Price and Quality

Essays on product differentiation

Jonas Häckner

STOCKHOLM SCHOOL OF ECONOMICS

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Foreword

The Industrial Institute for Economic and Social Research (IUI) has a long tradition in applied and theoretical microeconomic research. This orientation is based on the belief that most macroeconomic phenomena require an understanding of microeconomic dynamics. This study by Jonas Häckner is theoretical and focuses on the nature of competition when products are heterogeneous and markets imperfect. In such markets firms compete, not only with price but also with product design.

The analysis is basically positive and aims at bringing more realism into the theory of market behavior. Studies of the market mechanisms in the absence of intervention also provide a useful reference when evaluating the effects of various policy measures and market institutions. For example, one of the chapters discusses whether the current organization of the market for phone-ordered taxicabs is likely to create the optimal availability of transport capacity. The study is a theoretical part of the large IUI project, "The Limits of the Market Economy".

This book has been submitted as a Ph.D. thesis at the Department of Economics at the Stockholm School of Economics. It is the 44th doctoral or licentiate dissertation completed at the Institute since its foundation in 1939. IUI would like to thank Karl Jungenfelt, Henrik Horn and Hans Wijkander of the dissertation committee and Kenneth Burdett for intellectual backing. Generous financial support from the Tore Browaldh Foundation and the Swedish Transport Research Board is gratefully acknowledged.

Stockholm in February 1993

Gunnar Eliasson

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Stockholm in February 1993

Jonas Häckner

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Chapter I

Introduction and summary

In economic theory, firms are normally assumed to compete in prices or quantities. Few would dispute that product design is an equally important variable in the strategic interaction between firms. On the other hand, allowing for competition in several dimensions makes modelling difficult, so there are often good reasons for abstracting from such issues. The aim of this dissertation is to highlight two areas where product differentiation is likely to affect firm behavior significantly, especially the price mechanism. The first area, discussed in chapters II and III, concerns the interdependence between collusive pricing, cartel stability and product design, while the second area, discussed in chapters IV and V, treats oligopolistic pricing and capacity choices in the presence of negative and reciprocal consumption externalities.[1] In this chapter we present the research areas and give a brief summary of the thesis.

1. Collusive pricing, cartel stability and product design

Whenever it is possible to coordinate prices, firms are of course tempted to collectively raise prices above the non-collusive level. It is well known, however, that price cartels need not be stable since a cheating firm may capture a large fraction of the market by lowering its price unilaterally, thus making a substantial short-term gain. The theory of repeated games provides a set of conditions for when collusion is stable [Friedman (1971)]. It can be shown that any collusive prices can be sustained in equilibrium if

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[1]At the cost of some repetition, chapters II-V have been written in the form of independent articles that can be studied separately. Chapters IV and V are joint work together with Sten Nyberg.

- firms have an infinite time horizon,
- firms' strategies are to go back to the non-collusive prices forever if anyone cheats, and
- the discount factor is high enough.

If "tomorrow" is important enough, i.e. if the discount factor is high enough, the short-term gain from deviating will be outweighed by the reduction in future profit streams, and collusion is sustained. The higher the payoffs for deviating and for not colluding, the higher the discount factor has to be in order to keep collusion from breaking down. Conversely, the higher the collusive payoffs, the lower the discount factor may be. Payoffs, in turn, are likely to be affected by product differentiation. For example, when products are close substitutes, a small price differential will provide a cheating firm with a significant increase in demand, so cheating payoffs are likely to be large. On the other hand, competitive (i.e. non-collusive) payoffs are probably low since competition is fierce when products are similar. To sum up:

- The restriction that has to be put on the discount factor in order to keep collusion from breaking down depends on collusive payoffs, cheating payoffs and competitive payoffs.
- These payoffs, in turn, depend on product differentiation.

For a given discount factor, differentiation may therefore determine whether or not firms are in a position to collude successfully. Moreover, if firms can change design, the optimal degree of differentiation is likely to be a function of the discount factor. When the discount factor is high, any collusive agreement is sustainable, so rational firms would maximize profits with respect to both price and differentiation. Second, if the discount factor is low, firms may also want to use differentiation to improve cartel stability by making deviations less attractive. The central question of chapters II and III is whether differentiation facilitates collusive behavior, or if it makes collusion more difficult to sustain.

2. Oligopolistic competition and negative consumption externalities

The second main focus of the dissertation is oligopolistic pricing and capacity choices in the presence of negative and reciprocal consumption externalities. Studies on negative externalities have normally abstracted from strategic behavior on the production side. For many applications this is a natural assumption to make, for instance, when studying optimal capacity and fee structures for publicly provided goods, like street space. [See e.g. Vickrey (1969).]

Reciprocal externalities are also likely to be present on markets for private goods and services. One example is the transportation sector where flights are less likely to be overbooked the fewer the other passengers, and where the waiting time for taxicabs increases with per cab demand. Other examples are markets for prestigious brand-name goods where substantial output expansions may cause brand-name debasement. A common feature of these markets is that the perceived quality of the good or service is negatively affected by the total demand facing a firm. Hence, price and quality cannot be chosen independently. This, in turn, affects the strategic interaction between firms and specifically it influences price formation. Since an increase in demand results in more "congestion", thus reducing quality, price cuts tend to be relatively unattractive. This puts an upward pressure on price [See Scotchmer (1985)]. On the other hand, drawing from the theory of corrective taxes, we know that the socially efficient price must be above marginal cost in order to compensate for the negative externality [See e.g. Diamond (1973)]. Noting that policymakers historically have chosen to regulate the transportation sector heavily, both in terms of price and entry, it seems relevant to ask whether unregulated prices are likely to be high enough or if they are perhaps too high. This is the problem discussed in chapter IV.

When considering competing transportation services, the size of the externality is often affected by firms' capacity choices. For example, if the number of taxicabs in a city is very large, waiting time will not be an issue. Hence, if entry barriers are low, high industry profits will attract new capacity, thereby making the externality less important or even negligible. On the other hand, if a large externality is essential for keeping market prices high, established firms may want to restrict the inflow of new capacity by raising barriers to entry. In chapter V, we model a market for phone-ordered taxicabs using the theoretical framework developed in chapter IV, and discuss whether or not the market is likely to provide an efficient amount of capacity.

3. A summary of the thesis

In chapter II we study collusive pricing and cartel stability assuming horizontal product differentiation where quality is not "high" or "low" in an objective sense. Instead, as in markets for soft-drinks, toothpaste etc., each consumer has a favorite variety. The framework chosen is a model by d'Aspremont, Gabszewicz and Thisse (1979) which is very similar to the Hotelling (1929) model. When design is exogenous, collusion turns out to be more easily sustained if products are remote substitutes.[2] If firms can change design, the outcome will depend on the discount factor. When the discount factor is high, firms choose an intermediate degree of differentiation since joint profits are the highest possible to attain then. If the discount factor is lowered, firms will increase differentiation in order to sustain collusion. Consequently, there is a tendency for differentiation to relax competition and facilitate collusive agreements. The driving force behind this is that cheating payoffs increase to a very large extent when products become close substitutes.

Chapter III also deals with collusive pricing and cartel stability, but it is based on a model by Shaked and Sutton (1982) where products are vertically differentiated. This means that quality is indisputably higher for some products than for others. Although quality can be objectively ranked, differences in income make some people prefer expensive high-quality goods to cheap low-quality goods and vice versa. Limiting the analysis to the case of exogenous product design, we reach the following conclusion. When products are remote substitutes, the high-quality firm is well off already in absence of collusion which gives it weak incentives to collude. As this asymmetry is reduced, and competitive payoffs become lower for the high-quality firm,

[2]Independently of the author, Chang (1991) reaches the same conclusion.

reaching a collusive agreement is gradually facilitated. Hence, collusion is easier to sustain when products are similar. Chapters II and III therefore yield a negative conclusion. Unless products are differentiated in one dimension only (horizontally or vertically) there is no clearcut relationship between product differentiation and cartel stability.

In chapter IV, an oligopolistic model is developed that is characterized by reciprocal consumption externalities and price competition. Within that framework, we study price formation and economic efficiency. Depending on the size of the externality, it is shown that equilibrium prices can vary from marginal cost to the monopoly level, despite Bertrand competition and despite goods being homogenous in equilibrium. Moreover, prices always turn out to be too high from the social point of view, so welfare can be improved by means of a price-ceiling.

In chapter V, we study capacity decisions and social welfare in markets for phone-ordered taxicabs using the theoretical framework established in chapter IV. In a two-stage game, two competing radio dispatch services (RDSs) first choose capacities (i.e. the number of cabs hooked up) and then they compete in price. Two different organizational forms are compared. Under regime I, RDSs are cooperatives controlled by the cab drivers, with the objective to maximize per capita profits. Under regime II, they are privately-owned enterprises choosing connection fees to maximize firm profits. If fixed costs for entrant cabs are small, which we argue is the case, the capacities provided by the market will be insufficient under both regimes and prices will be too high. Thus, entry does not restore efficiency. Privately-owned RDSs will, however, be relatively more efficient as compared to cooperatively-run RDSs.

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Chapter II

Product differentiation and the sustainability of collusion

1. Introduction

This chapter examines the incentives to differentiate products horizontally in a collusive duopoly. The idea is that rational firms might want to use differentiation not only to increase profits, but also to facilitate collusion, even though these interests may conflict with each other.

Whenever it is possible to coordinate pricing decisions, it is naturally course tempting for firms to collectively raise prices above the non-cooperative level. This, however, creates incentives to cheat on the other members of the collusive club. By lowering its price unilaterally by a small amount, or by increasing output by a large amount, a cheating firm may capture a large fraction of the market and make a substantial short-term gain. Hence, for collusion not to break down, there must be some punishment mechanism for penalizing a cheater.

By applying the so-called "Folk theorem" it can be shown that any collusive outcome is sustainable if

(i) there is an infinite time horizon,

(ii) firms' strategies are to go back to the non-collusive prices forever if anyone cheats, and

(iii) the discount factor is high enough.

If "tomorrow" is important enough, i.e. if the discount factor is high enough, the short-run gains from cheating will be outweighed by the reduction in future profit streams and collusion is sustainable. The higher the payoffs for cheating and taking the punishment, the higher the discount factor has to be in order for collusion to hold. Conversely, the higher the collusive payoffs, the lower the discount factor may be allowed to fall.

Now, assume it is possible to choose certain product characteristics in eacl period. The optimal degree of product differentiation is then likely to be a function o the discount factor. As implied by the Folk theorem, almost any collusive agreemen is sustainable if the discount factor is high. Rational firms would thus want t(maximize profits with respect to both price and differentiation if this is the case Alternatively, if the discount factor is low, firms may also want to use differentiatioı in order to make deviations less attractive.

We want to study how rational colluding firms choose prices and product desigı at various discount factors in a repeated game. This is done within a specific theoretica framework, namely a version of the 1929 Hotelling model, by d'Aspremont Gabszewicz and Thisse (1979). Products are horizontally differentiated which mean that different consumers rank equally priced products differently. Hence, quality is nc "high" or "low" in an objective sense. At least potentially, most products are subje(to horizontal differentiation and in the markets for soft-drinks, toothpaste, detergent and soap etc, it is surely a key feature. The cost of changing product design is assume to be either zero or prohibitively high. Examples where redesigning costs are likely t be small are the markets for soft-drinks, newspapers, magazines and cable televisio networks. On the other hand, when products are differentiated by geographical distanc it might be reasonable to expect relocations to be prohibitively expensive. For exampl(competing supermarkets and gasoline stations are often likely to treat locations ε exogenous variables given by history.

There has been some work done on the connection between cartel sustainabilit and product substitutability for horizontally differentiated products. Independently (the author, Chang has analyzed the properties of the d'Aspremont, Gabszewicz ar Thisse version of Hotelling's model assuming exogenous product design.[1] His resul are basically identical to those of section 5.1 in this chapter. In a mimeo, Chang h: also endogenized product design within a framework resembling that of section 6, b

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[1]See Chang (1991), D'Aspremont, Gabszewicz and Thisse (1979) and Hotelling (1929). Other referenc related to the topic are Deneckere (1983), Majerus (1988) and Ross (1992). The similar work conducted Chang was not known to the author until it was mentioned in a referee report concerning an earlier draft the chapter.

in his model the (fixed) redesigning cost can take intermediate values.[2]

For the case when changing design is prohibitively costly, we reach the following conclusions: Joint profit-maximization is easier to sustain, in terms of the discount factor, in markets where products are relatively differentiated. If joint profit maximization is not sustainable, lowering the collusive price will enable firms to collude successfully. Moreover, these constrained monopoly prices are lower, the greater the substitutability. When product design can be changed costlessly, and is therefore endogenous, the following results are reached: When the discount factor is high, firms want to choose an intermediate degree of differentiation since joint profits are the highest possible to attain then. For sufficiently low discount factors, firms will increase differentiation in order to sustain collusion. Unless the discount factor is very low, prices will be unconstrained monopoly prices.

This suggests a fairly general tendency within this framework for differentiation to relax competition and facilitate collusive agreements. The driving force behind the results is that deviation payoffs increase to a very large extent when products become more similar.

The chapter is organized as follows: In section 2, the repeated game framework is briefly discussed as well as firm strategies and timing. We end up with a general expression for the minimal discount factor at which collusion can be sustained. The basic model is presented in section 3. In section 4, the firms' pricing decisions are discussed. We ask what prices will maximize joint profits, what prices a deviator would choose given these collusive prices and finally, what prices would constitute an equilibrium when firms are not colluding. Using this input, explicit expressions for the minimal discount factor needed to sustain joint profit maximization are derived in section 5. This is done under two assumptions. First, product design is exogenous. This provides conditions for when collusion is easily sustained in case product modifications are very costly. Second, firms may change design costlessly.

[2] On the other hand, he makes the ad hoc assumption that colluding firms will always choose designs that permit unconstrained joint profit maximization which is in fact not obvious. The reason is that the profits possible to extract depends on the designs chosen. For instance, if unconstrained monopoly pricing can be sustained for some set of designs, A, while it is not sustainable for some other set, B, it may still be possible to charge relatively higher collusive prices in the latter case without triggering a price war.

Specifically, they are free to pick any design once collusion has broken down. In section 6, where product design is endogenized, we show that the second version of the discount factor restriction can be given a behavioral interpretation in that the optimal design will in fact permit firms to charge (unconstrained) joint profit maximizing prices. Finally, some concluding remarks are made in section 7.

2. The repeated game framework

An implicit collusive agreement can be thought of as a contract between firms which is not enforceable by the legal system. Therefore, it has to be a subgame perfect Nash equilibrium (SPE) in order to be sustainable. Collusion is typically dealt with in infinitely repeated game settings where there is an underlying one-period base game with one or more Nash equilibria (NE). One SPE of the repeated game is to play the competitive one-shot NE in each period. However, as mentioned above, collusion can be sustained as an equilibrium if the discount factor is high enough. This is possible if the one-shot NE is being used as the mechanism for punishment [See Friedman (1971)]. Then, the punishment strategies form a SPE of the entire game. No one will take advantage of the fact that the collusive solution is not a one-shot NE if the one-shot gain by deviating is smaller then the losses in terms of reduced future profit streams. Thus, making the discount factor, δ, arbitrarily large will also make the discounted stream of profit reductions arbitrarily large and no deviation will take place.[3]

Formally, let π^c be the per period payoff for a colluding firm. π^d is the one-shot gain from deviating by undercutting the rival, while π^p is the NE payoff following a deviation from the period after the deviation and henceforth. Then, for collusion to be sustainable,

[3]There are other, less grim, strategies that can be used to sustain noncooperative collusive behavior. For example, it is possible to sustain collusion also when the punishment phase has a limited number of periods after which firms go back to the collusion. When punishments are milder, it is more difficult to sustain collusion in terms of the discount factor. Hence, grim strategies yield necessary conditions for sustainability in general.

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta\pi^p}{1-\delta} \quad or \quad \delta \geq \frac{\pi^d-\pi^c}{\pi^d-\pi^p} \equiv \gamma \ .$$

The γ-function is increasing in π^p and π^d but decreasing in π^c. The larger γ, the smaller the set of discount factors that sustain collusion. Clearly, all payoffs are likely to be affected by product differentiation, and it should therefore be possible to derive a function γ(a), where a denotes the degree of differentiation. The goal of this chapter is to describe how rational collusive firms would maximize profits subject to

$$\delta \geq \frac{\pi^d(a)-\pi^c(a)}{\pi^d(a)-\pi^p(a)} \equiv \gamma(a) \ . \tag{1}$$

The assumption that firms coordinate designs and prices in order to maximize profits is clearly strong. Furthermore, it does seem to require some explicit negotiation. Whenever possible, however, firms are likely to strive towards a situation that is Pareto efficient from their own point of view. The analysis could therefore be thought of as a benchmark.

The game played is the following. The time horizon is infinite. In period τ, firms first announce the prices of period τ and then the product designs of period $\tau+1$, i.e. "next year's" designs. There are no costs associated with these decisions. There is a collusive agreement specifying collusive prices and designs. If a firm deviates with respect to price in period τ, the strategies are to play the one-shot NE prices and designs forever afterwards. If a firm deviates in period τ with respect to next period's design, the strategies are also to play the one-shot NE prices and designs forever afterwards.

In the short run, product characteristics are generally less flexible than prices. This is reflected in firms choosing designs one year in advance. We assume that colluding firms cannot keep next year's design a secret and that information is leaked instantly.

A simplified version of the game is considered in section 5.1. There, the discount factor restriction is derived assuming that product design cannot be changed.

3. The model

In contrast to Hotelling's original formulation (1929), the d'Aspremont, Gabszewicz and Thisse version always has a unique equilibrium in prices and designs.[4] Like most models of product differentiation, it uses some rather simplistic assumptions. Differentiation is one-dimensional, the number of players is restricted to two, there is no question of entry or exit and, finally, each firm is allowed to produce only one variety.

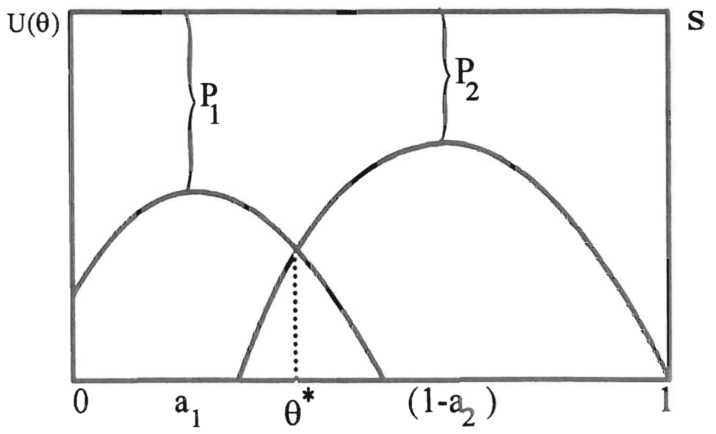
The two firms are indexed 1 and 2. Consumers are assumed to be uniformly distributed by taste along a line of unit length and the firms are producing varieties a_1 and $(1-a_2)$ in this one-dimensional product space. Each period, consumers buy at most one unit of an indivisible good that is homogeneous in all respects except for one: the distance between product design and consumer preference. There is a disutility cost associated with not being able to buy the favorite variety. The utility of a consumer with taste $\theta \epsilon [0, 1]$ is

$$U(\theta) = \begin{cases} s - t(\theta - a_1)^2 - P_1 & \textit{if buying from firm } 1 \\ s - t(1 - a_2 - \theta)^2 - P_2 & \textit{if buying from firm } 2 \\ 0 & \textit{otherwise} \end{cases} \qquad (2)$$

where s is the reservation price, t times the squared distance gives the disutility cos and P_1 and P_2 are the prices charged by the firms. The consumers' utility levels, give designs a_1 and a_2 and prices P_1 and P_2, are shown graphically in figure 1. Consumer purchase from the firm whose product characteristic and price give them the highe utility, or they refuse to buy at all if prices are too high. Firms have constant an identical marginal costs which are normalized to zero.

