compare the  $h/q$  for all the firms.

Of course, in order to obtain sensible results from these proceedings, the scatter of the points in the diagram about the regression line should not be significantly wide. If the concentration around the line is less than in Diagram 4, the method should be avoided.

When this method is applied to a firm, the measurements should be made more than once a year, and moreover, the variable traced against the x-axis should show variations from one period to another, sufficient that the diminutions in  $h/q$  caused by the application of scientific management techniques can be distinguished from those caused by modifications in the value of the variable. Diagram 5 shows this. In Diagram 5a, a situation has been depicted from annual data, where the  $h/q$  is progressively lowered, while the value of the variable slowly increases. The line connects the points in chronological order. There is no possibility whatsoever of distinguishing which is the cause of the decrease in  $h/q$ . On the other hand, Diagram 5b is computed from quarterly data, and it is quite easy to evaluate the influence of the alterations in the variable.

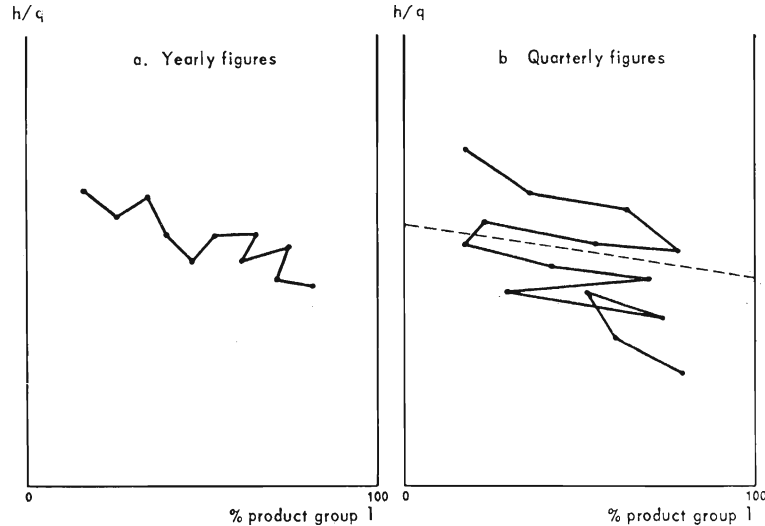


Diagram 5

The dotted line in the diagram has an inclination which roughly corresponds to short time changes of the curve. By reading again against 0 and 100 per cent, we obtain an evaluation of the  $h/q$  for both product groups. These can be used later as conversion figures.

If the data on man-hours cover more than two products or groups of products, resort has to be made to some more complicated method, as it is no longer possible to depict the situation with a simple graph. However, with multiple regression analysis it is still possible to obtain the corresponding result. The method has been used in a study on the Swedish building industry among others.<sup>1</sup> It has been pointed out that a simple application of regression analysis may yield deceptive results in some cases, due to non-fulfilment of the conditions of the analysis. For this reason, a much more complicated technique has been applied to a Danish survey in the shoe industry.<sup>2</sup>

In most cases, however, normal multiple regression analysis gives satisfactory results, if the variables are expressed as in the simpler example above, that is as an  $h/q$  calculated on total output and as the relative proportions of the different products. When using a time series for a firm, one should moreover include a time variable which receives value 1 for the first observed quarter, value 2 for the second, etc. With this time variable, some allowance is made for the continuous progress in scientific management techniques corresponding to the successive downpulls of the clusters of points in Diagram 5b.

#### MEASUREMENT OF OPERATIONS

The methods described in the above section may be difficult to apply in some firms where production consists principally of jobbing, where only one or a few units of every type of product are manufactured. In such cases, it is not meaningful to relate man-hours with the quantity of finished product. In other firms, similar difficulties may arise from the fact that the quality or the constitution of the products often changes, generally by means of modifications in the number of different operations in the finishing departments.

In both the above cases, a practical way to record changes in unit man-hours may be to measure the different operations instead of the products. Thus, man-hours in a certain operation are related to the volume of goods thus processed. A change in quality due to a modification in the number of operations per product does not then influence the index. Consequently, management action which assumes that the same quality of product can be obtained with fewer operations, does not affect the index. Instead, it is recorded as a reduction in quantity.

However the method—in various forms—has been used in a study of men's shoe factories in the U.K. and was the methodology recommended by EPA to the cotton industry.

In the basic computation of man-hours, it should be observed that it is in no way necessary to use the same quantity basis for all operations. It is preferable to express the quantity of product in each process in such a way that it corresponds as closely as possible to the amount of processing. Thus, in a weaving mill, a kilo of woven fabric may be a convenient quantity

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1. Mejse Jacobson, *Arbetsteknik vid egentliga byggnadsarbeten för bostadshus* (Work Methods Employed in Actual Construction Work for Houses), Stockholm, 1950, page 71.

2. G. Rasch, "A Method of Indirect Measurement in Productivity Studies", *Productivity Measurement Review*, No. 10, page 23 and No. 11, page 42.

basis for certain operations, while others depend rather on the number of metres woven. It can even happen that two different measures of quantity may be applied to the same process. This was the case in an English study of cold rolling mills for steel strip, where consideration was given partly to the length of rolled strip, and partly to the number of steel strip coils. The handling time for a small or a big coil is practically the same, while the time of processing is generally proportional to the length of the strip. It should be noted that in such cases the changes in efficiency which depend on changes in the lot sizes are eliminated, which is not always desirable.

The data on unit man-hours in different operations, obtained in this manner, may be combined with an index of the department or firm in exactly the same way as those for the products. The allocation and overhead methods described hereafter can be used, even in this case. However, an important difference is that at least some auxiliary departments can be treated similarly to the production departments. The only condition is that the services of the departments be measurable.

#### THE ALLOCATION METHOD

The allocation method amounts to referring all the man-hours in the firm to the finished products. For the base period, a figure for total man-hours per unit for every product is obtained which can be used directly according to Rule 2, page 28. This supposes that the different products can be distinguished during the whole course of processing, and that the accounts of the firm are in such a form that the allocation of the man-hours in the auxiliary departments can be made in accordance with the extent to which their services are used in the processing of the different products. For example, if the directly productive departments are debited with the services of such departments in every individual case, a good basis exists for an equitable allocation of the man-hours. Even in cases where a more schematic allocation of the costs of the auxiliary departments is made in the accounts, the same basis of allocation may be used for the man-hours.

From an accounting point of view, it may seem impossible or quite absurd to allocate to products such common services which generally cannot be related to any particular product. Neither is the partitioning of the base year man-hours of direct interest in calculating the  $h/q$  index. The question is rather: How many man-hours (in directly productive and in auxiliary departments) would have been spent during the base year, if the composition of production had been different from that which was recorded? One method of calculating this is to multiply the hypothetical production of every product by the equivalent man-hours per unit for the base period. The fact that such a calculation can be made accurately should assist in the evaluation of the number of hours spent by the auxiliary departments per unit of every product. Compare the discussion in Chapter 1.

After the allocation of the man-hours of the base period to the finished products has been made, Rule 2 can be applied directly to the calculation of the  $h/q$  index.

#### THE OVERHEAD METHOD

Where the product leaves the firm at different stages of processing or the semi-finished product is partly purchased and partly made by the firm itself, it is usually difficult to apply the allocation method. It is quite con-

venient, however, to consider every department in itself. It does not matter whether production is measured by the same measuring unit in all the departments or not. A total index for the whole firm can still be obtained by considering all the departments as independent from each other in the chain of production, and as working side by side even if they participate consecutively. It should also be noted that often, by choosing a measure of production convenient for every department, a more detailed allocation of the products is unnecessary. To make it possible, however, the measure used should reflect satisfactorily the man-hours used within the department. In raw material processing, weight may be the most convenient measure, while for the finishing operations it may be quantity or length (number of bottles, mileage of thread). In cases where the quantity of products in a department is largely dependent on a machine, it may be quite convenient to measure the volume of production in numbers of machine hours. Naturally, such proceedings can only be applied as long as the same machine is employed. When there is a change of machine, some means or other of assessing the relative hourly output of the old and the new machines is required, which is then multiplied by the number of machine-hours of the new machine.

When considering each department individually, it is generally acceptable to allocate to all the goods within the department the same overhead proportion of indirect man-hours. For a time series of man-hours per unit within the department, this overhead is unimportant. However, it exerts a certain influence on the sum of the results for all departments. Now, an absolute figure of labour input per unit can no longer be obtained for the whole firm, but only an index figure showing the changes by comparison with the base period.

Yet, when using this method, man-hours in the directly producing departments are treated differently from those in the auxiliary departments. In certain cases, e.g. transport, it may be hard to decide to which group a department or a worker should be allocated. As long as the purpose is not to compare different firms with each other, but only to establish a time series for the  $h/q$  of one firm, the criterion of the limit plays no great part, as long as it is not modified from one year to the other.

The method of calculation, expressed in symbols additional to the symbols used earlier (see page 27), is as follows:

$a_i$  = Number of man-hours in directly productive department  $i$ .

$A = \sum a_i$  = Total of man-hours in all directly productive departments.

$b_i$  = Number of man-hours in the auxiliary departments which are debited to or refer to directly productive department  $i$ .

