Prices under imperfect information
A theory of search market equilibrium

BY
BO AXELL

STOCKHOLM 1976

PRICES UNDER IMPERFECT INFORMATION.
A THEORY OF SEARCH MARKET EQUILIBRIUM

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av

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PREFACE

Early one Saturday morning some years ago I was awakened by an urgent telephone call from Lars Werin. Lars informed me that there remained some unsolved problems in the theory of pricing and allocation in the presence of information and transactions costs. This was the starting point for many, many months of research, of which this volume is the culmination.

Lars Werin and Claes-Henric Siven introduced me to this subject and have been my faithful supervisors throughout. I am indebted to them beyond my ability to repay. It is a fortunate student who draws either of them as supervisor --- and I have had them both.

Karl Göran Mäler and Harald Lang were especially helpful with the mathematical intricacies of Chapter V.

I want to thank the members of Lars Werin´s seminar: Kerstin Lövgren, Irma Rosenberg, Jan Herin, Thomas Franzén, Richard Murray, Bengt-Christer Ysander and Lars Svensson who supplied critical but constructive comments and advice. Lars Nyberg generously helped me with the computer program in Ch.IV.

Ulla Blomqvist and Marianne Bouveng were highly effective in providing typescripts of earlier versions of these chapters.

I want to thank Stanley Noval for making a final examination of the language. If the text reads more smoothly and with fewer than the usual number of perplexities, it is largely due to his efforts.

The final version, which was professionally typed by Kerstin Lindskog, fully exhibits her natural gifts for turning awkward-looking formulas into objects of beauty.

The research has been supported financially by the Swedish Council for Social Science Research and the Bank of Sweden Tercentenary Foundation.

Finally I wish to express my thanks to my wife Eva, my daughter Maria and my one-day old son for their patience and understanding during these months when so often my irregular working hours brought intrusions on a normal family life.

University of Stockholm
7th of April, 1976

Bo Axell

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CHAPTER I:

Introduction

I. INTRODUCTION

From the very beginning of research in the field of economics until today, nothing has been of more fundamental interest than the allocation of resources and the determination of prices. In the theory of prices from Smith via Walras and Arrow-Debreu, the main question has been whether or not the decentralized market economy is capable of attaining an equilibrium where a variety of goods is produced and consumed. A further question has been whether or not this production is optimal.

The general approach to the analysis of these questions has been to construct a reaction pattern for households and firms to events in the world outside them.

In the competitive analysis, the agents of both sides of the market are assumed to take the prices as given. For a single market this gives rise to the functions:

$$D = D(p) \qquad (1)$$
$$S = S(p) \qquad (2)$$

where D is demand, S is supply and p is price. To obtain a solution to this system of two equations and three unknowns, the equilibrium condition

$$D = S \qquad (3)$$

is added. This seems, perhaps, self-evident. However, a closer inspection shows that it is not. For an equilibrium to be of any interest the system must have a tendency to approach it, at least if the variables are near their equilibrium values. This is the same as to say that the equilibrium must be locally stable.

One way to guarantee this is to introduce the condition:

$$\dot{p} = f(D-S) \qquad (4)$$

where $f' > 0$ and $f(0) = 0$,

which says that if there is excess demand the price must rise. If the excess demand is negative, the price must fall.

Now, while this seems fairly logical and reasonable, it happens to be inconsistent with the assumption which gave rise to equations (1) and (2). If the price at any moment is below the price which equates demand and supply, there is excess demand in the market and the market price must increase. But (1) and (2) are derived under the assumption that all agents take price as given, households and firms deciding only how much to consume and produce, respectively. Firms in this model do not concern themselves with the setting of prices.

In the aggregate, however, there will be greater demand than supply if the price is below the equilibrium price. Some demand will be unsatisfied. There is thus a possibility for anyone firm to raise its price without risking loss of demand. But then producers are no longer price-takers in the market -- they no longer face an infinitely elastic demand curve, and the conditions, necessary to derive a supply curve [equation (2)], are no longer valid. When demand does not equal supply the firms in the market behave monopolistically, although all conditions for perfect competition are fulfilled. It is well known that supply curves do not exist in any case other than in perfect competition. But the truth is still more depressing than that. Supply curves do not exist even in perfect competition other than in equilibrium. As early as 1959 Arrow pointed this out in an article. [Arrow (1959)]

We may ask, what can save equation (1)-(4)? The answers, offered by those who have made great efforts in this field, are not encouraging. The invisible hand will have to become quite visible and more than that. Only the introduction of an auctioneer with more power than any imaginable price-controlling authority can save the theory of demand and supply. This auctioneer is assumed to anounce a price in the market. He then collects bids from individuals and households, comparing total desired supply with total desired demand. If the supply exceeds demand he decreases the price, and vice versa, and asks for a new round of demand and supply bids. This process continues until total demand equals total supply. He then permits agents to trade as desired at these prices. This is what we call a recontracting or a tâtonnement process.

There are no trades out of equilibrium. If the prices are disequilibrium prices, the auctioneer merely collects the wishes from the two sides of the market but does not permit trade to take place. Otherwise conditions will change and nothing guarantees that the tâtonnement process will converge.

Thus, the search for a story which tells how the competitive market ends up at an equilibrium leaves us with the auctioneer. This is quite upsetting, especially when the competitive analysis is thought to be the cornerstone in the theory of the market mechanism.

The desired process of price adjustment must result from an analysis of how prices are set and changed by those who actually set them, namely the agents of the market. Let us look a little closer at a disequilibrium situation. Let us say that all firms in a market charge the same price, but one which is below the market-clearing price, implying unsatisfied excess demand.Any one firm could, in this situation, raise price without losing all its demand. Firms face less than infinitely elastic demand curves and are thus monopolists in at least the sense that they have some choice in the setting of prices. Normally, a profit maximizing firm confronting a demand curve with finite elasticity aspires to set a price such that marginal revenue equals marginal cost. There are, however, several problems with the description of the price-setting behavior of this kind for a competitive firm out of equilibrium. A firm in the traditional pure competitive situation has to have knowledge only of a single price, the market price. But a competitive firm in a disequilibrium situation requires knowledge of a whole demand curve.

Furthermore, this demand curve is not independent of the behavior of the other firms in the market. There is a unique shape for the demand curve corresponding to each distribution of other firms' prices.

If there is excess demand in the market one firm could increase profit by means of increasing price. If this increase is sufficient to equalize marginal revenue and marginal cost, the market is cleared in some sense. But the market is self-evidently not in equilibrium. Any other firm could also profit from a similar price increase. The price adjustment process can then be described as a large number of monopolists trying to adjust their prices in order to increase profits. However, there is a great deal of information that each firm must have in order to make a correct decision. It must know not only a whole demand curve instead of just the market price, but also the price strategies of all other firms and their impact on its own demand curve. It is obviously quite unrealistic to believe that any firm could possess all this information. Assuming this would hardly be an improvement over the story of the auctioneer. The main reason why it is unrealistic to think that a firm could have all this information is that it is costly to collect information. It could not be optimal for a firm to try to obtain perfect information - even if this were possible.

In short, when firms charge prices below the competitive equilibrium price, they all try to make profits from their monopoly positions by means of price increases. The size of these increases will differ among firms, according to their beliefs on the shape of the demand curve and on their forecasting of what other firms will do.

Price changes will differ between firms, because different firms have different sets of information, as the information is incomplete and comes from stochastically governed market experiences, and furthermore different firms will have different expectations about other firms behavior, and the effects from this. As a result, one can expect considerable price dispersion among firms during the adjustment process mainly due to the situation of limited information. In addition, one must take into account the consumers' situation.
In the traditional view of the consumer in perfect competition, he or she buys at a given and constant market price. Any shop would charge exactly the same price as any other. However, in the disequilibrium situation, when different firms charge

different prices, there is a benefit from finding a firm charging a low price relative to the other firms in the market. A searching for such a low price is, however, not costless. It demands resources from the consumer, especially in the form of time needed for making contact with different firms.

In order to examine the stability of the competitive equilibrium, which is the same as constructing a theory of price adjustment in an atomistic market without introducing an auctioneer, one must analyze how small monopolistic firms adjust their prices when they have incomplete information about the demand function and when the consumers at the same time do not have full information about which firms are charging which prices.

Although on the surface there would seem to be forces that make the market's firms increase their prices if the price is less than the competitive equilibrium price and reduce it if the price is above, information costs for firms -- as well as for consumers -- will make the price adjustment process considerably different. In particular, the market may not converge to the competitive price, if consumer search costs are taken into consideration. Instead, if the process ever converges, the prices will approach either the monopoly price[1)] or a situation with price dispersion equilibrium.

In this study attempts are made to find the building stones for a theory of pricing when prices are set by the agents of the market who incur costs in connection with the collecting of information. We study the pricing in one particular market exclusively. The reason for this is not that interdependence among markets is unimportant, but that in order to construct a theory of general equilibrium (or more correctly of general interdependence), we must first understand better the interaction among agents within a single market. Presented now is a brief survey of the contents of the subsequent chapters.

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1) *Which is the price that a profit maximizing monopolist would charge if he controlled the whole market.*

Chapter II. Equilibrium in Markets with Imperfect Information -- a Presentation of some models

This chapter presents a collection of models which analyze the equilibrium properties and price adjustment mechanism for a market featuring search behavior. The models surveyed include those of Franklin Fisher, Peter Diamond, John Hey, and Michael Rothschild.

Chapter III. On the Concept of Stochastic Equilibrium

In this chapter the concept of equilibrium for a stochastic system is discussed. What equilibrium is for a model which is non-deterministic is far from self-evident. As an application of this analysis, which at the same time will serve to introduce stochastic consumer flow, we present a model of firm-size distribution in which consumers change firms randomly.

Chapter IV. Price Dispersion and Information -- an Adaptive Sequential Search Model

This chapter presents a model of search for the case when consumers have knowledge neither of the prices charged by individual firms nor of the shape and situation of the price distribution. In this case a price quotation has the double function of providing information not only of the existence of a store charging this price but of the shape of the whole distribution function. Consumers revise their opinion about the shape and situation of the price distribution after each price quotation, and at the same time, consider whether or not to accept this offer.

Consumers are assumed to behave in accordance with the reservation price rule, which means that they stop searching when the expected marginal revenue of further search no longer exceeds the marginal cost of search.

The frequency distributions of stopping prices are derived for different sets of search costs and initial subjective opinions about the variance of the distribution by means of Monte Carlo computer simulation. The resulting firm demand curve is also calculated for the different cases.

Chapter V. Price Adjustment and Equilibrium in Markets with Imperfect Information

The purpose of this chapter is to clarify the price adjustment process and the equilibrium for a market where both firms and consumers have incomplete information and pricing reflects the interdependence between them. The question of whether or not an equilibrium with price dispersion can persist is given special attention. Consumers with different search costs search for low-price firms, using sequential stopping rules. Firms, on the other hand, starting with no information about the shape of the stochastic demand curves they are facing, try to obtain information about these curves by experimenting with price changes. According to the information obtained they try to change prices in a profit-increasing direction. The prices in the market are thus endogenously determined. The behavior on one side of the market affects what the other side does and vice versa. This interdependence is summed up in a differential equation, describing how the distribution of prices will change over time. It is shown that if the distribution degenerates to a single price, the monopoly price is the only conceivable equilibrium price. Moreover, it is shown that there exists an equilibrium with price dispersion which satisfies the Nash condition. The relation between this equilibrium price distribution and the search-cost distribution is analyzed. The necessary and sufficient condition on the search cost distribution for a price dispersion equilibrium is derived.

Chapter VI. Stability of Price Dispersion Equilibrium

In this chapter the purpose is to analyze whether a price dispersion equilibrium in a market with limited information is stable. In the more general case of price dispersion equilibrium, which was derived in chapter V, with a continuous search cost distribution, the question of stability is very difficult to approach. A simplified model is therefore used in order to permit analysis of the stability properties. The search cost distribution is discrete and contains only two search costs. At the same time the price distribution consists of only two prices.

The equilibrium price distribution is derived and it is shown that a equilibrium with firms charging each of the two prices is a stable equilibrium. In other words, if the market is for one reason or another forced out of equilibrium, forces will operate to return it to the price dispersion equilibrium.

In the appendix the expansion to q different search costs is made. It is shown that in this case also the price dispersion equilibrium is stable.

Chapter VII. Search Market Equilibrium when the Probability of Finding a Firm is Dependent on Firm Size and on Advertising

In the previous chapters it is assumed that the firms make no special efforts to inform consumers of their existence and that all firms have an equal chance of being found. In this chapter these assumptions are relaxed. First we consider the case when the probability of finding a particular firm for a consumer during his search process is positively dependent on the firm's size. The model is then expanded to the case when the firms can affect the probability of being found by means of advertising. The condition for equilibrium as well as the stability of equilibrium are analyzed.

CHAPTER II:

Equilibrium in Markets with Imperfect Information -- A Presentation of Some Models

II. EQUILIBRIUM IN MARKETS WITH IMPERFECT INFORMATION - A PRESENTATION OF SOME MODELS

II.1. Introduction

During the past decade much attention has been paid to the problem of how prices change when -- in accordance with reality but in contrast with the assumptions of traditional theory -- they are set by agents of the market, when trade occurs out of equilibrium and, especially, when agents have to make decisions using incomplete information.

The fundamental work of Stigler (1961) has given rise to a growing body of literature concerning markets with limited information. There are three main directions of development. The first is the "micro-macro theory of the labor market" or "the generalized Phillips curve theory", represented by some of the articles in the Phelps volume (1970). By means of analyzing a typical firm and a typical individual, these models show how aggregated variables such as employment and the rate of wage inflation are influenced by sudden changes in aggregate demand. Although these models determine the wage-setting policy of a typical firm, they do not explain the presence of a distribution of wages.

The second direction of development is toward a richer theory of individual search behavior, in which optimality and reservation price properties can be examined under different search rules. Examples of work in this field are McCall (1965) and (1970), Rothschild (1974a), Siven (1974), Axell (1974), and Gould (1972).

A third category of models attempts to analyze the equilibrium properties of a market which in one way or another is subject to imperfect information and in which behavior on both sides of the market is taken into consideration. Examples of models in this category include Diamond (1971), Fisher (1970, 1972, 1973), Hey (1974), and to some extent Rothschild (1974b).

The present paper is an analysis and critique of the model of the last-named category. First we consider Fisher's model, in which he argues that a search market will converge to the competitive equilibrium. We contend that the equilibrium in his model does not fulfill the Nash conditions. Next, we review Diamond's model which shows that a search market converges to the monopoly price, a fundamental theorem which is now generally accepted. However, Diamond's assumptions rule out the possibility of all but a single-price equilibrium; the case of price dispersion in equilibrium cannot be analyzed within the framework of the model.

We then consider John Hey's model of price adjustment which again results in convergence to the monopoly price. The price adjustment process rests, however, on highly unrealistic assumptions. The Stigleresque search technique assumed by Hey can be shown to be far from optimal especially when the price distribution is degenerated. Finally, we consider briefly Rothschild's two-armed bandit theory, which shows how price dispersion can occur in a stochastic environment where firms choose objectively inferior prices despite optimal collection of information.

II.2. Fisher

Franklin Fisher has on three occasions (Fisher (1970), (1972), and (1973)) presented models for markets with limited information with the aim of establishing equilibrium properties for such markets. Since all the three are very closely related to one another, I shall confine myself here to a presentation of the first and last of the three.[1)]

In Fisher's world, all firms are assumed to have identical cost functions with increasing marginal costs. In each

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1) *The presentation of Fisher (1970) follows closely the survey in Rotschild (1973).*

period every firm sells as much as if it were a price taker at this price, i.e. it believes that it is confronting an infinitely elastic demand curve. It then sells such a quantity at which marginal cost equals price.

Each firm bases its opinion about the market price on its experiences from previous periods. If there was excess demand in the previous period, it raises price and increases supply for the current period. If there was excess supply, it lowers price and decreases supply.

Because different firms have different market experiences, there may be differences in prices. The consumer's reaction to a situation with price dispersion is to search for low-price firms.

Fisher does not describe consumer behavior in detail. He merely postulates that the search behavior is rational. From this follows that any firm charging a lower price than another firm will have at least the same number of customers.

All customers have identical individual demand functions and the quantity bought from a particular firm is dependent only on the price charged by this firm. It is not influenced by previous search activity. There exists an aggregate demand function $D(p)$ and an aggregate supply function $S(p)$. In accordance with assumptions, $S(p)$ is increasing and $D(p)$ is decreasing.

Assume that there exists a p^* such that $S(p^*) = D(p^*)$. If all stores charge the same price they will all experience excess demand or excess supply, unless the common price is p^*. The postulate of rational search behavior then implies that all firms will have the same number of customers because they charge the same price. If the market approaches this price, it will remain there. Fisher shows that the prices of all the firms converge towards this price.

The main arguments are the following: Let p_{min} be the lowest price in the market. If the market is out of equilibrium, the minimum price is either lower than the equilibrium price:

$$A: \quad p_{min} < p^* ,$$

or the minimum price is equal to or greater than the equilibrium price:

$$B: \quad p_{min} \geq p^{*}.$$

As a consequence of the assumption of rational search behavior, the firm with the lowest price will always have more customers than the average number of customers per firm. Then, if A is the case, the firm with the lowest price will experience excess demand and therefore raise its price. Fisher also shows that a rational search behavior implies that, if B is the case, the firm with the highest price will receive fewer customers than the average number of customers per firm. Such a firm will register negative excess demand and therefore reduce its price. Then, if the minimum price is lower than the equilibrium price, the minimum price will increase. If the minimum price is greater than the equilibrium price, the maximum price will decrease. This, accompanied by some assumptions about the speed of changes and continuity, is sufficient to assure that all prices converge toward p^{*}.

Following are some critics' views against this model (Rothschild (1973):

> *"The result is not surprising. The model can converge only to the competitive price as long as each firm sets its output by picking a point on its supply curve. The difficulty is that the behavior rule which firms are supposed to follow -- forecast equilibrium price and then produce output to the point where marginal cost equals predicted market price -- is not a reasonable rule for the environments this model is supposed to describe. Unless convergence to equilibrium is terribly speedy, firms will notice that their forecasts of market prices are often incorrect. The consequences of this failure to predict correctly are lost sales when price is underpredicted (and the firm experiences excess demand) and unsold stocks when it is overestimated. Fisher's suggestion that the firm will pick its quantity without considering this possible loss is unreasonable. There is a large literature on the effect of demand uncertainty on the firm summarized in Baron (1970), McCall (1971), and Leland (1972). Although the different models studied lead to an embarrassing diversity of conclusions, they agree on the basic proposition that uncertainty does affect firm decisions and thus short-run industry equilibrium.*

That firms in Fisher's model can hardly fail to notice that they have market power raises a more serious problem. Firms will observe that customers walk into their stores, ask the price, and walk out without buying. Although several explanations of this phenomenon may be possible, surely the most compelling is that the customers who inquire but do not buy would have bought had they faced a lower price. When a firm observes that it is able to sell some but not all of the product it wishes and reasons that some customers would have been willing to buy had the price been lower, it is hard for it to avoid drawing the conclusion that it faces a demand curve -- at least in the short run. What the firm's optimal rule should be in such a circumstance -- or even what a reasonable rule is -- is a difficult problem. However, it does not seem reasonable that the firm should go on pretending it is a helpless actor in a perfectly competitive market."

Even if the firm behavior during the adjustment is not quite reasonable, it is worse to assume that the firms behave as if they are confronting an infinitely elastic demand curve when the price distribution has degenerated to a single price. When the Fisherian equilibrium price p^* has been reached by all firms in the market, each firm produces and sells an amount such that marginal cost equals this price. If the firms behave in accordance with Fisher's rule, they will remain at this price, period after period. If, however, any one firm happens to test a price change, it would find that it has market power; a price increase would not reduce demand to zero and a price cut would not increase demand infinitely much. In other words, the firm is confronting a finitely elastic demand curve. In such a case a price equal to marginal cost is always a non-optimally low price, regardless of the size of the elasticity (provided the elasticity is not infinite). In other words, while one might defend the Fisherian rule of behavior as an acceptable rule of thumb for behavior during a very rapid process of adjustment, one can never accept it as an equilibrium rule. Such a rule would imply that each firm remains at a suboptimal price, period after period, without a single firm carrying out a profit-increasing price raise.

Thus, the equilibrium in Fisher's model fails to fulfill the most important condition for equilibrium in economics; -- namely that no one agent should have the possibility to improve his situation by means of changing any of the parameters he controls.

Aware of the deficiencies of this model, Fisher developed a third model [Fisher (1973)]. He says about his earlier model: "As Rothschild forcefully has pointed out, the assumed behavior of firms may not make much sense." Rothschild, however, did not underline strongly enough the fact that the unreasonable firm behavior was of less importance during the adjustment process than in the equilibrium. Even if Fisher were to use a different approach with respect to the firm behavior during the adjustment process, the basic defect still remains -- namely, that the equilibrium solution does not fulfill any acceptable equilibrium condition, for instance the Nash condition.

In Fisher's third model firms are aware of the fact that they are confronting a demand curve which, as a consequence of the consumers' search behavior, has negative slope. Fisher assumes that this demand curve is infinitely elastic at a low enough price: "... if a firm sets a price sufficiently low, it finds that its demand curve is flat over some range of outputs large enough that at the upper end of that range the firm's marginal cost exceeds the price in question."[1)] (See fig. 1 where also the marginal revenue curve is plotted).

This demand curve is not derived from assumptions concerning consumer search behavior. Fisher simply assumes that it would be a reasonable consequence of consumer search in a market with imperfect information.

Further, Fisher assumes that

$$\frac{\partial M^i}{\partial q_i} - \frac{\partial^2 C_i}{\partial q_i^2} < 0, \qquad (2.1)$$

where M^i is the marginal revenue curve, q_i = output and C_i = cost function for firm i.

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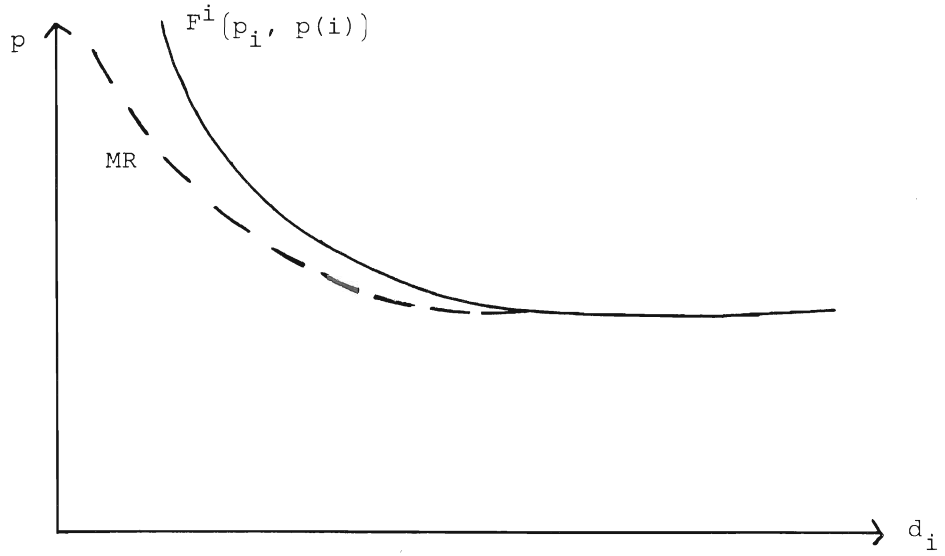
1) *Fisher (1973) page 456.*

Figure 1



The formula says that the marginal cost curve at any quantity has a smaller slope (if negative) than the marginal revenue curve, and the MC curve cuts the MR curve once from below.

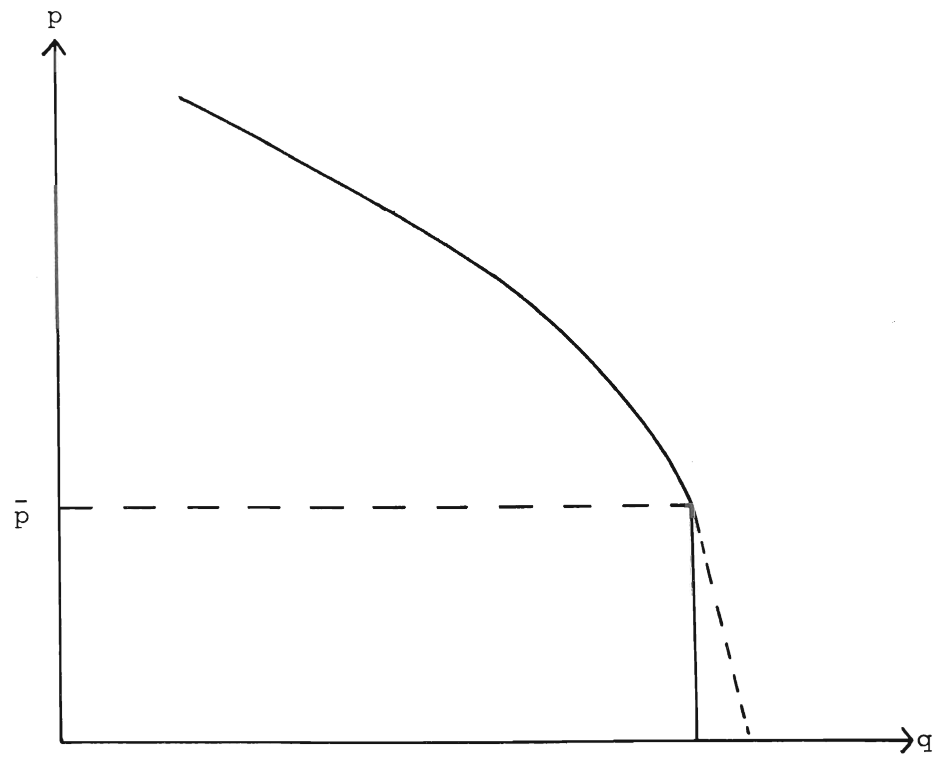
With these assumptions, together with some others, Fisher shows that the prices charged by all firms in the market will approach a certain value, that is, the distribution of prices will degenerate, and the common value will be the competitive price. In equilibrium, all firms charge one and the same price, with price equal to marginal cost.

It is self-evident that if the assumption about the shape of the marginal cost and marginal revenue curves is always valid, independent of the consumers' search behavior and the firms distribution over prices, one unique price must be profit maximizing. This price is that corresponding to the output which equates marginal cost and marginal revenue. If this price gives rise to excess profit, it will lead to entry and the consequent disappearance of the excess profit. The market approaches an equilibrium where marginal cost cuts the demand curve on its flat section.

But there can never exist a flat part of the demand curve if the demand schedule is derived from consumer search behavior under limited information, regardless of what kind of search behavior is assumed. The only situation that can generate an infinitely elastic demand curve is the case where all consumers have zero search cost, which is the same as saying that all consumers have perfect information in the market.

In contrast, it is possible to show that in a model with sequential search behavior,[1] a firm which sets a low enough price, will confront a zero elastic demand curve.

Figure 2



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1) *Which can be shown to be the only optimal behavior.*

This can easily be demonstrated. The demand for an individual firm's output can be regarded as product of three factors: 1) the number of consumers, denoted by μ, who come into contact with the firm: 2) the share (λ) of these who are willing to buy at the asking price, and 3) the individual demand (d), i.e.

$$q_i(p) = \mu \lambda d, \tag{2.2}$$

where q_i is the demand for firm i. q_i is a function of price in the extension μ, λ and d are.

It is hard to believe that μ is dependent on the firm's asking price. Rather, we can assume a given probability that a consumer will come into contact with firm i, a probability related to non-price factors. λ is dependent on the price. The lower the asking price, the greater the probability that it lies below the reservation price for an arbitrary consumer. Lowering the price, λ will rise until the lowest reservation price among the consumers is passed, whereupon it attains its maximum value equal to one. Because the lowest reservation price is certainly above the lowest price in the market, any firm setting the price among the lowest in the market will confront only the demand elasticity coming out from d, i.e. the individual demand elasticity.

Thus, if the demand which a firm is confronting is derived from optimally searching consumers, the result changes considerably. The price equal to marginal cost can never be a Nash equilibrium -- any firm could in this situation increase its profit by raising its price. The reason Fisher gets the competitive solution as the equilibrium solution in his model is that, instead of deriving the demand in a search market he simply assumes that the firm's demand curve is infinitely elastic at a sufficiently low price -- something that never could be the case in a search market.[1)]

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1) *One might however regard as the object of Fisher's analysis to see whether there is* any *faintly reasonable story that has a competitive ending without price dispersion. If there is none, then a lot of economic theory has to be revised from the ground up. The conclusion from the article may be that this revision will indeed have to take place, a conclusion supported in personal correspondence with Fisher.*

II.3. Diamond

Consider now a model by Peter Diamond [Diamond (1971)], which contains a demonstration of convergence to equilibrium for a market in which firms are aware of the difficulties consumers' experience in obtaining information, and in which firms make use of the market power which this situation gives them. In contrast to Fisher's model, Diamond's model does not converge to the competitive equilibrium, although it is conceded that the shape of the equilibrium, to a great extent, depends on the particular assumptions of the model. Diamond goes on to generalize "that models of this sort will not converge to competitive equilibrium". [Diamond (1973) p. 157.] The consumers have imperfect information about prices. They search by looking at one shop per period. At each shop they decide whether or not to buy according to the individual demand function $x(p)$, which is the same for all consumers. The commodity is of such a nature that the consumer will buy it only once. It is assumed that the number of consumers is so great that each firm during any period faces the same number and types of consumers. Each consumer has, in every period, a single-valued reservation price.[1] The level of the reservation price for any given consumer depends partly on the type of consumer and partly on how many periods have elapsed since this consumer originally entered the market. $q_t^{h\tau}$ is the notation for the reservation price in period t for a customer of type h who started searching in period τ. It is assumed that the reservation price for a given customer increases with each period that he fails to find a price below his reservation price. From this, it is apparent that Diamond regards consumers as having imperfect information about the shape of the price distribution. The increase in reservation price reflects an adaptive search behavior.