[4]For a discussion, see d'Aspremont, Gabszewicz and Thisse (1979).

Figure 1



One weak assumption is made concerning the size of the reservation price.

A1: $s \geq 5t/4$

If firms do not collude, the NE designs will turn out to be $a_1=a_2=0$, i.e. products are maximally differentiated. The inequality ensures that the largest collusive payoffs possible to attain when $a_1=a_2\geq 0$ are always higher than the non-collusive payoffs associated with $a_1=a_2=0$. It also ensures all consumers a strictly positive demand in (and out of) equilibrium.

In figure 1, θ^* denotes the consumer who is indifferent between varieties a_1 and $(1-a_2)$. Algebraically, θ^* is given by:

$$s - t(\theta^*-a_1)^2 - P_1 = s - t(1-a_2-\theta^*)^2 - P_2 .$$

Solving for θ^* and noting that the demand functions, D_i, are given by θ^* and $1-\theta^*$ respectively, we arrive at the following profit function for firm i:

$$\pi_i = P_i \left[\frac{1-a_j+a_i}{2} + \frac{P_j-P_i}{2t(1-a_i-a_j)} \right] , \qquad (3)$$

where $i,j \in \{1, 2\}$ and $i \neq j$.

4. Pricing strategies

In this section, the firms' pricing strategies are studied. We calculate the prices maximizing joint profits, the one-shot NE prices and the optimal deviation prices. Since the game is symmetric, the following assumption is made:

A2: The collusive agreement specifies equal prices and symmetric designs so that $a_1=a_2=a$.

Consequently, when $a=0$, products are remote substitutes and when $a=1/2$ they are identical.

4.1 The joint profit-maximizing price

The joint profit-maximizing price will be referred to as the unconstrained monopoly price or just the monopoly price. It is obtained by maximizing the joint profits of the two firms with respect to a uniform price, disregarding the question of sustainability. In sections 5.1 and 6 we also deal with optimal collusive prices in case monopoly pricing is not sustainable. These prices will be referred to as constrained monopoly prices.

A priori, it is not clear whether full market coverage is optimal under joint profit maximization. Intuitively, the higher the reservation price, the more profitable it is to cover the entire market. As it turns out, assumption A1 is sufficient to ensure full market coverage.

Lemma 1: Joint profit maximization implies full market coverage.

Proof: In the appendix

If $a \leq 1/4$, so that firms produce varieties close to the endpoints in product space, profits are maximized by raising prices until the consumer with preferences $\theta = 1/2$ is indifferent between buying and not buying. Charging a lower price would create no additional demand and charging a higher price would make consumers with preferences close to $\theta = 1/2$ choose not to buy, implying partial market coverage. Similarly, if $a \geq 1/4$, so that firms produce varieties close to the center in product space, the consumers with preferences at the endpoints will have zero utility at the profit-maximizing price. Let P^c and π^c denote the monopoly price and the corresponding per firm payoff. Solving for P^c from the utility function using the indifference conditions yields

$$\pi^c(a) = \frac{P^c(a)}{2} = \frac{1}{2}\left[s - t(1/2-a)^2\right] \qquad a \leq 1/4 \tag{4}$$

and

$$\pi^c(a) = \frac{P^c(a)}{2} = \frac{1}{2}\left[s - ta^2\right] \qquad a \geq 1/4 \ . \tag{5}$$

There is a strictly positive relationship between π^c and a when $a \leq 1/4$, and a negative relationship when $a \geq 1/4$. Consequently, monopoly profits are highest at $a = 1/4$. The intuition is that $a = 1/4$ minimizes average disutility costs, i.e. it maximizes the average willingness to pay.

4.2 The punishment price

The punishment price is the NE price of the one-shot base game. Each firm maximizes profits (expression (3)), taking design and the competitor's price as given. Since π_i is concave in P_i, straightforward differentiation yields the following reaction function for firm i:

$$P_i = \frac{1}{2}\left[t(1-2a_j+a_j^2-a_i^2) + P_j\right] .$$

The reaction functions are upward sloping, implying strategic complementarity. Solving for the equilibrium price, with p denoting punishment, we have:

$$P_i^p = \frac{t}{3}(3-4a_j-2a_i+a_j^2-a_i^2) .$$

Substituting P_i^p into (3) and rearranging, we end up with the following payoff for firm i during the punishment phase:

$$\pi_i^p = \frac{t(1-a_i-a_j)(3+a_i-a_j)^2}{18} . \tag{6}$$

Hence, by symmetry,

$$\pi_i^p = \frac{t(1-2a)}{2} . \tag{7}$$

Expression (7) is decreasing in product similarity. This is intuitive since when a is close to 1/2, the game is practically a standard Bertrand game with identical products which is known to yield marginal cost pricing and zero profits.

Finally we can state,

Lemma 2: There will be full market coverage in the punishment phase.

Proof: By lemma 1, we know that the market is covered under monopoly pricing. Hence, it must also be covered in a competitive situation with lower prices. □

4.3 The deviation price

Since monopoly prices are not one-shot equilibrium prices, it might pay to deviate from the collusive price. There are two possible deviation strategies. In both cases one firm lowers its price to make a short-run gain by stealing the competitor's customers. For some designs it might be optimal to steal only a fraction of the competitor's customer while for other designs, capturing the entire market may be more profitable.

When stealing the entire market, a deviating firm will have to lower its price until the consumer "disliking" its variety the most becomes indifferent between firms. Hence, if the deviator produces a variety close to $\theta=0$ he will have to make the consumer with preference $\theta=1$ indifferent, while if he produces a variety close to $\theta=1$ he will have to make the consumer with preference $\theta=0$ indifferent. Charging a lower price would create no additional demand and charging a higher price would make it lose some customers to the non-deviant firm.

Let subscripts w and f denote a "whole" theft and a "fractional" theft respectively and let superscript d denote deviation. In case the aggressive strategy is used, the deviation payoffs can easily be solved for from the utility function using the indifference conditions above.

$$P_w^d(a) = \pi_w^d(a) = P^c(a) - t(1-2a) \ . \tag{8}$$

From the definition of $P^c(a)$ in section 4.1, it follows that

$$\pi_w^d(a) = \frac{1}{4}\ (4s-5t+12at-4a^2t) \qquad a \leq 1/4 \tag{9}$$

and

$$\pi_w^d(a) = s-t+2at-a^2t \qquad a \geq 1/4 \ . \tag{10}$$

In the less aggressive case, a deviating firm faces the profit function

$$\pi_i = P_i \left[\frac{1}{2}+\frac{P^c(a)-P_i}{2t(1-2a)}\right] , \tag{11}$$

which is simply expression (3), firms having symmetric designs and the competitor charging $P^c(a)$. By profit maximization, the optimal deviation price equals

$$P_f^d(a) = \frac{1}{2}\ (P^c(a)+t(1-2a)) \ . \tag{12}$$

Substituting into (11) we have

$$\pi_f^d(a) = \frac{(2at-P^c(a)-t)^2}{8t(1-2a)} \;, \qquad (13$$

and inserting the monopoly prices of section 4.1, we end up with the followi deviation payoffs:

$$\pi_f^d(a) = \frac{[4s+3t-4at-4a^2t]^2}{128t(1-2a)} \qquad a \leq 1/4 \qquad (14$$

and

$$\pi_f^d(a) = \frac{[s+t-2at-a^2t]^2}{8t(1-2a)} \qquad a \geq 1/4 \;. \qquad (1$$

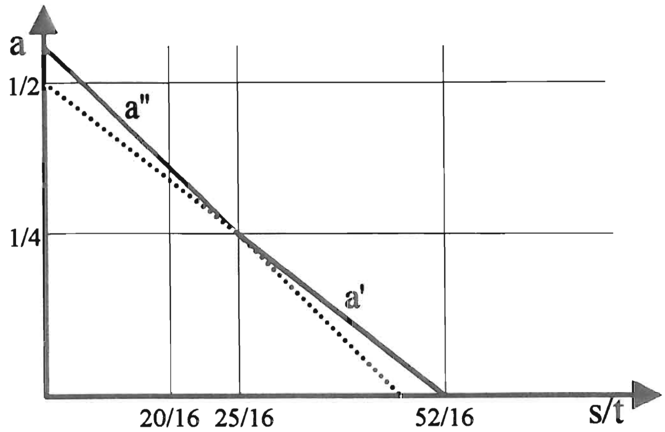
It remains to derive conditions for the relative profitability of the two deviat strategies.

Lemma 3: The aggressive deviation strategy dominates for a larger set of designs higher the reservation price, s. Moreover, if products are close substitutes, so th is close to 1/2, the aggressive strategy always dominates.

Proof: In the appendix

Lemma 3 is represented graphically in figure 2. For a's above the solid line, aggressive deviation strategy is preferred while for a's below it, the less aggres strategy is optimal. This is intuitive because when products are close substitute relatively small price differential will suffice when stealing the entire market. A l reservation price translates into a large price-cost margin which makes it profitabl trade off price reductions for quantity expansions.

Figure 2



Finally, we can note the following:

Lemma 4: There will be full market coverage when a firm deviates with respect to price.

Proof: By lemma 1, we know that the market is covered under monopoly pricing. Hence, it must also be covered in the case when one of the firms makes a price reduction. □

5. The discount factor restriction

Having derived the monopoly payoffs, the deviation payoffs and the payoffs in the punishment phase, we are in a position to derive expressions for the minimum discount factor needed to sustain monopoly pricing. First, we do this under the assumption that product design cannot be changed. Then we assume that firms can change des costlessly.

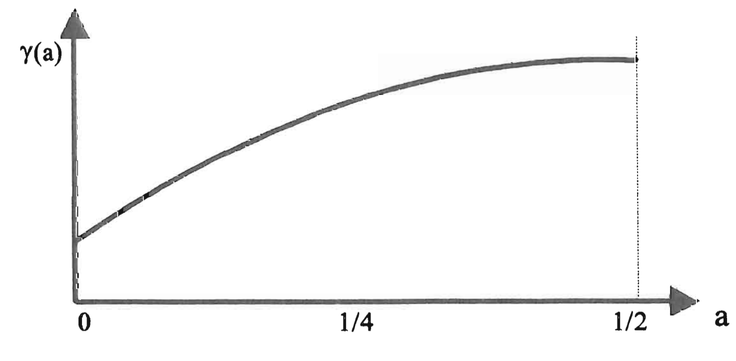
5.1 The case of fixed design[5]

In case changing product design is very costly, differentiation cannot be thought o an endogenous variable. It then becomes relevant to ask in what kind of ma monopoly pricing is most easily sustained. Is it when products are different or w they are similar? The answer is given by $\gamma(a)$ which can be derived by inserting relevant payoff functions into expression (1). As it turns out,

Lemma 5: $\gamma(a)$ is continuous and increasing in a. Moreover, $0<\gamma(a)\leq 1/2$.

Proof: In the appendix.

Figure 3



[5]The results in this section are identical to those obtained by Chang (1991).

Hence, for all parameters satisfying assumption A1, the discount factor restriction is less severe on markets where products are differentiated. Consequently, one could expect a greater amount of collusion on such markets. The intuition is the following. As products become more similar, deviation payoffs increase while the payoffs in the punishment phase decrease. Collusive payoffs increase for $a \leq 1/4$ and decrease for $a \geq 1/4$. However, the effect on deviation payoffs dominates the other effects, altogether making collusion more difficult to sustain in terms of the discount factor.[6] A typical $\gamma(a)$ is shown in figure 3.

When design is exogenous, firms naturally would like to charge unconstrained monopoly prices. If that is not possible, they can nonetheless choose a lower collusive price at which collusion is sustainable.

Lemma 6: If unconstrained monopoly pricing is not sustainable, firms will choose a lower price, P^* at which the discount factor restriction just binds. There will exist a P^* such that $P^p \leq P^* \leq P^c$ for every design and discount factor. Moreover, P^* is lower, the greater the substitutability.

Proof: In the appendix.

Thus, when unconstrained monopoly pricing is not sustainable, profits will be higher for remote substitutes. Increased differentiation raises the payoffs in the punishment phase but reduces the deviation payoff. The last effect is stronger, so duopolies with more differentiated products may charge higher prices without violating the discount factor restriction.

5.2 The case of variable design

In this section we derive conditions under which unconstrained monopoly pricing is sustainable when firms can change design costlessly. That is, we ask whether monopoly

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[6]The same result also holds when disutility costs are linear, as in the original formulation of the Hotelling model.

pricing is easier to sustain when the collusive agreement specifies a large amount differentiation or when it specifies a small amount of differentiation. Three addition problems will have to be addressed in this context.

First, it is no longer evident that firms would in fact want to susta unconstrained monopoly prices even if that were possible for some designs. As shov in section 4.1, unconstrained monopoly prices (and hence profits) are concave in a a maximal for $a=1/4$. Therefore, if unconstrained monopoly pricing is sustainable f designs close to $\theta=0$, but not for designs close to $\theta=1/4$, it still may be possible charge relatively higher prices close to $\theta=1/4$ without triggering a deviation. In secti 6, however, we prove that firms will in fact choose designs that permit unconstrain monopoly pricing.

Second, when firms are free to change design it could potentially be profital to deviate in product design rather than in price. However, this will not occur sin

Lemma 7: Deviations in price are more profitable than deviations in product desig

Proof: In the appendix

The intuition behind lemma 7 is straightforward. When a firm deviates with respec design, all subsequent periods are punishment periods. Firm profits in the punishm phase are increasing in differentiation so that the best a deviator could hope fo $a_1=a_2=0$. However, $a_1=a_2=0$ happens to be the equilibrium design following a p deviation. Since a price deviation in itself creates additional profits it must b dominant strategy.

Third, there is no reason to believe that firms would stick to the collusive de once price collusion has broken down. Differentiating (6) with respect to a_i, we h

$$\frac{\partial \pi_i^p}{\partial a_i} = \frac{-t(a_i-a_j+3)(3a_i+a_j+1)}{18} < 0 \ ,$$

so the NE design is $a_i=a_j=0$ during the punishment phase, which means $P^p=t$ and[7]

$$\pi_i^p = \pi^p = \frac{t}{2} \ . \qquad (16)$$

Therefore, the discount factor restriction will differ from that of section 5.1 in one important respect. The payoffs in the punishment phase are given by (16) instead of (7). Inserting the relevant profit functions it can be shown that

Lemma 8: $\gamma(a)$ is continuous and increasing in a. Moreover, $0<\gamma(a)\leq 1$.

Proof: In the appendix.

Since $\gamma(a)$ is increasing in π^p, allowing firms to adjust designs optimally in the punishment phase has the obvious implication of shifting the $\gamma(a)$-function upwards, thus making monopoly pricing more difficult to sustain in general. Except for that, the results remain basically unchanged. For all parameters consistent with assumption A1, unconstrained monopoly pricing is more easily sustained when the collusive agreement specifies a large amount of differentiation. The intuition is the following. As products become more similar, deviation payoffs increase while collusive payoffs increase for $a\leq 1/4$ and decrease for $a\geq 1/4$. The first effect always dominates, so collusion becomes more difficult to sustain in terms of the discount factor.

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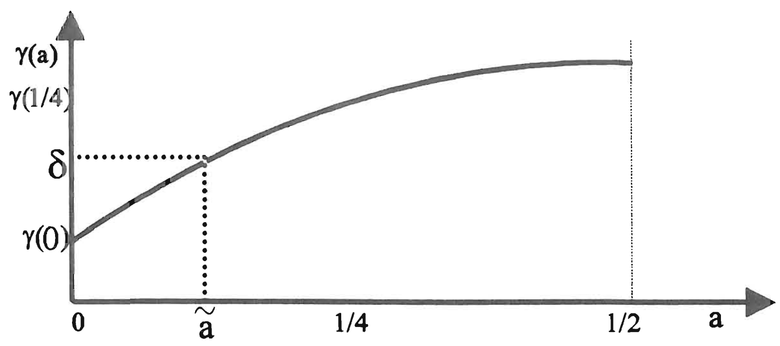
[7]Hence, in the base game, products are maximally differentiated in equilibrium. This is by no means a general result. There are basically two forces working in opposite directions. For given prices, a firm would want to move close to the competitor in order to steal customers. On the other hand price competition becomes more severe when products are similar. In our framework, the last effect dominates but, for example, in the original formulation of Hotelling's model, (with linear disutility costs) the opposite is true.

6. The optimal degree of differentiation

In this section, firms choose the collusive design in a rational way. Hence, they maximize joint profits with respect to design and price subject to the constraint that collusion must be sustainable. We show that the optimal design will in fact permit unconstrained monopoly pricing unless the discount factor is very low. Hence, the last version of the γ-function can be given a behavioral interpretation. Let $\tilde{a}$ define the minimum amount of differentiation needed to sustain unconstrained monopoly pricing. That is, $\tilde{a}$ is the solution to $\delta=\gamma(a)$.

Figure 4

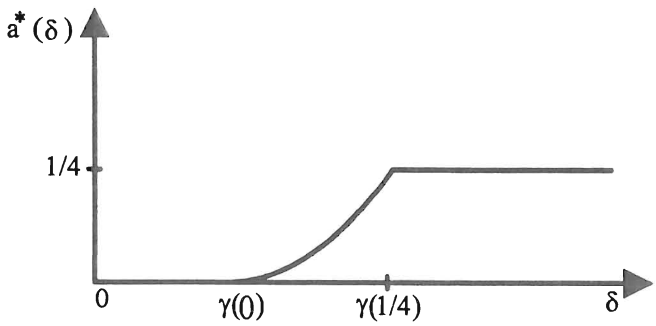


Let $a^*(\delta)$ denote the optimal design as a function of the discount factor. Then,

Theorem 1: If $\delta \geq \gamma(1/4)$, then $a^*=1/4$. If $\gamma(0)<\delta<\gamma(1/4)$, then $a^*=\tilde{a}$. If $\delta \leq \gamma(0)$, then $a^*=0$.

Hence, $a^*(\delta)$ looks like:

Figure 5



When the discount factor is high, firms are not constrained by $\gamma(a)$ so they maximize profits with respect to both price and design. Then $a^*=1/4$ and the price charged is the unconstrained monopoly price. When the discount factor is sufficiently low, firms increase differentiation, but the price charged is still the unconstrained monopoly price. Finally, for very low discount factors, $a^*=0$ so differentiation cannot increase further. In such case, firms choose a collusive price below the unconstrained monopoly price.

The rest of this section is used to prove theorem 1. The first part is straightforward. When $\delta>\gamma(1/4)$, monopoly pricing is sustainable for $a=1/4$. In section 4.1 we showed that unconstrained monopoly prices are maximized when $a=1/4$. In other words, $P^c(1/4)$ is sustainable and allows the maximal profit possible to be attained. This leads to the first observation.

Observation 1: When $\delta\geq\gamma(1/4)$, *then* $a^*=1/4$

When $\gamma(0)<\delta<\gamma(1/4)$ the problem is more complex. First, suppose firms are considering a design in the interval $a\in[0, \tilde{a}]$, where of course $0<\tilde{a}<1/4$. In that interval, unconstrained monopoly pricing is sustainable by definition. Moreover, unconstrained monopoly payoffs increase in a for $a<1/4$. Consequently,

Observation 2: When $\gamma(0)<\delta<\gamma(1/4)$, $a=\tilde{a}$ is the best choice among $a\in[0, \tilde{a}]$.