$B = \sum b_i$  = Total of man-hours in the auxiliary departments.

The definitions involve that  $A + B = H$ .

Further, denote by

$$u_i = \frac{a_i + b_i}{a_i} \quad U = \frac{A + B}{A} = \frac{H}{A}$$

$u_i$  is thus the co-efficient with which to multiply the man-hours in a department to obtain the total number of man-hours which refer to the corresponding production, that is

$$u_i = 1 + \frac{\text{overhead proportion expressed as a percentage}}{100}$$

If the symbol  $ij$  affixed to the different symbols represents product  $j$  within department  $i$ , the overhead method implies that  $h_{ij}^{(0)}$  is estimated by  $u_i^{(0)} a_{ij}^{(0)}$ . The  $h/q$  for the whole firm, expressed according to formula (2), will be:

$$H^{(1)} = \frac{\sum_i u_i^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}}{\sum_i u_i^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}} \quad (5)$$

Thus, the calculation of the  $h/q$  index for the whole firm simply requires that, in addition to the data concerning the base period, production data for all products in all departments and total of man-hours in the firm during every period of calculation be available.

An example will demonstrate better how the method works: A firm operates with four departments, numbered 1 to 4. It has five different products, but these products leave the firm at different stages of completion. The processing of two or more products is common to some departments, but separate in others. A certain amount of breakage occurs in every department, so that the delivered quantity of approved material is less than the received quantity. Purchased material comes in at two different stages. Diagram 6 shows the movement of the products in and out of the departments.

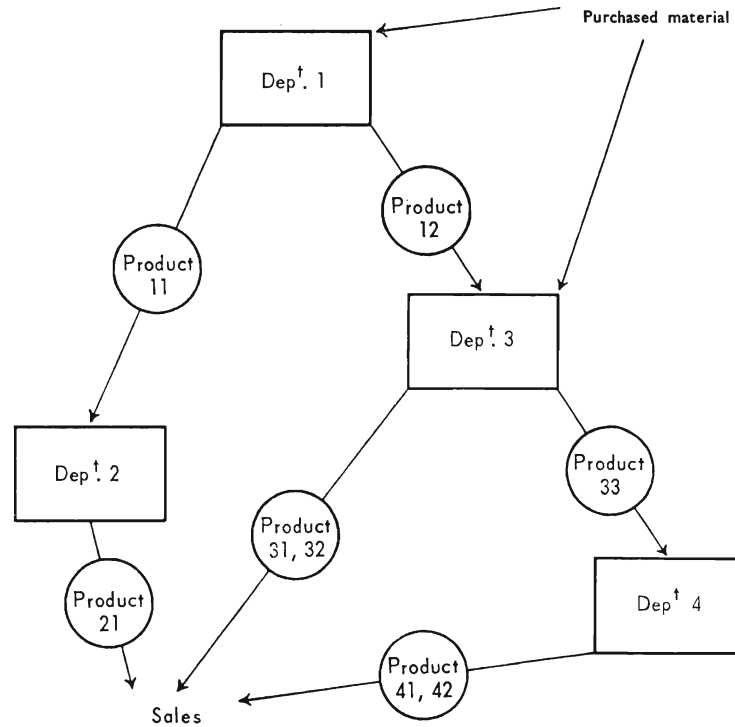


Diagram 6

The data needed for every product include the man-hours per unit during the base period (direct man-hours only) and the output during year 1. From this, a *calculated* number of direct man-hours for output during year 1 can be obtained.

TABLE 1

	YEAR 0		YEAR 1	
	MAN-HOURS PER TON	PRODUC- TION	CALCULATED NUMBER DIRECT HOURS	
	$\frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$	$q_{ij}^{(1)}$	$q_{ij}^{(1)}$	$\frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$
Dept. 1 Prod. 11 .....	3	8,000	24,000	
12 .....	4	5,000	20,000	44,000
Dept. 2 Prod. 21 .....	4	7,000	28,000	28,000
Dept. 3 Prod. 31 .....	10	2,000	20,000	
32 .....	5	3,000	15,000	
33 .....	8	9,400	75,200	110,200
Dept. 4 Prod. 41 .....	15	4,000	60,000	
42 .....	20	4,400	88,000	148,000

The calculations are carried on henceforth per department as shown in Table 2.

TABLE 2

	YEAR 0			YEAR 1	
	DIRECT MAN- HOURS $a_i^{(0)}$	MAN- HOURS FROM AUX. DEPTS $b_i^{(0)}$	$u_i^{(0)} =$ $\frac{a_i^{(0)} + b_i^{(0)}}{a_i^{(0)}}$	CALCULATED NR. DIRECT MAN-HOURS <sup>1</sup> $\sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$	$u_i^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$
Dept. 1 ..	36,000	20,000	1.56	44,000	68,600
2 ..	20,000	3,000	1.15	28,000	32,200
3 ..	101,000	42,000	1.42	110,200	156,500
4 ..	132,500	35,000	1.26	148,000	186,500
<i>Total</i> ..	<i>289,500</i>	<i>100,000</i>			<i>443,800</i>

1. From Table 1.

The total number of man-hours observed in the firm for year 1 was  $H^{(1)} = 412,000$ . The  $h/q$  index for the whole firm will thus be  $412,000/443,800 = 0.92$ .

In this respect, it should be observed that the only data for year 1 required for calculating the total index are the number of man-hours in the whole firm, and the output of the different goods. Should it be desirable to calculate, on top of this, an  $h/q$  index for each individual department,

the man-hours in each department will be required, as well as an allocation of the man-hours from the auxiliary departments to the directly producing departments. This allocation may be, of course, a rather laborious task. A Swedish steelworks, where calculations are performed in this manner, calculate a quarterly index for the firm, but departmental indices only once a year.

For the firm in this instance, the calculation of the index of each department will be as follows.

TABLE 3

	MAN-HOURS YEAR 1		TOTAL $h_i^{(1)}$	CALCULATED NO. OF DIRECT MAN-HOURS <sup>1</sup> $u_i^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$	$h/q$ INDEX $h_i^{(1)}$ $u_i^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$
	DIRECT MAN- HOURS $a_i^{(1)}$	FROM AUXI- LIARY DEPTS $b_i^{(1)}$			
Dept. 1 ..	42,000	26,000	68,000	68,600	0.99
2 ..	19,000	6,000	25,000	32,200	0.78
3 ..	98,000	44,000	142,000	156,500	0.91
4 ..	140,000	37,000	177,000	186,500	0.95
Total ..	299,000	113,000	412,000	443,800	

1. From Table 2.

#### SIMPLIFIED OVERHEAD METHOD

If the spreading of the overheads over each individual department runs into insuperable difficulties, the same overhead factor ( $U$ ) for all departments may be used as an approximation. This is the total number of man-hours in the firm divided by the number of man-hours in all the producing departments.

This involves the estimation of  $h_{ij}^{(0)}$  with the help of  $U^{(0)} a_{ij}^{(0)}$  and that of  $h_i^{(1)}$  with the help of  $U^{(1)} a_i^{(1)}$ .

Thus, the  $h/q$  index for the department  $i$  will be

$$\frac{U^{(1)} a_i^{(1)}}{U^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}} \text{ and for the whole firm } \frac{U^{(1)} A^{(1)}}{U^{(0)} \sum_i \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}} \quad (6)$$

The formulae show that the quotient of the overhead factors for year 1 and for year 0 is all that is required for the calculations. Since this quotient  $U^{(1)}/U^{(0)}$  is the same for all the departments and for the whole firm, it can be set apart and calculated once and for all. Further calculations are conducted on the simple basis of direct man-hours, after which the result is multiplied by the factor mentioned.

For the example above we get:

$$U^{(0)} = \frac{389,500}{289,500} = 1.35 \text{ and } U^{(1)} = \frac{412,000}{299,000} = 1.38 \text{ and } \frac{U^{(1)}}{U^{(0)}} = 1.02.$$



Calculations are performed exactly as in Table 1 above. Thereafter they are as follows:

TABLE 4

	DIRECT MAN-HOURS YEAR 1 $a_i^{(1)}$	CALCULATED NO. OF HOURS $\sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$	$\frac{a_i^{(1)}}{\sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}}$	$\frac{h/q \text{ INDEX}}{U^{(0)}} \cdot \frac{a_i^{(1)}}{\sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}}$ $\frac{U^{(1)}}{U^{(0)}} \cdot \frac{a_i^{(1)}}{\sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}}$
Dept. 1 .....	42,000	44,000	0.95	0.97
2 .....	19,000	28,000	0.68	0.69
3 .....	98,000	110,200	0.89	0.91
4 .....	140,000	148,000	0.95	0.96
Total .....	299,000	330,200	0.91	0.93

It may be seen that the results differ slightly from those obtained earlier, even for the index of the entire firm. This implies that the simplified overhead method should be avoided when one has reason to believe that the directly productive departments make use of the services of the auxiliary departments to varied extents. Even a very rough calculation of the overhead for every individual department generally gives a better result than the application of the same overhead to all departments.