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1) *Remember that a reservation price is a price such that the customer will always buy if the observed price is below this, and never if the observed price is above.*

There are problems in specifying the consumer's reservation price when he enters the market for the first time. One approach is to assume that new generations of consumers always enter the market with the same composition of initial reservation prices as earlier generations. Diamond rejects this possibility and chooses instead an assumption which implies that new consumers have learned something about the prices of the market already before they enter the market. If p is a price which has prevailed in the market for a long time, then a given consumer would be willing to buy at a slightly higher price $q^{*h}(p)$, rather than invest in another search step since he knows what the prices are elsewhere. For all h and p it is assumed that $q^{*h}(p) > p + \eta$ for some $\eta > 0$. q^{*h} is thus the ideal reservation price that a permanent degenerated price distribution at p would justify. Diamond assumes that if the price distribution degenerates to p and remains there, new generations of consumers will enter the market with initial reservation prices closer and closer to $q^{*h}(p)$. This means that either we have

$$q^{*h}(p) \leq q_{t+1}^{h\ t+1} \leq q_t^{ht}$$

or (2.3)

$$q_t^{ht} \leq q_{t+1}^{h\ t+1} \leq q^{*h}(p)$$

From this follows that, if p has been constant for a while, the reservation price for the new customers will be $q^{*h}(p)$ in their very first search step.

Diamond assumes that every firm has perfect knowledge about the demand curve it faces. The assumptions of the model permit us to derive this demand curve. The number of consumers entering the market in period τ with reservation price equal to or greater than p is denoted $N_t^{\tau}(p)$. The number of consumers in the market with reservation prices equal to or greater than p is then $\sum_{\tau} N_t^{\tau}(p)$. If the number of firms in the market is m and all firms get exactly the same share of the searching consumers each period, the number of buying consumers for a firm with the price p is equal to $\frac{1}{m} \sum_{\tau} N_t^{\tau}(p)$. The demand for this firm will then be

$$x(p) \frac{1}{m} \Sigma N_t^{\tau}(p). \tag{2.4}$$

The firm maximizes the profit function

$$\frac{1}{m} x(p) \Sigma N_t^{\tau}(p) p. \tag{2.5}$$

Firms are assumed to have no costs, but as Diamond points out constant costs would not affect the result if the profit function were quasi-concave.

It is assumed that $p\, x(p)$ is continuous, quasi-concave and has a unique maximum at the finite price p^*.

Thus, the only stable equilibrium price is p^*. According to the assumptions about consumers' behavior, a reservation price of $\bar{p} + \eta$ is possible if $\bar{p}$ has existed as the only price in the market for a long time. This means that if the price distribution degenerates to any price $\bar{p}$, any firm could raise its price slightly (less than η) without losing customers. Such a firm would, however, face a reduced quantitative demand from these customers, because $x(p)$ has finite elasticity. For a price $\bar{p}$ to be an equilibrium price it must be a price such that movements away from it would be unprofitable, the condition which fulfills the requirements for the Nash equilibrium. The only price which has this property is the monopoly price p^*, i.e. the price that maximizes $x(p)\, p$.

Diamond goes on to show what assumptions are needed to guarantee that the prices will converge to p^* within finite number of periods.

The models by Diamond and Fisher stand in clear contrast to one another. Diamond's market converges to the monopoly price while Fisher's market approaches the competitive price. It is obvious, however, that Fisher is wrong. The equilibrium solution in his model is not a Nash equilibrium, while Diamond's equilibrium is. No single agent has the power to improve his own situation by his own hand.

II.4. Hey

Another contribution to the analysis of the equilibrium and convergence properties of atomistic markets with limited information is that of John Hey (Hey (1974)). Hey analyzes the process of convergence under the assumption that firms have imperfect information about the market and act from successive estimations of the demand curve they confront.

The consumers in Hey's model are assumed to have a Stigleresque search behavior, i.e. they decide in advance how many search steps (k) they will go through, and will then buy from the firm which offers the lowest price among the k they have searched through. All consumers are assumed to have the same k. The market is assumed to one without repurchases (a market for consumer durables or a tourist market).

The firms are distributed with prices in accordance to the density function f(p) with the (cumulative) distribution function F(p). The density function for the minimum price of k search steps is:

$$g(p|k) = k\ f(p)\ \left[1 - F(p)\right]^{k-1} . \qquad (2.6)$$

If the quota of customers to firms is denoted by N, the number of purchasing customers per firm as a function of price is equal to:

$$R(p|k,\ N) = N\ k\ \left[1 - F(p)\right]^{k-1} . \qquad (2.7)$$

The elasticity of this function (which is the firms' demand curve if each consumer demands just one unit), is thus:

$$\frac{-\ (k - 1)\ p\ f(p)}{1 - F(p)} . \qquad (2.8)$$

If the firms are distributed with prices in accordance with a Pareto distribution,[1)] the elasticity will be a constant equal to $-\ (k - 1)\ \beta$ over all the price intervals.

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1) *See footnote on the next page.*

If all consumers have the same individual demand curve d(p) with constant elasticity θ, the firm's demand curve is:

$$D(p) = R(p|k, N)\ d(p). \qquad (2.9)$$

For this demand curve the elasticity is equal to the constant:

$$\theta - (k - 1)\ \beta. \qquad (2.10)$$

Firms are assumed to know nothing else about demand other than what they themselves face each period. In order to form an estimate of the elasticity, a firm has to change its price in two successive periods, thereby noticing two points on its demand curve.

If, however, all firms change their prices in two successive periods and do this in accordance with:

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1) *Then we have:*

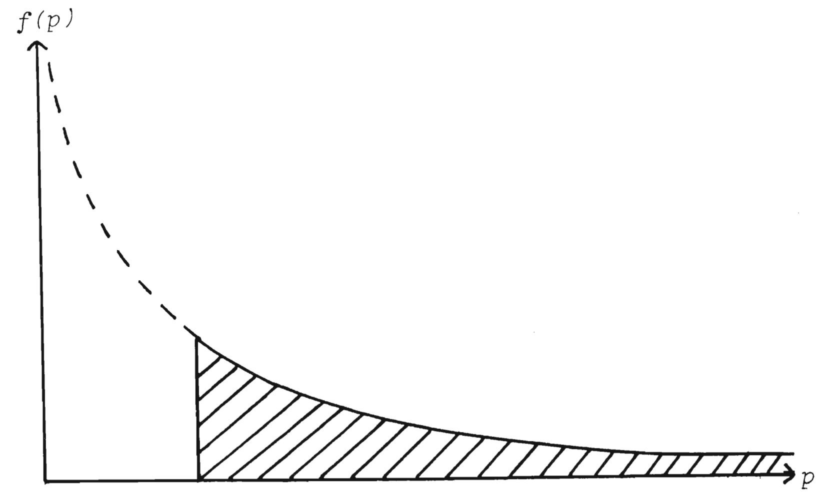
$$f(p) = \beta\ \alpha^{\beta}\ p^{-\beta-1} \qquad \alpha < p < \infty,\quad \alpha, \beta > 0$$

and

$$F(p) = 1 - \alpha^{\beta}\ p^{-\beta}$$

$f(p)$ has the shape as shown in the figure below.

Figure 3



$$p_{t+1} = a_t \, p_t^{b_t} \qquad (a_t, b_t > 0)^{1)} \tag{2.11}$$

and the firms are Pareto distributed in period t; they will *again* be Pareto distributed in period t+1. From this follows that all firms will *observe* an elasticity equal to θ instead of $\theta - (k-1)\,\beta$, the reason for this being that all firms remain in the same *relative* position after the changes if all follow the formula (2.11). The component $-(k-1)\,\beta$ in the elasticity formula is what follows from a change in relative price, i.e. a change in position in the price distribution relative to other firms. The fundamental assumptions which give this result are that the consumers have the inoptimal Stigleresque search behavior *and* that the desired number of search steps (k) is independent of the shape and situation of the price distribution.

Let us denote the factor of elasticity by $\frac{\theta}{\theta + 1} = \psi$. Then, the profit-maximizing price is $p = mc\,\psi$ (all the firms have the same marginal cost).

If the price, p_t, that a particular firm charged during a period had been optimal, this elasticity factor would necessarily have been ψ^*, where;

$$\psi^* = \frac{p_t}{mc}. \tag{2.12}$$

Hey assumes that firms change their prices as if they thought the true elasticity for the next period would correspond to a corresponding true elasticity factor ψ^*_{t+1} in accordance with:

$$\log \psi^*_{t+1} = \lambda \log \psi^*_t + (1-\lambda) \log \psi. \tag{2.13}$$

In other words, the expected true elasticity factor is assumed to be a definite proportion of the displacement between the logarithm for the estimated (ψ) and the logarithm for the hypothetical (ψ^*) elasticity factor.

Now if a firm changes its price in accordance with this rule (2.13), this price change will also correspond with rule

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1) *a_t and b_t are positive constants, the same for all firms.*

(2.11).[1]

In other words, if all firms use this price changing rule and have the same value of λ, there will again be a Pareto distribution only with smaller variance. If this continues period after period, the distribution will become more concentrated around the price mc ψ, i.e. the monopoly price. However, they will never reach this price because they always adjust by only a fraction of the remaining distance.

Now if firms had the opportunity to discover the true elasticity θ - (k-1) β, and adjust towards the optimal price which corresponds to this elasticity, the price distribution would instead converge to marginal cost - i.e. the competitive price.

Hey also shows that if the price distribution were normally distributed, the rule for updating the elasticity factor would have to be:

$$\psi^*_{t+1} = \lambda \ \psi^*_t + (1-\lambda) \ \psi \tag{2.14}$$

for the firms in period t+1 to still be normally distributed.[2]

Also in this case, the firms will estimate the elasticity θ in each period. A necessary and sufficient condition to estimate exactly θ is, as was mentioned earlier, that each firm be situated in the same relative position in the distribution. If the distribution is a normal distribution, this condition is fulfilled if all firms behave in accordance with (2.14) and have the same λ. Again in this case the distribution converges towards the monopoly price.

Hey lets his consumers use the Stigleresque search behavior, which means that they decide in advance to take k search steps and then buy at the lowest price found. As has been shown on several occasions [see for instance McCall (1965) and (1970)], such a search strategy is inoptimal. Assume for instance that a consumer finds the lowest price in the market

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1) *Actually* $p_{t+1} = (mc\ \psi)^{1-\lambda}\ p_t^{\lambda}$ *which agrees with (2.11).*

2) *But with smaller variance.*

in the very first search step. Then searching the remaining k-1 steps can only increase costs without improving revenue.

In my view, it is a more serious mistake in Hey's model that k is assumed to be independent of the shape and situation of the price distribution. This assumption is pecially troublesome if, as in this case, the main purpose of the analysis is to look for a degenerated equilibrium price. It is quite clear that a consumer, even if he must search à la Stigler, will search only once if he knows or think he knows that there is only one price prevailing in the market.

Firms are assumed to change prices in such a way that they preserve their original position in the distribution. This implies that there can be only one price-changing rule for each type of distribution. For a Pareto distribution, the firms must revise their prices in accordance with a somewhat far-fetched rule. They are not assumed to change the price a part of the distance between the original and estimated optimal price. Nor is the opinion of the elasticity of θ updated with λ. Nor is the opinion of the elasticity factor $\theta/(\theta+1)$ updated with λ. They update the logarithm of their elasticity factor by a fraction λ of the distance between what they think is the actual and the estimated.

But what happens if the firms do not follow this rule exactly and/or have different λ? It is immediately obvious that the distribution will no longer be Pareto in the following time period. The firms will change relative position in the distribution and will thus experience changes in demand (negative or positive) in addition to θ. From this follows that they will approach different prices, and there is no reason to believe that the distribution will converge at all. Even if the distribution were to converge, there is no way to predict to what price it would converge.

On the face of it, Hey's most important conclusion, as he says in the introduction: "It is shown that the price distribution converges to a degenerated distribution centered on the monopoly price." However, when we consider what specialized behavior is needed to establish this result, is it not more reasonable to conclude that convergence is highly *unlikely*?

Furthermore, the most serious objection to Hey's model is that the price that the distribution approaches is not necessarily an equilibrium price. This is so because the distribution approaches asymptotically the same price which individual firms approach in accordance with the updating rule. The assumptions of the model imply that firms diminish continuously the displacement to this price by a fraction of the remaining distance, however near they are. This implies that they can converge to any other price, dependent on the rule firms use. As mentioned earlier, the distribution will approach the competitive price if the firms discover the value of the true elasticity. But the competitive price is not an equilibrium price.[1)] Nor the monopoly price is an equilibrium price within the framework of Hey's model because consumers search inoptimally. If one changes consumer behavior in the model (so that, for instance, k diminishes when the variance diminishes), the distribution will not approach the monopoly price, if there are no simultaneous changes in any of the other assumptions.

II.5. Rothschild

The models described in this paper have all contained various assumptions about firms' behavior. There is, however, still a remarkable assymmetry between the firms' and the consumers' situation in the models. The consumers have to make their decisions in risky situations. They obtain price quotations by drawing randomly from an "urn". The firms, on the other hand, face a deterministic demand, constituted from the consumers who on average come to them, given the search rules. If we want to have symmetry in the model, and thus greater realism, we must instead confront firms with a stochastic demand curve, which would be the outcome if consumers search on the basis of randomly drawn prices.

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1) *An equilibrium must have the property that no one agent by his own hand could improve his situation, i.e. increase the profit, by means of changing a parameter he himself controls, e.g. the price.*

In order to form some idea of how the market reacts, and especially of what the equilibrium looks like, we must analyze the firm's behavior when a firm faces not only a stochastic demand curve but an *unknown* stochastic demand curve. The existing analyses of firm behavior under uncertainty have mainly been concerned with how a firm maximizes the expected utility of the profit when it faces a known demand function $D(p, \theta)$, where θ is a stochastic term.[1)]

Michael Rothschild (Rothschild (1974)) has analyzed a model in which one of the most important elements is that firms themselves have to collect information about the stochastic demand they confront. They can do this, for example, by making experiments with price changes. The cost for such experimentation, analogous with consumers' search behavior, is a decrease in expected profit. From this follows that it is not optimal for a firm to try to get perfect information about demand, just as it is not optimal for a consumer to search until he finds the very lowest price in the market.

The question that Rothschild puts is: Can there exist a price dispersion in a market even if some of the prices in the market yield less profit than others, if the firms behave in an optimal way? In mathematical statistics there is a problem of similar character, namely the problem of the two-armed bandit.

If a slot-machine has two arms, each yielding a different pay-off, is there then a probability greater than zero that a player following an optimal strategy will wind up choosing the less favorable of the two arms? The answer to this question is yes. Rothschild proves that there always exists a sequence of outcomes so discouraging that the inferior arm is choosen for all the future. Translated to the firm's choice of price, this means that if there is a finite number of prices to choose among, some better and others worse, there will at least be a probability greater than zero that some firms will remain at less favorable prices. This together with some

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1) *See for instance Baron (1970) and Leland (1972).*

assumptions about consumer behavior, makes it possible to show that a situation with price dispersion *might* occur in equilibrium.

There are, however, two weaknesses in this interesting analysis. First, it gives us an insight into whether or not an equilibrium with price dispersion *might* exist. Nothing can be said about what this equilibrium might look like. Secondly, the result stands in contrast with the concept of stochastic equilibrium for an atomistic market. If there are many firms and many consumers in the market, such that the law of large numbers is valid, we know that the mathematically expected distribution is extremely probable compared with any other. The fact that the probability for some other distribution might be greater than zero is thus of minor interest.

CHAPTER III:

On the Concept of Stochastic Equilibrium

III. ON THE CONCEPT OF STOCHASTIC EQUILIBRIUM

III.1. On Stochastic Equilibrium

In an economy where the agents do not have perfect information about everything, decisions (about trading, producing, collecting information, etc.) depend on the state of information the agent possesses. Because in general different agents have different market experiences their information states and hence their decisions will differ. The market experiences of an agent are a function of his previous contacts on the market. For sellers, information depends on what sort of customers have come into the store during previous periods, whether or not they have bought, how much they have bought and so on. The customers' information is based on what sellers they have come into contact with; the prices which were asked, the quality of the products etc. Common to agents on both sides of a market is that their action for successive periods is dependent on information collected up to that date. Thus, consumers decide whether or not to buy, whether or not to collect more information, how much to buy etc. on the basis of previous market experience. Firms decide on production technology, prices to be charged, quality, how much further information to collect (for instance by means of price and quality changes), on the basis of *their* contacts with customers during earlier periods.

The very contacts, whether or not they result in actual purchases (or contracts), will then govern the development of variables such as demand, supply, price and quality. The degree of information production (or collection) is of course governed by these factors too, but this is indeed a most intermediate good.

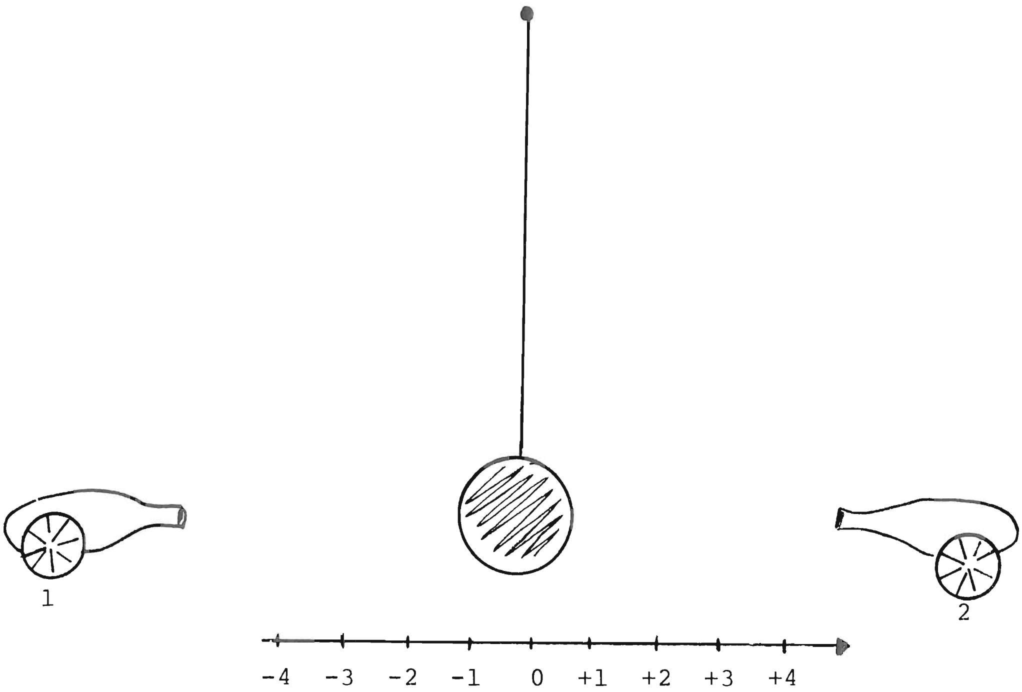
Because the contacts are stochastic, the development of the markets of an economy will depend on the stochastic events of previous periods. We can draw a comparison with the molecules in a gas. The position of a specific molecule in a gas is dependent on its contacts with other molecules during previous periods and their mass, speed and location.

We have to make a distinction between a micro state and a macro state. By a micro state we mean the exact position of each (named) individual with respect to all relevant variables in a period. By a macro state we mean the aggregate distribution over states, regardless of who is in what state. Thus every micro state is unique while a certain macro state could be associated with several different micro states. Usually we are interested in macro states: distribution of prices, size distribution of firms, income distribution etc. The analysis of macro states -- the probability of different macro states, the equilibrium macro state and so on -- must however be based on an analysis of micro states.

In this paper we wish to analyze the stochastic equilibrium of an economy (or a market). More precisely we ask: When is an economy in macro equilibrium if the micro states change according to certain stochastic processes? Equilibrium is then a situation in which a certain macro state is expected to repeat itself period after period. There are two cases when this can happen. One is when the micro state does not change over time. The other is when a micro state changes to another micro state resulting in the same macro state, i.e. the changes in the economy cancel out in aggregate.

However there are in a stochastic model elements which make the equilibrium concept somewhat awkward. Consider the following mechanical construction (Fig. 1). A ball hangs on a string. There are two guns situated on both sides of the ball which shoot bullets against the ball. If the bullets have the same speed and mass, and hit the ball always exactly at the same time, the ball will remain in the same position all the time. But what happens to the ball if the bullets are shot stochastically? Let us assume that the speed and mass of the bullets are always the same. Assume further that the expected energy (i.e. the expected number of bullets) per period is the same for each of the two guns. If the bullets are shot according to a stochastic process, for instance a Poisson process,

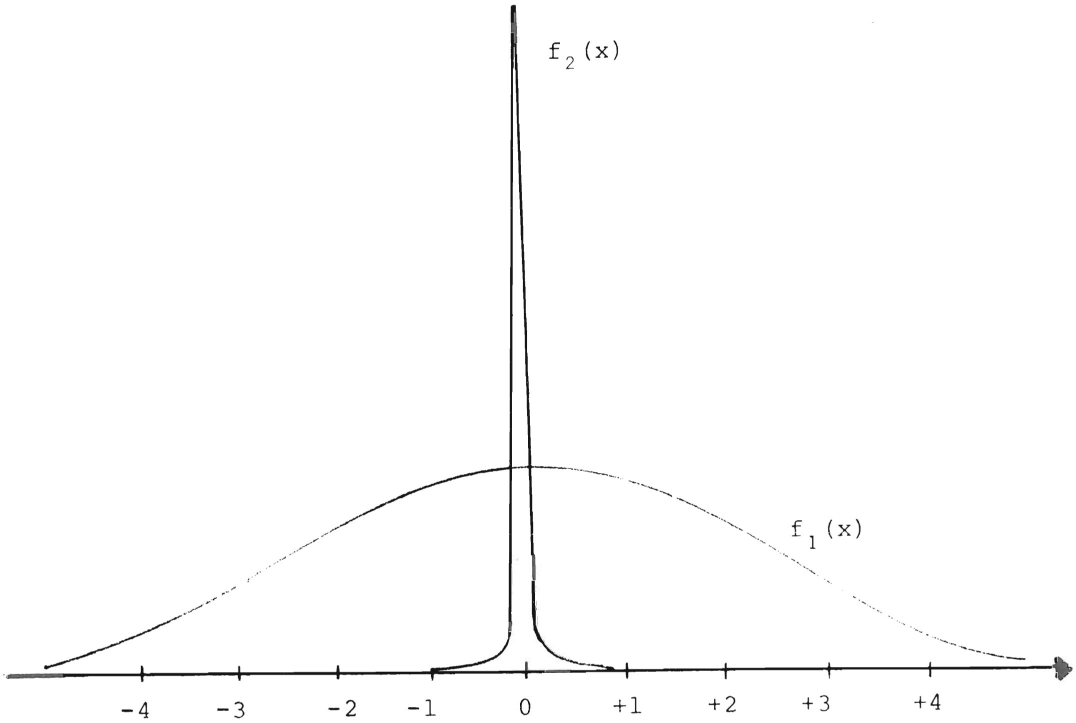
Figure 1



what is the probability density function for the position of the ball at an arbitrary time t? Obviously the shape (primarily the variance) of this probability density function is dependent on how many shots there are per period. If quite a long time on average elapses between two succeding shots, the probability density is like $f_1(x)$ in Figure 2. If on the other hand the shots come at frequent intervals the position of the ball can be described by $f_2(x)$.

Though the equilibrium position of the ball according to the definition of stochastical equilibrium (the mathematically expected position) is the same (namely zero) in the two cases, the probability of being in this position or near it differ widely in the two cases. The probability for the ball to be in the interval (-0.5, +0.5) is almost equal to one in case 2 $(f_2(x))$, while it is perhaps just 0.1 in case 1 $(f_1(x))$. Clearly a definition of equilibrium which does not take into consideration the probability of the system's being in or near

Figure 2



the equilibrium position, does not please us very much. In the example above the law of large numbers is sufficient to guarantee that the equilibrium position is very probable in case 2.

One possible definition of equilibrium for a stochastic model, which takes this problem into consideration, is:

$$\text{prob}\ (x - \delta \leq \bar{x} \leq x + \delta) \geq \psi \tag{1.1}$$

where δ is sufficiently small and ψ sufficiently large, i.e. if the probability of any variable (or all) in the system to be near a value $\bar{x}$ is sufficiently high, then $\bar{x}$ is the equilibrium values of x. This leads however to some other problems. First, in contradiction to the definition of mathematical expectation, $\bar{x}$ is not necessarily unique. Secondly, many stochastical models may have no equilibrium at all. Thirdly, how are the arbitrary values of δ and ψ to be chosen? However, where atomistic competition prevails, the law of large numbers assures that the mathematically expected configuration will agree with condition (1.1) for very small δ and very large ψ.

III.2. A Model of Firm Size Distribution

In order to illustrate the concept of stochastic equilibrium I now employ a simple model explaining the size distribution of firms.[1)]

Denote the size of a firm by S, which in this case measures the number of customers a firm has. F(S) is the size distribution function, i.e. the cumulative of the frequency distribution f(S). Then $F(S_0)$ is the frequency of firms with a size smaller than or equal to S_0. The market is characterized by a flow of customers from one firm to another. This flow is regarded as stochastically governed.

In this first approach, which is primarily supposed to be an example of the concept of stochastical equilibrium, we do not attempt any rigorous derivation of the nature of customer flow on the basis of behavioral assumptions. Instead we just describe the customer flow in a way that is perhaps not quite unreasonable.

To begin, let us consider a consumer whom we know to have left his earlier firm for another. We then ask; what is the probability that he will go to a firm of a specific size? i.e. given the fact that a consumer has left a firm of size B, what is the probability that he will go to a firm of size A? We assume that this probability is proportional to the frequency of firm of size A. If size A actually is the interval $[S_i, S_i + \Delta S]$, then this probability is $F(S_i + \Delta S) - F(S_i)$.[2)]

[1)] *This is a model similar to one developed in Murphy (1965) chapter 10. See also Näslund (1970).*

[2)] *What is most irritating with this assumption is the fact that each firm is assumed to attract customers with equal probability regardless of size.*

Secondly we ask: what is the probability that a customer, given the fact that he has changed to a firm of size A, came from a firm of size B?

We assume that this likelihood is proportional to the probability that there is a firm of size B.

If B is the interval $\left[S_j, S_j + \Delta S\right]$, then this probability is $F(S_j + \Delta S) - F(S_j)$.

The probability of one consumer changing from a firm of size $\left[S_j, S_j + \Delta S\right]$ to a firm of size $\left[S_i, S_i + \Delta S\right]$ is thus proportional to the product of the probabilities of finding a firm in the intervals in question, i.e.

$$\alpha\left\{\left[F(S_i+\Delta S) - F(S_i)\right]\left[F(S_j+\Delta S) - F(S_j)\right]\right\} \qquad (2.1)$$

where α is a positive constant.

If one firm of size S_i[1] gains a customer, the firm will increase in size: at the same time the firm of size S_j, which lost a customer, will decrease in size. The whole distribution $F(S)$ is then affected.

The equilibrium condition for a deterministic model is that there be no tendency for the variables of the model to change, given the data of the model.[2] The equilibrium condition for a stochastic model must of course differ slightly. Instead of the condition that the variables must stay the same in period t+1 as in period t, the condition is that the mathematically expected values of the variables must be the same in the following period as in the present. In other words all expected time derivatives of the variables must be equal zero.

In the present simple model of customer flow, we have shown how to derive the probabilities associated with a consumer's changing firms in a specific way. This change will alter the whole distribution function F to a small degree. For

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[1] *Let us say size S_i instead of size $\left[S_i, S_i+\Delta S\right]$ just for short. Of course there is no firm of a size exactly equal to S_i.*

[2] *For survey of the notion of equilibrium in deterministic models see Hansen, B.: Lectures in Economic Theory, Part I: General Equilibrium Theory. Studentlitteratur, Lund, Sweden 1966.*

the market to be in equilibrium, F must be of such a shape that for any change there exists, with equal probability, a compensating change such as to leave F unaffected. The change in this case is an increase in size of one firm from S_i to S_i+1, and a decrease of another firm from S_j to S_j-1. The compensating change then must be one consumer switching from a firm of size S_i+1 to a firm of size S_j-1. The probability of this change is then

$$\alpha\left\{\left[F(S_j-1+\Delta S) - F(S_j-1)\right]\left[F(S_i+1+\Delta S)-F(S_i+1)\right]\right\} \qquad (2.2)$$

Putting the two probabilities (2.1) and (2.2) equal gives us the equilibrium distribution F(S). Dividing (2.1) and (2.2) by (ΔS) and letting ΔS approach zero gives

$$f(S_j)\ f(S_i) = f(S_j-1)\ f(S_i+1) \qquad (2.3)$$

as the equilibrium condition.

The solution to (2.3) is $f(S) = \lambda e^{\mu S}$, where λ and μ are constants. Thus we have $F(S) = \frac{\lambda}{\mu} e^{\mu S}$ as the equilibrium size distribution of firms.

The values of the constants follow directly from the fact that f(S) is a density function and therefore its integral between 0 and ∞ must be equal to unity.

Now let us define

$$H = -\int_0^\infty f(S) \log f(S)\, dS \qquad (2.4)$$

which is the entropy of the market. Differentiation with respect to time gives us

$$\dot{H} = -\int_0^\infty \dot{f}(S) \log f(S)\, dS - \int_0^\infty \dot{f}(S)\, dS \qquad (2.5)$$

Because f(S) is a density function we have

$$\int_0^\infty \dot{f}(S)\, dS = 0$$

and hence

$$\dot{H} = - \int_0^\infty \dot{f}(S) \log f(S) \, dS \tag{2.6}$$

The flow of consumers was assumed to be from a firm of size S_j to a firm of size S_i and at the same time from a firm of size S_i+1 to a firm of size S_j-1.