Second, suppose firms are considering designs in the interval $a\in[\tilde{a}, 1/2]$. Then, unconstrained monopoly pricing is not sustainable, but some other price may allow firms to collude successfully. However, raising the collusive price above the monopoly level can only have a negative effect on sustainability since deviation payoffs increase while collusive payoffs decrease. On the other hand, lowering the collusive price reduces both deviation payoffs and collusive payoffs, and the net effect can be a mitigation of the discount factor restriction which allows firms to collude. To analyze this question we need a general expression for the discount factor restriction when prices are lower than the unconstrained monopoly price. Let $P\leq P^c$ denote the constrained collusive price and let g(a, P) be the general discount factor restriction. Thus, when $a\in[\tilde{a}, 1/2]$ firms would want to maximize profits subject to

$$\delta \geq g(a, P) = \frac{\pi^d(P, a)-\pi(P)}{\pi^d(P, a)-\pi^p} .$$

By symmetry, the collusive payoff, π, is equal to P/2. The payoffs in the punishment phase are independent of the collusive price so $\pi^p=t/2$ as before. Finally, we have already derived maximal deviation payoffs given an arbitrary collusive price, namely expressions (8) and (13) (replacing P^c by P). Plugging these into g(a, P) we have,

$$\delta \geq g(a, P) = \frac{4at+P-2t}{4at+2P-3t} \qquad P\epsilon P_w \tag{17}$$

and

$$\delta \geq g(a, P) = \frac{4a^2t^2+4at(P-t)+(P-t)^2}{4a^2t^2-4at(P-t)+(P-t)(P+3t)} \qquad P\epsilon P_f \ , \qquad (18)$$

where P_w denotes the set of constrained collusive prices for which the aggressive deviation strategy is most profitable. P_f is defined analogously for the less aggressive deviation strategy.[8] Clearly, lowering the collusive price will mitigate the discount factor restriction only if g(a, P) is increasing in P.

Lemma 9: $P\epsilon P_w$ is equivalent to $P>3t(1\text{-}2a)$ while $P\epsilon P_f$ is equivalent to $P<3t(1\text{-}2a)$. The right-hand side of expression (17) is increasing in P for $a<1/4$ and decreasing in P for $a>1/4$. The right-hand side of expression (18) is increasing in P for $P>t(2a+1)$ and decreasing otherwise.

Proof: In the appendix

Corollary 1: If $a>1/4$, then g(a, P) is decreasing in P for all $P\leq P^c$. If $a<1/4$, then g(a, P) is increasing in P for $t(2a+1)<P<P^c$ and decreasing otherwise.

Proof: Follows from $t(2a+1)<3t(1\text{-}2a)$ if and only if $a<1/4$. □

Hence, for $a\in[1/4,\ 1/2]$, lowering the collusive price will not mitigate the discount factor restriction so if unconstrained monopoly pricing is not sustainable, no other collusive price will be sustainable either. Consequently,

Observation 3: When $\delta<\gamma(1/4)$, $a^*\notin[1/4,\ 1/2]$.[9]

The remaining possibility is that firms consider a design in the interval $a\in[\tilde{a},\ 1/4]$. In that case we know from corollary 1 that the discount factor restriction can be mitigated

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[8]Intuitively, the higher the collusive price, the more profitable it is to capture the entire market when deviating.

[9]Collusion is trivially preferred to non-collusion.

by choosing $P < P^c$. Let us define P^* as the best collusive price at which collusion can be sustained. If such a price exists, it is given by lowering the collusive price until the discount factor restriction just binds. Hence,

$$P^*(a;\delta) = \frac{t[4a(1-\delta)+3\delta-2]}{2\delta-1} \qquad P^* \epsilon P_w \tag{19}$$

and

$$P^*(a;\delta) = \frac{t[(\delta+1)(1-2a)+2\sqrt{\delta(\delta-2a)(1-2a)}]}{1-\delta} \qquad P^* \epsilon P_f\ , \tag{20}$$

where P^* is solved from (17) and (18) assuming that equality holds.

Lemma 10: P^* is decreasing in a and continuous.

Proof: In the appendix

Hence, for $a \in [\tilde{a}, 1/4]$, collusion is either not sustainable, or if it is, profits can be increased by choosing a design closer to $\tilde{a}$. Consequently,

Observation 4: When $\gamma(0) < \delta < \gamma(1/4)$, $a = \tilde{a}$ is the best choice among $a \in [\tilde{a}, 1/4]$.

When $\delta < \gamma(0)$, unconstrained monopoly pricing is not sustainable for any design. Then, since P^* is decreasing in a, the best firms can do is to choose $a=0$ and $P^*(0;\delta)$. The lower δ is, the lower P^* has to be to make the discount factor binding. When δ approaches zero, $P^*(0;\delta)$ approaches t. This is intuitive since by then the game is practically a one shot game in which the unique Nash equilibrium is characterized by $a_1 = a_2 = 0$ and $P_1 = P_2 = t$. This leads to the last observation.

Observation 5: When $\delta < \gamma(0)$, $a^* = 0$ and the best price is $P^*(0;\delta)$.

Observations 1 to 5 prove theorem 1. The intuition is straightforward. Assume that monopoly pricing is not sustainable for a certain design, but that there exist a best collusive price lower than the monopoly price. Then, let firms increase differentiation keeping the price fixed. Since payoffs in the punishment phase are independent of collusive designs and the collusive price is fixed, the only thing happening is that deviation payoffs decrease, the reason being that a larger price differential is needed to accomplish the same increase in demand. This makes the discount factor restriction less severe, so that for a given discount factor there is now a slack allowing firms to raise collusive prices until it binds again. This process continues until $P^{*}=P^{c}$ (that is, $a=\tilde{a}$) or until $a=0$.

7. Conclusions

The Hotelling model has the following properties when extended into a repeated game: When changing design is prohibitively costly, monopoly pricing is easier to sustain in markets where products are relatively differentiated. In case monopoly pricing is not sustainable, lowering the collusive price will enable firms to collude successfully. Moreover, these constrained monopoly prices are lower, the greater the substitutability.

In case firms can change design costlessly, monopoly pricing is easier to sustain, the greater the amount of differentiation specified by the collusive agreement. Monopoly profits are maximized at an intermediate degree of differentiation. Consequently, if the discount factor is high, firms would choose this amount of differentiation. If the discount factor is low, rational firms will increase differentiation, still charging the unconstrained monopoly price. The lower the discount factor, the more differentiated the products will be.

These results suggest a fairly general tendency within this framework for differentiation to relax competition and facilitate collusive agreements. The driving force behind this is that deviation payoffs increase to a very large extent when products become more similar. However, since increased product similarity also reduces payoffs in the punishment phase, it is far from obvious, a priori, which effect should dominate. It would therefore be interesting to test the robustness of the results using slightly different models, for instance changing the quality concept to that of vertical differentiation. In that case, qualities are not just "different" but also "low" and "high" in an objective sense. Surely, this aspect of quality dissimilarity is equally important and one should be careful about drawing general conclusions before studying the possible implications of it.

Since the analysis is confined to a very specific framework, one can of course question the generality of the results also for other reasons. First, disutility costs are assumed to be quadratic in contrast to Hotelling's linear formulation. Nevertheless, the results are robust to a linear respecification, at least in the case of exogenous product design. Of course there may be other functional forms potentially altering the conclusions but linear and quadratic disutility costs both seem realistic.[10] Second, price and design are assumed not to be chosen simultaneously. Although this may not be unrealistic, the assumption does affect the game significantly. If the choices were made simultaneously, a price deviation would be combined with a deviation in design. This adds a significant amount of complexity although it is difficult to evaluate the importance of the assumption.

[10]It is difficult to imagine horizontally differentiated products for which disutility costs are convex, i.e the case when utility is most sensitive to changes in design when design is already close to the most favored variety.

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Appendix

Proof of lemma 1

It is obvious that the larger the reservation price, the more profitable it is to cover the entire market. We therefore want to derive a condition under which s is so large that full market coverage is always optimal.

Assume that the price that maximizes joint profits is so high that there is not full market coverage. Moreover, assume $a \leq 1/4$. Then the demand facing firm i will be given by θ such that

$$U(\theta) = s - t(\theta - a)^2 - P = 0$$

or

$$D_i = \theta = a + \frac{\sqrt{s-P}}{\sqrt{t}} .$$

Profit maximization then yields the following first order condition

$$a + \frac{\sqrt{s-P}}{\sqrt{t}} - \frac{P}{2\sqrt{t(s-P)}} = 0 .$$

Solving for P we have,

$$P^{**} = \frac{2}{9}\left[3s - a^2 t + a\sqrt{t(a^2 t + 3s)}\right] .$$

If, on the other hand, full market coverage is optimal, the joint profit-maximizing price is

$$P^c = s - t(1/2 - a)^2$$

from expression (4). Since $P^{**} = P^c$ is a permissible choice, partial market coverage is optimal for firms if $P^{**} > P^c$. This condition is equivalent to

$$s < \frac{t(4a^2 - 8a + 3)}{4} .$$

Since the right-hand side is decreasing in a, the inequality will never hold if $s > 3t/4$. This is always the case, due to assumption A1. Consequently, partial market coverage is not consistent with profit maximization.

Now, if $a \geq 1/4$, and profit maximization implies partial market coverage, the consumers with preferences closest to the endpoints will choose not to buy. The indifferent consumers have preferences θ and $1-\theta$ such that

$$U(\theta) = U(1-\theta) = s-t(a-\theta)^2-P = 0 \ .$$

Firm i will then faces the demand

$$D_i = \frac{1}{2}-\theta = \frac{1}{2}-a+\frac{\sqrt{s-P}}{\sqrt{t}}$$

and the first order condition is

$$\frac{1}{2} - a + \frac{\sqrt{s-P}}{\sqrt{t}} - \frac{P}{2\sqrt{t(s-P)}} = 0 \ .$$

Solving for P we have

$$P^{**} = \frac{(1-2a)\sqrt{t(12s+4a^2t-4at+t)}}{18} + \frac{12s-4a^2t+4at-t}{18} \ .$$

On the other hand, with full market coverage, the joint profit-maximizing price is

$$P^c = s-ta^2$$

from expression (5). Again, since $P^{**}=P^c$ is a permissible choice, partial market coverage is optimal if $P^{**}>P^c$ which is equivalent to

$$s < at(1+a) \ .$$

Since the right-hand side is increasing in a, the inequality will never hold since $s>3t/4$, due to assumption A1. Consequently, partial market coverage is not consistent with profit maximization in this case either. □

Proof of lemma 3

Consider a certain symmetric design and a corresponding monopoly price, $P^c(a)$. When choosing $P_f^d(a)$ rationally, $P_f^d(a)=P_w^d(a)$ is a permissible choice. Therefore, $P_f^d(a)>P_w^d(a)$ must imply $\pi_f^d(a)>\pi_w^d(a)$. Using expressions (8) and (12), we have

$$P_f^d(a) - P_w^d(a) = \frac{3t(1-2a)-P^c(a)}{2} \ .$$

In case $a \leq 1/4$, we know from expression (4) that $P^c(a) = s - t(1/2-a)^2$. Inserting this, letting $k \equiv s/t$, we have,

$$P_f^d(a) - P_w^d(a) = \frac{t}{8}\left[4a^2-28a-4k+13\right] ,$$

which is decreasing in a and equals zero at $a=a'=7/2-(k+9)^{1/2}$.

(i) When $5/4 \leq k \leq 25/16$, then $a' \geq 1/4$ so $P_f^d(a) \geq P_w^d(a)$ for all $a\epsilon[0, 1/4]$ implying that the less aggressive strategy is most profitable.

(ii) When $25/16 \leq k \leq 52/16$, then $0 \leq a' \leq 1/4$ so $P_f^d(a) \geq P_w^d(a)$ for $a\epsilon[0, a']$, implying that the less aggressive strategy is most profitable in that interval, while the aggressive strategy is most profitable for $a\epsilon[a', 1/4]$.

(iii) When $k \geq 52/16$, then $a' \leq 0$ so the aggressive strategy is most profitable for all $a\epsilon[0, 1/4]$.

In case $a \geq 1/4$ we know from expression (5) that $P^c(a) = s - ta^2$. Then,

$$P_f^d(a) - P_w^d(a) = \frac{t}{2}\left[a^2-6a-k+3\right] ,$$

which is also decreasing in a and equals zero at $a=a''=3-(k+6)^{1/2}$.

(i) When $5/4 \leq k \leq 25/16$, then $1/4 \leq a'' < 1/2$, so $P_f^d(a) \geq P_w^d(a)$ for $a\epsilon[1/4, a'']$, implying that the less aggressive strategy is most profitable in that interval, while the aggressive strategy is most profitable for $a\epsilon[a'', 1/2]$

(ii) When $k \geq 25/16$, then $a'' \leq 1/4$ implying that the aggressive strategy is most profitable for all $a\epsilon[1/4, 1/2]$ □

Proof of lemma 5

Define $k \equiv s/t$, which is larger than 5/4 by assumption A1. First, assume $5/4 \leq k \leq 25/16$. Then, by lemma 3, we can insert (4), (5), (7), (10), (14) and (15) into expression (1) yielding

$$\gamma(a) = \begin{cases} \dfrac{4k-4a^2+12a-5}{4k-4a^2-20a+11} & 0 \le a \le 1/4 \\[2ex] \dfrac{k-a^2+2a-1}{k-a^2-6a+3} & 1/4 \le a \le a'' \\[2ex] \dfrac{k-a^2+4a-2}{2k-2a^2+6a-3} & a'' \le a \le 1/2 \end{cases}.$$

In analogy, when $25/16 \le k \le 52/16$ we insert (4), (5), (7), (9), (10) and (14) into expression (1) arriving at

$$\gamma(a) = \begin{cases} \dfrac{4k-4a^2+12a-5}{4k-4a^2-20a+11} & 0 \le a \le a' \\[2ex] \dfrac{4k-4a^2+20a-9}{2(4k-4a^2+16a-7)} & a' \le a \le 1/4 \\[2ex] \dfrac{k-a^2+4a-2}{2k-2a^2+6a-3} & 1/4 \le a \le 1/2 \end{cases}.$$

Finally, for $k \ge 52/16$ we insert (4), (5), (7), (9) and (10) into expression (1). Then,

$$\gamma(a) = \begin{cases} \dfrac{4k-4a^2+20a-9}{2(4k-4a^2+16a-7)} & 0 \le a \le 1/4 \\[2ex] \dfrac{k-a^2+4a-2}{2k-2a^2+6a-3} & 1/4 \le a \le 1/2 \end{cases}.$$

Differentiating $\gamma(a)$ and inserting the relevant boundaries between segments yields the results. □

Proof of lemma 6

In the general case, with $P \le P^c$, the collusive per firm payoffs are P/2 by symmetry. The payoffs in the punishment phase are independent of the collusive price so they are given by (7) as before. Finally, we have already derived optimal deviation payoffs given an arbitrary collusive price, namely expressions (8) and (13) (replacing P^c by P). Thus, in the general case, the discount factor restriction is

$$\delta \ \ge \ g(a,\ P) \ = \ \frac{P-2t(1-2a)}{2P-3t(1-2a)} \qquad P \in P_w$$

and

$$\delta \geq g(a, P) = \frac{P-t(1-2a)}{P+3t(1-2a)} \qquad P \epsilon P_f ,$$

where P_w and P_f denote the sets of constrained collusive prices for which "whole" thefts and "fractional" thefts are most profitable. It is straightforward to show that the aggressive deviation strategy dominates for $P \geq 3t(1-2a)$ while the less aggressive strategy dominates for $P \leq 3t(1-2a)$. The expressions above equal 1/3 at $P=3t(1-2a)$ so g(a, P) is continuous. In addition, g(a, P) is increasing in P. Defining P^* as the maximal collusive price possible to charge without violating $\delta \geq g(a, P)$, it follows directly that P^* is the price that makes $\delta = g(a, P)$. Noting that $P=P^p=t(1-2a)$ implies $g=0$, it follows that there will exist a $P^p \leq P^* \leq P^c$ for any $0<\delta<\gamma(a)$. Solving for P^* and noting that $\delta<1/2$ is necessary for the discount factor to be a restriction, we have

$$P^* = \frac{t(2-3\delta)(1-2a)}{1-2\delta} \qquad \forall a \;\; if \;\; 1/3 \leq \delta \leq 1/2$$

and

$$P^* = \frac{t(3\delta+1)(1-2a)}{1-\delta} \qquad \forall a \;\; if \;\; 0 \leq \delta \leq 1/3 ,$$

since for $1/3 \leq \delta \leq 1/2$ both expressions are larger than 3t(1-2a) implying $P^* \epsilon P_w$ while for $0 \leq \delta \leq 1/3$ both expressions are smaller than 3t(1-2a) implying $P^* \epsilon P_f$. P^* is decreasing in a so when monopoly pricing is not sustainable, the constrained payoffs are lower, the more similar the products are. □

Proof of lemma 7

Assume that firm 1 deviates in period τ with respect to product design (but not with respect to price). Then the equilibrium strategies are to play the Nash equilibrium prices from $\tau+1$ to eternity. Since

$$\frac{\partial \pi_i^p}{\partial a_i} = \frac{-t(a_i-a_j+3)(3a_i+a_j+1)}{18} < 0 ,$$

it is evident that the best deviation design is $a_1=0$. However,

$$\frac{\partial \pi_i^p}{\partial a_j} = \frac{-t(a_i-a_j+3)(5-a_i-3a_j)}{18} < 0 ,$$

so in period $\tau+1$ and henceforth the deviator has a larger payoff, the smaller a_2. Since, $a_1=a_2=0$ would be the equilibrium designs (in period $\tau+1$ and henceforth) in case firm 1 instead deviated with respect to price (in period τ) it is obvious that a price deviation is more profitable. □

Proof of lemma 8

Define $k \equiv s/t$, which is larger that 5/4 by assumption A1. First, assume that $5/4 \leq k \leq 25/16$. Then, by lemma 3, we can insert (4), (5), (10), (14), (15) and (16) into expression (1) yielding

$$\gamma(a) = \begin{cases} \dfrac{16a^4-96a^3-8a^2(4k-23)+24a(4k-5)+(4k-5)^2}{16a^4+32a^3-8a^2(4k+1)-8a(4k-13)+16k^2+24k-55} & 0 \leq a \leq 1/4 \\ \dfrac{a^4-4a^3-2a^2(k-3)+4a(k-1)+(k-1)^2}{a^4+4a^3-2a^2(k-1)-4a(k-1)+(k-1)(k+3)} & 1/4 \leq a \leq a'' \\ \dfrac{k-a^2+4a-2}{2k-2a^2+4a-3} & a'' \leq a \leq 1/2 \end{cases} .$$

In analogy, when $25/16 \leq k \leq 52/16$ we insert (4), (5), (9), (10), (14) and (16) into expression (1) arriving at

$$\gamma(a) = \begin{cases} \dfrac{16a^4-96a^3-8a^2(4k-23)+24a(4k-5)+(4k-5)^2}{16a^4+32a^3-8a^2(4k+1)-8a(4k-13)+16k^2+24k-55} & 0 \leq a \leq a' \\ \dfrac{4k-4a^2+20a-9}{8k-8a^2+24a-14} & a' \leq a \leq 1/4 \\ \dfrac{k-a^2+4a-2}{2k-2a^2+4a-3} & 1/4 \leq a \leq 1/2 \end{cases} .$$

Finally, for $k \geq 52/16$ we insert (4), (5), (9), (10) and (16) into expression (1). Then,

$$\gamma(a) = \begin{cases} \dfrac{4k-4a^2+20a-9}{8k-8a^2+24a-14} & 0 \leq a \leq 1/4 \\ \dfrac{k-a^2+4a-2}{2k-2a^2+4a-3} & 1/4 \leq a \leq 1/2 \end{cases} .$$

Differentiating $\gamma(a)$ and inserting the relevant boundaries between segments yields the results. □

Proof of lemma 9

First, assume $P \epsilon P_f$. Then, $\pi_f^d(a) \geq \pi_w^d(a)$ by definition. For an arbitrary collusive price, these functions are defined by expressions (8) and (13) (inserting P instead of P^c). Solving for P it follows that $P \epsilon P_f$ is equivalent to $P < 3t(1-2a)$ and that $P \epsilon P_w$ is equivalent to $P > 3t(1-2a)$. Second, assume $P \epsilon P_w$. Differentiating (17), we have

$$\frac{\partial g(a,\ P)}{\partial P} = \frac{t(1-4a)}{(4at+2P-3t)^2} ,$$

which is negative for $a > 1/4$ and positive for $a < 1/4$. Third, differentiating (18) and denoting the denominator by D, we have

$$\frac{\partial g(a,\ P)}{\partial P} = \frac{4t(1-2a)}{D^2}\left[P^2-4a^2t^2-t(2P-t)\right] ,$$

which is positive for $P > t(2a+1)$ and negative otherwise. □

Proof of lemma 10

First, it will be shown that (19) and (20) are both decreasing in a. Let us begin with (19) so that $P^* \epsilon P_w$. Differentiation yields

$$\frac{\partial P^*}{\partial a} = \frac{4t(1-\delta)}{2\delta-1} .$$

If $P^* \epsilon P_w$, then surely $P^c \epsilon P_w$ as well. From lemma 3 we know that if $a < 1/4$, then we cannot have $P^c \epsilon P_w$ unless $k \geq 25/16$. But if $a < 1/4$ and $k \geq 25/16$, then $\gamma(a) \leq 1/2$ so for the discount factor to be a restriction in the first place, it must be the case that $\delta < 1/2$. Consequently, the derivative is negative.