#### SAMPLE OF PRODUCTS

Basing the calculations of the  $h/q$  index purely on a sample of products can simplify the calculation work in certain firms, but this, of course, introduces a new source of error. This method, however, is applied in several inter-firm comparisons of the U.S. Bureau of Labor Statistics and is used also by at least one large Swedish industrial firm.

Sampling can be used together with all the methods described above. But the principle of choosing the sample depends on circumstances. When using the allocation method, it is convenient to sub-divide the whole assortment of products into groups as homogeneous as possible, and to select one or several products from each group. On the other hand, with the overhead method, one or several products are selected and studied in every department.

The number of products to be singled out from every group or department depends to some extent on their importance, but above all on the estimated similarity in  $h/q$  trend of the products within the group or department. It does not matter though, if absolute man-hours per unit are at different levels. If all the products in a department undergo the same operations, and if improved techniques occur in the same process, the reduction in  $h/q$  may be equal for all products and one only is adequate for measurement purposes. The more heterogeneous the group of products, the more products should be selected.

As to the choice itself, one should first check on whether the group of products is dominated by one of them. If so, this main product should be chosen as the representative one. In other cases, it may prove possible to

find a product which can be considered as a kind of average of the products in the group. Otherwise, the selection must be random, and the probability of a product being selected should equal its part of the total production of the group.

Of course, man-hours for the representative products are preferably recorded permanently, but if this involves too many administrative staff, certain results can be obtained from measurements bearing on shorter periods, for example a month every year. Because of possible seasonal variations, the same period should be used each year.

If the calculations only concern one product at a time, the  $h/q$  index of the direct man-hours is obtained by dividing the man-hours per unit of the present period by that of the base period. The indices thus obtained for the products included in the sample can be used later as estimates of the  $h/q$  index of their respective group or department.

If several products are selected from the same department, an  $h/q$  index will be calculated with the help of the data on these products in the same manner as if they amounted to the total production of the department. An index for the firm, for example, can be calculated later, according to the allocation or to the overhead method.

When the  $h/q$  index is calculated according to the overhead method, and all the products are included, the formula for this calculation is (formula (5), page 52):

$$\frac{H^{(1)}}{\sum_i u_i^{(0)} \sum_j q_{ij}^{(1)} \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$$

This may also be written as:

$$\frac{H^{(1)}}{\sum_i u_i^{(0)} \sum_j a_{ij}^{(1)} \frac{q_{ij}^{(1)}}{a_{ij}^{(1)}} \cdot \frac{a_{ij}^{(0)}}{q_{ij}^{(0)}}$$

But  $\frac{a_{ij}^{(1)}}{q_{ij}^{(1)}} \cdot \frac{q_{ij}^{(0)}}{a_{ij}^{(0)}}$  is an index of man-hours per unit of product  $ij$  within the department for year 1 compared with year 0. If this index is represented by  $d_{ij}^{(1)}$  the formula may be written

$$\frac{H^{(1)}}{\sum_i u_i^{(0)} \sum_j \frac{a_{ij}^{(1)}}{d_{ij}^{(1)}}$$

Where product sampling is used, no data exists on  $d_{ij}^{(1)}$  for all the products. Instead, there is an estimate  $\bar{d}_i^{(1)}$  for the whole department, based on one or several selected products. This evaluation is used in such a way that  $\sum_j \frac{a_{ij}^{(1)}}{d_{ij}^{(1)}}$  is replaced by  $\frac{a_i^{(1)}}{\bar{d}_i^{(1)}}$ . Thus, the  $h/q$  index for the whole firm is calculated according to the formula:

$$\frac{H^{(1)}}{\sum_i u_i^{(0)} \frac{a_i^{(1)}}{\bar{d}_i^{(1)}}} \quad (7)$$

The following example illustrates the calculations involved when the overhead method is used together with "product sampling".

	YEAR 0			YEAR 1		
	DIRECT MAN- HOURS $a_i^{(0)}$	MAN- HOURS FROM AUX. DEPTS $b_i^{(0)}$	$u_i^{(0)} =$ $\frac{a_i^{(0)} + b_i^{(0)}}{a_i^{(0)}}$	DIRECT MAN- HOURS $a_i^{(1)}$	$h/q$ INDEX OF SELECT- ED PRO- DUCTS $d_i^{(1)}$	CALCU- LATED NO. OF MAN- HOURS $u_i^{(0)} \frac{a_i^{(1)}}{d_i^{(1)}}$
Dept. 1 ..	19,000	6,000	1.32	20,000	0.92	28,700
2 ..	25,000	13,000	1.52	21,000	0.96	33,200
3 ..	62,000	24,000	1.39	71,000	0.89	110,900
4 ..	35,000	12,000	1.34	36,000	0.94	51,300
<i>Total</i> ..						224,100

The total number of man-hours in the firm for year 1 was  $H^{(1)} = 208,300$ , and the  $h/q$  index therefore was  $208,300/224,100 = 0.93$ .

Of course, some degree of uncertainty attaches to this result due to the fact that calculations are based only on a sample of products. On top of the random error which always prevails in sampling studies, another factor of uncertainty intervenes here. Some influence may be exerted by the fact that the goods considered for the measurement of man-hours per unit have different characteristics from the other products in the department. For example, they may be more standardised, coming in long series, while the non-measurable part of the department's production is made up of jobbing work, with more restricted opportunities for improvements in technique. If this part of the output is not too big, one can perform an additional calculation in which the  $h/q$  for this part is assumed to be constant throughout the whole period. The exact value lies probably between the value obtained in this way and that calculated according to the methods explained above. If the proportion of jobbing work of such a type is large, the difference between the two index figures can be important and the results consequently insignificant. In such cases, it may be better to overcome this by measuring operations instead of products (compare page 49).

This kind of calculation is performed in the following manner for a department.

A sampling study indicates an  $h/q$  index of 0.92 for year 1 in this department. However, 25 per cent of the man-hours during year 0 could be related to the jobbing work products which could not be considered in the sample. If 0.92 is given as the  $h/q$  index for the whole department, this assumes that the  $h/q$  trend has been the same for these goods as for the others. Where no trend at all is related to the jobbing work, the  $h/q$  index for this department would be

$$0.75 \cdot 0.92 + 0.25 \cdot 1.00 = 0.94.$$

The correct  $h/q$  index is probably between 0.92 and 0.94.

## Chapter 4

### THE PROBLEM OF MEASUREMENT — OTHER INPUT FIGURES

To calculate the input-per-unit index of production factors other than labour, the methods described in Chapter 3 are generally applicable. However, difficulties arise of a different nature to those concerning manpower. Naturally, this concerns mainly input of capital. Labour input could be measured in man-hours which could be aggregated either for a department or for the whole firm. When capital is in question, there is no unit of measurement which can be considered as generally applicable.

This chapter will deal with the special problem of measuring raw material, energy and capital inputs.

#### THE PROBLEM OF MEASURING RAW MATERIAL AND ENERGY INPUTS

Industries which measure raw material input in their production with the greatest accuracy, are those which process a relatively uniform raw material in large quantities, e.g. steel works or pulp mills. In these cases, the measurement of raw material is no problem. The input can be expressed in a unit, e.g. tons of crude steel in a rolling mill or cubic metres of solid wood in a paper mill. The same applies generally to the consumption of energy, where one source usually predominates and is the only one whose input is worthy of measurement.

Difficulties arise from the moment it is necessary to consider two or more raw materials or sources of energy. There are then two ways to overcome such difficulties:

1. Calculate separate input figures for each raw material or source of energy. They can later be aggregated with labour and capital input according to the methods described in Chapter 5.
2. Reduce the total consumption of raw materials to a single measure (e.g. a quantity index) and later treat this as the measure of the quantity of raw material. All quantities of energy can be similarly reduced.

The first method creates no special problems of quantity measurement. The second method poses the question of how to establish the required quantity index.

There is no general method of determining the conversion figure for such a quantity index. Such a calculation should be performed only where the types of raw materials or energy can be substituted for each other. In the case of fuel, for example, one can calculate the whole fuel consumption

in tons of coal equivalent according to calorific value.<sup>1</sup> In other cases, one should attempt to find suitable equivalents. Thus, one can convert various raw materials in a steel mill into their iron content and aggregate tons of *Fe* instead of tons of material.

However, for raw materials which are not interchangeable but complement each other, as for example wood and fabric in a furniture factory, such a method cannot be used. In such cases, the convenient policy is either to study only the most important raw material or to calculate separate series for each and try to aggregate them according to one of the methods in Chapter 5.

When calculating the *r/q* index, there is usually no difficulty in applying Rule 2, page 28, directly, since it is generally possible to calculate the raw material input for every product during the base period. It is important to note here that the raw material input of the base year is the one to use as a conversion figure when aggregating the production. If the calculation of the *r/q* index is based on any otherwise computed production index, the erroneous conversion figures could introduce a bias whose annual variations could amount to more than the annual variation in the index.

Fuel or energy input is generally more difficult to allocate to the different types of products. As they are not greatly influenced by the minor changes in the product composition, input of it is usually obtained with sufficient accuracy by dividing the total amount of fuel used by an output index computed for another purpose.