Thus we have, according to previous assumption about the nature of consumer flow, the following time derivatives for f at those different sizes:

$$\dot{f}(S_i) = - \beta f(S_i) f(s_j) \tag{2.7}$$

$$\dot{f}(S_j) = - \beta f(S_i) f(S_j) \tag{2.8}$$

$$\dot{f}(S_i+1) = \beta f(S_i) f(S_j) \tag{2.9}$$

$$\dot{f}(S_j-1) = \beta f(S_i) f(S_j) \tag{2.10}$$

$$\dot{f}(S_i+1) = - \beta f(S_i+1) f(S_j-1) \tag{2.11}$$

$$\dot{f}(S_j-1) = - \beta f(S_i+1) f(S_j-1) \tag{2.12}$$

$$\dot{f}(S_i) = \beta f(S_i+1) f(S_j-1) \tag{2.13}$$

$$\dot{f}(S_j) = \beta f(S_i+1) f(S_j-1) \tag{2.14}$$

The total rate of change in entropy is then, inserting (2.7) to (2.14) into (2.6)

$$\begin{aligned} \dot{H} = {} & \beta f(S_i) f(S_j \log f(S_i) + \beta f(S_i) f(S_j) \log f(S_j) - \\ & - \beta f(S_i) f(S_j) \log f(S_i+1) - \beta f(S_i) f(S_j) \log f(S_j-1) + \\ & + \beta f(S_i+1) f(S_j-1) \log f(S_i+1) + \beta f(S_i+1) f(S_j-1) \log f(S_j-1 \\ & - \beta f(S_i+1) f(S_j-1) \log f(S_i) - \beta f(S_i+1) f(S_j-1) \log f(S_j) \end{aligned} \tag{2.15}$$

which can be simplified to

$$\dot{H} = \beta\left[f(S_i)\ f(S_j) - f(S_1-1)\ f(S_i+1)\right]\log \frac{f(S_i)\ f(S_j)}{f(S_j-1)\ f(S_i+1)} \qquad (2.16)$$

We have shown earlier that the condition for stochastical equilibrium is

$$f(S_i)\ f(S_j) = f(S_j-1)\ f(S_i+1)$$

Now we see that for the market to be in equilibrium the entropy of the market must be constant. However the most interesting feature with the entropy measure is that entropy of the market must always increase, as seen from (2.16). For any arbitrary pair S_i, S_j and any function f, which is a density function, $\dot{H} \geq 0$ because either the two factors in (2.16) are both positive or both negative. Thus we see that when the entropy of the market attains its maximum level, then the market is in equilibrium. Another way of finding equilibrium is therefore to search for the function f(S) that maximizes the entropy of the market.

The fundamental advantage of measuring the entropy and the change of entropy is that we thereby can draw important conclusions about the disequilibrium properties of the market. $\dot{H}$ in (2.16) is always positive, unless $f(S) = \lambda e^{\mu S}$, the equilibrium distribution, for some pair S_i, S_j. The entropy will always increase or, in other words, if the market is out of equilibrium it will always converge towards its equilibrium configuration. Further, if we know the actual disequilibrium firm size distribution function in a period, expression (2.16) shows the rate of convergence. Local stability properties follow of course immediately from the convergence property.

Now let us go back to the discussion of the equilibrium concept in a stochastically governed market. As mentioned the law of large numbers would be sufficient to guarantee that the expected value of a variable (or variables or function as in this case) would be a very good forecast of the actual value of the variable. This is so because the probability of the expected value in this case is extremely high compared to the probability of any other value. In the present model, fulfillment of the law of large numbers requires a very great number of consumers. If the probability for a firm of size S_0 to lose one particular consumer during a period is $\gamma (0 < \gamma < 1)$, then its expected total loss of consumers during a period is γS_0. The variance of the expected decrease is $\frac{\gamma(1-\gamma)\ S_0}{n}$ where n is the number of customers. If n is large this variance is very small. We can see that, given n and S_0, the variance is maximal if coefficient of variation $\gamma = 0.5$. More interesting, however, is the variance compared with the expected outcome. The ratio is $(1-\gamma)/n$ in this case. Here we see that not only a small n but also a small γ, i.e. a small probability for a given consumer to switch, will make the variance relatively great, thus making the expected change not so dominatingly probable compared to changes of other sizes. We can put this another way. Not only the total number of consumers must be large, but the number of flowing (changing) consumers must also be large.

Recall the expression of the rate of change in entropy (2.16).

$$\dot{H} = \beta\left[f(S_i)\ f(S_j) - f(S_j-1)\ f(S_i+1)\right]\log \frac{f(S_i)\ f(S_j)}{f(S_j-1)\ f(S_i+1)}$$

This is actually the *expected* rate of change in entropy. Thus we can say merely that the entropy of the market is *expected* to increase. It is of course a possibility that the consumer flow during a period is such that the actual change in entropy is negative. If however the period is sufficient long (the length that is sufficient depends on the number of consumers and the probabilities of change), the law of large numbers says that a development close to the expected will occur with a dominating probability.

The model presented in this section is a model of size distribution of firms when there is a stochastic flow of consumers in between firms. There are however some severe objections to the model as a market model. The most obvious deficiency is the fact that it makes no reference to prices. Further the activity in the market (there is only consumer activity) is not derived from assumptions about individual behavior (optimal or not). There must in fact be an implicit assumption that firms always charge a fixed price, which is the same for all firms. It is obvious that this can not in general be consistent with profit-maximizing behavior of firms. Firms will, as shown, be of different size, and optimal price may well differ as a result. This will on the other hand have important implications for the consumer behavior. We might accept that there is a flow of consumers between two particular firms in both directions if the firms charge the same price. If two firms charge different prices it is fairly easy to accept the existence of a flow of customers from the high price firm to the low price firm, while a flow in the opposite direction is hard to imagine. In the following chapters we will primarily be concerned with models of market allocation when the firms have active pricing policies -- when firms try to charge prices in order to maximize expected profits. The flow of consumers from one firm to another is motivated primarily by a desire to find a lower price than that of which they are currently aware.

CHAPTER IV:

Price Dispersion and Information - An Adaptive Sequential Search Model

PRICE DISPERSION AND INFORMATION—AN ADAPTIVE SEQUENTIAL SEARCH MODEL*

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Summary

This study attempts to determine whether imperfect information is sufficient to explain the occurrence of price dispersion on markets for homogeneous capital goods. Simulation technique is used to estimate the appearance of the distribution for accepted prices when the distribution of supply is given and the consumers search sequentially. The conditions under which the resulting distribution will be in equilibrium are discussed.

The prices actually charged for new motorcars in Sweden are surveyed each spring. These investigations have disclosed a substantial dispersion of prices paid by consumers from year to year. Moreover, the dispersion shows no tendency to narrow.

Table 1 illustrates the differences in prices actually charged for he most common makes surveyed in 1972 and 1973. The differences refer to the range of values in Swedish kronor lying between the lower and upper quartiles and between the 5th and 95th percentiles.

In some case the price dispersion appears to have narrowed between 1972 and 1973. Even so, the 1972 figures showed an increase compared with earlier years.

One of the explanations offered for a high price is that it may be due to the greater excess valuation put on a trade-in car. But that explanation is concentradicted by the portion of our data which relates to "straight" transactions, i.e. purchases not involving trade-in cars, show extremely small deviations from the total data.

How can there be such price dispersions for homogeneous products? In this paper I shall attempt to analyze the feasibiltiy of explaining price dispersion by means of imperfect information, an attempt that largely leads to negative results.

Information may be equated with other goods in that resources are used up to produce it. Firms may generate information, and here resources are used up because the information must be produced and distributed. Information

* I am indebted to Lars Werin, Jan Herin and Claes-Henric Siven for helpful discussions and comments on earlier drafts of this paper. I would also like to thank Lars Nyberg for his help with the computer program.

Table 1

Source: National Price and Cartel Office. Consumer prices of new cars, 1972 and 1973.

Make, model	Year of survey	Average price (Sw- kr.)	Difference between lower and upper quartiles	Difference between 5th and 95th percentiles
Mercedes 200	72	27 190	1 075	4 120
	73	30 582	943	2 277
Peugeot 504	72	21 610	1 610	3 985
	73	24 434	1 200	3 600
SAAB 99	72	22 115	1 030	3 050
	73	23 822	1 000	2 660
Opel Rekord	72	20 890	1 340	4 850
	73	23 034	1 875	3 676
Volvo 142	72	21 265	1 360	3 050
	73	23 062	600	2 551
Ford Taunus	72	17 715	845	2 360
1600 L	73	18 718	740	2 076
SAAB 96 V4	72	16 905	680	2 200
	73	18 022	520	1 750
Fiat 128	72	14 780	1 345	2 475
	73	15 374	960	2 415
Renault 4L	72	13 345	645	1 900
	73	14 289	600	2 478

may also be generated by consumers in consequence of their search activity, in which case their valuation of their time represents one of their resource inputs.

In the same way as for "ordinary" goods, information will be produced if incentives are present. Incentives to increase the information output will be present if the expected added benefit exceeds the marginal cost. Obviously, this is true both for consumers and producers. Every economic agent will attain his optimal information output when the marginal revenue, in the form of expected addition to total utility, equals the marginal cost.

Three types of information should be distinguished: (1) price information, (2) information about quality grades and properties, and (3) information about the usefulness of properties. The distinction between (2) and (3) may be respectively exemplified by two questions: Does the car model have overdrive? How do I feel about driving a car with overdrive?

A distinction of this kind is drawn by Nelson [6] in terms of "inspection goods" versus "experience goods". However, these are extreme cases where uncertainty exists for some goods concerning information of type (2) but not of type (3), whereas the reverse holds for other goods.

My aim with the present study will be to ascertain whether empirically observed price dispersions for durable consumer goods can be explained by the fact that generating information makes demands upon resources. Among the economists who so contend are Stigler [10] and Telser [11].

My focus here is on a market for homogeneous capital goods (for example, new motorcars of a specified make). The quality is assumed to be known. Prices, on the other hand, vary on the market from one firm to another. It is assumed that consumers do not initially know what prices different firms are asking. It is assumed that the firms do not generate any information. All information is the result of the activities of consumers in searching out different vendors.[1] The decisions that are taken—in this case buying decisions by consumers—must be considered rational in light of the information available at any one moment. Apparently erroneous decisions must then be explained by saying that the price data an individual has collected give a distorted picture of the real world. The model that is used must be able to reflect the fact that collecting information takes on the character of random sampling and may therefore result in a misleading basis for decision-making. That is not a sign of faulty behavior from the individual's point of view; he must always weigh the cost of improving the decisionmaking basis against the expected benefit of continued search activity.

This tendency to dispersion in consumer-held "pictures" of the true distribution of prices is very cleverly incorporated by Mortensen [5] into his model for the labor market. The perception of average price (actually wages) on the market held by N individuals shows the same dispersion as the mean of N random samples drawn from the true distribution. One problem, however, is to determine the size of the samples. When a search process is made adaptive (hardly any other approach can be counted on to reflect the real world from a practical point of view) its length is determined endogenously. It is the sequence in which the price data flow in which decides whether going ahead with searching will be worthwhile. Very short or very long search processes may be optimal in light of the information that flows in, but they do not tend to occur often.

Price Formation on Capital Goods Markets when Information is Imperfect

The model presented below refers to the behavior on a homogeneous capital goods market where the consumer does not *know* the mean or the variance of the price distribution, although he does have some prior views of a form to

[1] In this context, "imperfect information" implies that the generation of information requires resources.

be described. The individual must then make use of the information he receives in the course of searching to form his estimate of what the distribution looks like. When yet another price datum flows in on a certain date, the individual will revise his estimate of that distribution and form a new *a posteriori* distribution. He must then also consider whether it will be optimal to go on searching or to stop and accept the most favorable offer that is open to him. In other words the individual seeks out price data sequentially and estimates, after each new observation, the expected benefit from continued searching and compares that with the search cost. This gives us an *adaptive search model.*

The *a posteriori* Distribution

Let us assume that prices at which the firms in question offer homogeneous capital goods form a normal distribution $F(p)$, whose mean is $\bar{p}_F$ and variance is σ_F^2.

The individual can, we assume, obtain an address list of the relevant firms free of charge (for instance, by consulting the yellow pages in the telephone directory). He cannot know the price any particular firm will ask without first seeking out the firm and negotiating with it. However, activity of this kind will involve a search cost, c.

Even before any firm is contacted, the individual is assumed to have some idea of the variance on the market. This conception is based on experiences gained from similar markets or on earlier experiences of the same market. We call this initial subjective variance σ_i^2, to which the individual attaches a weight k, showing the degree of confidence he attaches to σ_i^2. The k may be regarded as a coefficient of inadaptability, and may be interpreted as the amount of imagined price data preceding the first actual datum that has provided information about σ_i^2.

After n price data, drawn at random from the address list, the individual forms an *a posteriori* distribution with the variance

$$\sigma_{in}^2 = \frac{k\sigma_i^2 + ns^2(p_1 \dots p_n)}{k+n-1} \tag{1}$$

where $s^2(p_1 \dots p_n)$ is the sample variance of the n price observations. The mean in this distribution is formed by the sample mean

$$\bar{p}_{in} = \frac{\sum_{j=1}^{n} p_j}{n} \tag{2}$$

Here I refrain for the time being from introducing any subjective conception of the mean. It follows that the individual will behave as though no other information about the distribution mean were to be had apart from the offers received during the search process. We can interpret this to mean that a very low weight is assigned to the subjective conception of the mean compared with the weight that is assigned to the information provided by the random sample.

Rule for Optimal Stopping

We assume that the individual knows that the prices are normally distributed. At this point either of two basic alternatives present themselves. The first is a search process whereby every offer must either be accepted or rejected and whereby a rejected offer cannot be recalled. The other is a process whereby a rejected offer can be recalled if it later appears advantageous. For a capital goods market such as the market for cars, the latter alternative would appear to be the more realistic, so I elect to analyze it.

After n drawings n price quotations have been obtained, whose sample variance is pooled with the subjective initial variance into σ_{in}^2 as indicated above. The mean on the market is expected to be identical with the sample mean p_{in}. We call this *a posteriori* distribution $f_{in}(p)$. One of the prices obtained is the lowest. Let us call this price p_m. The decision rule assumed to guide the individual's behavior is: If the expected benefit of carrying the search one step further is greater than the search cost he will keep searching; otherwise he stops to accept the lowest existing offer, i.e. p_m.

If we assume a linear utility function in the interval, i.e. the absence of risk aversion, this decision rule may be formulated:

$E(\Delta p) > c \rightarrow$ keep searching.

$E(\Delta p) \leqslant c \rightarrow$ stop and accept p_m.

The search cost, c, is assumed to remain the same for all search steps.

$E(\Delta p)$ is then p_m minus the expected probability that a price less than p_m will be drawn next, times the expected price given that a price less than p_m will be found, plus expected probabilities that a price greater than p_m will be found times p_m (which from now on will of course remain the most favorable).

$$E(\Delta p) = p_m - \left[\int_{-\infty}^{p_m} f(p) \frac{\int_{-\infty}^{p_m} pf(p)\,dp}{\int_{-\infty}^{p_m} f(p)\,dp} + p_m\left(1 - \int_{-\infty}^{p_m} f(p)\,dp\right) \right]^1$$

$$E(\Delta p) = p_m - p_m + \int_{-\infty}^{p_m} [f(p)\,p_m - f(p)\,p]\,dp$$

$$E(\Delta p) = \int_{-\infty}^{p_m} f(p)\,(p_m - p)\,dp$$

[1] To simplify notation, we designate $f_{in}(p)$ by $f(p)$ in these computations.

Insertion of a normal distribution transposes the foregoing into:[1]

$$E(\Delta p)=\frac{1}{\sigma_{in}2\pi}\int_{-\infty}^{p_m}\exp\frac{-(p-p_{in})^2}{2\sigma_{in}^2}(p_m-p)\,dp \tag{3}$$

where accordingly:

$$\sigma_{in}^2=\frac{k\sigma_i^2+ns^2(p_1\dots p_n)}{k+n-1}$$

and

$$p_{in}=\frac{\sum_{j=1}^{n}p_j}{n}$$

Number of Search Steps and the Acceptance-price Distribution

At this juncture two questions take on primary interest. First, how many steps will the search process be carried before being stopped? Second, what price will be accepted at cutoff point? Both these factors will be stochastically distributed because the optimal stopping date and hence the accepted price will depend on the sequence in which prices have been drawn.

We first investigate the condition for stopping after one drawing. The search process will be stopped if:

$$\frac{1}{\sigma_i 2\pi}\int_{-\infty}^{p_1}\exp\frac{-(p-p_1)^2}{2\sigma_i^2}(p_1-p)\,dp<c$$

where p_1 is the price obtained.

Solving for the integral results in:

$$\frac{1}{\sigma_i 2\pi}\left[\sigma_i^2\exp\frac{-(p-p_1)^2}{2\sigma_i^2}\right]_{-\infty}^{p_1}<c$$

which gives us:

$$\frac{\sigma_i}{2\pi}<c$$

as the condition for cutoff after one search step. In other words, stopping after one step will be independent of the price drawn. Only the initial conception of the variance and search cost will decide that.

Naturally, however, the accepted price is stochastic. The distribution of the acceptance price in this case is the same as the parent distribution.

If $\sigma_i > c\cdot 2\pi$, at least two search steps will be taken. The probability of stopping after two, three, four or more steps will depend on the sequence in which

[1] Cf. de Groot [4], pp. 341–349. There, however, the variance in the parent distribution is assumed to be known to the individual.

prices are drawn, which of course is a function of the parent distribution's mean and variance. The *a posteriori* distributions are dependent on σ_i and k. The stopping condition is obviously also dependent on c. As a result the probability density functions for N (number of steps) and p_{acc} (acceptance price) will be obtained which depend on σ_F, σ_i, k and c. In principle it should be theoretically feasible to find the distributions for N and p_{acc} and to determine how these depend on σ_F, σ_i, k and c, but this problem would seem to be unreasonably difficult to solve. Instead, I have opted for computer simulation to help me form an idea of the distributions which arise when the values for σ_i, c and k are ordered in sets.

Some Forks in the Road

I have chosen to employ an *adaptive* search process. Many studies dealing with search behavior among consumers or in the labor force assume that the distribution has an exactly known appearance for the searcher (see e.g. Siven [9] and Gronau [3]), or that subjective perception of the distribution is not affected while the search process goes on. The latter will arise if a very great value is set in this model on k. It is easy to demonstrate that the reservation price in this case will be invariable for the same search cost, and of the last price data. That is to say, different stopping prices can only be explained by different search costs. In an adaptive search process the stopping price will be determined both by the search cost and the sequence in which price data flow in.

I have assumed that any offer can always be recalled. Formulas which altogether disregard this option simplify the mathematical exercise. But by solving the problem with simulation, as is done here, I can use this more realistic approach. It would be even more realistic to insert a probability that a given offer still stands after a time and to let this probability diminish with time.

Let me dwell for a moment on the subjective elements in the form of σ_i^2, the subjective initial variance, and the weight k. When a stopping decision occurs in a certain situation, it may be explained by (1) the information the consumer has collected, (2) his search cost, (3) σ_i and (4) k. In principle, the available information can be measured as well a his search cost. σ_i^2 and k can then be calculated, but it will not be possible to separate the effects of σ_i^2 and k. So considered in light of the information available to the consumer, an apparently premature stopping may be explained by saying either that σ_i^2 deviates a great deal from the true variance and k is of moderate size, *or* that σ_i^2 does not deviate much from the correct value but weighs heavily combined with a big k. This is illustrated in Fig. 1.

By analogy with my assumption that there is no subjective conception of the mean I assume risk neutrality because I want to limit the number of parameters. The effect of risk aversion on the result is easy to see. Every

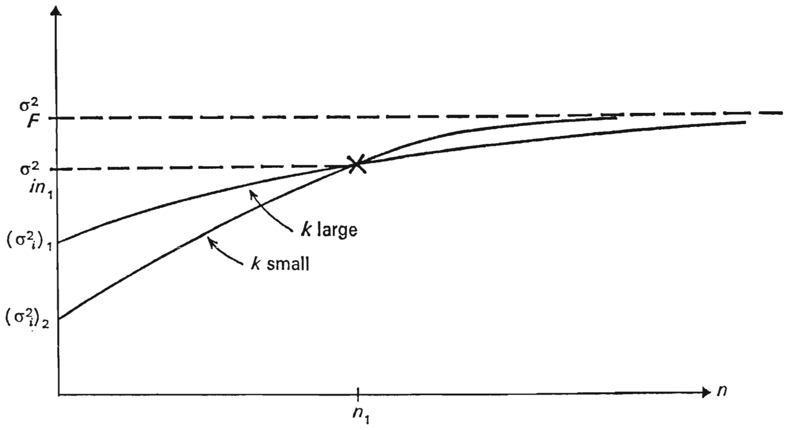

Fig. 1

decision whether to stop or to continue searching may be likened to a decision to buy or not buy a lottery ticket. The price of the ticket corresponds to c, the search cost, and the probability of winning a prize corresponds to the probability that a price lower than p_m will be drawn. The premium corresponds to the expected value of the price reduction. Thus it is obvous that a risk averter will tend to stop searching earlier than someone who is neutral to risks and the price he accepts will be higher on the average.

A Simulation

The stimulation was performed as follows. A random number was taken from a normal distribution with a mean of 20 000 (20 000 Sw. kr. is the approximate price of a new car) and standard deviation of 800. A Gaussian

Table 2

Trial	Mean p_F	Standard deviation σ_F	Subjective initial standard deviation σ_i	Weight for σ_i k	Search cost c	Average number of search steps $\bar{N}$	Standard deviation for number of search steps σ_N	Average acceptance price $\bar{p}_{acc}$	Standard deviation for acceptance price σp_{acc}
1	20 000	800	1 200	3	40	10.243	3.746	18 646	373
2	20 000	800	1 200	3	100	5.500	1.951	18 934	457
3	20 000	800	500	3	40	4.450	2.409	19 127	504
4	20 000	800	500	3	100	2.200	0.400	19 435	635
5	20 000	800	500	1	100	2.250	0.606	19 491	600

a posteriori distribution was formed with mean and variance as per (1) and (2). The value of continued searching was calculated according to (3) and compared with the search cost c. If $E(\Delta p)$ was greater than c, a new random number was drawn from the distribution. $E(\Delta p)$ was again compared with c. Whenever $E(\Delta p)$ worked out less than c the process was interrupted and the lowest of the prices drawn was written out together with the number of steps that had been required.

Five sets of parameter were used and 100 simulations were performed for each set. Table 2 sets out the means and standard deviations obtained for N and P_{acc} from the different trials. A series of histograms is appended to show the various distributions of N and P_{acc}.

Figs. 2*a–e* and 3*a–e* set out distributions for the acceptance prices and the number of search steps. The parent distribution, which we can call the supply distribution, is drawn in the diagrams and is normally distributed in all cases with a mean of 20 000 and a standard deviation of 800.

Thus we find that a given distribution of firms by price, $F(p)$, in combination with consumers who search rationally according to an adaptive process, generates a distribution which describes the frequency of purchases from firms that charge different prices. We can call the latter the "demand distribution", $E(p)$, which will depend not only on the appearance of $F(p)$ but also on the consumers' search cost, their subjective conception

Acceptance price distribution with different values for k, σ_i and c

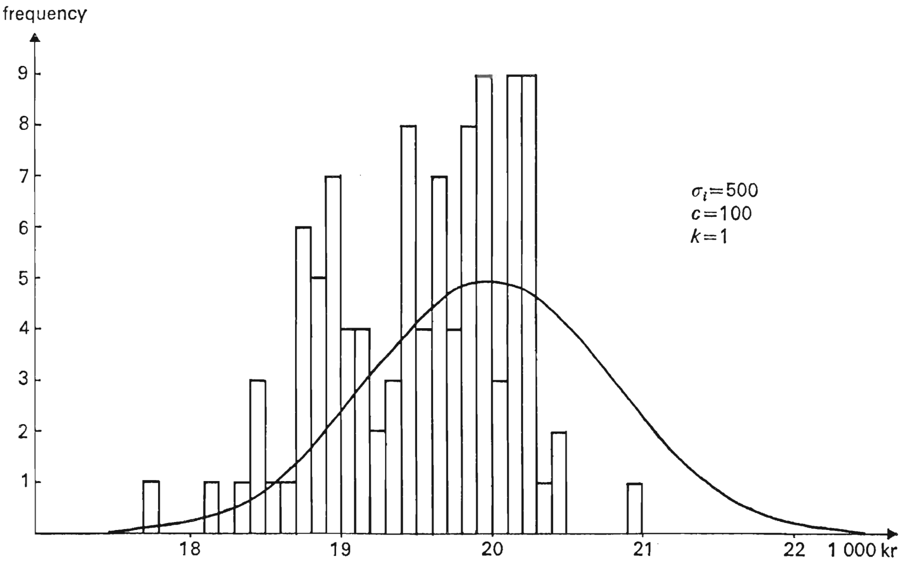

Fig. 2a

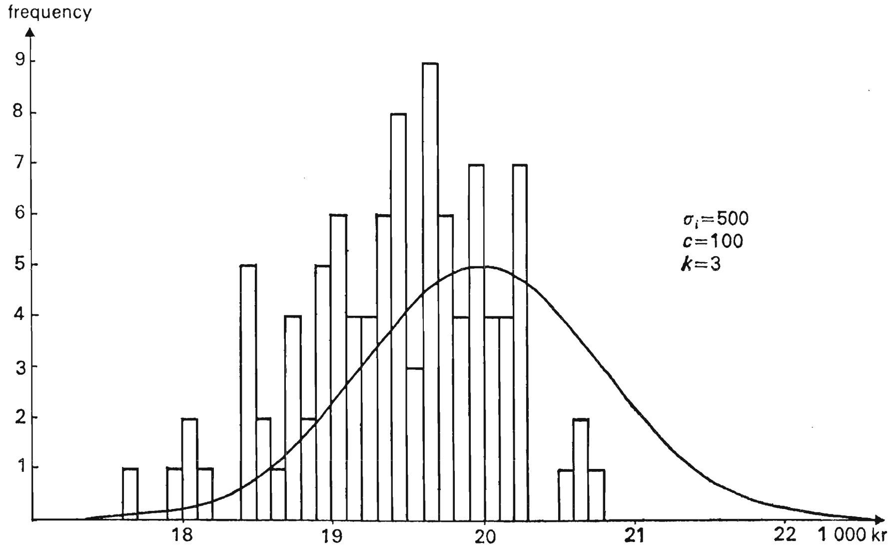
frequency
$\sigma_i=500$
$c=100$
$k=3$
18
19
20
21
22
1 000 kr

Fig. 2b

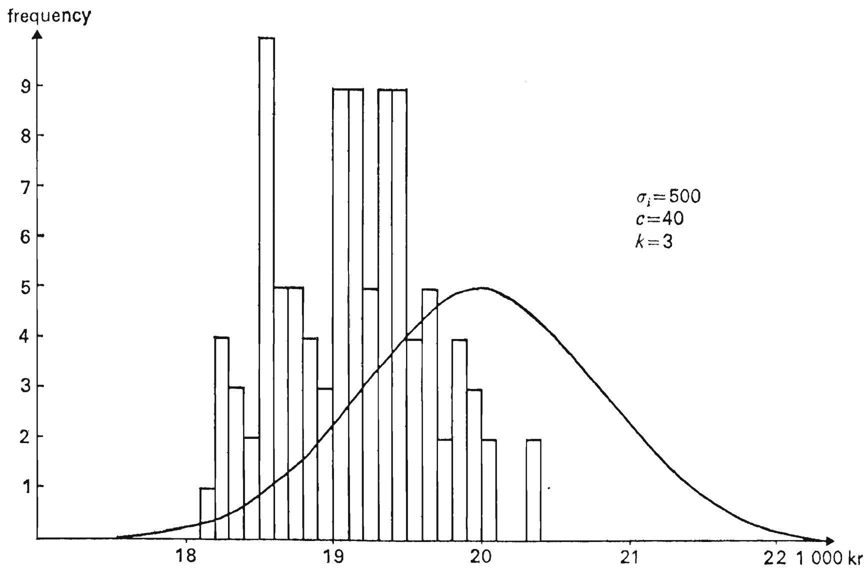
frequency
$\sigma_i=500$
$c=40$
$k=3$
18
19
20
21
22
1 000 kr

Fig. 2c

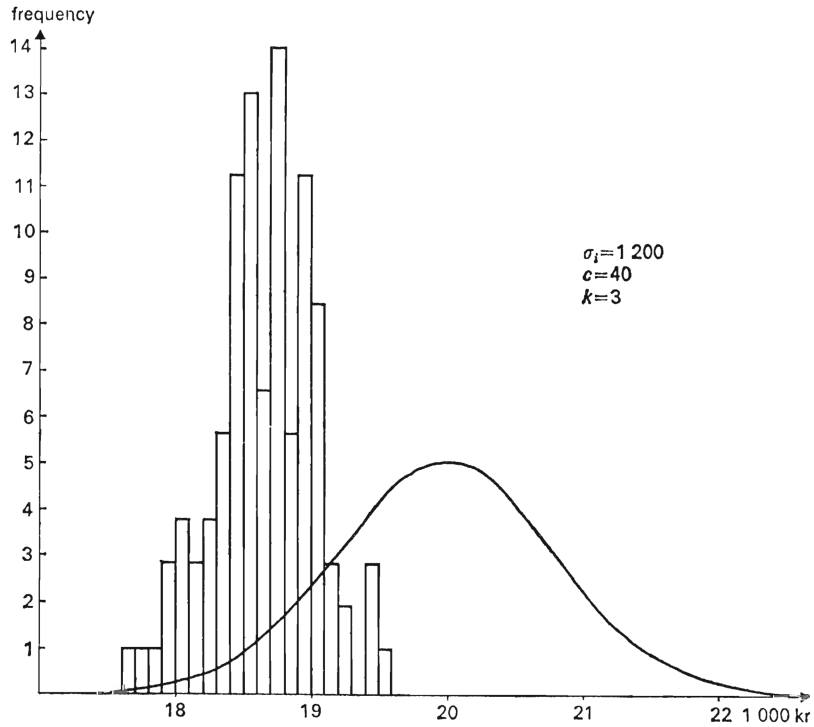
frequency
14
13
12
11
10
9
8
7
6
5
4
3
2
1
18
19
20
21
22 1 000 kr
σ_i=1 200
c=40
k=3