Now, consider (20). We know from corollary 1 that $\partial g(a, P)/\partial P \geq 0$ for $t(2a+1) \leq P \leq P^c$. Hence, $P = t(2a+1)$ minimizes the right-hand side of (18) yielding $g(a, t(2a+1)) = 2a$. Consequently, $\delta \geq 2a$ (or $a \leq \delta/2$), is a necessary and sufficient condition for the existence of a sustainable collusive price, $P^*(a;\delta)$. Differentiating (20), we have

$$\frac{\partial P^*}{\partial a} = \frac{2t[(\delta+1)\sqrt{(1-2a)(\delta-2a)}-\sqrt{\delta}(1-4a+\delta)]}{\sqrt{(1-2a)(\delta-2a)}(1-\delta)} .$$

The denominator is obviously positive and well defined if $\delta > 2a$. Denoting the numerator by N, we have

$$\frac{\partial N}{\partial \delta} = \frac{2t(4a-1-3\delta)}{2\sqrt{\delta}}\left[1 + \frac{\sqrt{\delta(1-2a)}}{\sqrt{\delta-2a}}\right] < 0 \qquad a \leq 1/4 ,$$

so N is maximal for $\delta=2a$. Inserting $\delta=2a$ we have $N<0$ so $\partial P^*/\partial a<0$ also when $P^*\epsilon P_f$. Finally, continuity follows from recalling lemma 9 and noting that (19) and (20) are both equal to 3t(1-2a) at

$$a = \frac{3\delta-1}{8\delta-2} .$$

□

Chapter III

Collusive pricing on markets for vertically differentiated products

1. Introduction

This chapter studies collusive pricing in markets for vertically differentiated products. The goal is to analyze the connection between product differentiation and cartel sustainability. Thus, we ask if price collusion is more likely to occur when products are close substitutes or when products are differentiated. It is assumed that changing product design is either impossible or prohibitively expensive. One can, for example, imagine a situation where some firms have incurred large sunk costs in the past in order to establish themselves as high-quality firms.

Whenever it is possible to coordinate pricing decisions, it is of course tempting for firms to collectively raise prices above the non-collusive level. This, however, creates incentives to cheat on the other members of the collusive club. By lowering its price unilaterally by a small amount (or by increasing output by a large amount), a cheating firm may capture a large fraction of the market and thus make a substantial short-term gain. Hence, for collusion not to break down, there must be some punishment mechanism for penalizing a cheater.

By applying the so-called "Folk theorem" it can be shown that any collusive outcome is sustainable if

(i) there is an infinite time horizon,

(ii) firms' strategies are to go back to the non-collusive prices forever if anyone cheats, and

(iii) the discount factor is high enough.

If "tomorrow" is important enough, i.e. if the discount factor is high enough, the short-run gains from cheating will be outweighed by the reduction in future profit streams and collusion is sustainable. The higher the payoffs for cheating and taking the punishment, the higher the discount factor has to be in order to keep collusion from breaking down. Conversely, the higher the collusive payoffs, the lower the discount factor may be allowed to fall. If products are not perfect substitutes, the payoffs will depend on the substitutability of products. Hence, product design may in fact determine whether firms are in a position to collude successfully or not.

Price collusion and product differentiation have previously been studied by Chang and, independently, also by Häckner.[1] Their commonly held quality concept is that of horizontal differentiation, which just means that different consumers rank equally priced products differently. At least potentially, most products are subject to horizontal differentiation and in the markets for soft-drinks, toothpaste, detergents and soap etc, it is surely a key feature. Modelling horizontal differentiation in line with the d'Aspremont, Gabszewicz and Thisse version of Hotelling's model, Häckner and Chang each find that monopoly pricing is easier to sustain, the more differentiated products are.[2]

Needless to say, horizontal differentiation captures only one aspect of product dissimilarity. Equally important is the case when quality is indisputably higher for one product than for another. For example, car manufacturers mostly specialize in one bracket on the quality ladder and few people would deny that Mercedes-Benz is a "better" make of car than Lada. Quality differences of this kind, which can be referred to as vertical differentiation, introduce an asymmetry which is not necessarily present in the horizontal case. Asymmetries, for instance cost differences, are often thought to discourage collusion, so it is not obvious that a positive relationship between differentiation and sustainability will hold also for vertically differentiated products.

In this chapter we examine the properties of a purely vertical model, namely a repeated game version of the (1982) Shaked and Sutton model. Here, different qualities can be objectively ranked, but due to income differences, some people will prefer

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[1]See Chang (1991) and chapter II in this book. Other references related to the topic are Deneckere (1983), Majerus (1988) and Ross (1992).

[2]See d'Aspremont, Gabszewicz and Thisse (1979) and Hotelling (1929). Häckner reaches similar results also for the case when product design is an endogenous variable.

expensive high-quality goods to cheap low-quality goods and vice versa. The reasons for choosing this specific framework are twofold. First, it is quite similar to the Hotelling (1929) model. This is important since we want to keep the framework as close as possible to Chang (1991) and Häckner (chapter II in this book) in order to high-light the importance of the quality concept used. Second, for mathematical reasons it is appealing to use a continuous quality variable. However, this also involves some rather simplistic assumptions. Differentiation is one-dimensional and we restrict the number of firms to two, thus ignoring entry and exit. Finally, each firm is allowed to produce only one specific variety.

The main finding is the following: In contrast to the conclusions of Chang (1991) and Häckner (chapter II), monopoly pricing is easier to sustain on markets where products are relatively similar. The incentives to deviate are always stronger for the high-quality firm. When products are remote substitutes, the high-quality firm is well off already in absence of collusion, and thus has weak incentives to collude. As this asymmetry is reduced, and competitive payoffs become lower for the high-quality firm, reaching a collusive agreement is gradually facilitated. In the Hotelling model there is no such asymmetry and the driving force is that differentiation makes deviation payoffs small.

The chapter is organized as follows: In section 2, the repeated game framework is briefly discussed. We end up with a general expression for the minimal discount factor at which collusion can be sustained. In section 3, the basic model is presented. In section 4, the firms' pricing decisions are discussed. We ask what prices will maximize joint profits, what prices a deviator would choose given the collusive prices and what prices would constitute an equilibrium when firms are not colluding. Using this input, explicit expressions for the minimal discount factor needed to sustain monopoly pricing is derived for each firm in section 5. Finally, some concluding remarks are made in section 6.

2. The repeated game framework

An implicit collusive agreement can be thought of as a contract between firms which is not enforceable by the legal system. Therefore, such a contract has to be a subgame perfect Nash equilibrium (SPE) to be sustainable. Collusion is typically dealt with in infinitely repeated game settings where there is an underlying one-period base game with one or more Nash equilibria (NE). One SPE of the repeated game is to play the competitive one-shot NE in each period but, as mentioned above, collusion can also be sustained as an equilibrium if the discount factor is high enough. This is possible if the one-shot NE is being used as the mechanism for punishment [See Friedman (1971)]. Then, the punishment strategies themselves form a SPE of the entire game. No one will take advantage of the fact that the collusive solution is not a one-shot NE if the one-shot gain by deviating is smaller then the losses in terms of reduced future profit streams. Thus, making the discount factor, δ, arbitrarily large will also make the discounted stream of profit reductions arbitrarily large and no deviation will take place.[3]

Formally, let π^c be the per period payoff for a colluding firm. π^d is the one-shot gain from deviating by undercutting the rival, while π^p is the NE payoff following a deviation and henceforth. Then, for collusion to be sustainable

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta\pi^p}{1-\delta} \qquad or \qquad \delta \geq \frac{\pi^d-\pi^c}{\pi^d-\pi^p} \equiv \gamma \; .$$

The γ-function is increasing in π^p and π^d but decreasing in π^c. The larger γ is, the smaller the set of discount factors that sustain collusion. Clearly, all payoffs are likely to be affected by product differentiation, and it should therefore be possible to derive a function $\gamma(\alpha)$ for each firm, where α denotes the degree of differentiation.

[3]There are other, less grim, strategies that can be used to sustain noncooperative collusive behavior. For example, it is possible to sustain collusion also when the punishment phase has a limited number of periods after which firms go back to the collusive solution. When punishments are milder, it is more difficult to sustain collusion in terms of the discount factor. Hence, grim strategies yield necessary conditions for sustainability in general.

Hence,

$$\delta \geq \frac{\pi^d(\alpha)-\pi^c(\alpha)}{\pi^d(\alpha)-\pi^p(\alpha)} \equiv \gamma(\alpha) \qquad (1)$$

The analysis is focused on exploring this expression in order to determine the connection between product differentiation and the restriction put on the discount factor to sustain collusion.

3. The model

There are two firms denoted firm h (for high quality) and firm l (for low quality) and subscripts h and l will refer to these firms. In each period, consumers purchase at most one indivisible good from the firm offering the best price-quality mix or they will refuse to buy at all if prices are too high. The utility of a consumer with income $V \epsilon [a, b]$ is

$$U(V) = (V-P_h)S_h \qquad (2)$$

when buying from firm h and

$$U(V) = (V-P_l)S_l \qquad (3)$$

when buying from firm l. If none of the goods are purchased, utility is normalized to zero. Hence, a positive utility level ensures a positive demand. The exogenous variable S_i, where $S_i \epsilon [S', S'']$ and $S_h \geq S_l$, denotes the quality of firm i while P_i is its price. $V-P_i$ can be interpreted as income left for other consumption. Moreover, there is a complementarity in consumption in that the utility of consuming other goods increases with the quality level of the differentiated good. Both firms have constant and identical marginal costs which are normalized to zero.[4] The consumers' utility levels are shown graphically in figure 1.

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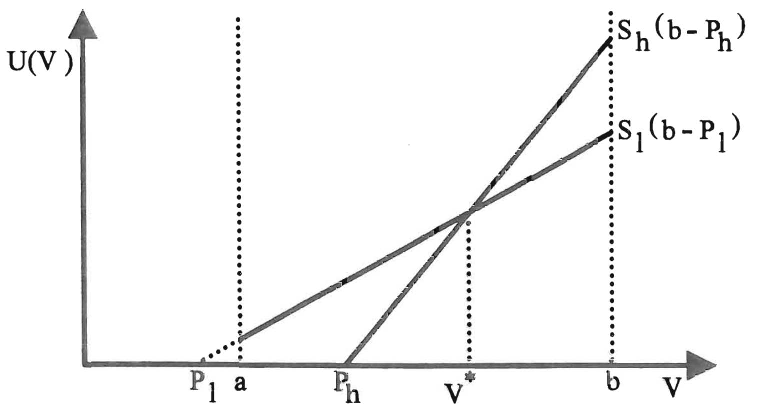
[4]Being a high-quality firm is often associated with large fixed costs, while marginal costs need not differ much across firms.

Figure 1



We make the same restriction on the income distribution as Shaked and Sutton:

A1: $\frac{1}{4} \leq \frac{a}{b} \leq \frac{1}{2}$.

In their paper, [Shaked and Sutton (1982)] A1 is shown to ensure full market coverage in the base game. However, an additional restriction is made here to ensure the poorest consumer a strictly positive utility level when buying the low-quality good in the base-game equilibrium. This also yields necessary and sufficient conditions for all consumers having a strictly positive demand in the base-game.

A2: $\frac{a}{b} > \frac{S_h-S_l}{2S_h+S_l}$

Letting (2) equal (3) and solving for the income level of the consumer that is indifferent between firms, V^*, we have

$$V^* = \frac{P_h S_h - P_l S_l}{S_h - S_l} \ . \tag{4}$$

Thus, with full market coverage, firm h will face demand b-V* while firm l faces demand V*-a. Consequently, profits are

$$\pi_h = P_h(b - V^*) \tag{5}$$

and

$$\pi_l = P_l(V^* - a) \ . \tag{6}$$

4. Pricing strategies

4.1 The punishment prices

The punishment prices are simply the NE prices in the one-shot base game. By assumptions A1 and A2, the market is covered at the equilibrium prices so firm profits are given by (5) and (6). Let $\alpha \equiv S_l/S_h$ (where $0 \leq \alpha \leq 1$) denote the degree of differentiation, so that a small α indicates a large amount of differentiation. Then, individual profit maximization yields the following reaction functions:[5]

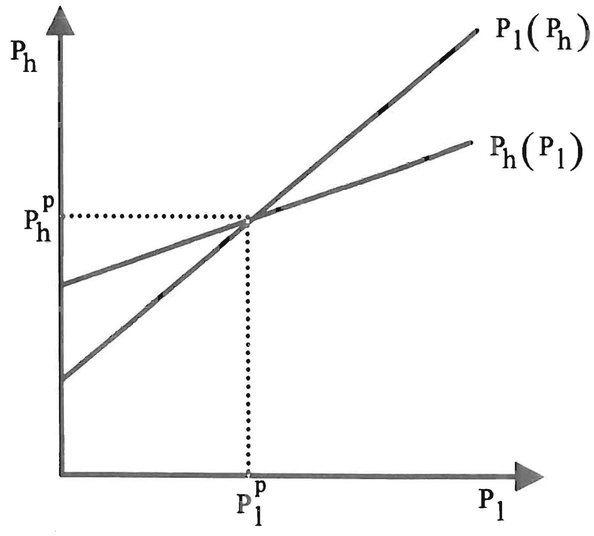
$$P_h(P_l) = \frac{1}{2}[\alpha P_l + (1-\alpha)b]$$

and

$$P_l(P_h) = \frac{1}{2\alpha}[P_h - (1-\alpha)a] \ .$$

[5]The second-order conditions have been checked for all optimization problems.

Figure 2



Hence, prices are strategic complements. The equilibrium prices, superscript p denoting punishment, are

$$P_h^p = \frac{(1-\alpha)(2b-a)}{3} \tag{7}$$

and

$$P_l^p = \frac{(1-\alpha)(b-2a)}{3\alpha} , \tag{8}$$

and the corresponding payoffs are

$$\pi_h^p = \frac{(1-\alpha)(2b-a)^2}{9} \tag{9}$$

and

$$\pi_l^p = \frac{(1-\alpha)(b-2a)^2}{9\alpha} \quad . \tag{10}$$

It should be noted that assumption A2 is equivalent to $P_l^p < a$, which ensures the poorest consumer a strictly positive utility level when buying the low-quality good.

4.2 The collusive prices

It is not obvious how to define collusive pricing in an asymmetric game. One way to handle the problem is to use an established criterion like the Nash bargaining solution.[6] Unfortunately, it proves analytically difficult to apply such a criterion. Instead we make a more or less ad hoc assumption that collusive prices maximize the size of the pie. It then turns out that both firms will benefit from collusion unless quality differences are very large. If one accepts the idea that firms are unwilling to forego joint profits in order to accomplish an alternative distribution, this assumption does not seem too far-fetched.[7]

First, assume that the prices maximizing joint profits are so high that the market is not entirely covered. It is evident that those who choose not to buy are the poorest consumers and that these consumers would have preferred the low quality good, had prices been lower. Then, from (3) it follows that people with incomes lower than P_l will choose not to buy. The demand facing the l-firm is therefore V^*-P_l so joint profits are,

$$\sum \pi = P_l(V^* - P_l) + P_h(b - V^*) \quad . \tag{11}$$

Since $\Sigma\pi$ is a concave function, setting the partial derivatives (wrt P_l and P_h) equal to zero yields necessary and sufficient conditions for a global maximum. The

[6]See Nash (1950).

[7]It does not seem realistic to assume that firms use trigger strategies to enforce very complex collusive contracts. Therefore, we do not allow for side-payments.

corresponding prices are[8]

$$P_h^c = \frac{2b}{3+\alpha} \tag{12}$$

and

$$P_l^c = \frac{b(1+\alpha)}{3+\alpha} \ , \tag{13}$$

where superscript c denotes collusion, and profits are

$$\pi_h^c = \frac{2b^2}{(3+\alpha)^2} \tag{14}$$

and

$$\pi_l^c = \frac{(1+\alpha)b^2}{(3+\alpha)^2} \ . \tag{15}$$

Now, let us instead assume that the entire market is served at the prices that maximize joint profits. Then

$$\sum \pi = P_l(V^*-a) + P_h(b-V^*) \ . \tag{16}$$

For $P_l^c > a$ the market would not be covered, which contradicts our assumption. And for $P_l^c < a$, both firms could raise prices by a small amount without taking a loss in terms of total demand. Joint profit maximization must therefore imply $P_l^c = a$. Differentiating (16) with respect to P_h then yields

$$P_h^c = \frac{a(1+\alpha)+b(1-\alpha)}{2} \ , \tag{17}$$

and the corresponding profit functions are

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[8]This maximization problem is conceptually equivalent to the problem of a monopolist producing two different qualities. It is interesting to note that both varieties are in fact produced even though quality is costless and valued by all consumers. This enables the monopolist to profitably price discriminate between the rich and the poor.

$$\pi_h^c = \frac{(b-a)[a(1+\alpha)+b(1-\alpha)]}{4} \tag{18}$$

and

$$\pi_l^c = \frac{a(b-a)}{2} \ . \tag{19}$$

Intuitively, full market coverage seems more likely when the income distribution is narrow, i.e. when consumers are fairly homogeneous. This intuition turns out to be true.

Lemma 1: The market is left partly uncovered if and only if

$$\frac{a}{b} \leq \frac{1+\alpha}{3+\alpha} \ .$$

Proof: Since $P_l^c = a$ is a permissible choice assuming the market is not covered, $P_l^c > a$ must imply that partial coverage is more profitable. This, however, is equivalent to the condition above. □

As we shall see, both firms benefit from collusion the way it is defined, unless differentiation is very large.