#### MEASURING CAPITAL INPUT

The rest of this chapter concerns capital input. Most of it will be dedicated to the discussion of the different possibilities of finding a convenient measure for aggregate capital investment. Thereafter, some thought will be given to the application of this measure of capital to the calculation of capital input per unit produced.

The establishment of a measure of capital input should start the same way as for the other input factors, that is to say by an effort to compare actual with minimum costs. All the elements of capital costs, therefore, should also influence the measure of capital input. Meanwhile, the effect of price changes should be eliminated.

Thus, accountancy data on capital costs are of no value here. Accountancy methods do not keep track of purely physical depreciation; what insurance companies call depreciation by wear and tear, which corresponds most closely to what we are interested in here. In the same spirit, interest should also be reckoned on all the remaining capital and not refer only to interest paid. This is also where maintenance costs come in. It is generally

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1. Official Swedish statistics apply the following conversion figures (metric tons):

1 ton coke	= 1.05 tons hard coal
1 ton peat	= 0.5 tons hard coal
1 hl. charcoal	= 0.017 tons hard coal
1 cubic metre wood	= 0.17 tons hard coal
1 ton petrol	= 1.60 tons hard coal
1 ton diesel oil	= 1.50 tons hard coal
1 ton kerosene	= 1.54 tons hard coal
1,000 cubic metres town gas	= 0.72 tons hard coal

difficult to calculate capital costs in this way and to obtain some idea of their quantitative contents. One must, therefore, resort generally to more summary proceedings.

It seems, therefore, natural to suppose that the quantitative contents develop on parallel lines with either the effective capital amount (for example, the number of machines of a certain type) or the use of capital (for example, the number of machine hours). But it is not certain that both of these yardsticks are proportional, as a result of two factors, one of which is the capacity in use and the other the life of the capital good. The former should be self-evident, but the latter may require some explanation.

Supposing a given machine can be offered in two different types both of which have the same overall properties of capacity and output, but that the first is worn out after five years, while the second lasts ten years. The second machine costs about twice as much as the first, so that the annual capital cost is the same in both cases. The higher interest payment is compensated by lesser entries. When passing from the first to the second type, the use of capital or, if preferred, *capital flow* is unchanged, while the average *capital stock* over the years is nearly double. It would appear that the first quantitative unit is closer to what we wish to measure. However, this represents an extreme case. Generally, capital flow should develop along fairly parallel lines with capital stock. Available data rather than principles should determine the choice of one of them as a starting point in measuring capital investment. However, the question of capacity deserves a special discussion.

There may be certain reasons for a machine not being used to full capacity during a certain period. The demand for the product, processed by that machine, may have decreased. Organisational disruption can cause longer lulls on the machine because material is not fed to it on time, or because no worker is available to set it, etc. The capacity of the machine may prove temporarily too high in relation to other processing stages.

The problem posed by the calculation of production efficiency is: does the higher capital cost per unit produced caused by an incomplete use of capacity correspond to lower efficiency? In other words: should the minimum cost per unit procured by full use of capacity be considered attainable when this capacity is not fully used?

The answer should be yes, when the cause is poor organisation, but less firmly so when the cause is a lack of demand for the product. In the latter instance, there is nothing the producing departments can do to remedy the situation in the short run, because it is not possible to attain the real minimum cost. In the long run, the real remedy is to adapt capacity to the expected demand. Since the responsibility for this adaptation should rest upon the head of the producing departments, it seems reasonable to burden production efficiency with a restricted use of capital.

Thus, there is a definite difference between short-term and long-term aspects. If there are possibilities of measuring capital consumption as well as capital stock, the long or short-term character of the efficiency measure will determine which one is preferable. However, it is normally impossible to measure capital consumption, so that there is no real choice.

Several methods of measuring capital input are now described. No order of preference is given for them—such preference will depend on which method is the simplest to apply in each particular instance.

Two of the methods in connection with capital flow concern the measurement of machine time and of machine rental. As far as measures of capital stock are concerned, the discussion will be mainly of a quantity index and value data, deflated by a price index.

#### CAPITAL FLOW

There are relatively few cases where it is possible to measure capital flow which takes into consideration the entire capital stock. However, one such case is when the departments are debited by a rental for the machines and buildings at their disposal, if such rents are calculated per unit of time. It does not matter then whether the firm really rents the machines as is the case with some punch-card equipment, or if a simple internal calculation is involved.

Even if the rent of the older machines is related to their replacement value, all that is necessary is to divide the total rent by a rent price index, which can be calculated as a chain index. Every link in this chain index is based on the price changes of the machines in existence, both at the beginning and at the end of the link period. The calculation of such a chain index is made in the same manner as when the problem was to take a new product into consideration for the  $h/q$  index (see page 36).

Now if the rent of machines is based on their purchase price, the rate of rent will only change when new machines are bought at prices which involve a higher (or lower) rent than the existing ones.

In this case the rent for every machine should be divided by an appropriate price index figure for the year when it was bought, so that the whole course of the rent is expressed in a common price rate. For the choice of indices and the discussion of the problem of quality, see below, measurement of capital stock, page 62.

A somewhat simpler method, which gives satisfactory results if purchases have been about the same every year, is to divide the total amount of rent by a price index figure which represents the average price of machines during the whole period the machines now in use were purchased. In practice, this can be performed by first assessing the average length of life of the machines in question—let us say it is ten years. The index figure to apply to a certain year will then be the average of the ten foregoing years of some convenient price indices.

If the study is restricted to some department with relatively uniform machine equipment, a quantitative measure may be used to spare the trouble of a conversion by means of price indices. In a Dutch study of the printing industry, the number of available working hours of the printing presses was used. In order to account for differences of speed, a conversion figure was calculated for every press and used to multiply available time. In this way, a real measure of total capacity was obtained.

The method of measurement of capital input, most often applied in efficiency studies, which really takes this factor into consideration, is the measurement of machine hours. An example of this is found in studies of cotton spinning mills in several countries, including Sweden. In this case, it is usual to measure the number of spinning hours spent per kilogramme of thread (or the reciprocal of this quotient). Thus, the measure concerns only a small part of the whole capital investment and one operation, spin-

ning proper. In detailed analyses, this method may be valuable, but it is not applicable to a whole firm. It should be noted that the latter method is the only one to consider consumed capital.

#### EXISTING CAPITAL STOCK

In most cases, a look into capital stock is necessary if an overall measure of capital investment is to be obtained. The point then is to find some way of adding up the amount of all types of invested capital into a single quantitative measure. In Chapter 1, the total of installed horsepower is used. Of course, this concerns only machine equipment and is a very incomplete measure at that, but it can be applied to a firm whose capital stock is mostly made up of motor-driven machines.

It should, in principle, be possible to compute quantity indices of the state of capital in different years by applying Rule 1 (page 27) in the calculation of the production index. The number of machines and other invested capital of different descriptions could thus be multiplied by pre-determined conversion figures before being added up. The series thus obtained represents the amount of capital. However, it would be very difficult to perform such calculations in practice because the problem of quality would be overwhelming. It is quite unusual for a type of machine to remain unchanged long enough to enable one to find out how much a machine purchased today would have cost in the base year. This state of things is further complicated by the fact that modifications generally involve improved efficiency and a higher hourly rate of production for the machine. Does this imply an increase of the amount of invested capital or a reduction of capital input per unit produced?

The difficulty of distinguishing changes in price from changes in quality may be illustrated by the following example.

A machine which can produce 1,000 units per hour costs 20,000 Swedish crowns in a given year. The following year, its makers release a new model of the same machine which can produce 1,500 units per hour. If it costs 30,000 crowns, this can be interpreted in the following ways, among others:

- a) The new machine is counted as  $1\frac{1}{2}$  old machines. The price of the machine and the capital input per unit produced are unchanged.
- b) The new machine is counted as an old one. The price of the machine has gone up 50 per cent but the capital input per unit produced has been reduced by one third.

One can also envisage interpretations somewhere between these two. The choice depends to a certain extent on the environment. Where market prices exist for the machines and both models are on sale together for some time, it is natural—although not necessarily correct—to consider the prevailing price difference between them as a measure of the difference in capacity. In such a case, the difference in performance is reflected by the change in capital input per unit produced.

Consequently, development in technical efficiency will partly depend on whether or not the suppliers of the machine are in a position to let a greater or a lesser rise in price accompany the improvements made to the machine. This is natural in itself, since we continuously strive to express



efficiency in terms of costs. It also means that if the firm makes its own machines, and thus only needs to account for the production cost in terms of machine cost, the change in efficiency can be different from that obtained when the machines are purchased from outside. This consequence may be irrelevant when the efficiency development of the whole industry is considered from a socio-economic point of view, but it is inevitable when the production efficiency of a firm is the point at issue.

The performance rate can seldom be expressed as simply as in the above example. The improvement of the machine may consist merely in closer tolerances for the product, so that there are less rejects upon reaching the next processing stage. In this and other cases, it is almost impossible to express the difference between the models, other than by using the relationship between their respective prices.