Fig. 2d

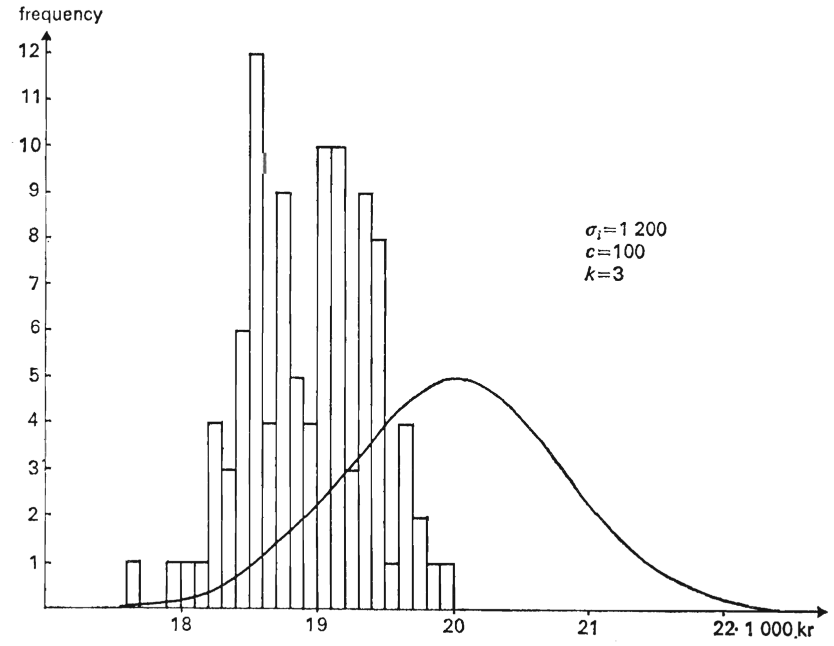
frequency
12
11
10
9
8
7
6
5
4
3
2
1
18
19
20
21
22· 1 000 kr
σ_i=1 200
c=100
k=3

Fig. 2e

Search-step distribution with different values for k, σ_t and c

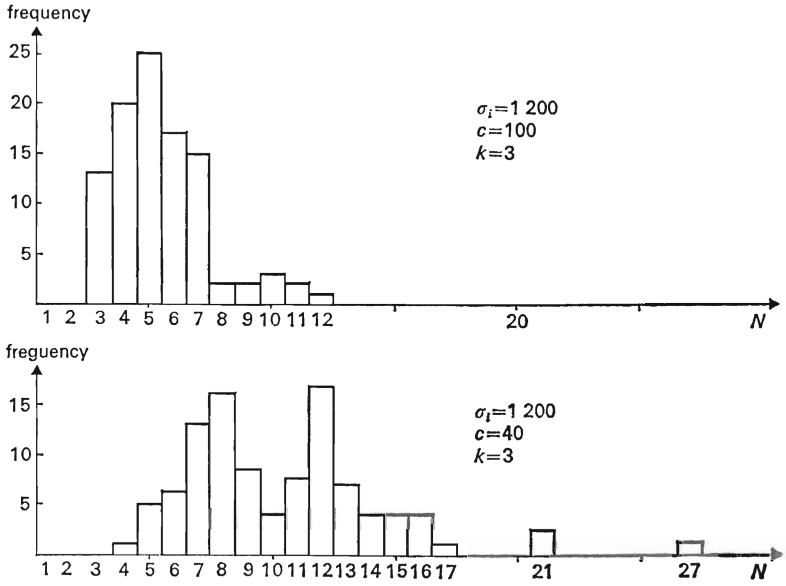

Fig. 3a, b

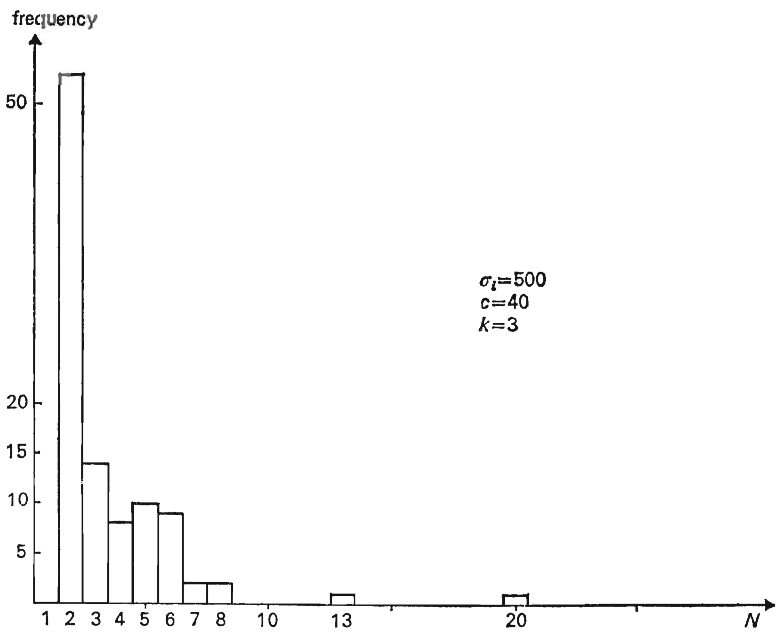

Fig. 3c

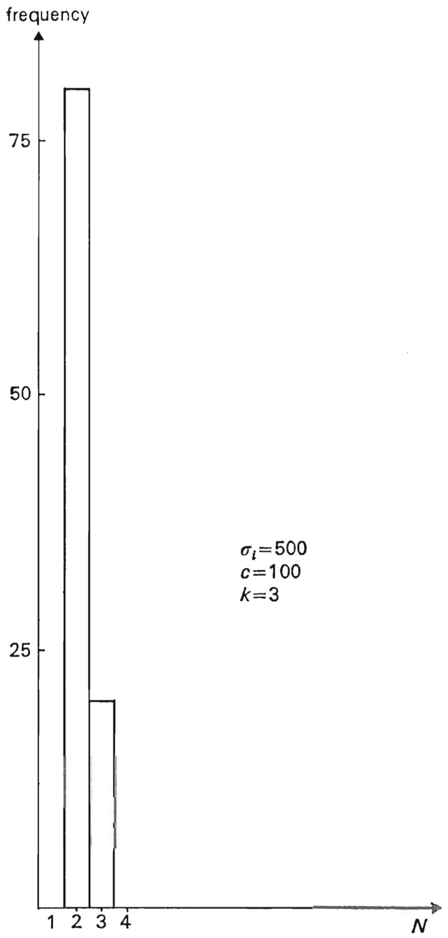

Fig. 3d

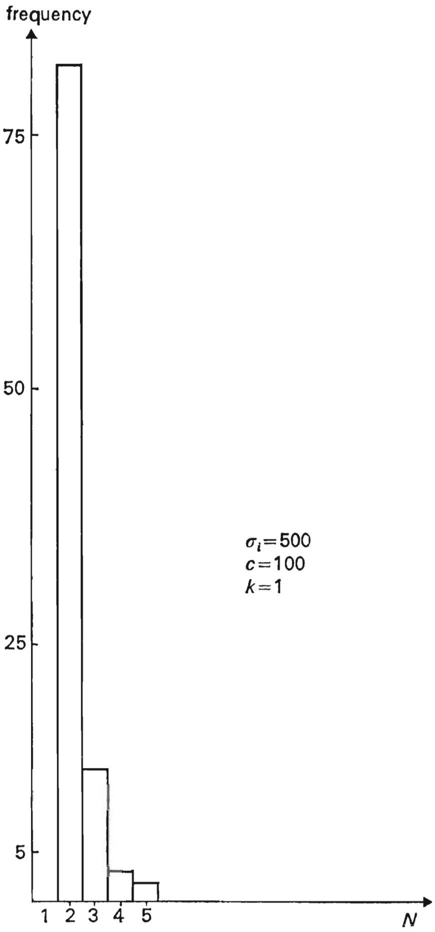

Fig. 3e

of the price dispersion and the weight they attach to this conception. $E(p)$ may be expressed in principle as

$$E(p) = g(c, \sigma_i, k, \sigma_F, \bar{p}_F)$$

If the mathematical expectation for $E(p)$ is designated $\bar{p}_E$ and the standard deviation σ_E, our material gives the following partial derivatives.

$$\frac{\delta \bar{p}_E}{\delta c} > 0 \text{ both for high and low } \sigma_i$$

$$\frac{\delta \sigma_E}{\delta c} > 0 \text{ both for high and low } \sigma_i$$

$$\frac{\delta \bar{p}_E}{\delta \sigma_i} < 0 \text{ both for high and low } c$$

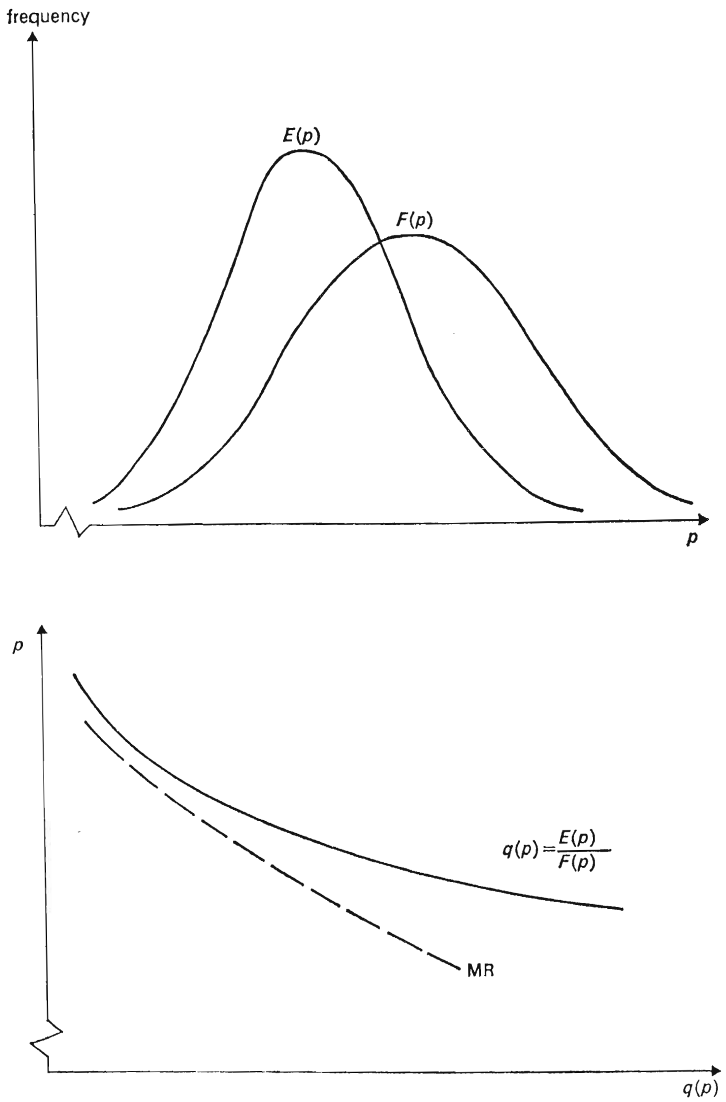

Fig. 4a, b

$\frac{\delta \sigma_E}{\delta \sigma_i} < 0$ both for high and low c

$\frac{\delta \bar{p}_E}{\delta k} < 0$ when $\sigma_i < \sigma_F$

The converse should hold if $\sigma_i > \sigma_F$.

$\frac{\delta \sigma_E}{\delta k} > 0$ when $\sigma_i < \sigma_F$

The converse should hold if $\sigma_i > \sigma_F$.

Table 3. *Below shows how the elasticities depend on the consumers' search costs and subjective standard deviations*

		Subjective standard deviation σ_i	
		500	1200
Search cost	40	− 24.977	− 35.8736
c	100	− 15.407	− 26.504

Mixed population
Elasticity: − 26.504

With a given distribution $F(p)$ and given values (or distributions) for c, σ_i and k, purchases will be made in each period from the different firms at a rate described by $E(p)$. Since $F(p)$ describes the distribution of firms, we can now see how large the share of the demand is per firm for different prices by means of the function

$$q(p)=\frac{E(p)}{F(p)} \text{ (see Fig. 4).}$$

Thus q, multiplied by a constant describes how demand would vary for one firm as it varies its price throughout the price interval, given that all other firms keep their prices unchanged. This corresponds to what is customarily referred to in price theory as the firm's demand curve. (Cf. the kinked demand curve in oligopoly.)

The method of least squares was used to adjust a demand curve of constant elasticity to the data obtained. This was done for every set of initial values, i.e. $\sigma_i=1\,200$ and $\sigma_i=500$ as well as $c=40$ and $c=100$. Additionally, a new population was used consisting of $\frac{1}{5}$ with $\sigma_i=1\,200$ and $c=100$, $\frac{1}{5}$ with $\sigma_i=1\,200$ and $c=40$, $\frac{1}{5}$ with $\sigma_i=500$ and $c=100$, $\frac{1}{5}$ with $\sigma_1=500$ nd $c=40$ and $\frac{1}{5}$ with $c>\sigma_i/\sqrt{2\pi}$, i.e. individuals who search optimally only once.

Demand functions take the form

$$q = kp^e$$

where e = elasticity.

Behavior of the Firms

The firms' prices have been assumed constant for the simulations, and thus $F(p)$ has remained constant. The consumers have generated a distribution of the resulting contracts, $E(p)$, with their search activity. Firms charging relatively high prices have landed a small share of the contracts, while the low-price firms have received a large share.

What are the conditions for the distribution to be an equilibrium distribution? By equilibrium I mean that $F(p)$ will remain unchanged until the next

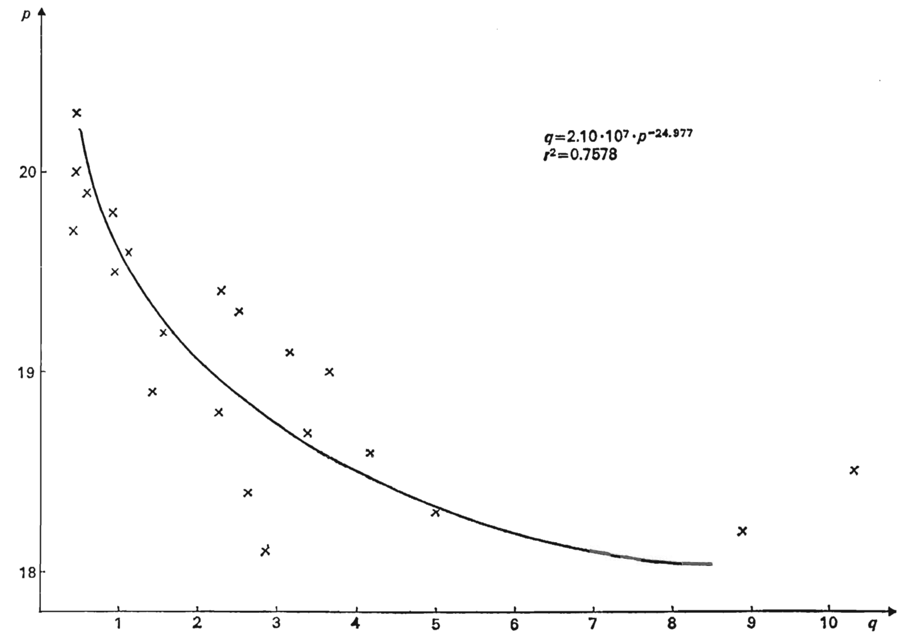

Fig. 5a

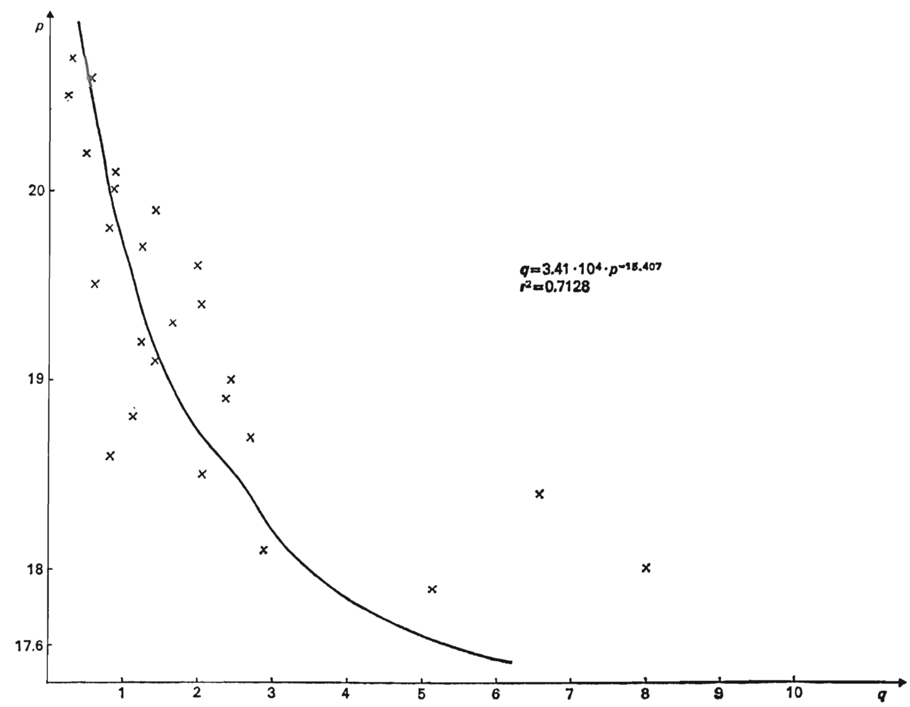

Fig. 5b

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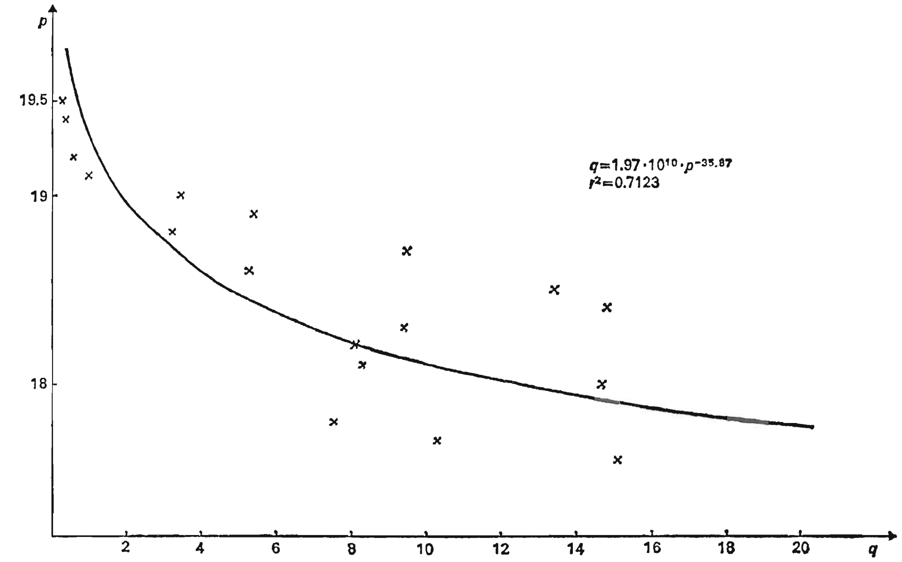
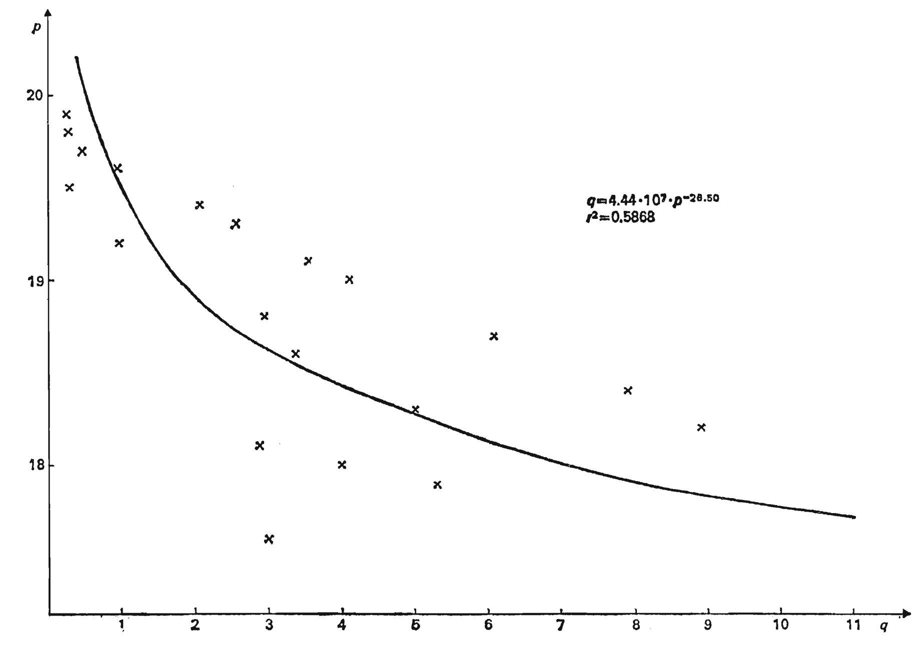


p
19.5
19
18
2
4
6
8
10
12
14
16
18
20
q
$q=1.97\cdot10^{10}\cdot p^{-35.87}$
$r^2=0.7123$

Fig. 5c



p
20
19
18
1
2
3
4
5
6
7
8
9
10
11
q
$q=4.44\cdot10^{7}\cdot p^{-28.50}$
$r^2=0.5868$

Fig. 5d

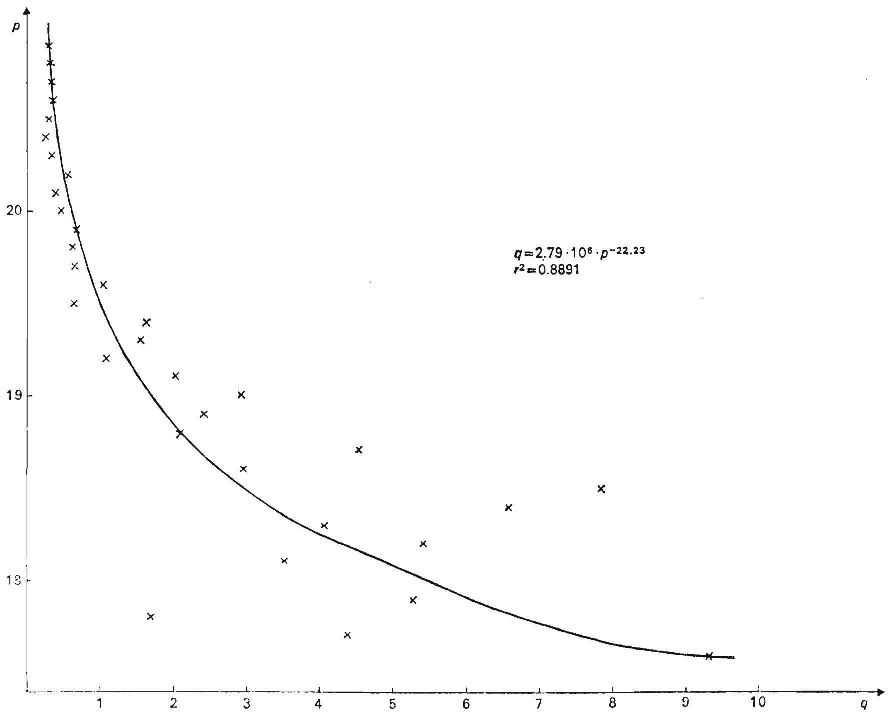


Fig. 5e

period. That will occur either if no firm changes its price or if price changes effected by different firms tend to cancel each other out. I shall now proceed to analyze ways and means of justifying equilibrium under price dispersion.

If all firms have the same costs and perfect information about the demand outcome following from each price after the first search round, and if every firm behaves as though all other firms intend to retain their prices from the previous period, the $F(p)$ distribution will collapse. Out of the demand function $q=kp^e$ and the cost function $c=f(q)$, the monopolistic profit-maximizing price p^* can be calculated in the usual manner. All firms on the market would thereupon change their price to p^*. The question of convergence is analyzed at greater length in Diamond [1] and Fisher [2].

But the real world, as illustrated in Table 1, shows a different picture. Substantial price dispersion tends to persist over time. Here I shall discuss four factors that may serve to explain persistent price dispersion: (1) Consumer search costs and *a priori* information. (2) The appearance of the cost function. (3) Corporate information about demand. (4) Price differentiation between customers.

If all consumers search only once, the stopping price distribution $E(p)$ will coincide in principle with the supply distribution $F(p)$.[1] As I have shown

[1] Apart from stochastic variations.

earlier, that will occur if search costs incurred by all individuals exceed their subjective perception of the distribution's standard deviation divided by 2π.[1] That would confront every firm with a demand curve of zero elasticity. It is quite obvious that pushing up prices is one way for the firms to increase their profits. If at least one consumer now calls on at least two firms and buys from the firm which asks the lowest price, $E(p)$ will tend to end up to the left of $F(p)$. The more people who search and the longer time they spend searching, the greater will be the deviation between $E(p)$ and $F(p)$. The elasticity (in absolute numbers) will then be greater than zero.

We may infer from the foregoing that a situation where all consumers accept the first price quotation does not lead to equilibrium. After all, that will give every firm an incentive to raise its price and the whole distribution $F(p)$ will be shifted to the right. When the prices go up across the board, the consumers will begin searching since, if the subjective distribution is based on the prices in a previous period, an average price offer from the new $F(p)$ will appear to lie far to the right on the subjective distribution. At that point it would seem worthwhile to collect at least one more price quotation.[2] The elasticity thereupon rises and will keep on rising as more search activity is generated. Once again we find a monopolistic profit-maximizing price around which all firms will tend to cluster.

How can equilibrium under price dispersion be explained on the basis of costs? The demand function I have estimated from the simulations made in the previous section was of constant elasticity. While that in itself was an arbitrary choice, let us stick to it.

$$q = kp^e$$

gives:

$$\pi = q \cdot \sqrt[e]{\frac{1}{k} q} - c(q)$$

$$\frac{d\pi}{dq} = 0 \quad \text{gives}$$

$$\frac{dc}{dq} = \frac{e+1}{e} \left(\frac{1}{k}\right)^{1/e} q^{1/e}$$

in order to satisfy the profit maximum condition for every price. The graphic counterpart (see Figs. 4*a* and 4*b*) is for MR to coincide with MC, which

[1] Subject, however, to the proviso that the subjective perception of the distribution's mean is weighted with zero after one price has been drawn. Otherwise a price found which exceeds the subjective mean would require a higher search cost for an optimal cutoff to occur and vice versa.

[2] Incidentally, this is the very mechanism which underlies derivations of the Phillips curve from microeconomic behavior in, say, Phelps [7]. An average wage offer from a wage distribution moved to the right is confused with an offer in the higher tail on the subjective wage distribution, which is based on wage levels of the previous period. Search activity diminishes and with that the duration of unemployment.

implies that the marginal costs are decreasing. However, the high elasticities (between -15 and -35 in my simulation) suggest a comparatively moderate rate at which MC diminishes with the scale of sales. Since the firms in our study are dealers, such a diminishing MC may be warranted by quantity discounts from the manufactuer (or general agent) as well as by economies of scale.

These arguments have been based on the assumption that the firms are familiar with the demand curve resulting from search behavior with perfect information. Any analysis of behavior where information is imperfect must of course also consider what firms do in that situation. Let us assume that an entrepreneur knows with certainty only the demand which confronted him in an earlier period at the price he then set. What is more, this demand is stochastic, as is clearly shown in Fig. 5. However, he has access to certain materials that enable him to gauge this demand. He has only a subjective idea of the demand at other prices. The decision he has to make is: Should I maintain the same price or change it? To answer that question he must ask himself two more: 1. What effect will a price change have on the expected profit? 2. How great is the expected value of the information increment?

Even though the price charged in the former period is meant to maximize expected profit with due allowance for the entrepreneur's subjective perception as to where his firm stands on the demand curve and its variance when prices differ, it may be optimal for him to change his price, since this would give him better information about segments of the demand curve unknown to him. Even so, such experimentation with an objectively better price might by ill luck turn out so badly that it would appear more advantageous to reinstate the original price even though it is less favorable on objective grounds. Of course, this is owing to the fact that the decision has to be made on the basis of incomplete information. That also includes an assessment of the value of collecting more information. Rothschild [8] shows that a firm which follows an optimal strategy will with positive probability charge the wrong price infinitely often and the correct price only a finite number of times. He goes on to say that "... if there are many firms, then some of them will charge some prices and some others. In other words, price variability will persist."

However, that is not possible on a market with adaptive, sequentially searching customers.[1] The crucial question is not whether a firm might stick to a wrong price, but rather what tendency holds for the majority of firms when it comes to changing prices. Most of the firms that try an objectively better price will also obtain information to prove that this price is actually better. At that point the supply distribution $F(p)$ will shift

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[1] Let us assume that all firms on this market have the same constant marginal cost.

decisively towards p^*. Since the search behavior of consumers will depend on the sequence in which they draw prices, and hence on $F(p)$, the stopping price distribution $E(p)$ will change and with that the demand which confronts the firms at different prices. The firms will then have new material on which to base their decisions. Some firms will have to reappraise their decisions to hold onto objectively wrong prices. For there to be equilibrium with price dispersion this process must lead to a situation in which no firm changes its price or where all price changes cancel each other out. If a firm experiments with an objectively better price, there will normally be a greater than 50 % probability that the outcome of the experiment will cause the firm to switch to this price. $F(p)$ will change if at least a few firms do experiment. Thus the question of whether there can be equilibrium with price dispersion may be said to be equivalent to this question: Will the condition for *not* performing the experiment be satisfied for each and every firm before the distribution collapses entirely? This problem requires further research.