Lemma 2: Assume that the collusive agreement specifies joint profit maximization. With partial market coverage, collusion will raise the profit level of both firms if the following condition holds:

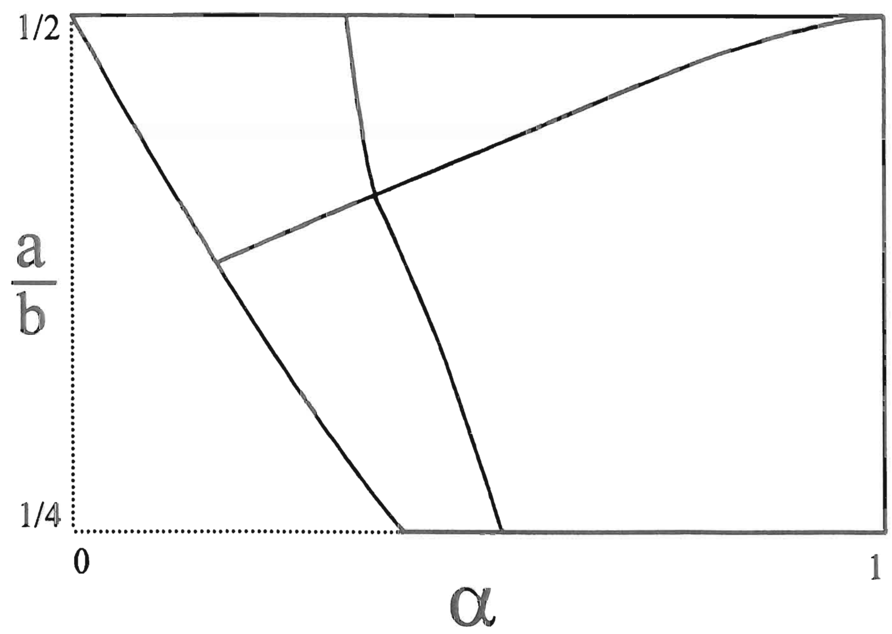
$$\frac{a}{b} \geq 2 - \frac{3\sqrt{2}}{(3+\alpha)\sqrt{1-\alpha}} \ .$$

If there is full market coverage under collusion, the corresponding condition is:

$$\frac{a}{b} \geq \frac{8+\alpha-3\sqrt{4\alpha^2+8\alpha-3}}{13+5\alpha} \quad .$$

Proof: Follows from comparing (14), (15) and (18), (19) with (9) and (10). The binding restriction turns out to be for the high-quality firm.□

Figure 3



Assumptions A1 and A2, and lemma 1 and 2 are shown graphically in figure 3. The area within solid lines is the parameter space satisfying A1 and A2, so that all consumers have a strictly positive demand in the base-game equilibrium. This area is divided horizontally and vertically by two lines. In the upper two regions, the income distribution is relatively narrow so the market will be fully covered at the profit-maximizing prices. In the lower two regions this is not the case. This is, of course, equivalent to lemma 1. In the two regions to the right, α is large, so products are close substitutes. This makes the market relatively competitive in absence of collusion, so both firms potentially have much to gain from forming a cartel. In the two regions to the left, products are remote substitutes meaning that the high-quality firm offers a significantly better product than the low-quality firm. Given the way collusive prices are defined, the high-quality firm prefers not to collude. This is equivalent to lemma 2. Naturally, we restrict our interest to parameters such that both firms find it worthwhile to collude, i.e. we study the two right-hand side regions that satisfy the inequalities of lemma 2. In other words, we assume that products are not too different.

4.3 The deviation prices

The optimal deviation strategy for the high-quality firm is computed straightforwardly. Since the h-firm by definition offers a better product than the l-firm, demand is quite sensitive to price reductions and this makes it profitable for the h-firm to steal the entire market. Once that is done, it faces the demand function of a monopolist which gives it weak incentives to cut back on price further. Consequently, the high-quality firm will steal the entire market from the low-quality firm, but it will never charge a price lower than P_l^c. Letting superscript d denote deviation, we have:

Lemma 3: The optimal deviation price for the high-quality firm is $P_h^d = P_l^c$.

Proof: In the appendix

The deviation payoffs for the h-firm are then

$$\pi_h^d = \frac{2b^2(1+\alpha)}{(3+\alpha)^2} , \tag{20}$$

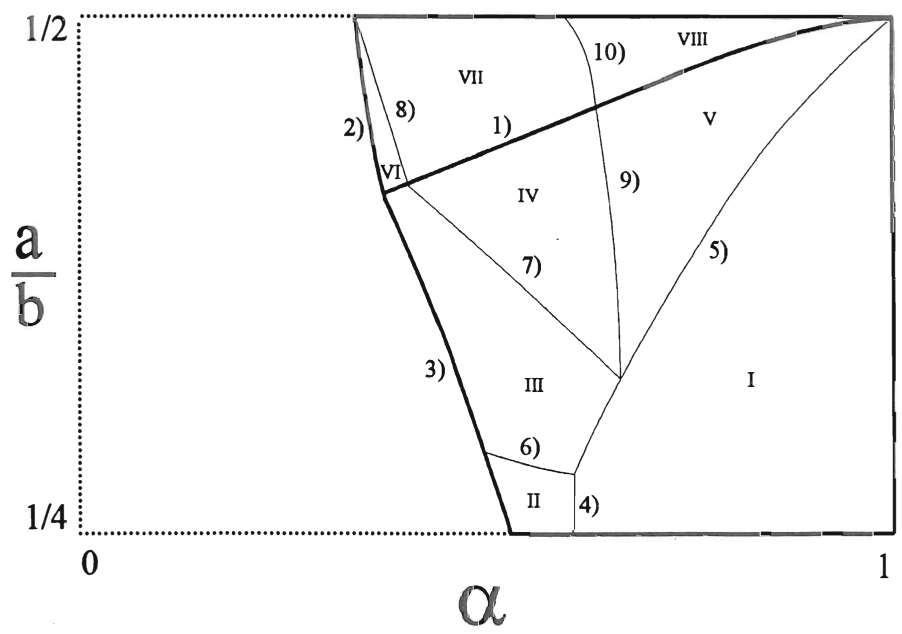
when market coverage is partial under collusion, and

$$\pi_h^d = a(b-a) \tag{21}$$

otherwise.

Regarding the l-firm, the situation is more complex. Figure 4 shows all possible combinations of income distributions, measured by a/b, and product differentiation, measured by α, that are permissible by assumptions A1 and A2, and where the h-firm gains from collusion (lemma 2). There are two regions within bold lines. In the upper region, incomes are relatively close; hence, collusive pricing results in full market coverage. In the lower region, there is partial coverage. Again, this is equivalent to lemma 1. Each sub-section in figure 4 represents a certain deviation profit function for the l-firm. The explicit functional forms of the lines 1)-10), dividing the sections, are provided in the appendix.

Figure 4



Let us begin by studying the sections in the lower region, so that there is partial market coverage under collusion. Then assume that the l-firm considers a price deviation. It has the choice between capturing all demand previously "belonging" to the h-firm and making a partial theft so that some consumers keep patronizing the (non-deviant) h-firm. Moreover, since the market was not fully covered in the collusive phase, the market may, or may not, be covered in the deviation period. Hence, there are basically four possible combinations.

MARKET COVERAGE

THEFT		PARTIAL	FULL
	PARTIAL	II	III IV
	TOTAL	I	V

First, if products are close substitutes, small price changes will result in large shifts in demand between firms and therefore a total theft seems natural. Indeed, in sections I and V a total theft is optimal, while in sections II, III and IV, a partial theft is optimal. A total theft is equivalent to making the richest consumer indifferent between firms. When the l-firm has captured all demand previously belonging to the h-firm, it in fact faces the demand function of a monopolist which means it has weak incentives to cut back on price further.

Second, the more narrow the income distribution, i.e. the more homogeneous the consumers, the more likely it is for the market to become covered when the l-firm deviates by lowering its price. In terms of figure 4, the market becomes covered in sections III, IV and V while it remains partly uncovered in sections I and II. Finally, in sections III and IV there is a partial theft and full market coverage in the deviation phase. However, when the income distribution is relatively dispersed (i.e. section III) there is a corner solution so that the poorest consumer is just indifferent between buying and not buying at the optimal deviation price.

To sum up:

Definition 1: *Section I is the set of parameters for which*

(i) *the collusive prices result in partial market coverage,*

(ii) *firm l's optimal deviation price results in a total theft, and*

(iii) *the market is left partly uncovered when firm l deviates.*

The deviation payoffs corresponding to section I are

$$\pi_l^d = \frac{b^2(1+\alpha)(\alpha^2+2\alpha-1)}{\alpha^2(3+\alpha)^2} \quad . \tag{22}$$

Definition 2: *Section II is the set of parameters for which*

(i) *the collusive prices result in partial market coverage,*

(ii) *firm l's optimal deviation price results in a partial theft, and*

(iii) *the market is left partly uncovered when firm l deviates.*

The deviation payoffs corresponding to section II are

$$\pi_l^d = \frac{b^2}{(3+\alpha)^2(1-\alpha)} \quad . \tag{23}$$

Definition 3:	*Section III is the set of parameters for which*
(i)	*the collusive prices result in partial market coverage,*
(ii)	*firm l's optimal deviation price results in a partial theft,*
(iii)	*the market is fully covered when firm l deviates, and*
(iv)	*the poorest consumer is indifferent between buying and not buying when firm l deviates.*

The deviation payoffs corresponding to section III are

$$\pi_l^d = \frac{a[2b-a(3+\alpha)]}{(1-\alpha)(3+\alpha)} \ . \tag{24}$$

Definition 4:	*Section IV is the set of parameters for which*
(i)	*the collusive prices result in partial market coverage,*
(ii)	*firm l's optimal deviation price results in a partial theft,*
(iii)	*the market is fully covered when firm l deviates, and*
(iv)	*the poorest consumer strictly prefers buying when firm l deviates.*

The deviation payoffs corresponding to section IV are

$$\pi_l^d = \frac{[a(\alpha^2+2\alpha-3)+2b]^2}{4\alpha(1-\alpha)(3+\alpha)^2} \ . \tag{25}$$

Definition 5:	*Section V is the set of parameters for which*
(i)	*the collusive prices result in partial market coverage,*
(ii)	*firm l's optimal deviation price results in a total theft,*
(iii)	*the market is fully covered when firm l deviates, and*
(iv)	*the poorest consumer strictly prefers buying when firm l deviates.*

The deviation payoffs corresponding to section V are

$$\pi_l^d = \frac{b(\alpha^2+2\alpha-1)(b-a)}{\alpha(3+\alpha)} \quad . \tag{26}$$

Now, consider the upper region in figure 4 where the market is covered under collusion. Then, obviously, the market is covered also when firm 1 deviates by lowering its price, so the question is reduced to whether firm 1 will choose a partial or a total theft. Again, if products are close substitutes, small price changes will result in large shifts in demand between firms; hence, a total theft becomes relatively profitable. Indeed, in section VIII there is a total theft, while in sections VI and VII there is a partial theft. In section VI, a deviating l-firm is not very fortunate. Differentiation is relatively large, meaning that the h-firm offers a significantly better product, and the income distribution is relatively narrow so consumers are fairly homogenous. It turns out that the optimal deviation price then coincides with the collusive price, so the l-firm cannot increase profits by deviating.

Definition 6: *Section VI is the set of parameters for which*

(i) *the collusive prices result in full market coverage, and*

(ii) *firm l's optimal deviation price is* $P_l^d = P_l^c = a$.

Clearly, in section VI, $\pi_l^d = \pi_l^c$.

Definition 7: *Section VII is the set of parameters for which*

(i) *the collusive prices result in full market coverage, and*

(ii) *firm l's optimal deviation price results in a partial theft.*

The deviation payoffs corresponding to section VII are

$$\pi_l^d = \frac{[a(3\alpha-1)+b(1-\alpha)]^2}{16\alpha(1-\alpha)} \quad . \tag{27}$$

Definition 8: *Section VIII is the set of parameters for which*

(i) *the collusive prices result in full market coverage, and*

(ii) *firm l's optimal deviation price results in a total theft.*

The deviation payoffs corresponding to section VIII are

$$\pi_l^d = \frac{[a(1+\alpha)-b(1-\alpha)](b-a)}{2\alpha} \quad . \tag{28}$$

Lemma 4: The parameter sets corresponding to sections I-VIII can be defined in a precise way.

Proof: In the appendix

5. The discount factor restriction

The discount factor restriction, $\gamma(\alpha)$, can now be computed for any given income distribution. Since this function is firm-specific, indices h and l are used from now on. Each section in figure 4 is characterized by a unique combination of collusive payoffs and deviation payoffs. Hence, for a given income distribution, the γ-function will consist of several segments. For example, when computing $\gamma_l(\alpha)$ for an income distribution close to a/b=1/4, collusive payoffs are given by (15) since we are in the lower region. Deviation payoffs are given by (23) for small α's (in section II) and by (22) for large α's (in section I). Finally, payoffs in the punishment phase are given by (10). All segments constituting $\gamma_l(\alpha)$ and $\gamma_h(\alpha)$ can be computed in a straightforward manner by inserting the relevant profit functions into expression (1). These functions, provided in appendix, are not very informative per se. However, it is possible to show the following:

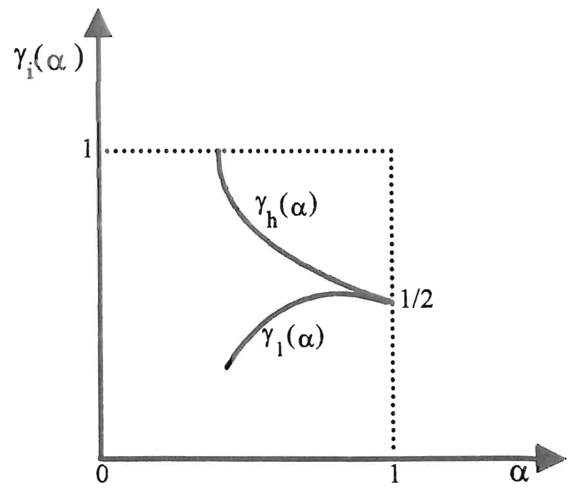
Lemma 5: The functions $\gamma_h(\alpha)$ and $\gamma_l(\alpha)$ are continuous.

Proof: Follows from inserting the relevant boundary functions 1)-10).□

Theorem: $\gamma_h(\alpha)$ is everywhere decreasing, approaching $\gamma_h(\alpha)=1/2$ for $\alpha=1$. $\gamma_l(\alpha)$ also approaches $\gamma_l(\alpha)=1/2$ for $\alpha=1$, but $\gamma_l(\alpha)<\gamma_h(\alpha)$ for all $\alpha<1$.

Proof: Follows from differentiation and inserting $\alpha=1$. $\gamma_l(\alpha)$ is increasing in α except for section I where it is concave, first increasing and finally decreasing. Taking the difference between $\gamma_h(\alpha)$ and $\gamma_l(\alpha)$ in section I it can be shown that $\gamma_l(\alpha)<\gamma_h(\alpha)$.□

Figure 5



Typical γ-functions are shown in figure 5. It is not surprising that the binding restriction turns out to be on the high-quality firm. Since its price-cost margin is relatively large, the profitability of a quantity expansion is relatively high which gives it a stronger incentive to deviate. In addition, the consumers' valuation of quality makes payoffs in the punishment phase relatively high for the h-firm. Since $\gamma_h(\alpha)$ is the binding restriction, and it is decreasing in α, it follows that collusive pricing is most easily sustained when products are identical. We could therefore expect to see more of collusive pricing in markets where products are relatively similar. The intuition is the following. As products become better substitutes, the payoffs in the punishment phase are reduced to such an extent that deviation becomes less attractive for the h-firm despite increasing deviation payoffs and decreasing collusive payoffs. This is clearly in total contrast to the conclusions of Chang (1991) and Häckner (chapter II). The explanation is that the payoffs of the punishment phase have a greater impact in the vertical case. When products are remote substitutes, the h-firm is quite well off already in absence of collusion, while the l-firm would agree to collude at almost any discount factor. As this asymmetry is reduced, and non-collusive payoffs become lower for the h-firm, reaching a collusive agreement is gradually facilitated.

6. Conclusions

Despite a similar structure of the models, there is a positive relationship between differentiation and sustainability in the Hotelling framework, while it is negative in the Shaked and Sutton framework. The asymmetry introduced by vertical differentiation gives the high-quality firm weak incentives to collude when products are remote substitutes. In the Hotelling model there is no such asymmetry and the driving force is that differentiation makes deviation payoffs small which facilitates collusion.

Since the results are extremely sensitive to the quality concept used, no clearcut prediction can be made unless products are differentiated in **one** dimension only (horizontally or vertically). If products are differentiated in both dimensions, theory seems to provide little guidance.

In this chapter we wanted to keep the analysis as close as possible to the work conducted by Chang and Häckner in order to facilitate a comparison.[9] There are at least two extensions that would follow naturally. First, since the results are conditional on a specific form of collusion, namely joint profit maximization, other coalitions specifying other collusive prices could also be sustainable. Hence, it may be possible to generate results that differ from ours. Re-examining the model for a more general class of coalitions would therefore be an interesting challenge. Second, it would be interesting to endogenize product design. This could alter the relationship but a priori it is difficult to predict in what way.

[9]See Chang (1991) and chapter II in this book.

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Appendix

The boundary functions of figure 4[10]

1) $$\frac{a}{b} = \frac{1+\alpha}{3+\alpha}$$

2) $$\frac{a}{b} = \frac{8+\alpha-3\sqrt{4\alpha^2+8\alpha-3}}{13+5\alpha}$$

3) $$\frac{a}{b} = 2 - \frac{3\sqrt{2}}{(3+\alpha)\sqrt{1-\alpha}}$$

4) $$\alpha = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

5) $$\frac{a}{b} = \frac{\alpha^2+2\alpha-1}{\alpha(3+\alpha)}$$

6) $$\frac{a}{b} = \frac{1}{3+\alpha}$$

7) $$\frac{a}{b} = \frac{2}{\alpha^2+4\alpha+3}$$

8) $$\frac{a}{b} = \frac{1-\alpha}{1+\alpha}$$

9) $$\frac{a}{b} = \frac{2(\alpha^2+2\alpha-2}{\alpha^2+2\alpha-3}$$

[10]Expression 1) is the boundary condition from lemma 1. Expressions 2) and 3) are boundary conditions from lemma 2. Finally, expressions 4)-10) are boundary conditions for the l-firm's deviation profit functions used in the proof of lemma 4.

10) $$\frac{a}{b} = \frac{3(1-\alpha)}{3-\alpha}$$

Proof of lemma 3

First, consider the case when collusive pricing results in partial market coverage. Assume $P_h^d > P_l^c$ so that the h-firm leaves a positive market share to the l-firm. This is equivalent to having an interior solution when maximizing $\pi_h = P_h(b\text{-}V^*)$ with respect to P_h after having inserted $P_l = P_l^c$. Then,

$$P_h^d = \frac{b(3-\alpha)}{2(3+\alpha)} \quad .$$

This, however, is smaller than P_l^c unless $\alpha < 1/3$. But if $\alpha < 1/3$, the first inequality of lemma 2 does not hold, so collusion would not have been sustainable in the first place. Now, assume $P_h^d < P_l^c$ so that all consumers with a positive demand strictly prefer the h-good. Then, profits are given by $\pi_h = P_h(b\text{-}P_h)$ which is increasing in P_h for $P_h < P_l^c$, hence $P_h^d < P_l^c$ cannot be an optimal deviation strategy. Thus, deviation payoffs are maximal for $P_h^d = P_l^c$.

Second, consider the case when the collusive prices result in full market coverage. Clearly, $P_h^d < P_l^c$ is not a good idea since then the deviation price could be increased without any loss in demand. $P_h^d > P_l^c$ is equivalent to having an interior solution when maximizing $\pi_h = P_h(b\text{-}V^*)$ with respect to P_h after having inserted $P_l^c = a$. Then,

$$P_h^d = \frac{a\alpha + b(1-\alpha)}{2} \quad .$$

This, however, is smaller than P_l^c unless,

$$\frac{a}{b} < \frac{1-\alpha}{2-\alpha} \quad ,$$

but then the second inequality of lemma 2 does not hold. Hence, collusion would not have been sustainable in the first place. Thus, deviation payoffs are maximal for $P_h^d = P_l^c$.□

Proof of lemma 4

SECTION I

If α is larger than the right-hand side of 4) profits are decreasing in P_l for all

$$P_l > P_l^d = \frac{b(\alpha^2+2\alpha-1)}{\alpha(3+\alpha)} ,$$

at which point consumer b is indifferent between firms. Moreover, if a/b is smaller than the right-hand side of 5), then $P_l^d \geq a$ implying partial market coverage. Lowering the deviation price further would not be profitable since then profits are $\pi_l = P_l(b-P_l)$ which is increasing in P_l for $P_l = P_l^d$. Finally, the right-hand side of 5) is smaller than the right-hand side of 1) so the collusive prices result in a partial market coverage. Since $\alpha \leq 1$ by definition and $a/b \geq 1/4$ by assumption A1, section I is fully characterized.