Because of these difficulties, the computation of a quantitative index for invested capital is not an advisable method. Generally it should be significantly simpler to use a value figure divided by a price index. However, it should be noted that the problem of quality does not vanish in this process—it is simply passed on to the compiler of the price index. If he calculates as in case (b) above and does not consider quality changes, most of the increased value will be attributed to the price increase, and the capital input per unit will appear lower than if he attempts to apply case (a). As the price indices cover a great number of goods, not all of which are modified at the same time, this only amounts to a difference in the long run without any significant influence on comparisons from year to year.<sup>1</sup>

In order to facilitate the choice of an index series and the interpretation of the ensuing result, we shall briefly describe the more usual Swedish price indices which may be adopted.

A price index for *industrial buildings* is calculated on a quarterly basis by the "Svenska Tarifföreningen" and published among other places in the paper "Industriförsäkringskydd" issued by that organisation. The index concerns a factory building in Stockholm with ground floor and garrets, with about 500 sq. metres floor space; the building is designed mainly for a mechanical workshop or similar use. There are different indices for the following types:

1. Wooden walls, carpentry and roofing.
2. Brick walls, wooden carpentry and roofing.
3. Brick walls, concrete frame between iron beams, wooden roofing.
4. Brick walls, concrete frame and re-inforced cement roofing.

All four types are indexed against three different base periods 1/1/1935, 1/9/1939 and 1/1/1950.

Price indices for *machines* are to be found in the index of wholesale prices of the Kommerskollegium and in the index of the Svenska Tarifföreningen.

*Kommerskollegii partiprisindex* is published monthly in "Kommerciella Meddelanden". It is calculated on a 1949 base, but in order to maintain a connection with an older series, this index is also published with 1935 = 100. Thereby, unbroken series have been obtained since 1929.

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1. For a thorough discussion of the problem of quality in price index calculations, see E. von Hofsten, *Price Indices and Quality Changes*, Stockholm, 1952.

The value figures divided by the price index (and then multiplied by 100) supply investment values with the prices of 1949 or of 1935. The index of wholesale prices aims at measuring price developments at wholesale trade level of all the goods sold in the country (not for export). One of the indices includes "machines and means of transportation" and should give a fairly accurate picture of price evolution in the machines purchased by the firm, even though its coverage is much broader.

As a rule, the wholesale price index relates to unchanged goods. When quality changes intervene, attempt is made to take them into account so that a change of price motivated by a change in quality does not influence the index. When capital value is divided by such an index, a machine with a higher performance rate will count as a higher quantity of capital than the preceding machine. The proceedings correspond to case (a) above.

*Svenska Tarifföreningens maskinprisindex* is computed quarterly in the same way as their index of building costs and can be procured by Tarifföreningen or by insurance companies. Five different indices are calculated.<sup>1</sup>

1. Light machinery.
2. Furnaces and heavy machinery.
3. Electric generators, motors and similar machinery.
4. Electric power and light installations.
5. Piping.

The indices mainly concern Swedish machines of standard quality. Even here, an attempt is made to evaluate quality changes so that the indices will measure the price evolution of machines of the same quality.

The indices are published for two different base periods, 1/1/1937 and 1/9/1939.

These indices may be applied to the conversion of value figures to constant prices. Such value figures can, in their turn, be obtained in different ways depending on the accounting system of the firm. Quite often, the fire insurance value can be used. If the firm has an insurance at up-to-date spot value, so that the insurance value is progressively adapted partly to the depreciation by wear and tear of invested capital, and partly to the changes in replacement prices, the insurance value simply has to be divided by the present index figure of a convenient price indices. In the case of an insurance at replacement value, there is no corresponding adaptation to the decrease in value, but the insurance value for, let us say, a machine—with few exceptions—is equal to the purchase value of an identical new machine throughout its duration. This value can also be used to obtain an index of capital amount after division by a price index. However, special calculations should be performed when a firm passes from one insurance system to the other.

Where the fire insurance value cannot be used, an approximate value of existing capital can be obtained by the simple addition of new investments over a convenient number of past years. The amount of investments for every year should then be divided by the corresponding index figure of

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1. Examples of machines to be included in the different groups are given in *Calculations of insurance values for buildings and machinery* [Instructions published (in Swedish only) by Svenska Tarifföreningen, Stockholm, 1956].

price indices. The number of years to be included should be chosen to correspond roughly with the length of life of the invested capital involved. Thus, the procedure corresponds to the use of the amount of an insurance at replacement value. It should prove convenient to treat machines and buildings separately on account of their unequal durability. If the life of a machine, for example, is established at ten years, and that of a building at forty years, investments in buildings for the period 1919 to 1958 and in machines for 1949 to 1958, divided by convenient index figures for every year, amount to a measure of the volume of capital at the end of 1958. In order to obtain the corresponding figures for 1959, building investments from 1920 to 1959 and machine investments from 1950 to 1959, etc., are used.

Until now, the discussion has referred only to the question of measuring capital investments in static installations. However, capital invested in stored goods can also prove important inasmuch as it replaces other input factors and should therefore be reckoned with. Thus, a stock of finished products can act as a cushion between many small orders and steady mass production. Larger batch sizes mean smaller unit labour and installation capital inputs, and controlled capital investment in stocks helps to increase the batch size. Similar reasoning applies to stocks of raw materials or of spare parts, so that all stocking costs must be included in capital costs as far as possible.

#### CALCULATING THE $c/q$ INDEX

As with the  $h/q$  index, a satisfactory theoretical calculation of the  $c/q$  index requires data on the total capital input for every period of calculation, and on capital input per unit for every individual product during the base period. The first set of data can usually be obtained by means of one of the methods described above, while only in exceptional cases is it possible to calculate the latter even for a single period. This means that the production index by which the total capital input should be divided cannot be calculated with the conversion figures which are appropriate in this case.

Since, after all, the measure of capital is generally rather insecure, there is no need for refined calculations at this point. Thus, the only rule for calculating the  $c/q$  index will be:

#### *Rule 4 — $c/q$ INDEX*

*Divide the index of total capital input by any production index, computed for whatever purpose and calculated with either some input figures or with prices as weights.*

This somewhat vague rule may be applied to a department as well as to the whole firm. As it seems impossible to measure capital input per unit produced with any greater accuracy, no great importance should be attached to minor variations of the  $c/q$  index. It should mainly be used together with the  $h/q$  index and eventually with other input indices for the calculation of a total index (compare Chapter 5).

## Chapter 5

### CALCULATING AN EFFICIENCY INDEX

Chapters 2 to 4 treated the problems which arise from attempts to calculate different input figures and their changes within the firm. If the calculation of series of indices for one or several inputs per unit of output was achieved by means of any of the methods already described, this is still not enough for a definite measure of technical efficiency as sought in Chapter 1. It fails, in one way or another, to account for several input figures at the same time.

The situation is then the same as was outlined in Diagram 1, page 15. Certain combinations of inputs seem to be of equal value from the point of view of efficiency—they correspond to the same degree of “technical knowledge”—and are represented there by a curve, the technical curve. The precise meaning of this curve has already been thoroughly discussed in Chapter 1, pages 14 and 15.

The comparison between two situations, each corresponding to one point in the diagram, must refer to the position of the curves on which the points lie. The simultaneous comparison of more than two inputs is similar, although this case cannot be illustrated so simply in a diagram. However, in order not to complicate the explanation more than necessary, the following discussion will bear, for the most part, on two inputs.

It is easy to see which of the two curves corresponds to the most efficient production. The closer to the origin, the lower its corresponding input per unit of all the input factors. But the knowledge that one position is better than the other is not enough; it is necessary to know in what degree. This requires a measure to determine the position of every curve. However, there is no univocal way of finding such a measure, and for this reason it is necessary to adopt some conventional method which supplies a meaningful measure. In the few papers published hitherto concerning these problems, two methods based on different principles have been proposed and applied. They can be described briefly as follows:

1. Find the point on the curve of the base period at which the relation between the inputs per unit is the same as that observed during year 1. A comparison of this point with year 1 will show that all the inputs have changed in the same proportion. Consequently, the  $h/q$ ,  $r/q$  and  $c/q$  indices are all equal and any one of them may be used as an indicator of development. Hereafter, we will call this common value the total index, since it concerns the total change in input per unit of output. The efficiency index is the reciprocal of the total index. Thus, the total index shows to what

extent the quantity of production factors required for a given production volume decreases, and the efficiency index shows to what extent the production volume which can be obtained from a given input of production factors increases.

2. The inputs of the present period are compared with the point on the curve of the base period at which all the inputs per unit but one are equal to the present ones. The index of this remaining input is used as the indicator of development. Thus, the whole change is reduced to a single input, usually that of labour. No efficiency index can be computed using this method.

The first of these methods seems to be the more satisfactory one, since it results in an index figure with a readily understandable content. If all prices remained unchanged, the total index also approximately shows the reduction of the total cost which corresponds to the decreased input figures. The result of the second method incurs some difficulty in interpretation, but in certain contexts its application is more versatile. Practical possibilities of the application of both methods are discussed below.