Another possibility, which I cannot discuss at greater length in the present context, may be that empirically observed price dispersions (as in Table 1) do not apply between firms but rather between consumers. That is to say, price differentials result rather from price discrimination. Firms may be thought of as abiding by some common rules of thumb as to which offer to give a consumer, based on certain outer signs (trade-in car, intensity of haggling, etc.). Such rules may be optimal and lead to discrimination against certain groups.

Conclusions

There is certainly a need to study how markets operate under conditions when information is not omnipresent at a zero price. My study is particularly concerned with the effects on market prices brought about by consumers who seek price information adaptively and behave sequentially. The question I have tried to answer first and foremost is how to explain persistent price dispersion on a homogeneous market. My approach has been to employ a model with adaptive, sequentially searching consumers. The effect of different search costs and subjective price distributions on their search behavior has been studied by means of computer simulation. An analysis of this problem must also include corporate behavior. One plausible explanation for a persistent price dispersion may be found in the cost side. If MC is diminishing, all prices may result in equally large profits even though consumer search behavior reduces the market shares of firms which charge higher prices.

Here I have merely attempted to describe how the market operates when information is imperfect. One of the main objectives has been to provide a basis for studying the normative aspects of information processes. What distortions can arise in the use of resources? How valuable are centralized collection of information and similar measures? What incentives exist for the privately organized dissemination of information and what effects do they have on the allocation of resources?

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CHAPTER V:

Price Adjustment and Equilibrium in Markets with Imperfect Information

V. *PRICE ADJUSTMENT AND EQUILIBRIUM IN MARKETS WITH IMPERFECT INFORMATION**

V.1. Summary and Introduction

V.1.1. Summary

What sort of equilibria exist in markets where it is necessary to use resources to acquire information? Specifically, the question has been raised whether the existence of information costs can lead to equilibria with price dispersion. In this chapter I analyze the interaction between consumers and firms when consumers with different search costs search for low-price firms, using sequential stopping rules. Firms, on the other hand, uncertain about the shape of the stochastic demand curves they are facing,try to obtain information about these curves by experimenting with price changes. According to the information obtained they try to change prices in a profit-increasing direction. The prices in the market are thus endogenously determined. The behavior on one side of the market affects what the other side does and vice versa. This interdependence is summed up in a differential equation, describing how the distribution of prices will change over time. It is shown that if the distribution degenerates to a single price, the monopoly price is the only conceivable equilibrium price. Moreover, it is shown that there might exist an equilibrium with price dispersion which satisfies the Nash condition. The relation between this equilibrium price distribution and the search-cost distribution is analyzed. The necessary and sufficient condition on the search cost distribution for a price dispersion equilibrium is derived.

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* *Forthcoming in* *The Scandinavian Journal of Economics*, *no 1 1977 in an abbreviated version.*

V.1.2. Introduction

The seminal work of Stigler (1961) has given rise to a growing body of literature concerning markets with limited information.

One category of models attempts to analyze the equilibrium properties of a market which in one way or another is subject to imperfect information and in which behavior on both sides of the market is taken into consideration. Examples of models in this category include Diamond (1971), Fisher (1970, 1972, 1973), Hey (1974), and to some extent Rothschild (1974b).

A common objection to Stigler's analysis is that he takes into account only one side of the market, that of consumers. Also it is obvious that some firms will receive higher profit than others. If firms have information (or if it is possible for them to collect it), a number of them will change their prices and thereby create a new search situation for the consumers.

Diamond assumes in his model that firms have perfect knowledge of the consumers and of their search strategies. A degenerated price distribution, containing the monopoly price only, will then be an equilibrium.

Fisher, on the other hand, assumes that firms change prices according to whether or not demand exceeds marginal-cost output. The price distribution then converges to the competitive equilibrium. However, any firm can increase its profits by means of a price increase. Therefore this could never be a Nash equilibrium.

Hey discusses the convergence of the price distribution when firms analyze demand changes due to price changes between periods. If the price-changing rules are of a quite special kind, the distribution will converge to the monopoly price.

In the model presented here the attempt has been to take into consideration consumers with different search costs who use sequential stopping rules in their search process. The firms, incompletely informed about the stochastic demand curves they are facing, try to compensate for this by experimenting with different prices. The main question is then what sort of equilibrium will persist in such a market. Of special interest is the question whether there is an equilibrium with persistent price dispersion, or whether all equilibria in markets of this kind are single-price equilibria.

V.2. The Model

V.2.1. Consumer behavior

We now specify the consumers' activity. Each period a number (k) of consumers enters the market with the intention of buying a single unit of the commodity in question. In this first approach k is regarded as exogenously given; and as for the good, it is regarded as an indivisible consumer durable. The consumers are assumed not to have any previous market experience. Each consumer faces a great number of firms, each charging its own price. The frequency distribution of firms over prices at time t is described by the density function $f_t(p)$, which is assumed to be continuous and differentiable. The consumers are assumed to have perfect knowledge about $f_t(p)$, but *do not know which firm charges which price*. They have to get information about this by undertaking some kind of search activity.

A consumer conducts his search by picking one firm at a time from $f_t(p)$, and he is equally likely to pick any of the firms. The cost to the consumer of each search step is c, assumed to be the same during the whole search process for a given consumer, but differing among consumers. Apart from direct outlays, this search cost also includes time disposal as well as various non-pecuniary elements (negative or positive) connected with search. The consumers entering the market each period are distributed according to a frequency distribution $\gamma(c)$ over search costs.

Given the distribution, $f(p)$, of firms, as well as the distribution, $\gamma(c)$, of consumers over search cost and their search rules, we are first interested in finding out *what the resulting distribution of actual purchases over prices actually charged will be.*

Consider a single consumer in period t. The most favorable price offer he has found up to that time is p_m, and he is assumed to be able to recall the offer. He believes that $f_t(p)$ will remain constant during his expected search process. As will be discussed later in more detail, f(p) will usually change between periods. It is assumed that actual changes will be observed immediately. In each period the consumer regards the current $f_t(p)$ as the best estimate of future f(p).[1)]

The consumer with p_m as the best offer so far now has to decide whether to continue searching or to accept p_m and buy. The expected benefit from searching once more is

$$\psi(p_m) = \int_0^{p_m} (p_m - p) \; f_t(p) \; dp \tag{2.1.1}$$

Integrating by parts gives

$$\psi(p_m) = \int_0^{p_m} F_t(p) \; dp$$

where

$$F_t(p) = \int_0^{p_m} f_t(p) \; dp$$

which is the cumulative distribution function.

The optimal rule for a risk-neutral consumer is to continue searching whenever the expected benefit from additional searching exceeds the cost; we thus have the following rules:

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1) *The assumption that a consumer will regard f(p) as constant may be defended by the fact that, in the cases analyzed within this model, he has no idea in what direction the changes will take and therefore uses the current distribution as the best estimate of future f(p).*

$\psi(p_m) > c \rightarrow$ keep searching,

$\psi(p_m) \leq c \rightarrow$ stop and accept p_m.

A consumer who, after having received each price quotation, decides whether to continue searching or to accept the quoted price according to the rules above is then following a sequential decision rule which can be proved to be optimal.[1)]

We can now see that there is a certain price, R, called the reservation price, such that the consumer will accept any offer below or equal to R, and reject any price above R. Clearly, R is the solution to the equation

$$\psi(R) = c.$$

Obviously, the function R(c) is monotonically increasing. In particular it is strictly monotonically increasing if F(p) is also strictly monotonically increasing, since, in this case, there is a unique reservation price for each search cost.

If the frequency distribution of search costs is $\gamma(c)$, what is then the frequency distribution of reservation prices?

The reservation price is the solution to

$$\int_0^R F(p)\, dp = c$$

If we denote $\int_0^R F(p)\, dp$ by $\tilde{F}(R)$, then $R = \tilde{F}^{-1}(c)$ is the inverse relation, which it is possible to use if $\tilde{F}$ is strictly monotonic.

The probability that a given consumer has a search cost less than or equal to $\bar{c}$ is

$$\text{pr } (c \leq \bar{c}) = \int_0^{\bar{c}} \gamma(c)\, dc.$$

Hence,

1) *See for example McCall (1970).*

$$pr(R \leq \bar{R}) = \int_{\tilde{F}^{-}(c) \leq \bar{R}} \gamma(c)\ dc.$$

Now define $\bar{c}$ by

$$\tilde{F}(\bar{R}) = \bar{c}.$$

$$R \leq \bar{R} \text{ if and only if } c \leq \bar{c}.$$

Then

$$pr(R \leq \bar{R}) = \int_0^{\bar{c}} \gamma(c)\ dc$$

and

$$g(\bar{R}) = \gamma(\bar{c}) \frac{d\bar{c}}{d\bar{R}},$$

where $g(\bar{R})$ is the distribution of reservation prices. We also have:

$$\frac{d\bar{c}}{d\bar{R}} = \tilde{F}'(\bar{R}) = F(\bar{R}) = \int_0^{\bar{R}} f(p)\ dp.$$

Then

$$g(\bar{R}) = \gamma\left(\tilde{F}(\bar{R})\right) \frac{d\bar{c}}{d\bar{R}} = \gamma\left(\tilde{F}(\bar{R})\right)\ F(\bar{R}), \qquad (2.1.2)$$

which is the frequency distribution of reservation prices we are looking for.

The next step is to derive the frequency distribution of prices at which the consumers actually buy. Let us call this distribution the *stopping price distribution*. This density function will be denoted by ω and the cumulative distribution function by Ω. Given the reservation price, R_i, a consumer will buy at the very first offer below R_i. The stopping price for consumer i will then be the first price he locates that is below R_i. Then, the probability distribution $\Omega_i(p)$, of the stopping prices for consumer i is given by

$$\mathrm{pr}(p \leq p_0 | R_i) = \int_0^{p} \frac{f(p)}{\int_0^{R_i} f(s)\, ds}\, dp = \frac{1}{F(R_i)}\, F(p_0) \quad \text{if } p_0 \leq R_i.$$

We also have

$$\mathrm{pr}(p \leq p_0 | R_i) = 1, \quad \text{if } p_0 > R_i.$$

If all consumers have the same reservation price, $\bar{R}$, then the frequency distribution of stopping prices $\omega(p)$ will be

$$\omega(p) = \begin{cases} \dfrac{f(p)}{\int_0^{\bar{R}} f(p)\, dp} & \text{if } p \leq \bar{R} \\ 0 & \text{if } p > \bar{R}. \end{cases}$$

However, the search cost is assumed not to be the same for all consumers, but distributed according to $\gamma(c)$. We can derive the distribution function, $\Omega(p)$, in this general case in the following way:

$$\Omega(p_0) = \mathrm{pr}(p \leq p_0) = \int_0^{\infty} \mathrm{pr}(p \leq p_0 | \bar{R})\, \mathrm{pr}(R = \bar{R})\, d\bar{R} =$$

$$= \int_0^{p} \mathrm{pr}(p \leq p_0 | \bar{R})\, \mathrm{pr}(R = \bar{R})\, d\bar{R} +$$

$$+ \int_{p_0}^{\infty} \mathrm{pr}(p \leq p_0 | \bar{R})\, \mathrm{pr}(R = \bar{R})\, d\bar{R} =$$

$$= \int_0^{p_0} \mathrm{pr}(R = \bar{R})\, d\bar{R} + \int_{p_0}^{\infty} \frac{F(p_0)}{F(\bar{R})}\, g(\bar{R})\, d\bar{R} =$$

$$= \int_0^{p_0} g(\bar{R})\, d\bar{R} + \int_{p_0}^{\infty} \frac{F(p_0)}{F(\bar{R})}\, \gamma\left(\tilde{F}(\bar{R})\right) F(\bar{R})\, d\bar{R} =$$

$$= \int_0^{p_0} \gamma\left(\tilde{F}(\bar{R})\right) F(\bar{R})\, d\bar{R} + F(p_0) \int_{p_0}^{\infty} \gamma\left(\tilde{F}(\bar{R})\right) dR$$

We note that $\Omega(\infty) = 1$. We can then derive the probability density $\omega(p)$ by differentiating $\Omega(p)$, and doing so, we get

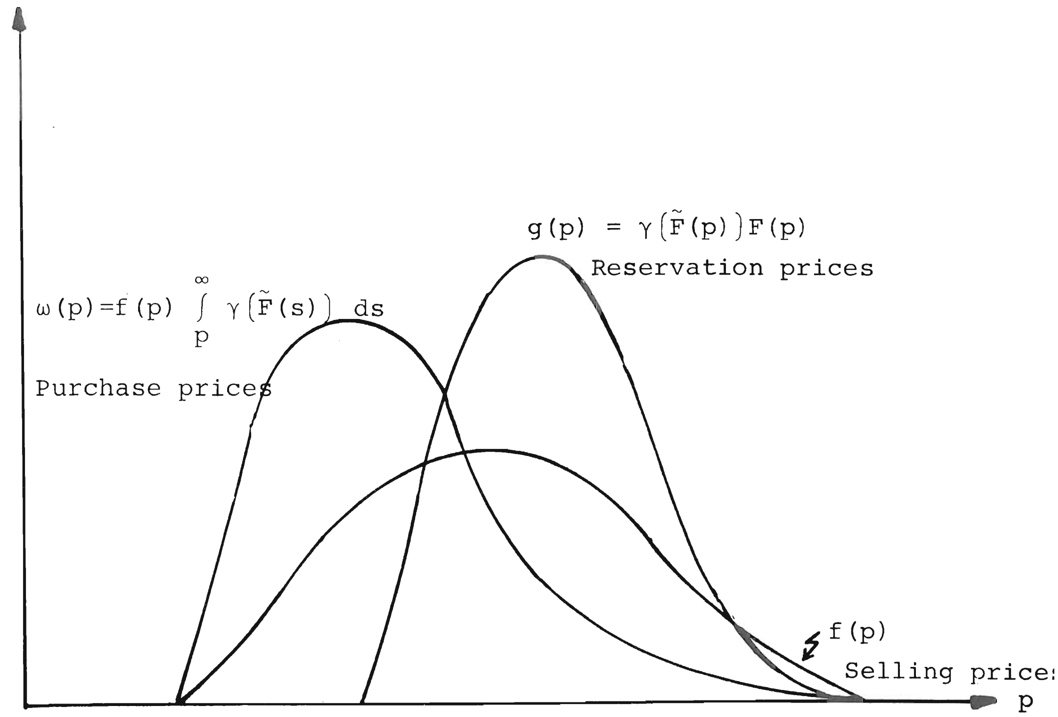
$$\omega(p_0) = \Omega'(p_0) = \gamma\left(\tilde{F}(p_0)\right) F(p_0) + F'(p_0) \int_{p_0}^{\infty} \gamma\left(\tilde{F}(\bar{R})\right) d\bar{R} -$$

$$- F(p_0) \gamma\left(\tilde{F}(p_0)\right) = f(p_0) \int_{p_0}^{\infty} \gamma\left(\tilde{F}(p)\right) dp, \tag{2.1.3}$$

if in the last step we denote the price variable by p instead of by $\bar{R}$.

We can now see how the distributions of firms, of search costs, reservation prices, and of stopping prices are related to each other. Figure 1 shows these distributions except for the search cost distribution.

Figure 1



Given the distribution, $f(p)$ of firms and the distribution, $\gamma(c)$, of search costs, there is a unique distribution, $g(p)$, of reservation prices. This will then generate a function $\omega(p)$, the stopping price distribution, showing how in their actual purchases consumers will be distributed over prices. Any change in either $f(p)$ or $\gamma(c)$ will then cause a change in $\omega(p)$.

V.2.2. The demand

We are now in position to derive the demand curve facing a firm. We will consider two cases, each involving a different assumption about the nature of the commodity purchased. First, we will assume that the good is of standard size and quality, and is purchased one unit per customer. In a price interval $[p_i, p_i + \Delta p]$ the number of consumers who actually buy in a given period is

$$k\left[\Omega(p_i + \Delta p) - \Omega(p_i)\right],$$

where Ω is the cumulative stopping price distribution. The number of firms in this interval is

$$m\left[F(p_i + \Delta p) - F(p_i)\right],$$

where m is the total number of firms. The number of customers *per* firm is then

$$\frac{k\left[\Omega(p_i + \Delta p) - \Omega(p_i)\right]}{m\left[F(p_i + \Delta p) - F(p_i)\right]} .$$

Letting Δp approach zero we then get

$$q_i = \frac{k}{m} \frac{\omega(p_i)}{f(p_i)} ,$$

which is the mathematical expectation of demand for a firm charging p_i.

Then using (2.1.3) we arrive at

$$q_i = \frac{k}{m} \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right) dp \tag{2.2.1}$$

which is the "firm's demand curve" if each consumer buys only one unit of the commodity. Note that this demand curve is the "cross sectional" one; it describes the demand confronting a firm hypothetically moving through the price interval, all other firms remaining at their original prices. Now, consider a second case, in which it is possible to buy different amounts of the commodity, or in different sizes. If all individuals have the same demand curve d(p), then the firm's demand curve will be

$$q_i = \frac{k}{m} d(p_i) \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right) dp \tag{2.2.2}$$

The elasticity of demand (η) is then in case of (2.2.1)

$$\eta = \frac{\gamma\left(\tilde{F}(p_i)\right) p_i}{\int_{p_i}^{\infty} \gamma\left(\tilde{F}(p_i)\right) dp} \; .$$

In case of (2.2.2) the elasticity of demand is η+e, where e is the elasticity of d(p). The case of finite individual demand elasticity will be treated in section V.3.2.

V.2.3. Firm behavior

Given the assumed behavior of the consumers and the associated demand curve, each firm has to decide what price to charge in order to maximize profit. Parallel with the consumers' situation, firms lack perfect information both about exact consumer behavior and the resulting demand curve. However, each firm knows its own demand at the price it charges during a period. However, it may obtain information about the shape of the demand curve by experimenting with price changes. The parallel with the consumer's search activity is obvious; firms risk losing profits by "searching out" the demand at other prices.

In this section we will derive an expression for the firms' price change based on assumptions of search behavior of the firms.

Each firm faces a stochastic demand curve in each period. There are two reasons for this. In the first place, the stopping distribution $\omega(p)$ is the *expected* distribution. Normally ω at p_i will differ from its expected value, causing the demand at p_i to be stochastic. In the second place, even if the number of buying consumers in the interval $[p_i, p_i + \Delta p]$ is equal to the expected number, the consumers need not be uniformly distributed among the firms in this interval, because the number of consumers *per firm* need not be large.

We can introduce this stochastic element into the firm's environment by adding a stochastic term to the demand function. We are then in position to derive the stochastic profit function. The stochastic term could in principle be derived from consumer search behavior. This, however, would be a very difficult task. For simplicity we assume instead that the stochastic environment of the firms can fairly well be described by adding a stochastic term u to the profit function. Then profit as a function of price is

$$\pi(p_i) = p_i\, q(p_i) - C\big(q(p_i)\big) + u, \qquad (2.3.1)$$

where the demand $q(p_i)$ is mathematically expected demand. $C\big(q(p_i)\big)$ is the cost function, which is taken to be the same for all firms in the market. u is a stochastic term which is added to expected profit.

Let us now describe the firm's experimental behavior. We assume that:

1. *All firms are risk neutral*
2. *A firm knows the expected demand (and thereby the expected profit) at the price it has charged itself during period t.*

3. *A firm does not know the demand at prices other than that which it has charged itself.*

Let us regard a particular firm i charging price p_i in period t. The firm will, during this period, register the demand q_i. The firm realizes that it is facing a finitely elastic demand curve, but it does not *know* whether p_i is the very best price or whether it could raise its profit by increasing or decreasing the price. However, the fact that it has chosen p_i reveals that it has no reason to believe that a lower price is likely to be better than a higher price.

We now assume that *if the firm undertakes an experiment with a price change, then it will be equally probable for it to raise as to lower its price.* Further, we make the simplifying assumption that *all firms are experimenting.*

Consider a firm charging p_i. It receives a profit of $\pi(p_i) + u$, where u is a stochastic term. Let us assume that u shows the variability in profit during relatively short periods (days for instance). We also assume that the profit function is homoscedastic, i.e. u has the same density function at all prices. If the firm remains at p_i for a longer period, say a month or two, it will get a fairly good picture of the expected profit $\pi(p_i)$. If during a short subperiod the firm tries another price, for instance $p_i + \Delta p$, then it will get the profit $\pi(p_i + \Delta p) + u$ at that price.

Given the experimental price increase Δp, a fundamental question is now what is the probability of an increase in profit, i.e. we ask what is the probability of the following relationship:

$$\mathrm{pr}\left(\pi(p_i + \Delta p) + u\right) \geq E\left(\pi(p_i)\right), \qquad (2.3.2)$$

where $\quad E\left(\pi(p_i)\right) = \pi(p_i).$

Let us call this probability ν_{i+1}. Making a Taylor expansion of $\pi(p_i)$ around p_i we get

$$\pi(p_i+\Delta p) = \pi(p_i) + \pi'(p_i)\,\Delta p + \frac{1}{2}\,\pi''(p_i)\,\Delta p^2 +$$

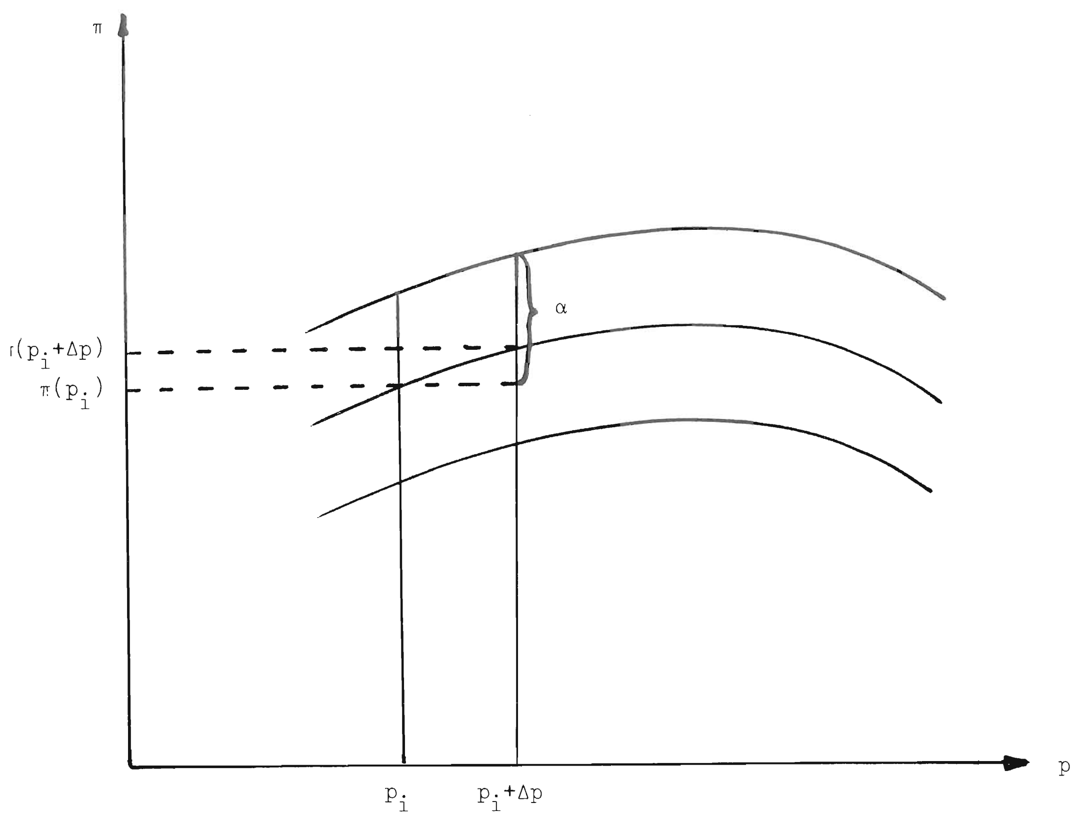
$$+ \;.....\; + \;.....$$

Linearizing in the interval, i.e. dropping terms of second degree and higher, the probability (2.3.2) is

$$\nu_i^+ = \mathrm{pr}\left(u \geq \pi'(p_i)\ \Delta p\right). \tag{2.3.3}$$

We see that ν_i^+ is the probability that the stochastic term does not reduce the profit at $p_i + \Delta p$ from its expected value more than the actual difference in expected profit, expressed by means of the slope of π at p_i times Δp. This is in figure 2 the probability of falling within α during the experiment with $p_i + \Delta p$.

Figure 2



If the profit function is homoscedastic, i.e. the stochastic term u has the same probability density function at all prices, where this density function is $\zeta(u)$ with the probability distribution $Z(u)$, we get

$$\nu_i^+ = 1 - \int_{-\infty}^{-\pi'(p_i)\,\Delta p} \zeta(u)\,du = 1 - Z\left(-\pi'(p_i)\,\Delta p\right).$$

Since $\zeta(u)$ is not derived from consumer search, we have to assume a reasonable shape for it. The normal density function is perhaps a good choice, but in a complicated interdependent analysis it will cause great analytical problems. Simple expressions will appear if we assume instead that the stochastic profit terms are uniformly distributed. Since the important thing is to introduce a stochastic element into the firm's environment, it would seem that this distribution is no worse than any other. Let us thus assume that $\zeta(u)$ is a rectangular distribution with limits -a and +a, i.e. $\zeta(u) = \frac{1}{2a}$. Then

$$Z(u) = \int_{-a}^{u} \frac{1}{2a}\,ds = \frac{u+a}{2a} \qquad -a \le u \le a.$$

The probability ν_i^+ is then

$$\nu_i^+ = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{\Delta p}{a}\,\pi'(p_i) & \text{if } -a \le \pi'(p_i)\,\Delta p \le a\,, \\ 1 & \text{if } \pi'(p_i)\,\Delta p > a\,, \\ 0 & \text{if } \pi'(p_i)\,\Delta p < -a. \end{cases} \qquad (2.3.4)$$

We see that this probability depends positively on the slope of the profit function and on the size of the price jump, but negatively on the variance of the stochastic term, as can be observed in figure 2.

If instead a firm tries the price $p_i - \Delta p$, then the probability of a profit increase is

$$\nu_i^- = \begin{cases} \frac{1}{2} - \frac{1}{2}\frac{\Delta p}{a}\,\pi'(p_i) & \text{if} \quad -a \leq \pi'(p_i)\,\Delta p \leq a, \\ 0 & \text{if} \quad \pi'(p_i)\,\Delta p > a, \\ 1 & \text{if} \quad \pi'(p_i)\,\Delta p < -a. \end{cases} \tag{2.3.5}$$

We see that $\nu_i^- = 1 - \nu_i^+$. Note that this follows from the approximation to a linear profit function in the interval $[p_i - \Delta p, p_i + \Delta p]$, evaluated at p_i.

V.2.4. Changes in the distribution of prices

In the previous section we derived the probability that a price change experiment will lead to increased profit at the experimental price. We now wish to study how firms actually change prices over time. Especially, we want to describe the aggregate effect of the behavior of the individual firms -- how the price distribution, i.e. the distribution of firms over prices, will change over time.

We assume the following behavior of firms (in addition to the earlier assumptions): *If a firm, charging the price p_i during a given period, experiments with the price $p_i + \Delta p$ during a sub-period and registers a higher profit at $p_i + \Delta p$, then it will charge the price $p_i + \Delta p$ during the next period; otherwise it will return to p_i.* From this follows that the probability that a firm charging p_i will raise its price to $p_i + \Delta p$ is $\frac{1}{2}\,\nu_i^+$.[1)]

We have assumed the market to be an atomistic market, i.e. one in which the number of firms is very great. Then the probability of changing the price from, for instance, p_i to $p_i + \Delta p$ will show the proportion of firms at p_i changing price in that direction.