SECTION II

In section II there must be an interior solution when maximizing $\pi_l = P_l(V^*-P_l)$ with respect to P_l letting $P_h = P_h^c$. This yields

$$P_l^d = \frac{b}{3+\alpha} ,$$

which is greater than a (implying partial market coverage) if α is smaller than the right-hand side of 4). Moreover, given this deviation price, $V^* < b$ (implying a partial theft) if a/b is smaller than the right-hand side of 6). Since the right-hand side of 6) is smaller than the right-hand side of 1), the collusive prices result in partial market coverage. Finally, since $a/b \geq 1/4$ by assumption A1, and a/b is larger than the right-hand side of 3) by lemma 2, section II is fully characterized.

SECTION III

First, assume $P_l^d > a$ so that $\pi_l = P_l(V^*-P_l)$ is the relevant profit function. Then, for a/b larger than the right-hand side of 6) payoffs are decreasing in P_l when evaluated at $P_l = a$. Assuming $P_l^d < a$ so that $\pi_l = P_l(V^*-a)$ is the relevant profit function, payoffs are increasing in P_l when evaluated at $P_l = a$ as long as a/b is smaller than the right-hand side of 7). Hence, payoffs are maximal for $P_l^d = a$. Given that $P_l = a$ and $P_h = P_h^c$, $V^* < b$ as long as a/b is larger than the right-hand side of 5). Hence, the theft is partial. Assuming that a/b is smaller than the right-hand side of 1) the collusive prices will result in partial market coverage. Finally, since a/b is larger than the right-hand side of 3) by lemma 2, section III is fully characterized. (It can be checked that assumption A1 is not violated)

SECTION IV

In this case there must be an interior solution when maximizing $\pi_l = P_l(V^* - a)$ with respect to P_l letting $P_h = P_h^c$. This yields

$$P_l^d = \frac{a(\alpha^2 + 2\alpha - 3) + 2b}{2\alpha(3+\alpha)} ,$$

which is smaller than a for a/b larger than the right-hand side of 7). This implies full market coverage. Moreover, $V^* < b$ for a/b smaller than the right-hand side of 9) which makes the theft partial. Assuming that a/b is smaller than the right-hand side of 1) guarantees partial market coverage at the collusive prices. Hence, section IV is fully characterized. (It can be checked that assumption A1 is not violated.)

SECTION V

Defining $\pi_l = P_l(V^* - a)$ and assuming a/b is larger than the right-hand side of 9), payoffs are decreasing in P_l for

$$P_l > P_l^d = \frac{b(\alpha^2 + 2\alpha - 1)}{\alpha(3+\alpha)} ,$$

at which point consumer b is indifferent between firms. However, lowering prices further would not increase demand since $P_l^d < a$ given that a/b is larger than the right-hand side of 5) which means that the entire market is already captured. Hence, deviation payoffs are maximal for $P_l = P_l^d$. Assuming that a/b is smaller than the right-hand side of 1) guarantees partial market coverage in the collusive phase. Hence section V is fully characterized. (It can be checked that assumption A1 is not violated.)

SECTION VI

Defining $\pi_l = P_l(V^* - a)$, letting $P_h = P_h^c$ and assuming a/b is smaller than the right-hand side of 8) profits are increasing in P_l at $P_l = a$. However, if profits instead are defined as $\pi_l = P_l(V^* - P_l)$ they are decreasing in P_l at $P_l = a$ as long as a/b is larger than the right-hand side of 2). Hence, the profit-maximizing deviation price coincides with the collusive price, $P_l^d = P_l^c = a$. Assuming that a/b is larger than the right-hand side of 1) guarantees full market coverage at the collusive prices. Hence, section VI is fully characterized. (It can be checked that assumption A1 is not violated.)

SECTION VII

In this case there must be an interior solution when maximizing $\pi_l=P_l(V^*-a)$ with respect to P_l letting $P_h=P_h^c$. This yields

$$P_l^d = \frac{a(3\alpha-1)+b(1-\alpha)}{4\alpha},$$

which is smaller than a for a/b larger than the right-hand side of 8) implying full market coverage. Moreover, $V^*<b$ for a/b smaller than the right-hand side of 10) implying a partial theft. Assuming that a/b is larger than the right-hand side of 1) guarantees full market coverage at the collusive prices. Finally, $a/b\leq 1/2$ by assumption A1. This fully characterizes section VII.

SECTION VIII

Defining $\pi_l=P_l(V^*-a)$ and assuming a/b is larger than the right-hand side of 10), payoffs are decreasing in P_l for

$$P_l > P_l^d = \frac{a(1+\alpha)-b(1-\alpha)}{2\alpha},$$

at which point consumer b is indifferent between firms. However, lowering prices further will not increase demand since $P_l^d<a$ implying that the entire market is already captured. Therefore, the price maximizing the deviation payoff is P_l^d. Assuming that a/b is larger than the right-hand side of 1) guarantees full market coverage at the collusive prices. Finally $a/b\leq 1/2$ by assumption A1. Hence, section VIII is fully characterized.□

The discount factor restriction

Defining $\beta\equiv a/b$ and $\alpha\equiv S_l/S_h$, the discount factor restriction consists of the following segments:
For the high-quality firm

$$\gamma_h(\alpha) = \frac{18\alpha}{\alpha^2(5+\alpha)(\beta^2-4\beta+4)+3\alpha(\beta^2-4\beta+10)-9(\beta^2-4\beta+2)}$$

in sections I-V, and

$$\gamma_h(\alpha) = \frac{9(1-\beta)[\alpha(1-\beta)+3\beta-1]}{4[\alpha(\beta^2-4\beta+4)-10\beta^2+13\beta-4]}$$

in sections VI-VIII. For the low-quality firm

$$\gamma_f(\alpha) = \frac{9(1+\alpha)(2\alpha-1)}{\alpha^4(4\beta^2-4\beta+1)+2\alpha^3(10\beta^2-10\beta+7)+6\alpha^2(2\beta^2-2\beta+5)+36\alpha\beta(1-\beta)-9}$$

in section I,

$$\gamma_f(\alpha) = \frac{9\alpha^3}{(\alpha^4+4\alpha^3-2\alpha^2+9)(4\beta^2-4\beta+1)-3\alpha(16\beta^2-16\beta+7)}$$

in section II,

$$\gamma_f(\alpha) = \frac{9\alpha[(1-3\beta)^2-\alpha^2(1-\beta^2)-2\alpha\beta(1-3\beta)]}{[(3+\alpha^3)(4\beta^2-4\beta+1)+\alpha^2(13\beta^2-4\beta+1)+\alpha(7\beta^2+2\beta-5)](3+\alpha)}$$

in section III,

$$\gamma_f(\alpha) = -\frac{9[\alpha^4\beta^2+4\alpha^3(1+\beta^2)+2\alpha^2\beta(2-\beta)-4\alpha(3\beta^2-2\beta+1)+9\beta^2-12\beta+4]}{\alpha^3(4+\alpha)(7\beta^2-16\beta+4)-2\alpha^2(7\beta^2+2\beta+4)-12\alpha(7\beta^2-10\beta+4)+9\beta(7\beta-4)}$$

in section IV,

$$\gamma_f(\alpha) = -\frac{9[(3-\alpha^3)(1-\beta)+\alpha(1+\alpha)(5\beta-4)]}{[\alpha(\alpha+2)(4\beta^2-13\beta+10)-3(4\beta^2-7\beta+4)](3+\alpha)}$$

in section V,

$$\gamma_f(\alpha) = 0$$

in section VI,

$$\gamma_f(\alpha) = \frac{9[(\alpha^2(\beta^2+2\beta+1)-2\alpha(1-\beta^2)+(1-\beta)^2]}{\alpha^2(17\beta^2+10\beta-7)+2\alpha(37\beta^2-28\beta+7)-55\beta^2+46\beta-7}$$

in section VII, and finally,

$$\gamma_f(\alpha) = \frac{9(1-\beta)(1-\alpha-\beta)}{\alpha(\beta^2+8\beta-11)+17\beta^2-26\beta+11}$$

in section VIII.

Chapter IV

Vanity and congestion: a study of reciprocal externalities[1]

1. Introduction

The pleasure derived from consuming a good is sometimes affected by the consumption patterns of other people. Such consumption externalities may be of a one-way type, as when a living-room view is obstructed by neighboring houses, or it may be reciprocal, as when driving a car reduces the street space available for other drivers, making driving less enjoyable. In this chapter we study welfare aspects of negative reciprocal externalities, of which congestion is a special case.

Negative externalities have long been a favorite topic of economic inquiry, but studies have normally abstracted from strategic behavior on the production side. For many applications this is a natural assumption to make, for instance when studying optimal capacity and fee structures for publicly provided goods, like street space [See e.g. Vickrey (1969)].[2]

Reciprocal externalities are, however, likely to be important also in markets for private goods and services in that they affect the strategic interaction between firms. In the literature on clubs, Bertrand competition is shown not to ensure marginal cost pricing in the presence of congestion.[3] The reason for this is that increased demand

[1]This chapter is joint work together with Sten Nyberg.

[2]A discussion of the more general problem of designing corrective taxes in the presence of externalities can be found in Diamond (1973) or Green and Sheshinski (1976). For the case of equal and reciprocal externalities Diamond shows that a uniform price, in excess of marginal cost by the value of the externality, permits the competitive equilibrium to be Pareto optimal.

[3]In Scotchmer (1985a), private clubs subject to congestion, like golf courses and sports clubs, are shown to choose membership fees above marginal cost in a Bertrand game. In contrast to our framework, consumer demand is assumed to be perfectly inelastic, so the question of price efficiency cannot be addressed. This assumption is relaxed in Scotchmer (1985b) but instead firms choose a two-part tariff consisting of membership fees and user charges. In equilibrium, firms tend to set low charges in order to increase the consumer surplus captured by the membership fee. In this paper, it will become clear that competition in linear prices have quite different implications.

results in more congestion which, in turn, reduces consumers' willingness to pay for the good. Hence, price cuts tend to be undesirable. On the other hand, the socially efficient price ends up being higher than the marginal cost in order to compensate for the negative externality. The question is whether prices are high enough or too high. Another example of reciprocal consumption externalities is given by markets for prestigious brand-name goods where substantial output expansions may cause brand-name debasement. For instance, if everyone wore Rolex watches, wearing one yourself would do little to enhance your prestige. [See e.g. Veblen (1899) and Hirsch (1976)]

Historically, policymakers have been inclined to thoroughly regulate some congested markets. The transportation sector is perhaps the best example. In most countries practically all transportation services, (airlines, the trucking industry, railroads, taxis, etc.), have been subject to extensive regulation, both in terms of price and entry. It is easy to see that congestion is a real issue in such markets. For instance, flights are less likely to be overbooked the smaller the number of passengers. And the availability of taxis decreases, i.e. the waiting time increases, when per cab demand increases. Whether negative consumption externalities provide a rationale for regulatory intervention depends on the strength of the externalities relative to the costs of regulation. Such costs would seem to depend on the context (availability of information etc.), and optimal regulation is used only as a benchmark in the analysis.

The aim of this chapter is to study price formation and economic efficiency on oligopolistic private goods markets characterized by reciprocal consumption externalities and price competition. The chapter is organized as follows: In section 2, the basic model is presented and the price equilibrium is characterized. In section 3, we examine welfare issues. Endogenous entry is discussed in section 4, and the chapter concludes in section 5 with some final remarks.

2. The model

There are two types of goods. One type of good is available in a number of different brands of identical intrinsic quality, and the other type is a composite good representing consumption of everything else. For the brand-name good, consumer utility is assumed to be increasing in the amount consumed, but at a decreasing rate. Furthermore, brands can be differentiated in terms of exclusiveness (i.e. total sales) and utility is increasing in exclusiveness (decreasing in the volume of sales of a certain brand). The marginal utility from consuming the composite good is assumed to be approximately constant for reasonable ranges of income. The utility function of a consumer j purchasing brand name good i can be written

$$U_{j,i}=U(y_{j,i},\ q_{j,i},\ Q_i)\ , \tag{1}$$

where $y_{j,i}$ and $q_{j,i}$ denote consumption of the composite good and of the brand name good respectively and Q_i represents total sales of brand i. By assumption, U_1, $U_2>0$, $U_3<0$, $U_{11}=0$ and $U_{22}<0$.

We assume that there is a continuum of identical consumers. The demand of an individual consumer patronizing firm i is derived by maximizing (1) with respect to $y_{j,i}$ and $q_{j,i}$ given that consumers correctly anticipate the equilibrium Q_i, and subject to the budget constraint $p_i q_{j,i} + y_{j,i} \leq I$, where the price of the composite good is normalized to unity. Furthermore, consumers do not perceive their own demand to influence the price-setting behavior of the firms. Nor do they take into account the effect of their own demand on exclusiveness.[4]

Let there be n firms each producing one brand-name good, possibly differentiated by exclusiveness. Consumers, being utility maximizers, would never buy from a firm unless it is the best deal around. Thus, for given prices, market shares, m_i, will adjust so that customers are indifferent between buying from different firms in equilibrium.[5] Consequently, expressed in terms of indirect utility,

$$V(p_1,\ Q_1,\ I) = V(p_2,\ Q_2,\ I) = \ldots = V(p_n,\ Q_n,\ I)\ , \tag{2}$$

which amounts to n-1 equations. The demand of a representative consumer patronizing

[4]A notable exception to this, however, is Groucho Marx's famous remark about joining clubs.

[5]Consumers being indifferent between firms of course introduces the need for some invisible hand guiding demand so that indifference actually holds. For example, if prices are equal and all "indifferent" consumers happen to patronize the same firm, they would not be indifferent any longer but rather realize that they all made a mistake.

firm i is derived using Roy's identity, yielding another n equations. Finally, the market shares add up to one, so there are 2n equations altogether. The total number of consumers being normalized to one, the aggregate demand facing a firm thus equals individual demand times the market share,

$$Q_i = q_{j,i} m_i \ , \tag{3}$$

which can be solved for explicitly using the 2n equations.

For the sake of tractability consumer preferences are assumed to have the simplest possible functional form consistent with the assumptions made. Consumer j's utility function is given by

$$U_{j,i} = y_{j,i} + (1-\alpha q_{j,i})q_{j,i} - \beta Q_i q_{j,i} \ . \tag{4}$$

The first term on the right-hand side is consumer j's consumption of the composite good, $y_{j,i}$, while the second term gives the quadratic gross utility from consuming the differentiated good, $q_{j,i}$. The last term reflects individual j's disutility of the consumption of others, Q_i, which is assumed to increase in his own consumption of variety i. Hence marginal utility and individual demand depend on exclusiveness. The decrease in utility of additional consumption is parameterized by α while β measures the impact of the negative externality.[6] The individual demand and the indirect utility function are given by

$$q_{j,i} = \frac{1-p_i-\beta Q_i}{2\alpha} \tag{5}$$

and

$$V(p_i, Q_i, I) = \frac{(1-p_i-\beta Q_i)^2}{4\alpha} + I \ . \tag{6}$$

[6]For positive externalities, $\beta<0$, the equal utility condition, expression (7), will not hold (or be unstable) and there is a tendency towards natural monopolies. The strategic implications of positive externalities are discussed in the literature on networks. See for example Katz and Shapiro (1985) and (1986).

Hence, in this case expression (2) implies

$$p_1 + \beta Q_1 = p_2 + \beta Q_2 = \ldots = p_n + \beta Q_n \ . \tag{7}$$

Let p be the vector of prices charged by the firms. The marginal willingness to pay for one unit is at most one so $p_i \leq 1$ and **p** is a point in the price simplex $P=[0,1]^n$. The demand facing firm i can now be expressed as a function of **p**.

Lemma 1: The aggregate demand facing firm i is

$$Q_i = \frac{2\alpha(\sum_{j\neq i} p_j - (n-1)p_i) + \beta(1-p_i)}{\beta(2\alpha n + \beta)} \ .$$

Proof: In the appendix.

Firms maximize profits with respect to price while taking into account the strategic interdependence between price choices. Consequently, the appropriate equilibrium concept is the Nash equilibrium. Marginal production costs, c_i, are assumed to be constant and strictly less than one. The profit function of firm i is

$$\pi_i = (p_i - c_i)Q_i \ . \tag{8}$$

Having characterized consumer behavior and firm behavior, the next step is to characterize the market equilibrium. Substituting the demand of firm i into its profit function and maximizing with respect to p_i, while taking the other firms' prices as given, yields the best response function, φ_i, of firm i.[7]

$$\varphi_i(p_{-i}) = \frac{1}{2}\left[c_i + \frac{2\alpha\sum_{j\neq i} p_j + \beta}{2\alpha(n-1) + \beta}\right] \ . \tag{9}$$

The reaction functions are linear and upward sloping in the competitors' prices implying strategic complementarity. Furthermore, cost differences affect the intercepts,

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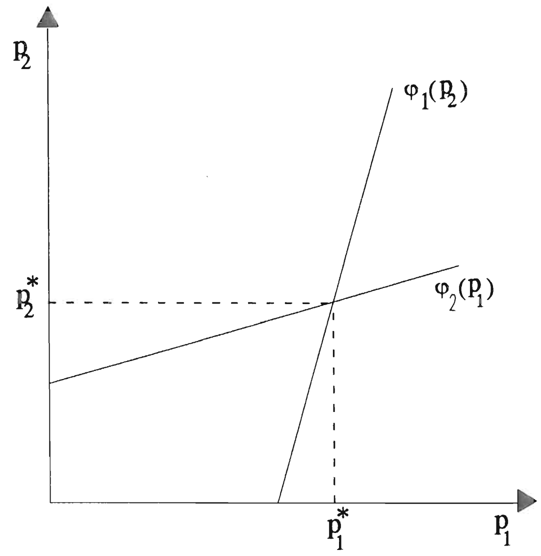
[7]Assuming Cournot competition instead does not change the analysis much. Equilibrium prices would be somewhat higher, but qualitatively all results would hold.

but not the slopes, i.e. the responsiveness to other firms' actions are unaffected. In figure 1, which illustrates the duopoly case, $c_1 > c_2$ making $p_1 > p_2$ in equilibrium.

Figure 1



If ß is large relative to α, the influence of the other firms' prices is very limited and the optimal price will be close to $(c_i + 1)/2$, i.e. the price that would be chosen by a profit maximizing monopolist. Nevertheless, this does not mean that extreme congestion is likely to be desirable from the perspective of the firms. On the contrary, if ß approaches infinity, consumers' valuation of the good is reduced to such an extent that firm demand goes to zero.

For a duopoly market, the existence of a unique and symmetric price equilibrium is intuitively clear and it can easily be established also in the n-firm case.

Proposition 1: There exists a unique equilibrium.