#### CONSTANT RELATION BETWEEN INPUTS PER UNIT

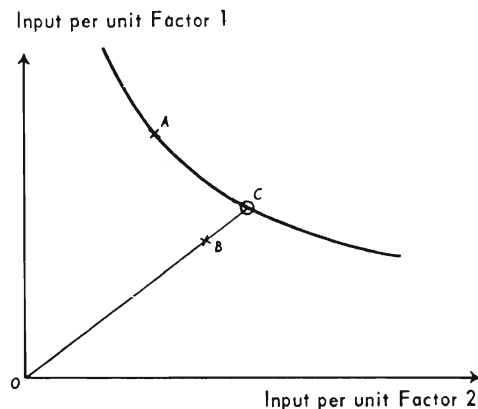


Diagram 7

The procedure concerning the first of the above methods is illustrated by Diagram 7 for the case where only two production factors are involved. Point *A* corresponds to the inputs observed for the base period, and point *B* to the present period. The curve which runs through *A* represents the technically possible combinations of inputs per unit for the base period which are equivalent to *A*. A line traced from the origin *O* through *B* crosses the curve at *C*. At this point, the relation between inputs is the same as in *B*, so that *C* should be compared to *B*. The equivalent change in both inputs is then represented by the relationship between the distances *OB* and *OC*. This relationship is the total index sought.

The difficulty remains that there are generally no direct data on the shape of the technical curve, and because of this the position of point *C* is uncertain. Several methods have been offered in the literature and employed in various studies to ascertain the position of *C*. The principal methods are described below, *viz.*:

- a) The shape of the curve is obtained by several observations.
- b) The curve is assumed to have a certain mathematical form.
- c) A straight line is used as an approximation of the curve.

#### THE SHAPE OF THE CURVE IS OBTAINED BY SEVERAL OBSERVATIONS

This method assumes that many observations can be made concerning various relationships between the factors used. The result is represented in a chart and the points which lie closest to the origin are connected by straight lines as in Diagram 8. Lines parallel to the axes are drawn from the extreme points. The result is an uninterrupted curve. All the points are either on the curve or higher to the right.

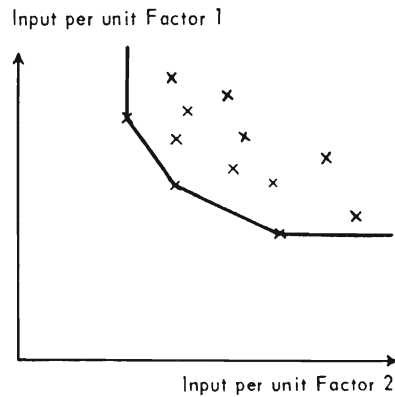


Diagram 8

This method is seldom applicable to an isolated firm, because the relation between the inputs of production factors changes seldom to the extent required by a clear conception of the shape of the curve. But it applies more easily to an industry study where every firm is represented by a point. The points thus determined are more scattered, and the curve obtained with them may be conceived as an approximation of the *T*-line (compare page 20), since only the most efficient firms influence the position of the curve. How the curve is used in sector comparisons will be further discussed in Chapter 6.

#### THE CURVE IS ASSUMED TO HAVE A CERTAIN FORM

Several forms of mathematically expressed curves may be considered as approximations of the technical curve. Since generally only one point on the curve is known, namely that of the combined inputs observed during

the base period, the choice of curve should originate from general considerations. An important thing to keep in mind here is not to complicate the computation of an efficiency index.

One type of curve, often used in similar circumstances, the Cobb-Douglas function, will be considered here. When it involves only labour and capital inputs, the function, turned into its simplest form, can be transformed so as to suit our purpose:

$$\left(\frac{h}{q}\right)^k \left(\frac{c}{q}\right)^{1-k} = \text{constant} \quad (8)$$

where  $k$  is a figure between zero and 1. By means of theoretical speculation on how a firm, which can choose its position along such a curve, should best combine its inputs, one finds that  $k$  is:

$$\frac{\text{wages cost}}{\text{wages cost} + \text{capital cost}}$$

The value of the constant in the right hand member of (8) depends on the choice of the measuring unit for  $c/q$  and  $h/q$ . If we use indices for both inputs,  $c/q = h/q = 1$  for the base period. Then, the curve passing through this point should have its constant in the right-hand member = 1.

The calculation of a total index now appears very simple. For a period with an  $h/q$  index  $I_{h/q}$  and a  $c/q$  index  $I_{c/q}$ , it can be shown that the total index which corresponds to the relationship between distances  $OB$  and  $OC$  in Diagram 7 can be calculated as follows:

$$\text{total index} = I_{h/q}^k I_{c/q}^{1-k}$$

In other words, the total index is a weighted geometric mean between the two input indices.

The practical calculation is made with the aid of logarithms:

$$\log \text{total index} = k \log I_{h/q} + (1 - k) \log I_{c/q}$$

For a Swedish paper mill, an  $h/q$  index of 0.77 and a  $c/q$  index of 0.89 were obtained for 1957 (1951 = 100). Total wages for the base year were 1.2 million crowns and capital costs 1.0 million crowns.  $k$  was thus =  $1.2/2.2 = 0.55$  and the total index:

$$0.77^{0.55} \cdot 0.89^{0.45} = 0.81$$

Thus, according to these calculations, the average saving of production factors amounted to 19 per cent. The efficiency index is  $1/0.81 = 1.24$ .

This method can easily be extended to several production factors. If the index of raw material input is represented by  $I_{r/q}$ , the total index of labour, capital and raw material inputs is calculated as follows:

$$I_{h/q}^k I_{c/q}^l I_{r/q}^m \text{ where } k + l + m = 1$$

and where  $k$ ,  $l$  and  $m$  stand for the shares of total cost attributable to wages cost, capital costs and cost of raw materials.

For further examples of this method of calculation, which is most highly recommended for use inside the firm, we can use the figures of Diagram 2 (page 21) for the whole Swedish industry as the basis of calculation of the efficiency index. However, it should be noted that the method has been purposely developed for an individual firm desiring to measure its technical efficiency against its self-determined objective. The result is

therefore difficult to interpret when figures for the whole industry are used and it should be considered mainly as a numerical example.

In the following table of calculations, consideration was given to labour input expressed in man-hours, to salaried employee input expressed in number of men, and to capital input expressed in horse-power installed, all quoted per unit of output. Exponents corresponding to  $k$ ,  $l$  and  $m$  above as used in the calculations are 0.55 for labour, 0.20 for salaried employee and 0.25 for capital. These were procured partly from the cost data of industrial statistics and partly from the official profit statistics.<sup>1</sup>

YEAR	INPUT-PER-UNIT INDEX (1946 = 100) FOR			LOG TOTAL INDEX = 0.55 log $I_{hlq}$ <sup>+</sup> + 0.20 log $I_{llq}$ <sup>+</sup> + 0.25 log $I_{clq}$	TOTAL INDEX	EFFI- CIENCY INDEX
	LABOUR $I_{hlq}$	SALARIED EMP- LOYEES $I_{llq}$	CAPITAL $I_{clq}$			
1946 ....	100.0	100.0	100.0	2.0000	100	100
1947 ....	96.5	102.7	100.7	1.9945	99	101
1948 ....	92.7	100.1	98.6	1.9805	96	105
1949 ....	88.8	99.8	101.2	1.9727	94	106
1950 ....	85.7	98.9	103.1	1.9654	92	108
1951 ....	82.7	98.7	105.4	1.9592	91	110
1952 ....	82.1	102.0	114.6	1.9694	93	107
1953 ....	78.4	101.4	120.3	1.9631	92	109
1954 ....	76.1	100.3	122.3	1.9569	91	110
1955 ....	73.6	98.4	120.8	1.9459	88	113
1956 ....	70.8	98.8	122.9	1.9389	87	115
1957 ....	69.7	98.7	124.3	1.9362	86	116

If these figures were those of a firm, they would infer that such a firm had, in 1957, a production volume higher by 16 per cent than it would have attained using the same quantities of the different production factors and with techniques unchanged since 1946. Measured in this way, efficiency has increased during the whole period except in 1952, when there was a slight set-back.

Here we must note a point of computation technique. It may be correct to include a relatively high number of digits as in the above calculations in order to minimize the effect of errors in rounding up the figures, but the final result must be expressed without decimals, as they might convey an accuracy too exaggerated for what the indices and the computation method can bear.

#### A STRAIGHT LINE USED AS AN APPROXIMATION OF THE CURVE

The simplest method from the computation point of view is to use a straight line as an approximation of the curve. This supposes that the total index is a weighted arithmetic mean of the input per unit indices. The slope of the line depends upon which weights are used.

1. *Income, Expenditure and Profits of Business Enterprises*, published yearly since 1953 by the Statistiska Centralbyrån (in Swedish with summary in English).



For several reasons, it is appropriate to choose the total costs for each production factor during the base period as weights for the different input indices. In this way, the weighted total index shows the amount of the relative cost reduction, assuming that prices have not changed. As expressed in terms of the Diagram, the selected straight line is the tangent to the curve of the base period at point *A*—assuming that the firm has chosen the cheapest possible combination of inputs for the base period.