The frequency of firms charging p_i at time t is $f_t(p_i)$. The frequency of firms charging p_i at time t+1 is the share

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1) Remember that we have assumed that the probability it will experiment with a price increase is one half.

of those at $p_i - \Delta p$ at t which experimented with a price increase (i.e. one half) *and* obtained positive information (i.e. profit increase) plus the share of those at $p_i + \Delta p$ which experimented with a price decrease *and* obtained positive information, plus those at p_i that experimented with a price decrease or a price increase *and* obtained negative information. We thus get[1)]

$$f_{t+1}(p_i) = \frac{1}{2} f_t(p_i - \Delta p) \ \nu_i + \frac{1}{2} f_t(p_i + \Delta p)(1-\nu_i) +$$

$$+ \frac{1}{2} f_t(p_i) \ \nu_i + \frac{1}{2} f_t(p_i)(1-\nu_i),$$

which is

$$f_{t+1}(p_i) = \frac{1}{2} f_t(p_i - \Delta p) \ \nu_i + \frac{1}{2} f_t(p+\Delta p)(1-\nu_i) + \frac{1}{2} f_t(p_i) \quad (2.4.1)$$

Writing $f_t(p_i - \Delta p)$ with help of Taylor expansion we get

$$f_t(p_i - \Delta p) = f_t(p_i) - f_t'(p_i) \ p + \frac{1}{2} f_t''(p_i) \ p^2 - \dots + \dots .$$

In a corresponding way we have for $f_t(p_i + \Delta p)$:

$$f_t(p_i + \Delta p) = f_t(p_i) + f_t'(p_i) \ \Delta p + \frac{1}{2} f''(p_i) \ \Delta p^2 + \dots + \dots .$$

Disregarding terms of second degree and higher, we can write expression (2.4.1) as

$$f_{t+1}(p_i) - f_t(p_i) = \frac{1}{2}\Big[f_t(p_i) - f_t'(p_i)\Delta p\Big] \ \nu_i +$$

$$+ \frac{1}{2}\Big[f_t(p_i) + f_t'(p_i)\Delta p\Big] \ (1-\nu_i) - \frac{1}{2} f_t(p_i),$$

which can be simplified to

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1) *Note that we have changed notation of ν_i slightly. Here we think, for simplicity, that the linearization around the experimental price and the ordinary price does not differ too much.*

$$f_{t+1}(p_i) - f_t(p_i) = f_t'(p_i)\ \Delta p(\tfrac{1}{2} - \nu_i).$$

Converting from discrete time intervals to continuous time, approximating the difference with the derivative, we have

$$\dot{f}(p_i,\ t) = f'(p_i)\ \Delta p(\tfrac{1}{2} - \nu_i).$$

If we now substitute for the expression for ν_i derived earlier we have

$$\dot{f}(p_i,t) = f'(p_i)\ \Delta p\left(\tfrac{1}{2} - \tfrac{1}{2} - \tfrac{1}{2}\,\tfrac{\Delta p}{a}\ \pi'(p_i)\right),$$

which is

$$\dot{f}(p_i,t) = -\tfrac{1}{2}\ f'(p_i)\ \pi'(p_i)\ \frac{\Delta p^2}{a}. \tag{2.4.2}$$

The expression for the profit function is

$$\pi(p_i) = p_i\ q_i(p_i) - C\left(q_i(p_i)\right),$$

where

$$q_i(p_i) = \frac{k}{m}\int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right)\, dp.$$

We get

$$\frac{d\pi}{dp_i} = \frac{k}{m}\left[\int_{p_i}^{\infty} \gamma\left(\tilde{F}_t(p)\right)\, dp - p_i\ \gamma\left(\tilde{F}_t(p_i)\right)\right] + \frac{dC}{dq_i}\ \frac{k}{m}\ \gamma\left(\tilde{F}_t(p_i)\right).$$

Rearranging terms we get

$$\frac{d\pi}{dp_i} = \frac{k}{m}\left[\int_{p_i}^{\infty} \gamma\left(\tilde{F}_t(p)\right)\, dp + \gamma\left(\tilde{F}_t(p_i)\right)\left[\frac{dC}{dq_i} - p_i\right]\right]. \tag{2.4.3}$$

The complete expression for the change of the price distribution will then be

$$\dot{f}(p_i,t) = -\frac{\Delta p^2}{2a} f_t'(p_i) \frac{k}{m} \left[\int_{p_i}^{\infty} \gamma(\tilde{F}_t(p))\, dp + \right.$$

$$\left. + \gamma(\tilde{F}_t(p_i)) \left[\frac{dC}{dq_i} - p_i\right]\right]. \qquad (2.4.4)$$

V.3. Market Equilibrium

V.3.1. Competitive equilibrium

In previous sections we have shown how consumers search for low-price firms using sequential stopping rules, and how firms experiment with price changes in order to increase profits. The behavior of these two sides of the market, and expecially their interaction, can be described by a differential equation (2.4.4), showing how the frequency of firms charging different prices changes over time.

In this section we will examine the sort of equilibrium that can exist in a market as we have described. Let us for a moment dwell on the concept of equilibrium. What should be meant by equilibrium in a market of this sort? It is apparent that a straightforward application of any definition of equilibrium such as that of the Arrow-Debreu model is not possible. Bent Hansen (1966) points out that "equilibrium is essentially a dynamic concept". He quotes the definition of equilibrium in classical mechanics: "Any configuration of a rigid body, or of a system of bodies, is said to be one of equilibrium if the body or the system can remain indefinitely in this configuration under the forces acting upon it."

When prices can be treated parametrically, as in an Arrow-Debreu world, any reasonable corresponding dynamic model will give rise to a situation where, if the Arrow-debreu equilibrium conditions are fulfilled, the forces are in balance, or "the system can remain indefinitely in this configuration under the forces acting upon it".

In a dynamic model equilibrium conditions require all time derivatives to be equal to zero. In the present model the equilibrium condition thus is that $\dot{f}(p_i, t)$ equal zero for all p_i. This is the stationary solution of the dynamic model. In this chapter we do not analyze the question of whether or not the market will attain this stationary solution. We merely say that *if* a situation like this appears (by chance), then it will repeat itself period after period. In other words, nothing is done about the question of convergence to equilibrium and corresponding stability properties.

A fundamental question is whether or not the competitive equilibrium can be an equilibrium solution to a market with imperfect information. We find that we must answer in the negative. We then ask, given that the price distribution has degenerated to a single price, whether there is any price that can be an equilibrium price. We are able to show that, within this model, the only price which can be an equilibrium price is the monopoly price. Further, we ask if there is a distribution of firms over prices for a given search-cost distribution that will fulfill the equilibrium conditions although there is price dispersion. We find that there does exist such a distribution. Furthermore, we derive necessary and sufficient conditions for the search cost distribution to fulfill equilibrium requirements.

The expression for $\dot{f}$ is

$$\dot{f}(p_i,t) = -\frac{\Delta p^2}{2a} f_t'(p_i)\ \pi'(p_i) \tag{2.4.2}$$

The equilibrium condition is

$$\dot{f}(p_i, t) = 0.$$

If any of the factors in (2.4.2) is equal to zero the equilibrium condition will be fulfilled. $\Delta p = 0$ or a (the limits of the stochastic term) approaching infinity corresponds to a situation where the experimental jump is zero and, respectively, where the variance of the stochastic term is infinitely large. Both cases are uninteresting.

If $f_t'(p_i) = 0$, then the distribution of firms is rectangular. The hitch in this case is that this $f_t'(p_i)$ is undefined at the end-points of the distribution. The only remaining possibility, then, is that π' be zero, or more explicitly:

$$\pi'(p_i = \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p_i)\right)\, dp + \gamma\left(\tilde{F}(p_i)\right)\left[\frac{dC}{dq_i} - p_i\right] = 0^{1)}. \qquad (2.4.3b)$$

Competitive equilibrium is characterized by the following:

1. All firms charge a single price, i.e. the price distribution is degenerate.
2. At this price, price equals marginal cost.

Rearranging terms in the equation (2.4.3b) we get

$$\frac{dC}{dq_i} = p_i \left[1 - \frac{1}{\dfrac{\gamma\left(\tilde{F}(p_i)\right)\ p_i}{\int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right)dp}} \right]$$

where

$$\frac{\gamma\left(\tilde{F}(p_i)\right)\ p_i}{\int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right)dp} \qquad (3.1.2a)$$

is the elasticity of demand of the firm's demand curve.[2)]

A necessary condition for fulfilling condition 2 above, is that (3.1.2a) approach infinity when the variance of the

---

1) *For simplicity we disregard the constant* $\frac{k}{m}$.

2) *Note that* $\int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right)\, dp$ *is the demand curve and* $\gamma\left(\tilde{F}(p_i)\right)$ *is the price derivative of this.*

distribution of firms approaches zero.[1)]

Let us see if this is possible. We know the relations between $f(p)$, $F(p)$ and $\tilde{F}(p)$. $F(p)$ is the integral of $f(p)$, while $\tilde{F}(p)$ is the integral of $F(p)$. $f(p)$ is a density function. If p_1 and p_2 are the lower and upper end-points of $f(p)$, then $F(p)$ increases from zero at p_1 to unity at p_2, and is equal to unity above p_2. $\tilde{F}(p)$ then increases in the interval $[p_1, p_2]$ with a slope of less than one. Above p_2 the slope of $\tilde{F}(p)$ is equal to one. Then we see that if the variance approaches zero, i.e. p_1 and p_2 approach a price $\bar{p}$ between them, $\tilde{F}(p)$ will approach a straight line with unity slope starting from $\bar{p}$. Then, in this case, we have

$$\tilde{F}(p_i) = p_i - \bar{p}.$$

Remember that $\tilde{F}(\bar{R}) = \bar{c}$, i.e. $\tilde{F}(\bar{R})$ is the search cost for an individual with reservation price $\bar{R}$. Then, in this case,

$$p_i - \bar{p} = c_i$$

or

$$p_i = \bar{p} + c_i ;$$

i.e. an individual with search cost c_i has the reservation price $\bar{p} + c_i$, given that the prices all firms charge are concentrated about $\bar{p}$.[2)]

We can rewrite the expression for the elasticity (3.1.2a) as

$$\frac{\gamma(p_i - \bar{p})\, p_i}{\int_{p_i}^{\infty} \gamma(p - \bar{p})\, dp} \tag{3.1.2b}$$

Let $c_1 > 0$ be the lowest of all the search costs registered among the consumers. Consider a firm charging p_i.

---

1) *Note that condition 1 is a restriction in this case. More specifically we put the question: "If 1 is the case, is it then possible that 2 can be fulfilled?"*

2) *Note that in the present formulation all individuals demand exactly one unit of the commodity, which thus is regarded as indivisible.*

If $f_t'(p_i) = 0$, then the distribution of firms is rectangular. The hitch in this case is that this $f_t'(p_i)$ is undefined at the end-points of the distribution. The only remaining possibility, then, is that π' be zero, or more explicitly:

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1. All firms charge a single price, i.e. the price distribution is degenerate.
2. At this price, price equals marginal cost.

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$$\frac{dC}{dq_i} = p_i \left[1 - \frac{1}{\dfrac{\gamma(\tilde{F}(p_i))\ p_i}{\int_{p_i}^{\infty} \gamma(\tilde{F}(p))dp}} \right]$$

where

$$\frac{\gamma(\tilde{F}(p_i))\ p_i}{\int_{p_i}^{\infty} \gamma(\tilde{F}(p))dp} \qquad (3.1.2a)$$

is the elasticity of demand of the firm's demand curve.[2)]

A necessary condition for fulfilling condition 2 above, is that (3.1.2a) approach infinity when the variance of the

---

[1)] *For simplicity we disregard the constant* $\frac{k}{m}$.

[2)] *Note that* $\int_{p_i}^{\infty} \gamma(\tilde{F}(p))\ dp$ *is the demand curve and* $\gamma(\tilde{F}(p_i))$ *is the price derivative of this.*

distribution of firms approaches zero.[1)]

Let us see if this is possible. We know the relations between $f(p)$, $F(p)$ and $\tilde{F}(p)$. $F(p)$ is the integral of $f(p)$, while $\tilde{F}(p)$ is the integral of $F(p)$. $f(p)$ is a density function. If p_1 and p_2 are the lower and upper end-points of $f(p)$, then $F(p)$ increases from zero at p_1 to unity at p_2, and is equal to unity above p_2. $\tilde{F}(p)$ then increases in the interval $[p_1, p_2]$ with a slope of less than one. Above p_2 the slope of $\tilde{F}(p)$ is equal to one. Then we see that if the variance approaches zero, i.e. p_1 and p_2 approach a price $\bar{p}$ between them, $\tilde{F}(p)$ will approach a straight line with unity slope starting from $\bar{p}$. Then, in this case, we have

$$\tilde{F}(p_i) = p_i - \bar{p}.$$

Remember that $\tilde{F}(\bar{R}) = \bar{c}$, i.e. $\tilde{F}(\bar{R})$ is the search cost for an individual with reservation price $\bar{R}$. Then, in this case,

$$p_i - \bar{p} = c_i$$

or

$$p_i = \bar{p} + c_i \ ;$$

i.e. an individual with search cost c_i has the reservation price $\bar{p} + c_i$, given that the prices all firms charge are concentrated about $\bar{p}$.[2)]

We can rewrite the expression for the elasticity (3.1.2a) as

$$\frac{\gamma(p_i - \bar{p})\ p_i}{\int_{p_i}^{\infty} \gamma(p - \bar{p})\ dp} \qquad (3.1.2b)$$

Let $c_1 > 0$ be the lowest of all the search costs registered among the consumers. Consider a firm charging p_i.

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1) *Note that condition 1 is a restriction in this case. More specifically we put the question: "If 1 is the case, is it then possible that 2 can be fulfilled?"*

2) *Note that in the present formulation all individuals demand exactly one unit of the commodity, which thus is regarded as indivisible.*

If $p_i < \bar{p} + c_i$, then the numerator in (3.1.2b) equals zero and the denominator equals one. $p_i < \bar{p} + c_1$ is then fulfilled for all firms charging $\bar{p}$ if $c_1 > 0$. Then, we see that if all firms charge the same price, and all consumers have positive search costs, the elasticity of the firm's demand curve cannot approach infinity. Hence $\pi' \neq 0$ if $\bar{p}$ equals marginal cost, and thereby $\dot{f} \neq 0$. We thus find that the competitive solution cannot be an equilibrium.

V.3.2. Non-competitive equilibrium

If there can be no competitive equilibrium in this model, can there be any other kind of equilibrium? Let us first see if there is any other equilibrium with a degenerate price distribution. Let us relax the assumption that the good is indivisible. Denoting the individual demand curves by $d(p)$, and assuming them to be identical for all consumers, the demand curve facing a firm charging p_i is

$$q(p_i) = \frac{k}{m} d(p_i) \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right) dp$$

The profit function is then

$$\pi(p_i) = p_i \frac{k}{m} d(p_i) \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right) dp - C(p_i)$$

so that

$$\frac{d\pi}{dp_i} = \frac{k}{m} \left\{ \left[p_i \frac{dd}{dp_i} + d(p_i) \right] \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right) dp - \right.$$

$$- p_i\, d(p_i)\, \gamma\left(\tilde{F}(p_i)\right) + \frac{dC}{dq_i} \left[d(p_i)\, \gamma\left(\tilde{F}(p_i)\right) - \right.$$

$$\left. \left. - \frac{dd}{dp_i} \int_{p_i}^{\infty} \gamma\left(\tilde{F}(p)\right) dp \right] \right\}$$

As has been shown earlier $\int_{p_i}^{\infty} \gamma\left((\tilde{F}(p)\right) dp = 1$ and $\gamma\left(\tilde{F}(p_i)\right) = 0$ if all consumers have search costs greater than zero and all firms charge the same price. Then

$$\frac{d\pi}{dp_i} = \frac{k}{m}\left[d(p_i) + \frac{dd}{dp_i}\left(p_i - \frac{dC}{dq_i}\right)\right] \qquad (3.2.1)$$

A price that will make $\frac{d\pi}{dp_i}$ equal to zero, and thus $\dot{f} = 0$, is then

$$p_i = \frac{dC}{dq_i}\frac{e_{ki}}{e_{ki} + 1}, \qquad (3.2.2)$$

where e_{ki} is the elasticity of the individual demand curve $d(p)$ at p_i.

Thus, there exists a price that will be an equilibrium price for a degenerate price distribution. The price which fulfills the equilibrium condition is the monopoly price with respect to the individual demand curves.[1)]

The similarity to the solution of monopolistic competition is appealing. There is, however, one important difference. In monopolistic competition the elasticity in the "mark-up factor" between price and marginal cost is determined from both the individual elasticity and the flow of customers among firms with similar products. Thus the elasticity is normally fairly large and the deviation between price and marginal cost thereby small. In the present model the elasticity is the same as a pure monpolist would face. If the individual demand curves are iso-elastic and marginal cost is constant, the equilibrium price in the atomistic market of the present model would be the same as the price charged by a profit-maximizing monopolist controlling the whole market.

We can explain this equilibrium situation in the following way. If one firm tries to increase its price a little in a situation where all other firms charge the same price, it

1) *This is the same result as in Diamond (1971).*

will decrease its sales only by the amount that corresponds to the decreased demand from the same number (on average) of consumers it had when it charged the same price as other firms. The reason is that consumers who find this firm in their first search step will not search further if the difference in price relative to other firms does not exceed the search cost. If instead the firm lowers its price slightly it will face the same elasticity, since no consumer buying elsewhere will invest in searching for the firm, especially when the probability of finding it is almost zero. Only those who by chance find the firm in their first search step will buy from it. They will however buy a larger amount corresponding to their demand curves. Marginal revenue at a price p_i is then $p_i(1 + \frac{1}{e_{ki}})$, where e_{ki} is the individual price elasticity at p_i.

V.3.3. Equilibrium with price dispersion

We have been able to show that *if* the price distribution degenerates to a single price, this price has to be the monopoly price for the market to be in equilibrium. We now ask whether there exists an equilibrium with price dispersion. More specifically we ask if there is a distribution $f(p)$ such that, at given distribution of search costs $\gamma(c)$ and given cost function $C(q)$, the differential equation $\dot{f}(p_i, t)$ equals zero for all prices. The condition is thus that $\pi'(p_i)$ (equation 2.4.3b) equal zero for all prices, when the variance at $f(p)$ is positive.

Let $\int_{p_i}^{\infty} \gamma(\tilde{F}(p))\, dp$ be denoted by $q(p_i)$ and $\gamma(\tilde{F}(p_i))$ by $\frac{dq}{dp_i}$.

Further let us assume constant marginal cost: $\frac{dc}{dq} = mc =$ const. Condition (2.4.3b) then becomes

$$q(p_i) - \frac{dq}{dp_i}\left[mc - p_i\right] = 0 \qquad (3.3.1)$$

Rearranging terms and solving for the elasticity of demand we get

$$e = \frac{dq}{dp_i}\frac{p_i}{q} = \frac{p_i}{mc - p_i}, \qquad (3.3.2)$$

which shows the change in elasticity necessary for (3.3.1) always to be fulfilled.

Examining (3.3.2) we see that

$$\lim_{p \to mc^+} e = -\infty$$

and

$$\lim_{p \to \infty} e = -1$$

and

$$\frac{de}{dp_i} = \frac{mc}{(mc - p_i)^2} > 0 \text{ for all } p_i \neq mc.$$

If $p < mc$ the elasticity must be positive except when $p_i < 0$. Then it follows that only $p_i > mc$ is economically meaningful.

Let us solve the differential equation (3.3.1). It can be written

$$dq(p_i - mc) + q\, dp_i = 0$$

Dividing by $q(p_i - mc)$ we get

$$\frac{dq}{q} + \frac{dp_i}{p_i - mc} = 0$$

The solution to this expression is

$$\int \frac{1}{q}\, dq + \int \frac{1}{p_i - mc}\, dp_i = A,$$

i.e.

$$\log q = A - \int \frac{1}{p_i - mc}\, dp_i.$$

Then we have

$$q(p_i) = \exp\left[A - \int \frac{1}{p_i - mc}\, dp_i\right].^{1)}$$

Putting $e^A \equiv B$ we get

$$q(p_i) = B \exp\left[- \int \frac{1}{p_i - mc}\, dp_i\right],$$

and so

$$q(p_i) = B \exp\left[- \log\,(p_i - mc)\right] = \frac{B}{p_i - mc}\,, \qquad (3.3.3)$$

which is the shape of the demand curve, that fulfills the condition that the profit derivative be zero at all prices, when marginal cost is constant.

Now it remains to analyze which combinations of distribution of firms and distribution of search costs will give raise to a demand curve like that of (3.3.3). Reinserting $\int_{p_i}^{\infty} \gamma(\tilde{F}(p))\, dp$ for $q(p_i)$ we get

$$\int_{p_i}^{\infty} \gamma(\tilde{F}(p))\, dp = \frac{B}{p_i - mc}. \qquad (3.3.4)$$

Remember that $\tilde{F}(p) = \int_0^p F(s)\, ds$ and $F(p) = \int_0^p f(s)\, ds$. Equation (3.3.4) is then the implicit form of the relation between a search-cost distribution $\gamma(c)$ and a distribution of firms that will fulfill the condition that the profit derivative be equal to zero at all prices and consequently make the differential equation (2.4.4) equal to zero at all prices.

1) *We use the common notation* $\exp\,[f(x)]$ *for* $e^{f(x)}$.

V.3.4. Necessary and sufficient condition

We are now in position to examine the condition for the existence of a price dispersion equilibrium in a search market where consumers search in accordance with optimal stopping rules. The problem is the following: Given a density function $\gamma : R \to R$ and two positive constants B and mc, search a continuous probability distribution, with a continuous density function f, on (mc,∞) such that

$$\int_p^\infty \gamma(\tilde{F}(s)) \;\; ds = \frac{B}{p - mc}, \qquad p > mc$$

where $\tilde{F}$ is given by:

$$F(p) = \int_0^p f(s)\, ds$$

$$\tilde{F}(p) = \int_0^p F(s)\, ds$$

The question is: for which functions γ is this solvable?

Proposition: A necessary and sufficient condition on γ for the problem above to have a solution is the following:

i) γ is defined on $(0, \infty)$,

$\gamma \in C^2$, i.e. γ twice differentiable,

$\gamma' < 0$,

$\gamma'' > 0$,

$\gamma(c) \to 0$ when $c \to \infty$,

$\gamma(c) \to \infty$ when $c \to 0+$

ii) $\dfrac{\gamma^{\frac{3}{2}}(c)}{\gamma'(c)}$ is decreasing.

iii) $$\lim_{c\to\infty} \frac{\gamma(c)^{\frac{3}{2}}}{\gamma'(c)} = -\frac{\sqrt{B}}{2}$$

iv) $$\lim_{c\to 0+} \frac{\gamma(c)^{\frac{3}{2}}}{\gamma'(c)} = 0.$$

Proof for the necessary condition:

$$\int_p^\infty \gamma(\tilde{F}(s))\, ds = \frac{B}{p-mc}, \qquad p > mc$$

$$\Rightarrow$$

$$\gamma(\tilde{F}(p)) = \frac{B}{(p-mc)^2}, \qquad p > mc$$

$\tilde{F}(p)$ is increasing and $\tilde{F}((mc,\infty)) = (0,\infty)$ so that γ *has to be defined on* $(0,\infty)$. Further γ has to be strictly decreasing because $\frac{B}{(p-mc)^2}$ is. By letting $p \to mc+$ and $p \to \infty$ respectively we see that

$$\gamma(c) \to 0 \text{ when } c \to \infty$$

and

$$\gamma(c) \to \infty \text{ when } c \to 0+$$

Put $\Gamma = \gamma^{-1}$. Then we have

$$\tilde{F}(p) = \Gamma\left(\frac{B}{(p-mc)^2}\right),$$

or

$$\Gamma(c) = \tilde{F}\left(\sqrt{\frac{B}{c}} + mc\right)$$

from which we see that $\Gamma \in C^2$, $\Gamma' < 0$, and $\Gamma'' > 0$. From this we get $\underline{\gamma \in C^2}$, $\underline{\gamma' < 0}$, $\underline{\gamma'' > 0}$ and besides:

$$\Gamma'(c) = \frac{-\sqrt{B}\, F\left(\sqrt{\frac{B}{c}} + mc\right)}{2c^{\frac{3}{2}}}$$

or

$$c^{\frac{3}{2}}\, \Gamma'(c) = \frac{-\sqrt{B}}{2}\, F\left(\sqrt{\frac{B}{c}} + mc\right)$$

or

$$\frac{\gamma(c)^{\frac{3}{2}}}{\gamma'(c)} = \frac{-\sqrt{B}}{2}\, F\left(\sqrt{\frac{B}{\gamma(c)}} + mc\right).$$

Now we know that $\gamma(c)$ is strictly decreasing from ∞ to 0 when c goes from 0+ to ∞, and since $F(p)$ increases from 0 to 1 when p goes from mc to ∞, we obtain from this the properties ii), iii), and iv).

Thus the proof of the necessary condition is complete.

Sufficiency:

Because of the presumptions in i) there exists $\Gamma \overset{\text{def.}}{=\!=} \gamma^{-1} \in C^2$, and Γ is defined on $(0,\infty) = \gamma(0,\infty)$. Put

$$F(p) \overset{\text{def.}}{=\!=} \begin{cases} \Gamma'\left(\dfrac{B}{(p-mc)^2}\right) \dfrac{-2B}{(p-mc)^3}, & p \geq mc \\ 0 & p < mc \end{cases}$$

Then it is obvious that $F \in C^1$ for $p > mc$. A reformulation gives:

$$F\left(\sqrt{\frac{B}{\gamma(c)}} + mc\right) = \frac{-2\,\gamma(c)^{\frac{3}{2}}}{\sqrt{B}\,\gamma'(c)}, \qquad c > 0$$

The presumtpions on γ then imply that F is increasing, $F(mc+) = 0$ and $\lim_{p\to\infty} F(p) = 1$.

Consequently $f = F'$ solves the problem, for we have

$$F(p) = \frac{d}{dp}\,\Gamma\left(\frac{B}{(p-mc)^2}\right), \qquad p > mc$$

i.e.

$$\tilde{F}(p) = \Gamma\left(\frac{B}{(p-mc)^2}\right), \qquad p > mc$$

(note that $\Gamma(s) \to 0$ when $s \to \infty$)

i.e.

$$\gamma\left(\tilde{F}(p)\right) = \frac{B}{(p-mc)^2}, \qquad p > mc$$

which is equivalent to

$$\int_p^\infty \gamma\left(\tilde{F}(s)\right)\, ds = \frac{B}{p-mc}, \qquad p > mc$$

(note that $\gamma\left(\tilde{F}(p)\right) \to 0$ when $p \to \infty$)

Thus the proof is complete.

An example:

In order to illustrate the equilibrium, we now show an example.

The function

$$\gamma(c) = \frac{B}{\left(\frac{c}{2} + \sqrt{c + \frac{c^2}{4}}\right)^2}$$

satisfies the conditions i) - iv).

The equilibrium density function f is thus:

$$f(p) = \frac{2}{(p-mc+1)^3}, \qquad p \in (mc, \infty)$$

because:

$$F(p) = 1 - \frac{1}{(p-mc+1)^2}, \qquad p > mc$$

$$\tilde{F}(p) = p-mc-1 + \frac{1}{p-mc+1}, \qquad p > mc$$

$$\tilde{F}^{-1}(c) = mc + \frac{c}{2} + \sqrt{c + \frac{c^2}{4}}, \qquad c > 0$$

Then

$$\gamma(c) = \frac{B}{\left(\tilde{F}^{-1}(c)-mc\right)^2}, \qquad c > 0$$

and consequently

$$\gamma\left(\tilde{F}(p)\right) = \frac{B}{(p-mc)^2}, \qquad p > mc$$

i.e.

$$\int_p^\infty \gamma\left(\tilde{F}(s)\right)\, ds = \frac{B}{p-mc} \qquad p > mc$$

The results are thus that there are some restrictions on the shape of the distribution of search costs for a search market of the specified kind for an equilibrium with price dispersion to exist. The density function for the search cost distribution must be decreasing and convex. This is however not a particularly restrictive condition. As was mentioned earlier the most important search cost for an individual is the time cost, which implies that the search cost distribution will be closely associated with the income distribution. Most studies of income distribution show density functions which are decreasing and convex, for instance the Pareto distribution and the exponential distribution.

The implications of condition ii) are not immediately apparent. The condition requires that $\frac{\gamma(c)^{3/2}}{\gamma'(c)}$ must be a decreasing function. $\gamma(c)$ is always positive and decreasing, while $\gamma'(c)$ is always negative and increasing. Thus condition ii) says that $\gamma'(c)$ has to increase sufficiently fast when c increases. It seems that a fairly wide range of density functions fulfill this condition.

The conditions iii) and iv) concern the necessary shape of the search cost density function for extreme high and low search costs. These conditions seem quite restrictive. However, they arise from the equilibrium condition that *any* price above marginal cost should yield the same expected profit. A firm charging a price very much above marginal cost will then of course earn an extremely high profit per unit sold. In order to hold expected profit per period down (i.e. at the same level as at more "normal" prices) the probability of selling a unit must be very low. Therefore the probability that there exists a customer with a search cost high enough to buy at this high price must be very small. Thus the combined probability of firm charging an extremely high price and also of a consumer with a still higher reservation price is very small. In the normal case there are no firms actually charging extremely high prices, but there exists a small probability that there might be such firms. Although the condition for the extreme tail of the search cost distribution is quite restrictive, it is of little practical relevance, since the probability of finding a consumer with such high search costs is almost equal to zero.

In the "normal" price region, i.e. prices from a bit above marginal cost and up to less than "extreme" levels, there are no especially restrictive conditions for the shape of the search cost distribution. $\gamma(c)$ has to be decreasing and convex and furthermore $\frac{\gamma(c)^{3/2}}{\gamma'(c)}$ must be decreasing. There are no reasons to believe that a "true" search cost distribution does not fulfill these conditions.

V.3.5. Conclusions

In this chapter the main purpose has been to predict what prices will appear in a market for a homogeneous commodity when the agents of both sides of the market have incomplete information but still have the opportunity to collect further information which however is costly.

Perhaps the most important difference from a perfectly competitive market is the absence of the auctioneer. The prices are set, as in real life, by the sellers of the market in accordance with the information they have at a certain moment. Another important difference is that there exists trade out of equilibrium. It is actually the trade in disequilibrium which is the source of information for the firms' price setting strategy.

In the dynamic adjustment model, firms change prices in a direction they think will increase profit, based on demand information obtained from the prices they actually charge. The demand is at the same time derived from the consumers' search for low price firms. The consumers search sequentially from the true price distribution constituted by the firms' price setting, and are assumed to search in accordance with a reservation price rule. That means that a consumer will stop searching when the expected gain from further search is less than the marginal cost of search.

The chain of behavior is the following:

Firm behavior is derived on basis of the outcome of actual sales at different prices, which depends on consumer behavior, that is on consumer search processes, which in turn depend on the price distribution, i.e. the firms' price setting. From this it follows that the distribution of prices is endogenously determined from the interaction of firms and consumers in the market.

The development of the price distribution in disequilibrium situations can be described by a partial differential equation. Because of the complexity of this differential equation very little qualitative can be said about the market in disequilibrium from a general point of view. In order to get answers about the development of prices out of equilibrium we need further specifications about the parameters of the model.

The analysis in the rest of the chapter is concentrated on the equilibrium of the market. The equilibrium condition for a dynamic model is simply a situation in which the time derivatives of all endogenous variables are equal to zero. However, the equilibrium condition commonly used in neoclassical theory, the Nash condition, is in general a more demanding condition. An equilibrium solution to a model then must have the property that the behavior of each agent is optimal given the environment specified. In other words, no one agent can in equilibrium improve his situation by changing the value of any parameter he controls.