Proof: First, the price simplex, P, is a non-empty, compact and convex set. Furthermore, the vector-valued best-response function, $\Phi(\mathbf{p})$, is linear and thus u.h.c. and convex. Finally, it can easily be shown that $\Phi(\mathbf{p}) \subset P$ and thus Kakutani's theorem guarantees a fixpoint. Uniqueness then follows directly since $\Phi(\mathbf{p})$ is a contraction. □

Corollary 1: If $c_i = c$ for all i, then the equilibrium will be symmetric with $p_1 = p_2 = \ldots = p^*$.

$$p^* = \frac{2\alpha(n-1)c + \beta(c+1)}{2\alpha(n-1) + 2\beta}$$

Proof: Identical costs yield symmetric reaction functions ensuring a symmetric equilibrium. Solving (9) for $p_i = p_j$ yields p^*. □

Proposition 2: The equilibrium price, p^*, is increasing in β and for $\beta = 0$, $p^* = c$. When β approaches infinity, p^* approaches the monopolistic price, $(c+1)/2$.

Proof: Follows from differentiating p^*. □

Hence, equilibrium prices are above marginal cost despite the fact that firms compete in prices and products are undifferentiated in equilibrium, costs being equal. The undercutting strategy becomes unattractive since output expansions affect quality negatively. Technically speaking, in a standard Bertrand game, firm demand and profits are discontinuous at the lowest price charged by the competitors. This discontinuity is smoothed out by reciprocal externalities allowing a price differential to be compensated for by differences in quality. Hence, it is not possible to capture the entire market by undercutting the rival slightly. If β is small, the situation is nevertheless very similar to the standard Bertrand game with prices close to marginal cost and basically no profits. This suggests that there may be incentives for firms to deliberately try to influence the impact of congestion on consumer utility.[8]

Proposition 3: The equilibrium price, p, is decreasing in n and it approaches c as n approaches infinity.*

Proof: Follows from differentiating p*. □

Not surprisingly, an increase in the number of firms induces a more competitive market structure leading to lower prices.

3. Social welfare implications

Consumers do not take into account the negative externality they inflict on their fellow consumers in the sense that buying the product makes it less exclusive and hence less desirable for others. Thus, the equilibrium consumption of exclusive items, given a certain price, can be expected to be too high from the consumers' point of view. Indeed, this can easily be demonstrated to be the case. As shown above, the externality affects the strategic interaction between producers, thereby generating an equilibrium price that is above marginal cost. But the question is whether this price is sufficiently high to compensate for the externality, or whether it is really too high from a social point of view.

To facilitate the comparative static analysis we examine a symmetric price equilibrium where individuals choose the same q. Since consumers are identical, social welfare can be measured by the utility of the representative individual minus the per capita cost of production. Letting W denote social welfare,

[8] If, for example, transportation firms could commit to lower their capacity, it could be interpreted as an increase in ß, possibly leading to higher profits. These issues are discussed more thoroughly in chapter V.

$$W = y + (1-\alpha q)q - \frac{\beta q^2}{n} - cq \ . \tag{10}$$

Differentiating W with respect to q gives the socially optimal individual consumption

$$q^{**} = \frac{n(1-c)}{2\alpha n+2\beta} \ . \tag{11}$$

Hence, the more severe the externality, the lower is the socially optimal consumption level. Moreover, this can be seen to be higher than the equilibrium quantity, derived by inserting the equilibrium price (Corollary 1) into aggregate demand. It thereby follows that the price that maximizes social welfare, p^{**}, is lower than the equilibrium price.

Proposition 4: The socially optimal consumption level can always be obtained by means of a price-ceiling, the ceiling being

$$p^{**} = \frac{2\alpha cn+\beta(c+1)}{2\alpha n+2\beta} \ .$$

Proof: Solving for the price that makes individual demand equal to q^{**} yields p^{**}. The difference between the equilibrium price, p^{*}, and p^{**}, is strictly positive for all $\beta>0$. □

Note that p^{**} approaches marginal cost as ß approaches zero. This is true for p^{*} too so for an arbitrarily small ß, p^{*} will be arbitrarily close to p^{**} yielding an arbitrarily small welfare loss. It is not surprising that a negative consumption externality raises optimal prices above marginal cost. The important social welfare conclusion is that the anti-competitive feature of the market, also caused by the externality, will be too strong, thus motivating a price ceiling.[9]

Another interesting conclusion concerns empirical estimates of consumer surplus in the presence of negative externalities. Comparing (11) with the actual demand

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[9]Of course, policy implications of this kind make most sense in cases of physical externalities such as those found in competing transportation systems. It seems difficult to argue convincingly for regulating the prices of Cartier and Rolex watches.

function of lemma 1, it is clear that the area below the demand function will be larger than the true consumer surplus. Consequently, any conventional method to estimate consumer surplus will yield biased results.

4. Entry

Until now, the number of firms has been exogenous. In absence of fixed costs or other entry barriers, a free entry equilibrium would be characterized by an infinite number of firms, each producing an infinitely small amount. Prices would be driven down to marginal cost, despite the externality, completely eroding firm profits. However, entry may involve substantial initial costs on many markets. For example, in the transportation sector large fixed investments in capacity, as well as in marketing, are generally required when entering the market.[10] We therefore introduce a fixed cost, K, keeping the assumption of equal marginal costs across firms.

Proposition 5: Firm profits increase in market concentration and decrease in industry cost level.
Proof: In the appendix.

Hence, the larger the fixed cost, and the larger the marginal cost, the smaller the number firms that could enter profitably.

Proposition 6: Firm profits are quasiconcave in β and increases (decreases) in β for low (high) values of β.

Proof: Follows from simple differentiation of the profit function.

[10] In markets for exclusive brand-name goods, marketing expenses are often very large when new products are introduced. If the simplifying assumption is made that marketing has only an informational value, and does not influence preferences directly, marketing may readily be thought of as a sunk cost.

Thus, given a certain K, the equilibrium number of firms will be largest for some intermediate value of ß. The explanation is that for low values of ß, the market will be fairly competitive, implying low profits and no opportunity for a large number of firms to cover their fixed costs. On the other hand, if ß is large, aggregate demand will be very low since the marginal utility from consuming the good will be reduced to a great extent. Hence, only a small number of firms would be able to enter profitably.

We may conclude that if fixed costs are not negligible, it is reasonable to expect a small number of incumbent firms to charge prices above marginal costs without being threatened by new entrants.

5. Conclusions

The introduction of consumption externalities into a standard Bertrand oligopoly model has several important implications. First, as would be expected, they induce over-consumption from the consumers' perspective, at any given price. Second, they change the incentives of firms, thus dampening competition. Firms may charge prices well above marginal cost despite Bertrand competition and despite goods being homogenous in equilibrium. In fact, if the externality is substantial, equilibrium prices may be close to the monopoly level. The anti-competitive effect dominates the over-consumption effect which translates into a market price that is too high from a social point of view. Thus, welfare can be improved by means of a price-ceiling, which should be noted is commonly practiced in markets for transportation services. Furthermore, we may note that any standard estimate of consumer surplus based on observed demand functions will be positively biased in the presence of negative externalities.

These conclusions are of course based on a specific parameterization of the utility function. However, in most cases linear demand functions and linear "crowding" costs are probably good approximations of real conditions.

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Appendix

Proof of lemma 1

In equilibrium, equation (7) must hold. Using (3) and (5) we then have

$$p_1 + \frac{\beta m_1(1-p_1)}{2\alpha+\beta m_1} = p_2 + \frac{\beta m_2(1-p_2)}{2\alpha+\beta m_2} = \ldots = p_n + \frac{\beta m_n(1-p_n)}{2\alpha+\beta m_n} ,$$

which implies that,

$$\frac{2\alpha p_1+\beta m_1}{2\alpha+\beta m_1} = \frac{2\alpha p_2+\beta m_2}{2\alpha+\beta m_2} = \ldots = \frac{2\alpha p_n+\beta m_n}{2\alpha+\beta m_n} = k .$$

Thus, the number of customers buying from i can be written in the form

$$m_i = \frac{2\alpha(k-p_i)}{\beta(1-k)} ,$$

which summing over all i yields an expression for k. Substituting for k results in

$$m_i = \frac{\beta(1-p_i)-2\alpha(np_i-\sum_{j=1}^{n} p_j)}{\beta(n-\sum_{j=1}^{n} p_j)} .$$

Recalling equations (3) and (5) and substituting for m_i yields the desired result. □

Proof of proposition 5

Differentiating the profit function yields

$$\frac{\partial\pi}{\partial n} = \frac{-\alpha\beta(1-c)^2[2\alpha^2(n(2n-3)+1)+4\alpha\beta(n-1)+\beta^2]}{2(2\alpha n+\beta)^2(\alpha(n-1)+\beta)^3)} < 0$$

and

$$\frac{\partial\pi}{\partial c} = \frac{-\beta(1-c)(2\alpha(n-1)+\beta)}{2(2\alpha n+\beta)(\alpha(n-1)+\beta)^2} < 0 ,$$

which establishes the proposition. □

Chapter V

Deregulating taxi services: a word of caution[1]

1. Introduction

This chapter studies the performance of a market for phone-ordered taxi cabs which is subject to negative waiting time externalities. Using the Bertrand oligopoly framework established in chapter IV we examine the role of firm types, private vs cooperative, in determining the market outcomes.

In most countries the taxicab industry is subject to various types of regulation such as entry restrictions and price controls. A common rationale for regulating the industry has been to make transportation available at times when demand is low and in areas where population is dispersed. For example, in return for agreeing to serve relatively thin markets a firm could be granted a monopoly position. Another alleged reason for regulating the market is that a policymaker can maintain a price level that is "reasonable" in the eyes of consumers while producers are ensured a "reasonable" profit level by means of entry restrictions. Critics of regulation would argue that such arguments are thinly veiled excuses for catering to interest groups.[2]

The poor performance of regulated industries in general initiated a wave of deregulation during the 1980s. Whether deregulating a taxi market improves its performance depends on many factors. One of the most important factor is the presence or absence of inherent market failures that give rise to inefficiencies in the absence of regulation.[3] Essentially two types of distortions have been discussed in the literature, one arising from imperfect information about prices and the other caused by negative

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[1]This chapter is joint work together with Sten Nyberg.

[2]When deciding on the appropriate number of licenses, regulators in Sweden saw fit to seek guidance from incumbent taxi firms, since they would be best informed about demand conditions. Not surprisingly this resulted in insufficient capacity and long waiting times, not unlike a monopoly situation.

[3]Some evidence of excessive prices can be found in Teal and Berglund (1987). They compare the effects of deregulation in six US cities and find that rates increased after deregulation. Entry was substantial on the cab level, but few radio dispatch services were established. Furthermore, taxicab productivity declined resulting in lower earnings for taxi drivers.

externalities in consumption of taxi-services. The former avenue of research, drawing on search theory, is probably best suited for analyzing the market for street-hailed cabs where price information is more likely to be incomplete.[4] In this chapter we focus on markets for telephone-ordered taxicabs, where price information is easier to come by and where waiting time presumably is an important determinant of product quality.

The externality argument was first brought up by Orr (1969)[5] who noticed that demand is likely to depend not just on prices but also on waiting time. Waiting time, in turn, depends on capacity as well as on the equilibrium demand for taxi services. Hence, there is a negative externality in the sense that one consumer's demand will increase waiting time for all other consumers making the service less valuable to them. In a perfectly competitive market this leads to an over-consumption of taxi services, or in other words, excessively low prices.

Although several authors have stressed the interdependence between demand, price and capacity, the economic implications have not been thoroughly analyzed. Prices have been assumed to be competitive, monopolistic [Foerster and Gilbert (1979)] or exogenously given by regulation [De Vany (1975) and Schroeter (1983)]. In the absence of regulation it seems reasonable to assume that prices are set by the Radio Dispatch Services (RDSs), rather than by individual cab owners [Douglas (1972) and Williams (1980b)]. The analysis requires an explicit oligopolistic framework because when they set prices, firms take into account the pricing decisions of their competitors as well as the effects of the waiting time externality. The latter circumstance makes unilateral price cuts less attractive since, for a given capacity, increased demand means longer waiting time and thus a lower willingness to pay.[6]

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[4]Using search theoretical arguments, Douglas (1972) and Schreiber (1975) claim that prices would be excessively high on an unregulated market. The reason being that unilateral price increases are relatively profitable if price information is scarce and search costs high. Williams (1980a), (1980b) and Coffman (1977) criticize Schreiber's analysis noting that it is confined to the market for cruising cabs while 70-80% of the US taxi demand consists of telephone ordered trips for which price comparisons are relatively easy. Furthermore, most taxi firms have large fleets making price advertizing worthwhile. Finally, on the cruising cab market, the presence of cabstands facilitates price comparisons, further reducing search costs.

[5]Assuming price-taking behavior, Orr characterized equilibria under various price- and entry regulations. Although he found it unlikely, he concluded that an increase in capacity might in fact stimulate demand to such extent that profits per cab increase.

[6]That such a mechanism may put an upward pressure on price has in fact been shown in a quite different context, namely in the theory of clubs [Scotchmer (1985)].

Ceteris paribus, the externality may in fact help firms sustain a higher profit level than otherwise would have been possible. This, in turn, suggests that there might be incentives to cut back on capacity in order to increase waiting time.

The chapter is organized as follows: The basic model is presented in section 2 and some results concerning price-setting behavior and social welfare are derived. For the sake of expositional clarity the analysis is confined to a duopoly. All results in section 2 can however be generalized to the n firm case. In section 3 the model is extended to allow for entry. Finally, some concluding remarks are made in section 4.

2. The model

Taxi firms, by which we mean radio dispatch services (RDSs), choose fares and decide on fees for drivers wishing to hook up to their service. Fares are assumed to be linear in the quantity of services consumed, q, and each driver can at most be hooked up to one RDS. The expected waiting time when ordering a taxi from a certain firm is assumed to depend on the demand facing that firm divided by the size of their taxi fleet. The fleets are initially assumed to be of equal size and are normalized to one.

Consumers value two things. First, their utility is assumed to be linearly increasing in the consumption of a composite good, y, representing "everything else." Second, consumer utility is assumed to increase, at a decreasing rate, in the amount of taxi services consumed, e.g. the number of (equally long) trips demanded, and decrease in waiting time. To make the welfare analysis tractable we specify a simple utility function with the above properties. Assuming a continuum of identical consumers, the utility of consumer j patronizing firm 1 is given by

$$U_{j,1} = y_{j,1} + (w - \alpha q_{j,1})q_{j,1} - \beta Q_1 q_{j,1}\ , \tag{1}$$

where the last term reflects the disutility of waiting, caused by others' consumption, Q_1. The marginal utility of the first unit of good q consumed is denoted by w. The diminishing utility of additional consumption and waiting time is parameterized by α

and ß respectively.[7] Waiting time is assumed to become more important, the more taxi trips consumed, thus affecting marginal utility and individual demand. Furthermore, consumers disregard the effect of their own demand on the price-setting behavior of firms. The demand for taxi services by a single consumer patronizing firm 1 is derived from the individual consumer's utility maximization subject to the budget constraint, $I = y_{j,1} + p_1 q_{j,i}$, where the price of the composite good is normalized to one. That is,

$$q_{j,1} = \frac{w-p_1-\beta Q_1}{2\alpha} . \tag{2}$$

The aggregate demand of firm 1, normalizing the number of consumers to unity, is simply $Q_1 = q_{j,1}m$, where m is firm 1's market share. Consumers will choose to ride with the firm offering the best price - waiting time tradeoff. In equilibrium customers are indifferent with respect to the different firms, i.e. in terms of their indirect utility functions, $V(p_1,Q_1,I) = V(p_2,Q_2,I)$. For our specific utility function this yields

$$p_1 + \beta Q_1 = p_2 + \beta Q_2 . \tag{3}$$

Solving for the market shares satisfying the above condition for given prices and letting $m_2=(1-m)$ be firm 2's market share we have

$$m = \frac{2\alpha(p_2-p_1) + \beta(w-p_1)}{\beta(2w-p_1-p_2)} \tag{4}$$

and thus the aggregate demand for firm 1's services is given by

$$Q_1 = \frac{2\alpha(p_2-p_1) + \beta(w-p_1)}{\beta(4\alpha + \beta)} . \tag{5}$$

Firm 2's demand is derived analogously. The marginal cost of producing taxi services is assumed to be constant and the profit of firm 1, given there are no fixed costs, is given by

[7]β can actually be given two structurally indistinguishable interpretations. The first, and most obvious, interpretation is that it reflects consumers' aversion toward spending time waiting. However, it may also be thought of as a technology parameter that relates capacity to waiting time.

$$\pi_1 = (p_1 - c_1)Q_1 \,. \tag{6}$$

The best-response function for firm 1 is obtained by differentiating profits with respect to p_1:

$$\varphi_1(p_2) = \frac{1}{2}\left[c_1 + \frac{2\alpha p_2 + \beta w}{2\alpha + \beta}\right]. \tag{7}$$

Thus, prices are strategic complements. Furthermore, the slope being less than one ensures a unique equilibrium. The symmetric case, where firms face equal marginal costs, c, not surprisingly yields a symmetric equilibrium with $p_1 = p_2 = p^*$, where

$$p^* = \frac{1}{2}\left[c + \frac{\alpha c + \beta w}{\alpha + \beta}\right]. \tag{8}$$

It is easy to see that the equilibrium price, p*, is increasing in β. If consumers are infinitely patient, $\beta = 0$, firms face true Bertrand competition and prices are driven down to marginal cost. If waiting time does matter, firms will earn positive profits. In fact, as ß approaches infinity prices are close to the monopoly level, (c+w)/2. Equilibrium profits are however highest for intermediate values of ß. For low ßs, the market will be fairly competitive and for high ßs aggregate demand is greatly reduced by the impact of the negative externality.

In contrast to the standard Bertrand equilibrium, prices are above marginal cost despite price competition and homogeneous products in equilibrium, costs being equal.[8] [9] Moreover, while the socially efficient price on a market with negative externalities is higher than marginal cost it can be shown that the externality weakens competition to such an extent that the equilibrium price level is actually higher than optimal. As shown in chapter IV, social welfare can thus be improved by means of a price-ceiling given by

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[8]A similar result can be found in Scotchmer (1985).

[9]In fact, it suffices for a fraction of all consumers to have an aversion towards waiting time for all firms to profitably charge prices above marginal cost. It is fairly easy to construct examples of asymmetric equilibria assuming two consumer groups consisting of "businessmen" with a high willingness to pay for transportation but a large queue aversion and "ordinary people" with a low willingness to pay for transportation and a moderate queue aversion.

$$p^{**} = \frac{1}{2}\left[c + \frac{2\alpha c+\beta w}{2\alpha+\beta}\right], \tag{9}$$

where p** approaches marginal cost as ß approaches zero. This holds true for p* as well. Hence, if ß can be made arbitrarily small, efficiency losses will also become arbitrarily small. As will be discussed in section 3, an inflow of new cabs can be interpreted precisely as a reduction in ß.

3. Entry

The findings in section 2 suggest that price competition alone may not suffice to ensure efficient pricing on the market for taxi services. However, the results were derived under the assumption of fixed capacity. Insofar as regulated capacity is the real culprit, removing the institutional barriers to entry may go a long way in improving conditions.

The natural entry barriers on the cab level are likely to be very low. There is a reasonably efficient market for used cars and leasing may also be a viable option. The only element of sunk cost would appear to be the time and money spent in getting the taxi driving-license. Hence, high industry profits would soon attract new capacity thereby reducing waiting time. Prices would be driven towards marginal costs and industry profits dwindle but the social cost of negative consumption externalities would be negligible. However, this also suggests that RDSs have an incentive to try to restrict the inflow of new cabs.

Entry can, of course, take place on the RDS level as well. Establishing an RDS may, however, entail substantial fixed costs.[10] [11] First, office staff, marketing costs

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[10]The airline industry may serve as an interesting comparison. Airline business was widely held to be essentially contestable for many of the same reasons put forward in the discussion about the taxi industry. The experience following airline deregulation in the US was however somewhat disappointing in that factors like gate access and computerized booking systems tended to impede entry, or at least make entry less attractive [Levine (1987)]. There may be incumbency advantages for established radio dispatch companies that are in some respects parallel to that of the computerized booking systems.