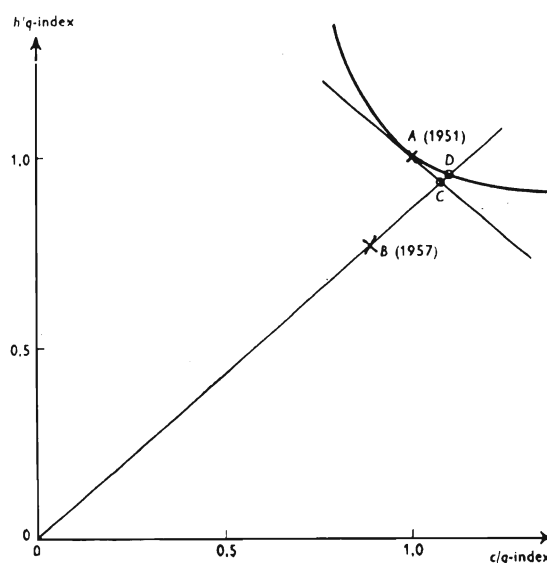


Diagram 9

In the above-mentioned paper mill, if the sum of wages during the base period amounted to 1.2 million crowns and capital costs to 1.0 million crowns, the weighted index figure will be:

$$\frac{1.2 \times 0.77 + 1.0 \times 0.89}{2.2} = 0.82$$

The difference between this result and that of the preceding section, obtained as geometric means, is not very important.

If the price relationship between the production factors has been modified since the base period, it can be argued that calculations performed in this way give an upward bias to the index figures, that is to say a downward bias in the rate of improvement. This is shown by Diagram 9. Since the technical curve is convex towards the origin, distance *OD*, to be compared to *OB*, is longer than *OC* and therefore the quotient *OB/OC* is more than *OB/OD*. The index figure calculated in this manner can therefore be considered as the upper limit to the desired total index.

Meanwhile, it may be argued that, if present costs were the chosen weights, the index would be lower than it ought to be. However, the lower limit of the total index calculated in this manner is not as definite as the

upper one as some suppositions concerning the shape of the technical curve must be fulfilled in order that a total index calculated with full knowledge of the curve should not fall below the limit. In spite of this, it may prove interesting to calculate the index by using the weights of the base year as well as the present. If the results are too far apart, this is a warning against granting too much consideration to the figures.

If the present costs are to be used as weights, the whole comparison should be reversed, so that year 1 becomes the base period. The index of year 0 for the different inputs is then computed with year 1 as a base, which is done by taking the reciprocal value of the index for year 1. Thus, if the cost of wages during the later period of comparison in the above-mentioned paper mill was 1.8 million crowns and capital cost at 1.6 million crowns, the total index for year 0 according to this method of calculation will be:

$$\frac{1.8 \times \frac{1}{0.77} + 1.6 \times \frac{1}{0.89}}{3.4} = 1.22$$

Again, by taking the reciprocal of the result, the index of year 1 is 0.82. Thus, in this case, the result was the same for both alternatives, so that the value appears reasonable.

Another possible method offered in literature is to weight the input index with the sum of the standard costs of the respective production factors. In this way, occasional fluctuations in prices exert no influence on the index; neither do they influence the policy of the firm.

In some special cases, a weighted sum of the absolute inputs per unit may be used instead of a weighted sum of the indices, which somewhat simplifies calculations and may supply further data. This proves especially convenient when the different input figures to be used can be expressed in the same unit and may be quite simply aggregated. Two examples of this procedure are given below.

The first aims at adding labour and salaried employee inputs.

The total amount of employee salaries is divided by the average hourly wages of the workers for the same period. Thereby, one obtains a quantitative measure for salaried employees expressed in average labour man-hours. Changes in the quality of the staff of salaried employees, due perhaps to a relative increase in the number of lower-grade salaried staff are brought out as quantity changes. The number of salaried employee hours thus calculated can then be treated in the same manner as man-hours in the auxiliary departments. Employee input per unit may be directly added to the  $h/q$ , from which a measure of total manpower input is obtained.

If the development of wages has followed parallel lines for both workers and salaried employees, this method gives exactly the same result as that of weighting the input indices with the costs of the base period. In Sweden, the method has also been applied to textile industries (MAB & MYA) as well as to iron and steel industries (Fagersta Bruks AB).

As another example of direct adding up of input figures, we shall quote the case where it may be convenient to express raw material input in terms of man-hours and to aggregate it to labour input. This concerns firms where raw materials undergo several processing stages, in the last of which it is possible to influence raw material consumption by modifying breakage, etc. For a primary analysis of the inputs in these departments, the number of

man-hours spent on the material in preceding departments within the firm may be used as its price.

A simple numerical example will illustrate the procedure. Suppose department B of a mechanical firm processes one product only from a semi-finished product supplied by department A, and that department A also delivers part of its production to other finishing departments and to the market (which makes it impossible to apply the allocation method). Here are the available figures:

	YEAR 0	YEAR 1
Department A: $h/q$ .....	4.0	3.5
Department B: $h/q$ .....	6.0	5.0
$r/q$ .....	1.2	1.1

Let us now study department B, where we obtain the  $h/q$  index of year 1:  $5.0/6.0 = 0.83$  and the  $r/q$  index:  $1.1/1.2 = 0.92$ . A comparison with the help of  $h/q$  year 0 in department A shows the total change in manpower requirements on the basis of improvements in department B. This is due partly to direct economy of manpower in the department and partly to indirect economy through a reduced requirement for the semi-finished product of department A. The calculation can be made by extracting the total number of required hours:

$$\frac{5.0 + 1.1 \cdot 4.0}{6.0 + 1.2 \cdot 4.0} = \frac{9.4}{10.8} = 0.87$$

Naturally, this type of calculation can generally be made, even with several products in department B, each of which originates from one or several raw materials or semi-finished products processed in other departments. Calculations in this case follow the formula:

$$\frac{\sum_j \left[ a_{Bj}^{(1)} + q_{Bj}^{(1)} \sum_g c_{jg}^{(1)} \frac{a_{Ag}^{(0)}}{q_{Ag}^{(0)}} \right]}{\sum_j \frac{q_{Bj}^{(1)}}{q_{Bj}^{(0)}} \left[ a_{Bj}^{(0)} + q_{Bj}^{(0)} \sum_g c_{jg}^{(0)} \frac{a_{Ag}^{(0)}}{q_{Ag}^{(0)}} \right]}$$

where  $c_{jg}$  = consumption of product  $g$  per unit of product  $j$  processed in department B. The lower index  $A$  here refers to all departments in processing stages prior to B.

The advantage of this method is that it refers efficiency changes to the right department. On the other hand, it does not allow the construction of an index for the whole firm which would incorporate  $h/q$  changes in the departments processing semi-finished products.

It must be pointed out here that a modified relationship between production in department B and the supply to it of products processed in department A are not necessarily caused by an increase in the efficiency of department B as represented above. It may well be that department A processes a better product; one which causes less breakages in department B. In such a case, the changed consumption of raw material can also be reflected to a certain extent by an  $h/q$  index calculated according to the allocation method, where all man-hours refer to the finished product. A detailed look into raw material consumption may also be fruitful. This can be made in a similar way to the above method, namely by expressing the  $h/q$  of department A as man-hours per unit of raw material ( $h/r$ ) multiplied by raw material consumption ( $r/q$ ).

A fruit cannery in Israel established that the  $h/q$  index for the process of peeling and slicing grapefruit was 0.65 in 1958 (1957 = 1). A closer analysis proved that this excellent result had been obtained partly by a reduction in man-hours per grapefruit—the  $h/r$  index had become 0.73—and partly by reduced wastage. At the same time, the number of grapefruits required per can went down by 11 per cent, that is to say the  $r/q$  index became 0.89. The total result was then:

$$0.89 \cdot 0.73 = 0.65.$$

In the same way, it may prove convenient in machine processing to split the  $h/q$  into two factors, one expressing labour input per machine-hour ( $h/c$ ) and the other expressing machine-hours per unit produced ( $c/q$ ). This technique has been used in several studies on cotton spinning mills.

#### ALL INPUTS BUT ONE REMAIN UNCHANGED

In the methods reported so far, in one way or another a composition of inputs equivalent to the total input of the base period were found which have been considered to bear the same proportionate relationship to each other as during the year of comparison. Thus, the total index expressed the same change for all input figures from the base year position to that of the present period. The difficulty has been to find points of comparison which would enable us to say that they were equivalent to the real base year point.

A method in which another point of comparison has been applied to several investigations is one recommended by EPA in connection with cotton spinning mills. Here the point of comparison sought is one that is equivalent to the base year point and represents the *same* input of capital as during the present period. The comparison is then made between the labour input given by the comparison point and the actual one.

The principle will be evident from Diagram 10.

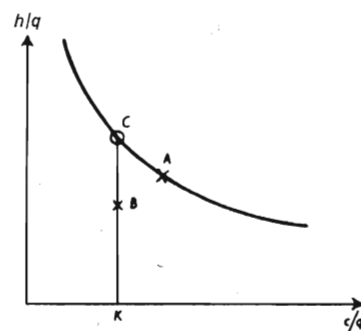


Diagram 10

In the same way as before,  $A$  stands for the combination of inputs observed during the base year. The curve through  $A$  links up all the points which correspond to the best conceivable production methods for the base period. Point  $B$ , observed during period 1, is now compared to the point of the curve which has the same  $c/q$  value as  $B$ , namely  $C$ . The index figure sought will be  $KB/KC$ .