In the present model an equilibrium of the dynamic model will automatically fulfill the Nash condition because the behavior of the agents is optimal in all situations.

CHAPTER VI:

Stability of Price Dispersion Equilibrium

VI. *STABILITY OF PRICE DISPERSION EQUILIBRIUM*

VI.1. *Introduction*

In the previous chapter it was shown that a situation with price dispersion was possible as an equilibrium situation. However, we did not examine whether or not this price dispersion equilibrium was stable. If a certain distribution of firms f(p) related to the actual search cost distribution $\gamma(c)$ according to (3.3.4) in chapter V occurred, then this distribution would repeat itself period after period. But if this situation is disturbed by some exogenous shock, nothing guarantees that F(p) will go back to the equilibrium shape. This is a question of the stability of equilibrium. The ordinary way of analyzing the stability properties of a system to study the effect of a small deviation from equilibrium. In this chapter we shall use a simpler version of the search model to examine its stability properties.

VI.2. *The Model*

In the simpler model, we assume that in a market for a homogeneous consumer durable there exist only two possible prices p_1 and p_2. Any firm in this market is bound to charge either p_1 or p_2. No other price is permitted, a very restrictive assumption, to be sure. However, we intend this only as a simplification of the more reasonable assumption that there exists only a finite number of attainable prices, which in reality is actually the case. The smallest technically possible price difference is 1 cent. If there exists an upper and lower bound on price then there can exist only a finite number of attainable prices. If the number of prices which can be charged is finite, then the simplification to a two price case is not more restrictive than that in the theory of international trade from n countries to two countries.

Second, we assume that consumers are divided into two groups, those with search cost c_1 and those with search cost c_2. We can interpret this as discussed in previous sections, as though consumers are divided into two income classes, with time costs being the most important search cost.

As in the general model of the previous section we adopt the following assumptions.

1. The market is an atomistic market, i.e. the firms are so small relative to the market that their own behavior does not affect market conditions. Individual consumers are in the same way small relative to the market.

2. Consumers are rational in the sense that they seek to maximize their expected utility at every instant in time.

3. In this primary formulation, consumers are assumed to know the exact distribution of firms over prices, i.e. p_1 and p_2.

4. The market is a market for a consumer durable. A consumer buys exactly one unit and then leaves the market. There are always new generations of consumers, replacing those who have made their purchases and left the market.

We use the following notation: two prices p_1 and p_2; two search costs c_1 and c_2. The frequencies of firms at the two prices at time t are $f_t(p_1)$ and $f_t(p_2)$. These will be denoted by f_{t1} and f_{t2} for simplicity or just f_1 and f_2 when time does not enter into the model.

The frequencies of consumers at the two search costs are $\gamma(c_1)$ and $\gamma(c_2)$ or simply γ_1 and γ_2. Further we have $f_1 + f_2 = 1$ and $\gamma_1 + \gamma_2 = 1$.

The consumers collect price offers from the firms by searching at one firm at a time. They know the frequency distribution but they do not know which firm charges which price. Each search step has a cost of c_1 or c_2 depending on which group the consumer belongs to. In the previous chapter we showed that the expected gain from one more search step given the best previous quotation p_i was $\int_0^{p_i} f(p)(p_i-p)\,dp$. In this case, with just two prices, the expected gain from one more search step is $(p_2-p_1)\ f_1$ if p_2 is the best current offer. If p_1 has been found already then the expected gain from one more search step is of course zero. If a consumer has found a firm with the price p_2, then it is always optimal to continue searching if the search cost is lower than $(p_2-p_1)\ f_1$. In this case we then assume that the consumer has the opportunity to reach decision in between each search step.

The marginal cost of search, i.e. the cost of taking an additional search step, might vary during the search process. It is most natural for the search cost to increase. There are at least two reasons for this: 1) Search costs will take more and more from the budget. If the consumer has decreasing marginal utility with respect to all other commodities (e.g. a composite commodity), and the marginal utility function is convex,[1] then the cost measured in utility, even with constant money cost, will increase; 2) if the consumer is getting impatient with searching, then this will increase the disutility of the time spent in searching.

Nevertheless, we assume here constant marginal cost of search. From the formula for the expected gain from search above we have for a consumer with search cost c the following decision rule:

$$\begin{array}{ll} \text{if } (p_2-p_1)\ f_1 > c & \text{keep searching,} \\ \text{if } (p_2-p_1)\ f_1 \leq c & \text{stop and accept } p_2, \end{array}$$

if p_2 is the price offer found. If p_1 is found it is always optimal to stop.

Now it is easy to derive the stopping price distribution. By stopping price distribution we mean the frequency distribution of actual purchases.

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1) *Convexity implies that the consumer has greater risk aversion with lower income.*

If $c_2 > (p_2 - p_1) f_1$ then consumers with search cost c_2, those in γ_2, will always accept the very first offer attained. These consumers (with search cost c_2) will then be divided into p_1-firms and p_2-firms in proportions to f_1 and f_2.

Consumers for whom $c_2 < (p_2 - p_1) f_1$ will always search until they find a firm charging price p_1. Of course the same holds for consumers with search cost c_1.

These results give us three cases:

1. $c_1 > (p_2 - p_1) f_1$
   $c_2 > (p_2 - p_1) f_1$

2. $c_1 < (p_2 - p_1) f_1$
   $c_2 > (p_2 - p_1) f_1$

3. $c_1 < (p_2 - p_1) f_1$
   $c_2 < (p_2 - p_1) f_1$

It is obvious that consumers described by the third case will always end up at the firms charging the lower price p_1.

The first case implies that consumers in this group accept the very first offer and thereby distribute themselves over p_1 and p_2 according to f_1 and f_2.

In the second case the consumers with the search cost c_1 will all search until they find the price p_1, but those with search cost c_2 stop at their very first offer.

The frequency distribution of stopping prices will in these three cases be:

Table 1. Stopping price distribution

	p_1	p_2
case 1	f_1	f_2
case 2	$\gamma_1 + f_1\gamma_2$	$1-(\gamma_1 + f_1\gamma_2) = f_2\gamma_2$
case 3	1	0

We are now in position to analyze the situation of the firms. If k is the total number of consumer per firm per period, then the expected number of consumers per firm per period at the two prices in the three cases are:

Table 2. Consumers per firm

	p_1	p_2
case 1	k	k
case 2	$k(\frac{\gamma_1}{f_1} + \gamma_2)$	$k\gamma_2$
case 3	$\frac{k}{f_1}$	0

If the consumers buy exactly one unit irrespective of whether the price is p_1 or p_2, then the table above shows the demand too. Consumer demand might, however, have an elasticity different from zero. If a consumer pays p_2 he might choose a smaller TV-set or automobile than for p_1. If this is expressed by an individual demand curve d(p), and if all consumers have the same demand curve, then table 2 is instead:

Table 3. Demand

	p	p_2
case 1	$K\ d(p_1)$	$K\ d(p_2)$
case 2	$K\ d(p_1)(\frac{\gamma_1}{f_1} + \gamma_2)$	$K\ d(p_2)\gamma_2$
case 3	$\frac{K\ d(p_1)}{f_1}$	0

We can now calculate the expected profit. Given a cost function C(q), we have the following profits in the different cases:

Table 4. Profits

	p_1	p_2
case 1	$p_1Kd(p_1) - C(kd(p_1))$	$p_2Kd(p_1) - C(kd(p_2))$
case 2	$p_1Kd(p_1)\ (\frac{\gamma_1}{f_1} + \gamma_2) - C(kd(p_1))(\frac{\gamma_1}{f_1} + \gamma_2)$	$p_2Kd(p_2)\ \gamma_2 - C(kd(p_2)\ \gamma_2)$
case 3	$\frac{pKd(p_1)}{f_1} - C(\frac{Kd(p_1)}{f_1})$	$C(0)$

We are now ready to examine the equilibrium properties of this simplified system.

VI.3. Equilibrium

The concept of equilibrium has been discussed in more detail in earlier chapters. We shall now examine if this market has a Nash equilibrium, by which we mean a situation in which no single agent in the market can improve his situation (utility or profit) by means of changing any parameter under his control.

Where consumers are concerned, the only parameter under their control is the number of search steps. Since we assume that they search according to an optimal stopping rule, they obviously cannot improve their situation, and therefore, for consumers, the Nash conditions are automatically fulfilled. It remains only to show that firms cannot obtain higher profits at any other values of the parameters under their control.

The only parameter the firms control is the price parameter. Then the profit must not be higher at p_2 than at p_1 if there are firms at both p_1 and p_2. Further, the profit must not be higher at p_1 than at p_2. Hence, the equilibrium condition is $\pi(p_1) = \pi(p_2)$, where π is profit.

Imposing this equilibrium condition on the present model we get:

$$\text{case 1} \quad p_1 Kd(p_1) - C\big(kd(p_1)\big) = p_2 Kd(p_2) - C\big(kd(p_2)\big) \qquad (3.1.1)$$

$$\text{case 2} \quad p_1 Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - C\left(kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right)\right) =$$

$$= p_2 Kd(p_2)\ \gamma_2 - C\big(kd(p_2)\ \gamma_2\big) \qquad (3.1.2)$$

$$\text{case 3} \quad \frac{p_1 Kd(p_1)}{f_1} - C\left(\frac{Kd(p_1)}{f_1}\right) = -\ C(0) \qquad (3.1.3)$$

Now let us determine if there exists a distribution of firms, f_1, f_2, over the two prices which fulfill this condition. Let us first examine the case with a linear cost function $a_1 q + a_2$ (where a_1 is marginal costs) and zero elastic individual demand, i.e. $d(p) = 1$. Note that we are now considering a short run equilibrium, in which profit can be at any level. Long run equilibrium is maintained via entry and exit. Consequently, in the long run K is adjusted so that profits become zero.

In case 1 we have the equilibrium condition:

$$p_1 K - a_1 K - a_2 = p_2 K - a_1 K - a_2$$

which is fulfilled if $p_1 = p_2$.

That is, if the difference between the two prices is so small and/or the search costs so high that no consumer searches more than once, the equilibrium is necessarily a single price equilibrium. Note that this is a necessary condition but not a sufficient one. Sufficiency requires that this price gives maximum profit. If $d'(p) \equiv 0$, there exists no such price, which is of no interest because it arises from a simplifying assumption soon to be removed. The reason why there must be a single price equilibrium when all consumers search only once is that each firm always gets its fair share of the consumers (K) independently of the price charged. Then higher prices will always give higher revenues for the same total cost.

If we have d(p) with elasticity different from zero we have the following equilibrium condition:

$$p_1 Kd(p_1) - a_1 Kd(p_1) - a_2 =$$
$$= p_2 Kd(p_2) - a_1 Kd(p_2) - a_2 \qquad (3.2)$$

Rearranging the terms we get:

$$a_1 = p_1 \left(\frac{e+1}{e} + \frac{p_2 - p_1}{p_1}\right) \qquad (3.3)$$

where

$$e = \frac{\left(d(p_2) - d(p_1)\right) p_1}{(p_2 - p_1)\ d(p_1)}$$

Further we have as a contingency

$$(p_2 - p_1)\ f_1 < c_1 < c_2 \qquad (3.4)$$

From this we see that $p_1 = p_2 = p^*$ is an equilibrium if and only if

$$p^* = mc\ \frac{e}{e+1} \qquad (3.5)$$

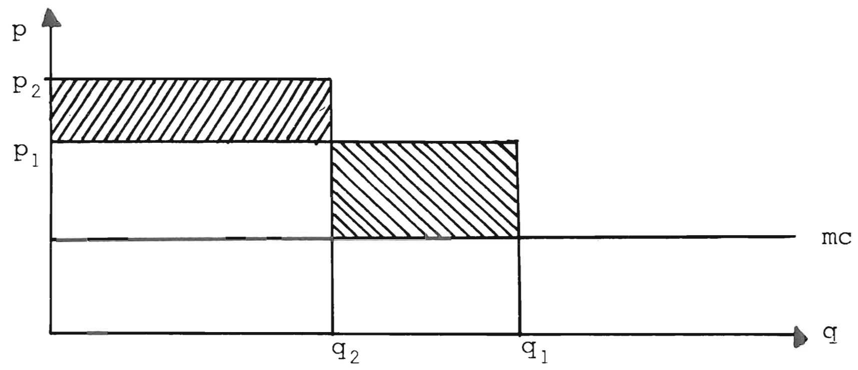
i.e. if this price is the monopoly price.

To get an equilibrium when p_1 differs from p_2 the necessary conditions are first that the deviation is smaller than $\frac{c_1}{f_1}$, where c_1 is the smaller search cost. Secondly d(p) must have such a shape that the change in demand exactly compensates the change in price with respect to profit. Rearranging terms in (3.2) we get

$$-\left(d(p_2) - d(p_1)\right)(p_1 - mc) = d(p_2)(p_2 - p_1) \qquad (3.6)$$

which says that the shaded areas in figure 1 must be equal. The necessary shape of d(p) fulfilling this condition is analyzed in greater detail in the previous section.

Figure 1.



Further we see that f_1 and f_2 do not enter into the equilibrium condition. This means that if the equilibrium condition is fulfilled it does not matter how many firms there are charging p_1 and p_2 respectively. The reason for this is that consumers never search more than once and therefore all firms get their fair share. However, the size of f_1 and f_2 will affect the validity of the no-search case. Recall the no-search condition $(p_2-p_1)\ f_1 < c_1 < c_2$, the greater f_1 the smaller the maximal possible deviation between p_1 and p_2. This is because the expected gain from search is dependent on how probable it is to find a low price firm. This probability is f_1.

The equilibrium analyzed is a short run equilibrium. In the long run profits have to be non-negative or, if possible, zero. In case 1 profits are

$$\pi = p_1Kd(p_1) - a_1Kd(p_1) - a_2 = p_2Kd(p_2) - aKd(p_2) - a_2 \quad (3.6)$$

The condition for long run equilibrium $\pi = 0$ gives

$$K = \frac{a_2}{d(p_1)(p_1-a_1)} = \frac{a_2}{d(p_2)(p_2-a_1)} \quad (3.7)$$

K is the ratio of consumers per firm. If the number of firms is m and the number of consumers per period is ℓ then $K = \frac{\ell}{m}$. Then we have

$$m = \frac{\ell d(p_1)(p_1-a_1)}{a_2} = \frac{\ell d(p_2)(p_2-a_1)}{a_2}$$

as the long-run equilibrium number of firms.

Let us next analyze case 3. The equilibrium condition is

$$\frac{p_1 Kd(p_1)}{f_1} - a_1 \frac{Kd(p_1)}{f_1} - a_2 = - a_2 \tag{3.8}$$

This yields

$$p_1 = a_1 = mc$$

The reason for this is that all consumers search until they find p_1. Thus the firms at p_1 will get all the demand and those at p_2 will get no demand at all. Firms at p_2 will have a loss of a_2. p_1 then must equal marginal cost to make the same loss for the firms at p_1 as the firms at p_2. This is the only short run equilibrium attainable in this case. It is obvious that this could never be a long run equilibrium. In this case the presumption was that $(p_2-p_1)\ f_1 > c_2 > c_1$. Without knowing the firm behavior we cannot say if the market will change to case one or two or just disappear.

Let us finally analyze the most interesting case, namely case 2. The equilibrium condition is

$$\begin{aligned} p_1 Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_1 Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_2 = \\ = p_2 Kd(p_2)\ \gamma_2 - a_1 Kd(p_2)\ \gamma_2 - a_2 \end{aligned} \tag{3.9}$$

which is

$$\frac{d(p_1)(\gamma_1 + f_1\gamma_2)}{d(p_2)\ \gamma_2} = \frac{p_2-mc}{p_1-mc} \tag{3.10}$$

which says that the rate of demand between p_1 and p_2 shall equal the inverse rate of marginal profit.[1] This rate of de-

---

1) *Note however that this is not marginal profit in the ordinary sense. It is actually the mark-up profit, the difference between price and marginal cost.*

mand is dependent on the frequency distribution of search costs γ, the frequency distribution of firms f and the shape of the individual demand curve d(p). Let us call the rate of marginal profit, which is constant, RMP, i.e.

$$\frac{p_2-mc}{p_1-mc} = RMP_{21} \tag{3.11}$$

Then we ask; what is the necessary distribution of firms fulfilling the equilibrium condition for a given search cost distribution γ_1, γ_2?

The answer is:

$$f_1 = \frac{\gamma_1 d(p_1)}{\gamma_2 (RMP_{21}-1)\ d(p_2)} \tag{3.12}$$

$$f_2 = 1 - \frac{\gamma_1 d(p_1)}{\gamma_2 (RMP_{21}-1)\ d(p_2)} \tag{3.13}$$

If the demand is completely inelastic then the equilibrium distribution is simply

$$f_1 = \frac{\gamma_1}{\gamma_2 (RMP_{21}-1)} \tag{3.14}$$

$$f_2 = 1 - \frac{\gamma_1}{\gamma_2 (RMP_{21}-1)} \tag{3.15}$$

The search costs then, according to the presumption for case 2, have to fulfill

$$c_1 < (p_2-p_1)\ \frac{\gamma_1 d(p_1)}{\gamma_2 (RMP_{21}-1)\ d(p_2)} < c_2 \tag{3.16}$$

Because f is a frequency distribution we have the restriction

$$0 \leq f_1,\ f_2 \leq 1$$

which gives

$$0 \leq \frac{\gamma_1 d(p_1)}{\gamma_2 (RMP_{21}-1)\ d(p_2)} \leq 1 \qquad (3.17)$$

The condition is thus

$$\frac{p_2-mc}{p_1-mc} = RMP_{21} \geq \frac{\gamma_2\big(d(p_2) - d(p_1)\big) + d(p_1)}{\gamma_2 d(p_2)}{}^{1)} \qquad (3.18)$$

In the zero-elastic case this condition is

$$\frac{p_2-mc}{p_1-mc} = RMP_{21} \geq \frac{1}{\gamma_2} \qquad (3.19)$$

A numerical example: If $\gamma_1 = 0{,}6$ and $\gamma_2 = 0{,}4$ we have

$$\frac{p_2-mc}{p_1-mc} \geq \frac{1}{0.4}$$

or

$$p_2 - mc \geq 2.5(p_1-mc)$$

If for instance (in the zero elastic case) $p_1 = 1.2mc$, p_2 must be greater than 1.5mc.

If the demand is unit elastic we must have instead $p_2 \geq 1.39mc$ if $p_1 = 1.2mc$.

Now let us look at long run equilibrium.

$$\begin{aligned} \pi &= p_1 K d(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_1 K d(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - \\ &- a_2 = p_2 K d(p_2)\ \gamma_2 - a_1 K d(p_2)\ \gamma_2 - a_2 = 0 \end{aligned} \qquad (3.20)$$

gives

$$K = \frac{a_2}{d(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right)(p_1-a_1)} = \frac{a_2}{d(p_2)\ \gamma_2 (p_2-a_1)} \qquad (3{:}21)$$

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1) *The left hand inequality is always valid because all the factors are greater than or equal to zero.*

$$K = \frac{\ell}{m}$$

gives

$$m = \frac{\ell d(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right)(p_1 - a_1)}{a_2} = \frac{\ell d(p_2)\gamma_2(p_2 - a_1)}{a_2} \tag{3.22}$$

as the long run equilibrium number of firms.

VI.4. *Stability of Equilibrium*

It was shown in the previous section that a search market with incomplete information and only two possible prices and two search costs may have an equilibrium fulfilling the Nash condition. There are only three possible search situations if the consumers follow optimal stopping rules in a sequential search process: 1) Both groups' reservation prices are above the two prices. Then no consumer will search more than once and thus stop at the first price quotation. 2) One group's reservation price is in between the two prices and the other's is above the highest. Then consumers in one group, the low search cost group, will search until they find a low price firm and the other, the high search cost group, will stop after one search step. 3) All consumers have search costs such that their reservation prices are in between the two prices charged. Then consumers will continue to search until they find a low price firm.

We have shown that case 3 can never be a long run equilibrium so long as f_1, $f_2 \neq 0$. Case 1 may have an equilibrium solution if the individual demand curve has exactly the necessary shape. The solution is then independent of the firm distribution. In case 2 we have shown that there is a distribution f fulfilling the Nash condition for a given search cost distribution γ and individual demand curve d(p).

We shall now concentrate our analysis on case 2. Case 3 is uninteresting because it cannot generate an equilibrium.

The only interesting thing with case 1 is that it leads to a single-price equilibrium.

Although we have shown that the Nash conditions for long-run equilibrium are fulfilled in case 1 and 2, nothing has been said about whether the equilibrium is stable in those cases. In this section we will analyze the stability properties of the equilibrium.

A necessary condition for an equilibrium to be stable is that the market returns to the equilibrium situation if it is forced out of equilibrium, for instance by a random shock. Let us note the equilibrium distribution of firms by f_1^* and f_2^*. A distribution which deviates from equilibrium is then for instance $f_1^* - \Delta f$, $f_2^* + \Delta f$. The random shock may be for example a changing by a number of firms of their prices from p_1 to p_2, sufficient to affect the market. The question is; does the market return to the equilibrium situation or not?

Let us first analyze case 2, in which one group of consumers keep searching until they find a low price firm. The other group searches only once. The profit for the low price firms is:

$$\pi_1 = p_1 Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_1 Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_2 \qquad (4.1)$$

The profit for the high price firms is:

$$\pi_2 = p_2 Kd(p_2)\ \gamma_2 - a_1 Kd(p_2)\ \gamma_2 - a_2 \qquad (4.2)$$

The stability of equilibrium then depends on the sign of $\frac{d\pi_1}{df_1}$ and $\frac{d\pi_2}{df_2}$.

$$\frac{d\pi_1}{df_1} = p_1 Kd(p_1)\left(\frac{f_1 \frac{d\gamma_1}{df_1} - \gamma_1}{(f_1)^2} + \frac{d\gamma_2}{df_1}\right) - $$
$$- a_1 Kd(p_1)\ \left(\frac{f_1 \frac{d\gamma_1}{df_1} - \gamma_1}{(f_1)^2} + \frac{d\gamma_2}{df_1}\right) \qquad (4.3)$$

$$\frac{d\pi_2}{df_2} = p_2 Kd(p_2) \frac{d\gamma_2}{df_2} - a_1 Kd(p_2) \frac{d\gamma_2}{df_2} \tag{4.4}$$

γ_1, γ_2 constant gives

$$\frac{d\pi_1}{df_1} = -\ (p_1 - a_1)\ Kd(p_1)\ \frac{\gamma_1}{(f_1)^2} \tag{4.5}$$

$$\frac{d\pi_2}{df_2} = 0 \tag{4.6}$$

We can see that $\frac{d\pi_1}{df_1}$ is always negative when $p_1 > mc$. *Thus we can draw the important conclusion that the equilibrium is stable.* To explain this, let us start from an equilibrium distribution f_1^*, f_2^*. A number of firms at p_1 increase their prices to p_2 giving rise to a new distribution $f_1^* - \Delta f$, $f_2^* + \Delta f$ which is not an equilibrium distribution. According to (4.5), profits will increase for firms at p_1 but remain unchanged at p_2. Thus $\bar{\pi}_1 > \bar{\pi}_2$ where we denote the profit at the distribution $f_1^* - \Delta f$, $f_2^* + \Delta f$ by $\bar{\pi}$. Thus it is profitable for any firm at p_2 to decrease price to p_1. Price cutting continues to be profitable until the original distribution f_1^*, f_2^* is realized.

This stability is realized because the original decline in the frequency of firms at p_1 causes profits to rise for remaining firms, while profits per firm at p_2 are unaffected. Now, it remains to explain why profits at p_1 change, but not profits at p_2. For this, we must look at the frequency distribution of stopping prices. This is $\gamma_1 + f_1\gamma_2$, $f_2\gamma_2$. A decrease of f_1 will then cause a decrease in the frequency of consumers but not at the same rate as the decrease in the frequency of firms. The firms at p_1 will still get all the consumers with search cost c_1 but these consumers will then be divided among fewer firms. The consumers with search cost c_2 who come to firms charging p_1 will diminish in number, but because they search only once, this decrease will be proportional to the decrease in f_1.

The consumers at p_2 all come from the high search cost group. Therefore, because they search only once, the number of consumer stopping at p_2 will increase exactly in proportion to the increase in number of firms, thus leaving demand per firm and profit unaffected.

Now let us examine the stability properties of the other cases. Case 3 has no equilibrium with non-negative profit, making further analysis unnecessary. We have

$$\frac{d\pi_1}{df_1} = \frac{-(p_1-a_1)\ Kd(p_1)}{(f_1)^2} \tag{4.7}$$

$$\frac{d\pi_2}{df_2} = 0 \tag{4.8}$$

$\frac{d\pi_1}{df_1}$ is negative if $p > mc$ but the equilibrium condition requires $p = mc$ in this case. Thus the equilibrium cannot be described as either stable or unstable. This is obvious from the analysis in the previous section. We showed that any distribution f will fulfill the equilibrium condition, then a change in f cannot affect the situation.

In case 1 we have

$$\frac{d\pi_1}{df_1} = 0 \tag{4.9}$$

$$\frac{d\pi_2}{df_2} = 0 \tag{4.10}$$

Again, the equilibrium is neither stable nor unstable. The reason is the same as in case 3 -- the equilibrium is independent of f.

VI.5. The Convergence to Equilibrium

Let us return to case 2. We showed that the equilibrium is stable, i.e. if the market is forced out of equilibrium a bit by some exogenous disturbance, then it will go back to the original equilibrium situation. But what happens if there is a great disturbance -- a disturbance such that the market is not just a little way from equilibrium but far from it? Stability is no longer the object of analysis but rather the disequilibrium properties of the model. One of the most important questions in disequilibrium analysis is: Which disequilibrium situations will end up at an equilibrium and which will not? Or, put another way, how far from equilibrium can the market be forced and still come back to it? It is obvious that we then need a specification of the behavior of firms. In other words we need a dynamic model for a disequilibrium analysis.

Consider now various possible disequilibrium situations and the direction of the forces in these cases.

Whether or not the market is in or out of equilibrium it must fall into one of the three following categories:

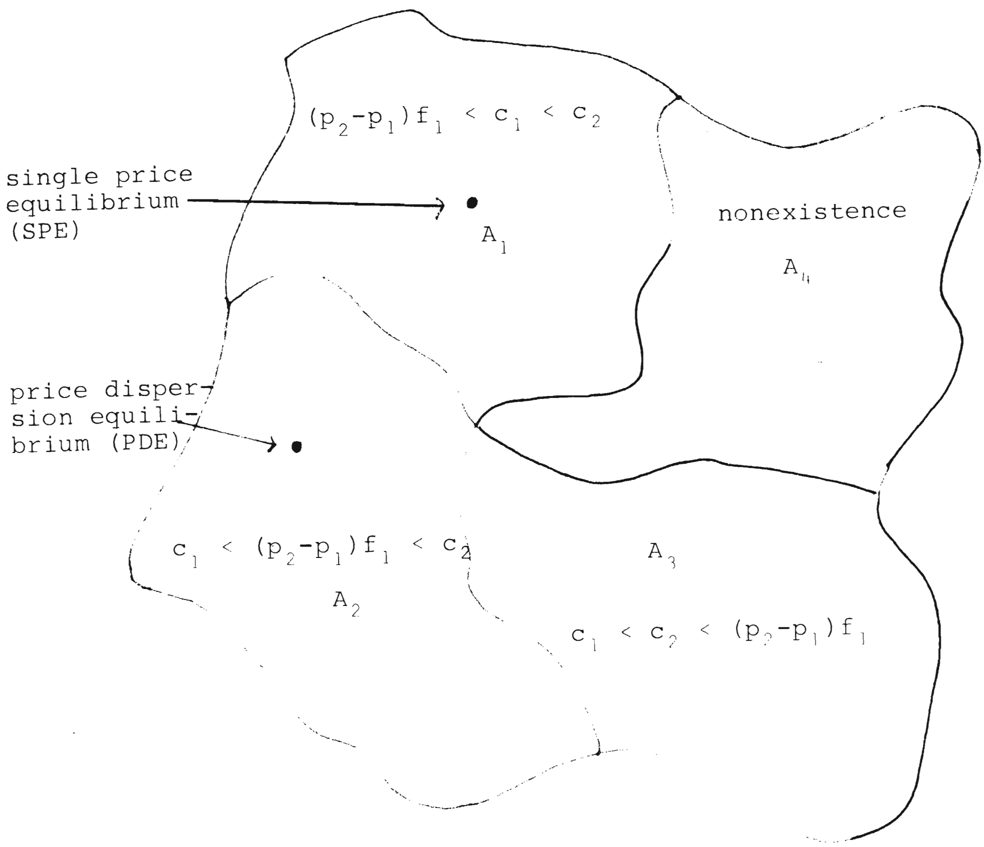
case 1	$(p_2-p_1)\ f_1 < c_1 < c_2$	(no search)
case 2	$c_1 < (p_2-p_1)\ f_1 < c_2$	(the low search cost group will search)
case 3	$c_1 < c_2 < (p_2-p_1)\ f_1$	(both groups will search)

There is a fourth case when nothing is produced at all; the market does not exist. There are two possible equilibria; one price dispersion equilibrium which follows from case 2 and one single price equilibrium which follows from case 1.[1)]

We illustrate the market in the diagram in figure 2.

1) *I disregard the price disperson equilibrium in case 1 because it is an uninteresting special case.*

Figure 2



Call the set A and the subsets A_1, A_2, A_3 and A_4. Single price equilibrium (SPE) is given by A_1 and price dispersion equilibrium (PDE) by A_2. Starting from an arbitrary point in A there are four possible developments; 1) Ending up at SPE, 2) Ending up at PDE, 3) Ending up in A_4, 4) The market will move around in disequilibrium forever.

Now we wish to derive the subsets of A which give rise to these four developments. Let us call these B_1, B_2, B_3 and B_4.

A situation that deviates slightly from PDE would, as shown, create forces that would tend to make the market go back to PDE. However, a change in f would change the situation of consumer search since the gains from search, $(p_2-p_1)\ f_1$, would be thereby affected. Thus a great change in f might change the situation from case 2 to case 1 or 3, i.e. from A_2 to A_1 or A_3.