[11]Although high fixed costs per se do not constitute entry barriers in a strict sense, they do limit the number of firms that can coexist on the market without running a loss. If prices adjust instantaneously to new market conditions (in contrast to the contestable market framework where hit and run entry is feasible) then, even in the absence of sunk costs, firms may earn positive profits in equilibrium.

and equipment costs are more or less independent of scale. Furthermore, it is inconvenient for a consumer to memorize more than a few phone numbers to different taxi firms. There may also be returns to scale in that expected waiting time is likely to decrease with fleet size even if demand per cab is kept constant. This is because the geographical distance between a (randomly located) customer and the nearest taxi can be expected to decrease with the size of the (randomly located) taxi fleet. These effects, benefiting incumbents, may to some extent be approximated by increasing returns to scale in the operation of a service. Some empirical evidence in support of this can be found in Teal and Berglund (1987) who report that US deregulations typically have resulted in massive entry on the cab level while the market structure on the RDS level has been more or less unaffected.

Assuming that entry is most likely to occur on the cab level, we now analyze the effects of entry, keeping the number of RDSs fixed. This is done by introducing an initial stage in which RDSs decide on capacities by taking into account the effect on equilibrium prices in the subsequent stage. Technically speaking, we solve for the subgame perfect Nash equilibrium of a two-stage game. Fleet sizes, equilibrium prices and quantities are compared under two different assumptions regarding the organizational structure of the RDSs, denoted regimes I and II. These structures may be thought to reflect different regulatory regimes or market practices. For the sake of tractability the analysis is confined to a duopoly market and RDSs are assumed to be symmetric in terms of organizational structure.

Under regime I, RDSs are cooperatives controlled by the cab drivers. Only members are allowed to vote when deciding on capacities so new memberships are refused (and old ones terminated) as benefits the majority of the members. Hence, RDSs choose fleet size to maximize per cab profits. In regime II, RDSs are privately owned enterprises choosing connection fees to maximize firm profits.

Firm capacity is modelled by making ß firm-specific letting, $\beta_i = b/f_i$, where f_i denotes the fleet size of firm i and b reflects aversion towards waiting time. Replacing β with β_1 and β_2 in expression (3) and proceeding as in section 2, the demand facing firm 1 becomes

$$Q_1 = \frac{f_1[2\alpha f_2(p_2-p_1)+b(w-p_1)]}{b[2\alpha(f_1+f_2)+b]}. \tag{10}$$

Straightforward differentiation implies that the gross equilibrium profit of RDS 1 is

$$\pi_1 = \frac{bf_1[w-c]^2[\alpha(2f_1+f_2)+b][2\alpha^2 f_2(2f_1+f_2)+\alpha b(2f_1+3f_2)+b^2]}{4[3\alpha^2 f_1 f_2+2\alpha b(f_1+f_2)+b^2]^2[2\alpha(f_1+f_2)+b]}. \tag{11}$$

It can be checked that the waiting time facing firm 1's customers, Q_1/f_1, is decreasing and convex in f_1 at equilibrium prices, which is reasonable since the first unit of capacity is likely to reduce waiting time to a greater extent than the hundredth unit.

Let K_c denote the fixed cost of an entrant cab and let K_r denote the fixed cost of an RDS.[12] Then $\bar{K}(f_i)=K_c+K_r/f_i$ is the average fixed cost of a cab hooked up to an RDS with fleet size f_i.[13] The marginal cost of running a RDS is assumed to be zero.

3.1 The fleet size equilibria

When the RDSs maximize profits per cab, $\pi_I \equiv \pi_i/f_i-\bar{K}(f_i)$, with respect to fleet size, there is a clear incentive to keep the fleet small. A privately owned RDS maximizes total profits, i.e. connection fees times fleet size minus costs. The highest connection fee possible to extract is $Z=\pi_i/f_i-K_c$ which yields a per cab profit amounting to $\pi_i/f_i-\bar{K}(f_i)$ just as under regime I. Hence, firms maximize $\pi_{II} \equiv f_i\pi_I=f_i(\pi_i/f_i-\bar{K}(f_i))$ with respect to f_i. For a given size of the competitor's fleet the relation between π_I and π_{II} is illustrated in figure 1.[14]

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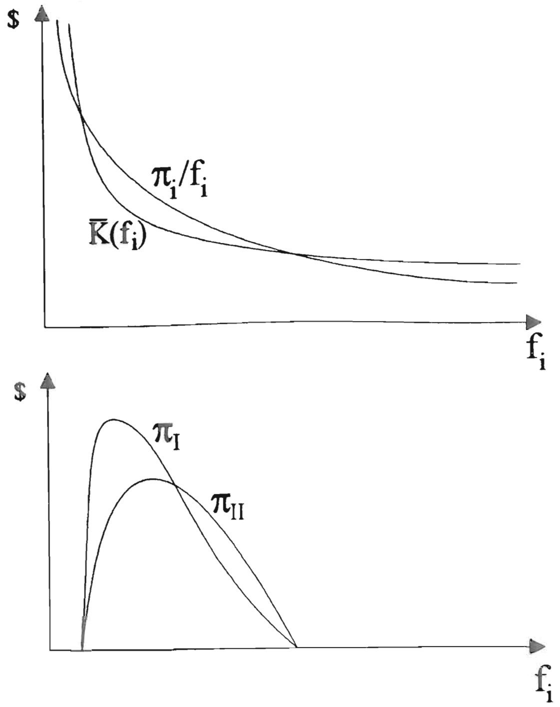
[12] K_r could include wages, marketing costs and capital costs while K_c could include leasing fees, and the driver's opportunity cost of working in the cab industry.

[13] The net RDS profit function can be shown to be single peaked for positive fleet sizes. Using equation (11) they can be written on the form; $\pi_1(f_1) - f_1K_c - K_r = f_1[\pi_1/f_1 - K_c] - K_r$ where π_1/f_1 is decreasing in fleet size. It follows that profits per cab are also single peaked.

[14] In figure 1 maximal profit per cab is higher than maximal profits per RDS. This is simply due to the optimal fleet sizes being smaller than one which, in turn, follows from normalizing the total number of consumers to one.

Figure 1



Lemma 1: Fleet sizes are strategic complements under regime I and strategic substitutes under regime II.

Proof: Profit per cab, π_I, is at least quasiconcave in f_i since π_i/f_i and $\bar{K}(f_i)$ are both decreasing and strictly convex in f_i and intersect twice. It is then obvious that π_{II} has the same property. Strategic complementarity (substitutability) follows from applying the implicit function theorem to the first order condition noting that the cross derivative of π_I (π_{II}) wrt fleet sizes is positive (negative). □

If firm 2 increases its capacity, firm 1 will lose some customers to firm 2, reducing Q_1 and hence waiting time. When demand is reduced, waiting time becomes less sensitive to changes in f_1 which also makes firm demand less sensitive. In turn, gross profits, π_1 and gross profits per cab, π_1/f_1, become more robust to changes in f_1. Under regime I, firm 1 can therefore increase its fleet size, spreading the fixed cost, K_r, over a larger number of cabs, incurring only a small loss in terms of π_1/f_1. Conversely, under regime II, firm 1 can reduce its fixed cost payments, $f_1K_c+K_r$, by reducing its fleet size, without affecting π_1 very much.

Proposition 1: Under both regimes, there exists a unique and symmetric equilibrium in fleet sizes.[15]

Proof: The reaction-functions, $f_i(f_j)$, are identical. Under regime I they are concave and upward sloping (by strategic complementarity) and under regime II they are downward sloping (by strategic substitutability). □

Proposition 2: (i) Under regime I, the equilibrium fleet size decreases in consumers' valuation of taxi services, w, and increases in marginal cost, c. (ii) Increases in w raise prices while the effect on quantity is ambiguous. Increased costs, c, have indeterminate effects on prices and quantities. (iii) Increased RDS fixed cost, K_r, increases f_i given any f_j, resulting in lower prices and larger equilibrium quantities. The fixed cost per cab, K_c, does not affect fleet sizes.

Proof: In the appendix

As consumers' valuation of taxi services increases, (or marginal cost decreases,) the firm will want to trade off some of this for a reduction in fleet size in order to increase per cab profits.

The direct effect of an increases in w is a rise in both price and quantity. However, firms benefit from cutting back on capacity, which increases prices and reduces quantities. Hence, only the effect on price is clear. Similarly, when c

[15]Since equilibrium taxi fleets are symmetric under all regimes, the assumption of identical RDSs in section 2 can in fact be rationalized.

increases, the direct effect is a rise in price and a reduction in quantity. As capacity increases, prices go down, and quantities go up, so the net effect is unclear. Finally, when the fixed cost of an RDS, K_r, increases, there is a tendency to spread it among a greater number of members, which lowers prices and increases equilibrium quantities. A policymaker could therefore induce lower prices through imposing a lump sum tax on RDSs which is a somewhat paradoxical result. Raising the fixed cost per cab, K_c, does not affect the maximization problem.

Proposition 3: (i) Under regime II, if consumers are patient, i.e. when b is small, the equilibrium fleet size decreases in consumers' valuation of taxi services, w, and increases in marginal cost, c. If consumers are impatient, i.e. when b is large, the opposite is true. (ii) For small b, increases in w raise prices while the effect on quantity is ambiguous. Increased costs, c, have indeterminate effects on price and quantity. When b is large, w has a positive effect on quantity while the effect on price is ambiguous. Increases in marginal cost raise prices and reduce quantity. (iii) Increased per cab fixed costs, K_c, reduces f_i given any f_j. This raises prices and reduces quantity. The RDS fixed cost, K_r, has no effect on capacities.

Proof: In the appendix

If consumers have a large aversion towards waiting, the willingness to pay for a reduction in waiting time will increase greatly when w increases, in which case, it is profitable to expand capacity. When consumers are patient, waiting time is not a major issue and increases in w are immediately traded off for reductions in capacity in order to reduce the fixed cost payments.

When b is small, price and quantity derivatives with respect to w and c are the same as in regime I and for the same reasons. Therefore, let us assume that b is large. The direct effect of an increase in w is a rise in both price and quantity. But since firms increase capacity, which tends to reduce price and increase quantity, the only clear effect is on quantity. When c increases, on the other hand, the direct effect is a rise price and a reduction in quantity. In this case, firms cut back on capacity, which tends to raise prices and reduce quantity so the effect in this case is unambiguous.

Finally, when the fixed cost of taxicabs, K_c, increases, firms naturally cut back on capacity which raises prices and reduces equilibrium quantities. Consequently, one way for a policymaker to induce lower prices is to subsidize the fixed cost of entrant cabs. Raising the fixed cost of an RDS, K_r, does not affect the maximization problem.

From a welfare perspective, it is interesting to compare the equilibrium fleet sizes. In figure 1, which is drawn for an arbitrary f_j, we can see that the equilibrium fleet size in regime II, f_{II}, is larger than that of regime I, f_I. Indeed, given any f_j it will be optimal to choose a higher f_i under regime II than under regime I. In terms of equilibrium prices and quantities, $p_I > p_{II}$ and $Q_I < Q_{II}$.

Of course one could also imagine a situation where a regime I firm competes with a regime II firm.[16] Assume that the market initially is in a regime I equilibrium. Then one firm, say firm 2, is reorganized as a regime II firm. Since the best response to a given f_1 is larger for a regime II firm than for a regime I firm its reaction function shifts out. Firm 1's reaction function is increasing in f_2 so at the new intersection both firms will have larger fleet sizes but firm 2 will have the largest one. Compared to a symmetric regime II equilibrium, firm 2 will have a larger fleet size in the hybrid equilibrium and firm 1 a smaller one. All drivers would of course prefer to belong to the cooperative firm but only a limited number of members are accepted.

3.2 Policy implications

The main conclusion from the last section is that market profits will be positive despite "free" entry of taxicabs. The reason is the endogenous entry barrier, in the form of high connection fees and exclusion, created by the RDSs.

If the fixed cost of entrant cabs, K_c, is low, it would be socially desirable to reduce entry barriers to a minimum since a large number of new cabs would drive ß towards zero, without incurring a great cost on society. Consumers' valuation of taxi services would increase and market prices be driven towards marginal cost. In other words, the market would become more and more similar to the standard Bertrand

[16]The two major firms on the Stockholm taxicab market are organized in this manner.

market with constant marginal cost pricing and almost no externalities. Clearly, the market outcome will not be efficient in this case, but regime II will be relatively more efficient than regime I. If the industry could be costlessly re-regulated, one therefore might want to prevent the RDSs from refusing to hook up new entrants. If costs are observable, the fees could also be subject to regulation.

However, if the fixed cost of entrant cabs is substantial, some entry barrier may be needed to prevent the positive price-cost margin from attracting too many cabs from the social point of view. More specifically, when a cab enters on the margin, the consumers' valuation of the price decrease and the waiting-time reduction may be smaller than the fixed cost. Regime I might then be relatively efficient since equilibrium fleet sizes are small.

4. Conclusions

The sunk cost of an entrant cab is likely to be small since cabs can be leased and there exist well-functioning markets for second hand taxi equipment. Also, the fixed costs are likely to be moderate, consisting mainly of a leasing fee and perhaps the opportunity cost of working in the industry. All this put together makes for a strong case for deregulation. However, price competition alone does not ensure efficiency. Cooperatively-run RDSs will be relatively less efficient compared to privately-owned RDSs. Since firms will not voluntarily choose large capacities, one could even argue for a regulation of the RDSs guaranteeing free access and, if costs are observable, low connection fees. Thus, a case could be made for stimulating competition between independent taxi firms, but to separate the production of the services from the ordering system which could be run as a regulated monopoly or be publicly operated. In such case, the costs of regulation must of course be taken explicitly into account. Specifically, information asymmetries may make it difficult to induce cost efficiency.

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Appendix

$$p_1^* = \frac{6\alpha^2 c f_1 f_2 + \alpha b(c(2f_1+3f_2)+w(2f_1+f_2)) + b^2(c+w)}{2(3\alpha^2 f_1 f_2 + 2\alpha b(f_1+f_2) + b^2)} \tag{A1}$$

$$Q_1^* = \frac{f_1(w-c)(2\alpha^2 f_2(2f_1+f_2) + \alpha b(2f_1+3f_2) + b^2)}{2(3\alpha^2 f_1 f_2 + 2\alpha b(f_1+f_2) + b^2)(2\alpha(f_1+f_2)+b)} \tag{A2}$$

Proof of proposition 2

(i) Follows from applying the implicit function theorem on the first order condition, noting that

$$\frac{\partial^2 \pi_I}{\partial f_i \partial w} < 0, \; \frac{\partial^2 \pi_I}{\partial f_i \partial c} > 0 \, .$$

(ii) Differentiating equilibrium price, equation (A1), wrt w yields

$$\frac{dp^*}{dw} = \frac{\partial p}{\partial f_1}\frac{\partial f_1}{\partial w} + \frac{\partial p}{\partial f_2}\frac{\partial f_2}{\partial w} + \frac{\partial p}{\partial w} \, ,$$

where fleet size affects price negatively. As w has a negative effect on fleet size and the direct effect of w is to increase prices, the total effect must be positive.

Differentiating equilibrium price, equation (A1), wrt c yields

$$\frac{dp^*}{dc} = \frac{\partial p}{\partial f_1}\frac{\partial f_1}{\partial c} + \frac{\partial p}{\partial f_2}\frac{\partial f_2}{\partial c} + \frac{\partial p}{\partial c} \, ,$$

where fleet size affects price negatively. As c has a positive effect on fleet size and the direct effect of c is to increase prices, the total effect is indeterminate.

Differentiating equilibrium quantity, equation (A2), wrt w yields

$$\frac{dQ^*}{dw} = \frac{\partial Q}{\partial f_1}\frac{\partial f_1}{\partial w} + \frac{\partial Q}{\partial f_2}\frac{\partial f_2}{\partial w} + \frac{\partial Q}{\partial w} \, ,$$

where fleet size affects quantity positively. As w has a negative effect on fleet size and the direct effect of w is to increase prices, the total effect is indeterminate.

Differentiating equilibrium quantity, equation (A2), wrt c yields

$$\frac{dQ^*}{dc} = \frac{\partial Q}{\partial f_1}\frac{\partial f_1}{\partial c} + \frac{\partial Q}{\partial f_2}\frac{\partial f_2}{\partial c} + \frac{\partial Q}{\partial c} ,$$

where fleet size affects quantity positively. As c has a positive effect on fleet size and the direct effect of c is to reduce quantity, the total effect is indeterminate.

(iii) The effect of fixed costs on fleet size is derived applying the implicit function theorem to the first order condition, noting that

$$\frac{\partial^2 \pi_I}{\partial f_i \partial K_c} = 0, \frac{\partial^2 \pi_I}{\partial f_i \partial K_r} > 0 .$$

Fleet size, in turn, affects equilibrium prices negatively and equilibrium quantities positively. This follows trivially from differentiating (A1) and (A2). □

Proof of proposition 3

(i) Follows from applying the implicit function theorem on the first order condition, noting that

$$\frac{\partial^2 \pi_{II}}{\partial f_i \partial w} < 0, \frac{\partial^2 \pi_{II}}{\partial f_i \partial c} > 0 ,$$

when b is small and

$$\frac{\partial^2 \pi_{II}}{\partial f_i \partial w} > 0, \frac{\partial^2 \pi_{II}}{\partial f_i \partial c} < 0 ,$$

when b is large. In the first case price and quantity derivatives with respect to w and c are the same as under regime I, and for the same reasons. Therefore. assume b is large.

(ii) Differentiating equilibrium price, equation (A1), wrt w yields

$$\frac{dp^*}{dw} = \frac{\partial p}{\partial f_1}\frac{\partial f_1}{\partial w} + \frac{\partial p}{\partial f_2}\frac{\partial f_2}{\partial w} + \frac{\partial p}{\partial w} ,$$

where fleet size affects price negatively. As w has a positive effect on fleet size and the direct effect of w is to increase prices, the total effect is indeterminate.

Differentiating equilibrium price, equation (A1), wrt c yields

$$\frac{dp^*}{dc} = \frac{\partial p}{\partial f_1}\frac{\partial f_1}{\partial c} + \frac{\partial p}{\partial f_2}\frac{\partial f_2}{\partial c} + \frac{\partial p}{\partial c} ,$$

where fleet size affects price negatively. As c has a negative effect on fleet size and the direct effect of c is to increase prices, the total effect must be positive.

Differentiating equilibrium quantity, equation (A2), wrt w yields

$$\frac{dQ^*}{dw} = \frac{\partial Q}{\partial f_1}\frac{\partial f_1}{\partial w} + \frac{\partial Q}{\partial f_2}\frac{\partial f_2}{\partial w} + \frac{\partial Q}{\partial w},$$

where fleet size affects quantity positively. As w has a positive effect on fleet size and the direct effect of w is to increase prices, the total effect is must be positive.

Differentiating equilibrium quantity, equation (A2), wrt c yields

$$\frac{dQ^*}{dc} = \frac{\partial Q}{\partial f_1}\frac{\partial f_1}{\partial c} + \frac{\partial Q}{\partial f_2}\frac{\partial f_2}{\partial c} + \frac{\partial Q}{\partial c},$$

where fleet size affects quantity positively. As c has a negative effect on fleet size and the direct effect of c is to reduce quantity, the total effect is must be negative.

(iii) The effect of fixed costs on fleet size is derived by applying the implicit function theorem to the first order condition, noting that

$$\frac{\partial^2 \pi_{II}}{\partial f_i \partial K_c} < 0, \frac{\partial^2 \pi_{II}}{\partial f_i \partial K_r} = 0 .$$

Fleet size, in turn, affects the equilibrium price negatively and the equilibrium quantity positively. This follows trivially from differentiating (A1) and (A2). □

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