In order to determine the position of *C*, the shape of the curve should be known. The methods used in the preceding section for an approximation of the curve can also be applied here. The present method is, however, of greater interest when it is possible to calculate an hypothetical  $h/q$  for the base year from the data of experience (for example from other firms) and under the assumption that capital equipment was the same as that in year 1. Input of capital does not then need to be translated into figures. It is, however, essential that it is as big at point *C* as at point *B*.

The final result of this method is not a total index of the type previously described. In fact, it shows the change in  $h/q$  provided  $c/q$  (and other input figures) remains *unchanged* and the entire improvement refers to the single factor  $b/q$ . The change will always be bigger than in a total index calculated above, which was a kind of average index of all the production factors. Thus in a way a measure of the change calculated according to this method exaggerates the improvement achieved. The result can also vary widely, depending upon to which production factor the change refers. To which question in particular an index calculated according to this method provides an answer is difficult to say, and for this reason the method is not recommended apart from exceptional cases.

## Chapter 6

### SURVEYS OF AN INDUSTRY OR SECTOR

Most of the methods discussed in Chapters 2 to 5 are primarily for the use of firms desiring to estimate the development of their production efficiency over a period of time. We shall discuss below in what way these methods can also be applied to a comparison between firms in the same industry.

The purpose in comparing several firms in an industry by means of some measure of efficiency is to provide the different firms with a basis for judging their own position vis-à-vis the others. Many studies of this kind have been performed in various countries and were moreover often combined with a certain advisory activity destined to help the least successful firms to make use of the result. Meanwhile, experience has shown that even the firms with the best efficiency figures have something to learn from such a comparison. Generally, the firm with the lowest overall input figure does not show the lowest input figure in every department considered separately.

As a result of the interest awakened in a large number of countries by these types of comparisons, a very comprehensive literature describes different studies made and supplies methodological advice on their realisation. For this reason, it seemed superfluous to discuss here in detail the organisation of such inter-firm comparisons. Instead, we refer you to the manual on Productivity Measurement published in January 1956 by the European Productivity Agency (Vol. II. "Plant level measurements, methods and results").

Let us note however that most studies concentrate entirely on labour input per unit. The usual procedure is to group the firms in various ways according to other variables in order to find at least a partial "explanation" of the differences in unit labour input. For such a method to be efficient, most of the variables should be considered simultaneously, in order not to analyse the same differences several times and to refer them to different causes. Thus, it seems considerably simpler to consider directly, in an efficiency index, the differences in capital input and in other factor inputs. The remaining differences in efficiency indices are then easier to analyse.

#### INDEX FOR A FIRM AND INTER-FIRM COMPARISON — DIFFERENCES

The objective of an index for a firm was formulated in Chapter 1 to show that the index would reflect changes in the efficiency of the producing departments. The measures compared in an inter-firm study have a similar objective and may consequently be based on the same numerical data.

However, some circumstances which presented difficulties for the index for a firm are of minor importance in a comparison between firms, and vice versa. We shall mention two essential differences between both types of comparisons.

The conversion of the output of the different firms into a standard product in inter-firm comparisons is made with the aid of co-efficients which may be procured in one of the firms only, either by means of a theoretical study or by indirect calculation. This last method is much simpler and safer to use in an inter-firm comparison than within an individual firm (see page 46).

Since an inter-firm comparison usually relates to a single moment in time, price changes play no part. Generally, equal price conditions may be relied upon for all the firms. This permits and indeed renders essential a much larger use of prices in inter-firm comparisons than in time series for one firm. Thus, if the different inputs per unit are multiplied by the prevailing prices of the production factors and then aggregated, an indicator of the relative cost conditions is obtained for the different firms, where the influence of differences in the composition of production has been eliminated. This is the correct measure of the relative production efficiency of the firms. It is thus obtained directly, which was not possible in a study within an individual firm. If the lowest observed cost is divided by the total cost of any particular firm, the rate of production efficiency for that firm is obtained.

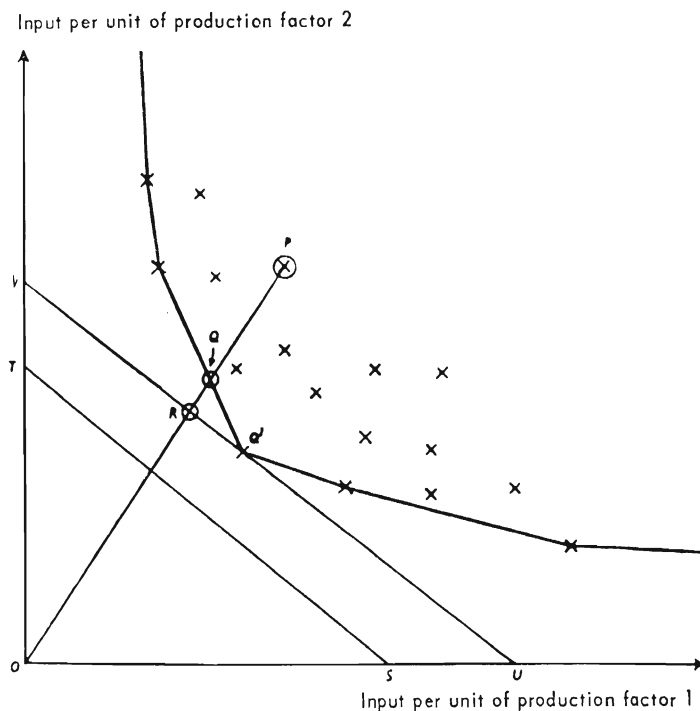


Diagram 11

If the inter-firm study includes enough firms, it may be possible to distinguish price efficiency from production efficiency for the different firms. This can be interesting, since price efficiency depends on price relationships and its benefit is more restricted in time than that of production efficiency. The method is based upon a technical curve, or rather a *T*-line, obtained from the figures for the firm as described on page 68.

The polygonal line drawn to connect the points which lie closest to the origin is an evaluation of the *T*-line of the industry. Since the price relationship between the production factors is the same for all the observations—the study is supposed to relate to a single point of time—even this relationship can be represented in the diagram.

Diagram 11 shows the proceedings.

In addition to the *T*-line, a straight line *UV* is drawn on the diagram. The slope of this line is such that all the points which lie upon it represent combinations of the production factors which are equivalent as far as costs are concerned. This is obtained by choosing a convenient amount, for example 100 crowns, and by making the distance *OS* equal to the number of units of production factor 1 afforded by this amount. In a corresponding manner, *OT* will be the number of units of production factor 2 which this amount can provide. The line *ST* now has the desired slope. *UV* is then drawn parallel to *ST* and tangent to the *T*-line.

For a firm, whose input is represented by point *P*, two different quotients can now be calculated. According to Farrell's terminology, quotient *OQ/OP* is the index of the firm's technical efficiency. In contrast to the case of the corresponding time series index for a single firm, technical efficiency is measured here in relation to attainable minimum values of inputs, namely those attained by the best firms in the branch. Moreover it is possible to calculate the quotient *OR/OQ*, which depicts the price efficiency of the firm. Even if technical efficiency was equal to unity, which would position the firm at point *Q* instead of *P*, as a result of faulty adaptation to prices, total costs would stand higher than necessary by as much as distance *QR*. The point on the *T*-line which gives the lowest total costs is really *Q*<sup>1</sup>, where costs are the same as in *R*.

Of course, the *T*-line being estimated as it is, this efficiency index should not be endowed with too much accuracy. The result can only be considered reliable when the number of firms included is so large that the curve contains many points.

Still, both quotients are interesting in principle. Their multiplication gives *OR/OP*, which is the index figure of the firm's production efficiency on the basis of total costs. It clearly appears here that this notion is a compound of price efficiency and technical efficiency, which was somewhat vaguely explained when first introduced in the discussion of principles in Chapter 1, page 14.

Thus, the inter-firm comparison renders possible the making of some calculations which could not be achieved on the basis of data from a single firm. Later on, the result of these calculations may profitably be used by the firm when establishing the firm index.

From different standpoints, however, an inter-firm comparison is more difficult to realise than an index for a firm. The main difficulties concern the demarcations required to ensure the similarity between firms from several points of view. For example, firms in the same branch may have different exten-



sions in the chain of processing. Steel works may have a finishing department, a mechanical firm its own foundry, a paper mill may produce its own pulp, etc. Thus, in a comparison between firms, common departments are the only ones that can be included. This generally involves no special difficulties as to production volume, raw materials, direct man-hours or investments in the producing departments. But the allocation of the services of the auxiliary departments is now a bigger problem, since part of them refer to departments which are not included in the comparison. Therefore, the result of inter-firm comparisons depends on the apportionment. The allocation method described on page 50 should be preferred here but is often difficult to apply. The overhead method (page 50) may be used if it is based on a real sharing out of the services of the auxiliary departments among the producing departments. The use of the simplified overhead method (page 54) is not recommended in an inter-firm comparison, because in this case the allocation is simply proportional, and can entail misleading results.

In this respect, the importance must be emphasised of the limits between producing and auxiliary departments and between workers and employees, which should be drawn equally in all the firms.

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## FROM THE CATALOGUE

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