Now let us go through the different possible starting points. If the market is in A_2, then it will always converge to PDE. The proof follows immediately from the proof of stability of PDE when there is no change in the condition for the case. If the market is forced out of equilibrium (case 2) then the condition of search will be affected also. The expected gains from search are $(p_2-p_1)\ f_1$. If f_1 diminishes then this expected gain will decrease. The critical point is when the conditions for case 2 are no longer fulfilled. These conditions are $c_1 < (p_2-p_1)\ f_1 < c_2$. The critical values of f_1 are then $f_1 = \frac{c_1}{p_2-p_1}$ and $f_1 = \frac{c_2}{p_2-p_1}$ for which case 2 becomes case 1 and case 3 respectively. We then go from A_2 to A_1 or A_3 in figure 2. If then the disturbance is a decrease in f_1 smaller than $f_1^* - \frac{c_1}{p_2-p_1}$ or an increase smaller than $\frac{c_2}{p_2-p_1} - f_1^*$, then the market will go back to the original price dispersion equilibrium.

But what happens if the disturbance is so great that a change from A_2 to A_1 or A_3 occurs?

If the market changes to A_1 there will be no search beyond the first step. The profits at p_1 and p_2 are then independent on the distribution f. Which one of p_1 and p_2 that will give higher profit is of course dependent on the actual figures of costs and on the shape of d(p). If p_1 yields higher profit than p_2, f_1 will increase without change in either absolute or relative profit, until f_1 again is greater than $\frac{c_1}{p_2-p_1}$. The market is then back in A_2 (case 2) and will revert to the PDE-situation.

If, on the other hand, profit is higher at p_2; then f_1 will decrease. The decrease will continue until $f_1 = 0$, i.e. all firms are at p_2, i.e. the market goes to SPE.

If the disturbance is an increase in f_1 greater than $\frac{c_2}{p_2-p_1} - f_1^*$, the market will change from case 2 to case 3. If

$p_1 > mc$ the profit is higher at p_1 than at p_2. f_1 will then increase until $f_1 = 1$ and $f_2 = 0$. Then we are again at a single price equilibrium.[1)]

We can illustrate the stability properties of the model in a phase diagram. We have the following function for the profit difference:

$$\pi_1-\pi_2=\begin{cases} p_1Kd(p_1) - a_1Kd(p_1) - a_2 - p_2Kd(p_2) + \\ + a_1Kd(p_2) + a_2 \qquad \text{if } (p_2-p_1)\, f_1 < c_1 < c_2 \\ \\ p_1Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_1Kd(p_1)\left(\frac{\gamma_1}{f_1} + \gamma_2\right) - a_2 - \\ - p_2Kd(p_2)\,\gamma_2 + a_1Kd(p_2)\,\gamma_2 + a_2 \\ \qquad \text{if } c_1 < (p_2-p_1)\, f_1 < c_2 \\ \\ \frac{p_1Kd(p_1)}{f_1} - a_1\frac{Kd(p_1)}{f_1} \\ \qquad \text{if } c_1 < c_2 < (p_2-p_1)\, f_1 \end{cases} \tag{5.1}$$

We have not adopted so far any specification of the firm's disequilibrium behavior. However, the Nash equilibrium condition states implicitly that if a firm can obtain a higher profit at another price, then sooner or later it will change to that price. This is of course one of the vaguest assumptions we could adopt. In this model, the assumption becomes:

$$\operatorname{sgn} \dot{f}_1 = \operatorname{sgn} (\pi_1-\pi_2) \tag{5.2}$$

that is, f_1 changes in the same direction as the difference between π_1 and π_2.

---

1) Of course there is no search (beyond the first step) in this equilibrium situation. One could argue that the market is in A_1 (case 1). However the reason for no search is that there is only one price in the market. There is however potential search -- any firm trying to raise price to p_2 will immediately create search. Then it is just a semantic question if the market is in A_1 or A_3.

Because this function is impossible to draw in a figure we assume the special shape

$$\dot{f}_1 = \alpha(\pi_1 - \pi_2) \tag{5.3}$$

where α is a positive constant.

We then have the system of equations:

$$\begin{cases} 5.1 \\ 5.3 \end{cases}$$

or

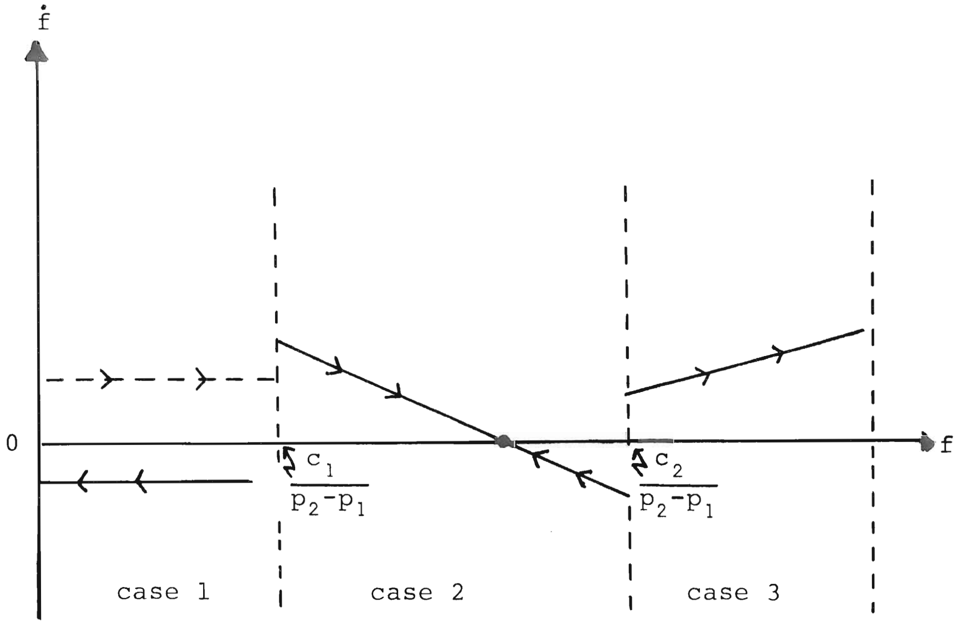
$$\begin{cases} \pi_1 - \pi_2 = G(f_1) \\ \dot{f}_1 = \alpha(\pi_1 - \pi_2) \end{cases} \tag{5.4}$$

where $G(f_1)$ is the right hand side of (5.1). From (5.4) we get the first order differential equation

$$\dot{f} - \alpha\, G(f_1) = 0 \tag{5.5}$$

We analyze the stability properties of the model in the phase diagram in figure 3.

Figure 3



We have for the three cases

case 1 $\dot{f}_1 - \alpha A_1 = 0$

case 2 $\dot{f}_1 - \alpha (A_2 \frac{1}{f_1} + A_3) = 0$

case 3 $\dot{f}_1 - \alpha A_4 \frac{1}{f_1} = 0$

where $A_i (i=1,\dots,4)$ are constants.[1)] A_1 is positive if the profit at p_1 is higher than at p_2, negative otherwise. A_2 and A_3 are always positive if $p_1 > mc$. A_4 is always positive.

If the market is in case 2 it will always converge to the price dispersion equilibrium. If the market is in case 1, it will converge to a single price equilibrium if the profit is higher at p_2. Otherwise f_1 will increase until the search conditions change the market to case 2 (the dotted line in fig. 2). Then the market will converge to the price dispersion equilibrium. If the market is in case 3 f_1 will increase and the market goes to a single price equilibrium. The cases in which the market goes to a single price are consistent with a long run equilibrium if and only if this price is the monopoly price.

VI. 6. Conclusions

Although it is possible to show that there are some conditions under which an equilibrium with price dispersion can exist in a search market where consumers have different search costs,

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1) $A_1 = K\left[(p_1-a_1)\, d(p_1) - (p_2-a_1)\, d(p_2)\right]$

$A_2 = K\, (p_1-a_1)\, d(p_1)\, \gamma_1$

$A_3 = K\, \gamma_2\, \left[(p_1-a_1)\, d(p_1) - (p_2-a_1)\, d(p_2)\right]$

$A_4 = K\, (p_1-a_1)\, d(p_1)$

a fundamental question is whether this equilibrium is stable. In the general case with continuous search cost distribution and price distribution, the stability properties are extremely difficult to analyze. In this chapter it is shown that when these distributions are taken to be discrete and furthermore to contain only two prices and two search costs, the price dispersion equilibrium is stable.

VI.7. Appendix

In this appendix we will generalize from the two price, two search cost case to an q search cost case.

Let us assume that there are still only two feasible prices p_1 and p_2. The frequency of firms are f_1 and f_2 respectively. There are however q groups of consumers with search costs c_1, $c_2,\ldots,c_i,\ldots,c_q$. The frequency of consumers at these different search costs are $\gamma_1,\gamma_2,\ldots,\gamma_i,\ldots,\gamma_q$. Denote the cumulative search cost distribution by H. Then we have

$$H(c_\psi) = \sum_{i=\psi}^{q} \gamma_i,$$

that is $H(c_\psi)$ is the frequency of consumers with search cost greater than or equal to c.

We derive the stopping price distribution (which of course is a two point distribution):

$$1 - (1-f_1)\, H\big[(p_2-p_1)\, f_1\big] \quad , \quad (1-f_1)\, H\big[(p_2-p_1)\, f_1\big]^{1)} \qquad \text{(A.1)}$$

---

1) *The consumers stopping at p_1 consist of a) those who search until they find a firm charging p_1 and b) those who search just once and happen to find a firm charging p_1 in their first step. The frequency of the first category is $\{1-H[(p_2-p_1)f_1]\}$ and of the second $H[(p_2-p_1)\, f_1]$. The consumers stopping at p_1 are; all of category (a) and f_1 times those of category (b). The consumers stopping at p_2 are f_2 or $1-f_1$ times category (b).*

the demand per firm at the two prices:

$$K\left\{\frac{1}{f_1} - \left(\frac{1}{f_1}\right)H\left(p_2-p_1\right)\right)\right\} \quad , \qquad K\left\{\left(\frac{1}{f_1} - 1\right)H\left((p_2-p_1)f_1\right)\right\}; \tag{A.2}$$

as well as the distribution of profit:

$$\begin{aligned} p_1: \qquad & p_1K\left\{\frac{1}{f_1} - \left(\frac{1}{f_1} - 1\right)H\left((p_2-p_1)f_1\right)\right\} - c\left\{\ \right\}, \\ p_2: \qquad & p_2K\left\{H\left((p_2-p_1)f_1\right)\right\} - c\left\{\ \right\} \end{aligned} \tag{A.3}$$

The equilibrium condition $\pi_1 = \pi_2$ then gives:

$$\begin{aligned} &(p_1-a_1)\left\{\frac{1}{f_1} - \left(\frac{1}{f_1} - 1\right)H\left((p_2-p_1)f_1\right)\right\} - a_2 = \\ &= (p_2-a_1)\left\{H\left((p_2-p_1)f_1\right)\right\} - a_2 \end{aligned} \tag{A.4}$$

or

$$\frac{p_2-a_1}{p_1-a_1} = \frac{1 - (1-f_1)\ H\left((p_2-p_1)f_1\right)}{f_1\ H\left((p_2-p_1)\ f_1\right)} \tag{A.5}$$

which, as in the two search cost cases, states that the frequency of firms at the two prices must be such that the rate of demand at the two prices equals the inverse of the rate of marginal profit at those prices.

Let us now analyze the stability of this equilibrium. The profit at p_1 is

$$\pi_1=K(p_1-a_1)\left[\frac{1-H\left((p_2-p_1)f_1\right)}{f_1} + H\left((p_2-p_1)f_1\right)\right] - a_2 \tag{A.6}$$

Then

$$\frac{d\pi_1}{df_1} = -\ K(p_1-a_1)\left[\frac{1+f_1\ \frac{dH}{df_1} + H}{(f_1)^2} + \frac{dH}{df_1}\right] \tag{A.7}$$

The profit at p_2 is

$$\pi_2 = K(p_2-a_1)\ H\left((p_2-a_1)f_1\right) \tag{A.8}$$

Then

$$\frac{d\pi_2}{df_1} = K(p_2-a_1)\ \frac{dH}{df_1} \tag{A.9}$$

(A.9) is always negative, i.e. the profit will always increase at p_2 if f_2 increases. (A.7) is negative if

$$\left[\frac{1 + f_1 \frac{dH}{df_1} + H}{(f_1)^2} + \frac{dH}{df_1}\right] \text{ is positive.}$$

The condition for stability of equilibrium is;

$$\frac{d\pi_1}{df_1} < \frac{d\pi_2}{df_1}$$

We then examine

$$-\ (p_1-a_1)\ \frac{1+f_1(1+f_1)\ \frac{dH}{df_1} + H}{(f_1)^2} \overset{?}{\gtrless} (p_2-a_1)\ \frac{dH}{df_1} \tag{A.10}$$

If the change in f is so small that H is not affected, then we have

$$-\ (p_1-a_1)\ \frac{1 + H}{(f_1)^2} < 0 \tag{A.11}$$

i.e. the equilibrium is stable.

If on the other hand H is affected by the change in f, which is the case if there are vary many different search costs, the stability condition becomes:

$$\frac{dH}{df_1} > \frac{-\ RMP_{12}(1+H)}{f_1 RMP_{12} + f_1^2(RMP_{12} + 1)} \tag{A.12}$$

The right hand side is always negative. $\frac{dH}{df_1}$ is always negative or zero. Thus the condition for stability is that H must not fall too rapidly. This corresponds to the case in the main text when all consumers with search cost c_2 become searchers because of an increase in f_1, i.e. the market has changed from case 2 to case 3.

Substituting the equilibrium solution of f_1, which is

$$f_1 = \frac{1 - H}{(RMP_{21} - 1)\ H}, \tag{A.13}$$

into (A.12) we can solve for the shape of H necessary to give rise to stable equilibrium with price dispersion.

CHAPTER VII:

Search Market Equilibri
when the Probability of
Finding a Firm is Depen
ent on Firm Size and on
Advertising

VII. *SEARCH MARKET EQUILIBRIUM WHEN THE PROBABILITY OF FINDING A FIRM IS DEPENDENT ON FIRM SIZE AND ON ADVERTISING*

VII.1. Firm Size Dependent Probabilities

We have in the previous chapter derived the equilibrium distribution of firms under the assumption that all firms have an equal chance of being found by a consumer taking another search step. In this section we relax this assumption. First we consider the case in which the probability of coming into contact with a firm is dependent on the firm size. Secondly we consider the case in which firms can affect the probability of being found by means of, for instance, advertising.

We will throughout use the model with two prices p_1 and p_2 and two search cost C_1 and C_2. The frequency of firms is f_1 at p_1 and f_2 at p_2. The frequency of consumers with search cost C_1 is γ_1 and γ_2 with search cost C_2.

As was shown in the previous section there may be three different cases. The reservation price of both groups is above p_2; one reservation price is above p_2 and the other between p_1 and p_2; both reservation prices are between the two prices. The interesting case, at least if we are interested in price dispersion equilibrium, is the middle case; some consumers are searching more and some less. Therefore we will in this section concentrate the analysis on the situation when the presumptions for this case are fulfilled.

First let us relax the assumption that firms have an equal chance of being found, independent of size. If consumers get information about the existence of other firms through other consumers, it is natural to think that the frequency of signals emanating from one particular firm will increase monotonically with the number of consumers buying at that firm a given moment, i.e. with the firm size. Let us therefore assume that the probability of drawing a particular firm in a search step is proportional to its size. Further assume that consumers, when they buy, buy one unit of standard size independently of the price, i.e. the individual demand is zero elastic. Denote the frequency of consumers stopping (purchasing) at the two prices ω_1 and ω_2, respectively. In contrast to the case described in the previous section, the conditions of search for one consumer are now not only dependent on the distribution of firms and of prices, they are also affected by the search behavior of other consumers.[1)]

The expected gain from one more search step, given that p_2 has already been found is $\omega_1(p_2-p_1)$.[2)] We assume $c_1 < \omega_1(p_2-p_1) < c_2$ and thereby that the low search cost group will search until they find a firm changing p_1 while the high search cost group will stop after one step.

The frequences of consumers at the two prices are thus

$$\omega_1 \text{ at } p_1 \text{ and } \omega_2 \text{ at } p_2, \text{ respectively}$$

or

$$\gamma_1 + \gamma_2\omega_1 \quad \text{and} \quad \gamma_2\omega_2$$

But in equilibrium we must have

$$\omega_1 = \gamma_1 + \gamma_2\omega_1$$

and

$$\omega_2 = \gamma_2\omega_2,$$

and consequently

$$\omega_1 = 1$$

and

$$\omega_2 = 0$$

---

1) Of course the search behavior of other consumers is of indirect importance in the previous model too, because the equilibrium distribution of firms was determined by the behavior of all consumers.

2) The probability of finding one particular firm with the price p_1 is ω_1/f_1. The probability to find any firm charging p_1 is thus ω_1.

if $\gamma_1 > 0$.

Thus there could never be a price dispersion equilibrium in this case. There will be no demand at the higher price in equilibrium and therefore all firms have to charge p_1. The lower of the two prices will become the equilibrium price if the market is in a disequilibrium situation where $c_1 < \omega_1(p_2-p_1)$. Now, if all firms are charging the price p_1 and are allowed to change to another price p_0 lower than p_1, will then all firms change to p_0? No, if no firms or just a few firms change to p_0 the condition $C_1 < \omega_0(p_1-p_0)$ is apparently not fulfilled. It is not worthwhile for a consumer with positive search cost to search for p_0 if the probability to find a firm at that price is very small, which is the case if there are few firms at p_0. The firms changing to p_0 will get only their fair share of the consumers, i.e. f_0.

Changing the price slightly up or down is profitable only if the common price (p_1 in this case) does not fulfill the necessary condition for monopolistic profit maximum. Again we have the monopolistic price as the only single price equilibrium.

If the probability of finding a firm is proportional to its size, then we could not have a price dispersion equilibrium. However the assumption about probabilities proportional to the size of firm is not quite realistic, for it implies that a firm with no customers, although it exists, would have absolutely no probability of being found. So we relax this assumption a bit and instead assume that the probability of being found is linearily increasing with the size but with a positive intercept, i.e. the probability of finding a firm with no consumers is greater than zero but of course smaller than the probability of finding a bigger firm.

We then have the following frequency distributions of consumers over the two prices:

$\underline{p_1}$	$\underline{p_2}$
$\gamma_1+\gamma_2(\alpha f_1+\beta\omega_1)$	$\gamma_2(\alpha f_2+\beta\omega_2)$

α and β are positive constants with the property $\alpha+\beta=1$ since $\alpha f_1+\beta\omega_1$ and $\alpha f_2+\beta\omega_2$ must sum up to unity because they are probabilities.

In equilibrium we then have the following necessary frequency distributions of consumers:

$$\omega_1 = \gamma_1 + \gamma_2(\alpha f_1 + \beta\omega_1)$$

and

$$\omega_2 = \gamma_2(\alpha f_2 + \beta\omega_2)$$

which have the following solutions in ω_1 and ω_2:

$$\omega_1 = \frac{\gamma_1 + \gamma_2 \ \alpha f_1}{1 - \gamma_2\beta}$$

and

$$\omega_2 = \frac{\gamma_2\alpha f_2}{1 - \gamma_2\beta}$$

The demand per firm is thus respectively

$\underline{p_1}$	$\underline{p_2}$
$\dfrac{\gamma_1/f_1 + \gamma_2\alpha}{1 - \gamma_2\beta}$	$\dfrac{\gamma_2\alpha}{1 - \gamma_2\beta}$

If the cost function is $G(q) = a_1 q + a_2$ the profit per firms is respectively

$\underline{p_1}$	$\underline{p_2}$
$(p_1-a_1)\dfrac{\gamma_1/f_1 + \gamma_2\alpha}{1 - \gamma_2\beta} - a_2$	$(p_2-a_1)\dfrac{\gamma_2\alpha}{1 - \gamma_2\beta} - a_2$

A necessary condition for Nash equilibrium is that the profit at one of these prices must not be higher than at the other, i.e. $\pi_1 = \pi_2$.

The equilibrium condition is thus

$$(p_1 - a_1)\ \frac{\gamma_1/f_1 + \gamma_2\alpha}{1 - \gamma_2\beta} - a_2 = (p_2 - a_1)\ \frac{\gamma_2\alpha}{1 - \gamma_2\beta} - a_2$$

or

$$RMP_{12}^{\;1)} = \frac{\gamma_2\alpha}{\gamma_1/f_1 + \gamma_2\alpha}$$

This gives the equilibrium distribution of firms over prices:

$$f_1 = \frac{RMP_{12}\ \gamma_1}{(1 - RMP_{12})\gamma_2\alpha}$$

and

$$f_2 = 1 - \frac{RMP_{12}\ \gamma_1}{(1 - RMP_{12})\gamma_2\alpha}$$

We see from this that the equilibrium solution here differs from the original equal-probability case only by the coefficient α in the denominator, which makes the frequency of firms at p_1 in equilibrium smaller as the probability of finding a firm increases with firm size. α=1 will of course make both the assumptions and solution identical with the previous case.

VII.2. Stability of Equilibrium

We now examine the stability of the equilibrium solution of the present formulation of the model in the same way as we did in chapter IV, section 4.

We have

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1) *Where* $RMP_{12} = \frac{p_1 - a_1}{p_2 - a_1}$

$$\frac{d\pi_1}{df_1} = -(p_1 - a_1)\ k \frac{\gamma_1}{(f_1)^2\ (1 - \gamma_2\beta)}$$

and

$$\frac{d\pi_2}{df_1} = 0$$

We can see that $\frac{d\pi_1}{df_1}$ is always negative if $p_1 > mc$. As $\frac{d\pi_2}{df_1}$ is always zero, the equilibrium is stable also in this case.

VII.3. A Model with Advertising

For the simplified search model with only two prices and two search costs we have shown that there exist price dispersion equilibria, furthermore that these equilibria are stable, both when all firms are equally likely to attract consumers and when the probability of coming into contact with a particular firm increases with firm size. However, there was no activity from the firm's side to try to alter the probability of being found. In this section we will allow firms to affect the probabilities of being found through advertising. Because goods do not differ in quality as among firms there is of course no advertising about quality. The propose of advertising is to inform people about the existence of the firm (store) and about its price. Pure price advertising is with some exceptions very rare. The yellow pages never carry price advertisments, and the papers only seldom, even if the commodity is well-defined and well known. (Note that I refer primarily to consumer durables.) The reason for this perhaps calls for a closer analysis, but we will not consider that problem here.

Let us assume that firms inform consumers of their existence by means of advertising. Advertising is here a way of affecting the probabilities that a given consumer will draw any particular firm during his next search step. Advertisment of course will demand resources. Let us assume that the cost of advertising increases along with its effect, i.e. the probability of being drawn increases monotonically with advertising costs. A linearly increasing function is obviously inappropriate since this probability could equal unity at a finite cost. Let us therefore assume that the probability of being drawn is equal to the amount of advertising coming from this particular firm in relation to the total amount of advertisment in the market. If the cost (amount) of advertising per period for a firm at p_1 is b_1 and b_2 for a firm at p_2, then the probability of finding any firm at p_1 is

$$\frac{f_1 b_1}{f_1 b_1 + f_2 b_2}$$

and thus the probability of finding a firm charging p_2 is

$$\frac{f_2 b_2}{f_1 b_1 + f_2 b_2}$$

We have the following stopping distribution of consumers over the two prices:

$$\begin{array}{cc} \underline{p_1} & \underline{p_2} \\[6pt] \gamma_1 + \dfrac{b_1 f_1}{b_1 f_1 + b_2 f_2}\gamma_2 & \dfrac{b_2 f_2}{b_1 f_1 + b_2 f_2}\gamma_2 \end{array} \qquad (3.1)$$

The profit per firm at these two prices is:

$$\begin{array}{cc} \underline{p_1} & \underline{p_2} \\[6pt] (p_1 - a_1)\left(\dfrac{\gamma_1}{f_1} + \dfrac{b_1}{f_1 b_1 + f_2 b_2}\gamma_2\right) - & (p_2 - a_1)\,\dfrac{b_2}{f_1 b_1 + f_2 b_2}\gamma_2 - \\ - (a_2 + b_1) & - (a_2 + b_2) \end{array} \qquad (3.2)$$

From this follows that a necessary condition for equilibrium is:

$$(p_1-a_1)\left(\frac{\gamma_1}{f_1} + \frac{b_1}{f_1b_1+f_2b_2}\gamma_2\right) - (a_2+b_1) =$$

$$= (p_2-a_1)\frac{b_2}{f_1b_1+f_2b_2}\gamma_2 - (a_2+b_2) \tag{3.3}$$

i.e. the profit per firm must be equal at the two prices.

Further b_1 and b_2 must be chosen in accordance with profit maximization for firms at p_1 and p_2 respectively. $\frac{d\pi_1}{db_1} = 0$ and $\frac{d\pi_2}{db_2} = 0$ gives the following necessary conditions:

$$(p_1-a_1)\ \gamma_2 f_2 b_2 = (f_1b_1+f_2b_2)^2 \tag{3.4}$$

and

$$(p_2-a_1)\ \gamma_2 f_1 b_1 = (f_1b_1+f_2b_2)^2 \tag{3.5}$$

Equations (3.3), (3.4), and (3.5) together with the conditions, $f_1 + f_2 = 1$, $\gamma_1 + \gamma_2 = 1$ and $p_2 > p_1$ are then sufficient to solve the equilibrium values for f_1, f_2, b_1 and b_2 when γ_1, γ_2, p_1, p_2, a_1 and a_2 are exogenously given.

If we put $(p_1-a_1) = A_1$ and $(p_2-a_1) = A_2$ we have

$$b_1 = \frac{A_1\gamma_2}{\left[\frac{A_2}{A_1} + 1\right]^2 f_1} \tag{3.6}$$

and

$$b_2 = \frac{A_1\gamma_2}{\left[\frac{A_1}{A_2} + 1\right]^2 f_2} \tag{3.7}$$

The solution for f_1 and f_2 is then:

$$f_1 = \frac{\gamma_1\left(-\frac{A_1^3}{A_2} + 2\,\frac{A_1^2}{A_2} + A_1\right) + \frac{A_1^3}{A_2^2}\left(\frac{A_1}{A_2} - 2\right)}{\gamma_1\left(-\frac{A_1^3}{A_2^2} + 2A_1 - A_2\right) + \frac{A_1^4}{A_2^3} - 2\,\frac{A_1^3}{A_2^2} + 2\,\frac{A_1^2}{A_2} + A_2 - A_1} \tag{3.8}$$

and

$$f_2 = \frac{\gamma_1\left(-2\,\frac{A_1^2}{A_2} + A_1 - A_2\right) + 2\,\frac{A_1^2}{A_2} + A_2 - A_1}{\gamma_1\left(-\frac{A_1^3}{A_1^2} + 2A_1 - A_2\right) + \frac{A_1^4}{A_2^3} - 2\,\frac{A_1^3}{A_2^2} + 2\,\frac{A_1^2}{A_2} + A_2 - A_1} \tag{3.9}$$

VII.4. Stability of Equilibrium

Now it remains to examine the stability of the price dispersion equilibrium. As in the previous cases we examine what happens if some firms, in an equilibrium situation, change (by chance) from p_2 to p_1. Does the market then go back to the price dispersion equilibrium? As before we assume that the forces on the firms work in a profit-increasing direction.

We have

$$\frac{d\pi_1}{df_1} = (p_1 - a_1)\,\frac{-1}{(f_1)^2} - \frac{b_1}{(b_1 f_1 + b_2 f_2)^2} \tag{4.1}$$

$$\frac{d\pi_2}{df_1} = (p_2 - a_1)\,\frac{-\,b_1}{(b_1 f_1 + b_2 f_2)^2} \tag{4.2}$$

and thus, defining Δ as the difference in the profit derivative:

$$\Delta = \frac{d\pi_1}{df_1} - \frac{d\pi_2}{df_1} = \frac{-\,(p_1 - a_1)}{(f_1)^2} + (p_2 - p_1)\,\frac{b_1}{(b_1 f_1 + b_2 f_2)^2} \tag{4.3}$$

which, with the help of (3.6) and (3.7) can be simplified to

$$\Delta = \frac{1}{(f_1)^2} \; \frac{-(p_1-a_1)^2 (p_2-a_1) \; \gamma_2^2 + p_2-p_1}{(p_1-a_1)(p_2-a_1) \; \gamma_2^2} \tag{4.4}$$

For the equilibrium to be stable, a necessary condition is that Δ be negative. $\frac{1}{(f_1)^2}$ is apparently positive. Then it remains to examine whether or not the second factor in (4.4) is negative.

A necessary condition for Δ to be negative is:

$$p_2 < \frac{p_1-a_1 (p_1-a_1)^2 \; \gamma_2^2}{1 - (p_1-a_1)^2 \gamma_2^2} \tag{4.5}$$

i.e. for the advertising equilibrium to be stable, the difference between p_1 and p_2 must not be too large. App rently there always exist prices p_1 and p_2 fulfilling the inequality (4.5) if $p_1 > a_1$, thus making the equilibrium stable.

Assuming $a_1 = 0$, i.e. zero marginal cost, and putting $p_1 = 1$ (which implies no loss of generality), (4.5) becomes:

$$p_2 < \frac{1}{1 - \gamma_2^2} \tag{4.6}$$

We see that the upper constraint on p_2 for a stable solution is always a figure greater than unity, which for all $\gamma_2 < 1$ implies that a p_2 can always be found such that $p_2 > p_1$. Further we see that the maximal possible difference between p_1 and p_2 for a stable solution is a decreasing function of γ_2, i.e. the frequency of consumers which because of high search cost search only once.

Thus, we see that introducing firm size dependent probabilities to attract consumers or the possibility for firms to advertise does not change the fundamental stability properties of the model.

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