Hierarchical Modelling of Private Demand in Sweden

Kent Rune Sjöholm

The Industrial Institute for Economic and Social Research

is an independent non-profit research institution, founded in 1939 by the Swedish Employers' Confederation and the Federation of Swedish Industries.

Objectives

To carry out research into economic and social conditions of importance for industrial development in Sweden.

Activities

The greater part of the Institute's work is devoted to long-term problems, especially to long-term changes in the structure of the Swedish economy particularly within manufacturing industry.

Board

Address

Industriens Utredningsinstitut
Box 5501, S-114 85 Stockholm, Sweden
Tel. 08-783 80 00

Hierarchical Modelling of Private Demand in Sweden

Kent Rune Sjöholm

Distribution: Almqvist & Wiksell International
Stockholm, Sweden

Licentiate dissertation
Department of Economics,
The University of Stockholm, 1993

IUI Dissertation Series, No. 4
ISSN 1102-6057
ISBN 91-7204-425-X
Printed by AB Grafiska Gruppen
Stockholm 1993

Foreword

Private consumption is the largest single component of final demand, amounting to almost 55 per cent of GDP in 1991. The second most important item, exports, contributed 37 per cent. Its relative size makes it important to understand the behavior of private consumption.

IUI has a long tradition of household demand analysis beginning with the study by Ragnar Bentzel and Kurt Eklöf in 1957, followed in 1971 by the consumption expenditure systems analysis by Anders Klevmarken and Johan Dahlman. Over the years, many studies of special demand categories such as automobiles and graphical products have been carried out at IUI.

This study reestimates the earlier expenditure system, and then focuses on private demand for transport and communications services in general, and telecommunications services in particular. It is part of a larger IUI project on the supply and demand of communications and information services in the Swedish economy.

This study was defended on January 28, 1993, as a licentiate thesis at the Department of Economics at the University of Stockholm. It is the 46th doctoral or licentiate dissertation completed at the Institute since its founding in 1939.

The Institute would like to thank the thesis advisor, Professor Lars Werin at the University of Stockholm, for his encouragement and support and also the discussant Lennart Flood for valuable comments.

Financial support from Televerket is gratefully acknowledged.

Stockholm in March 1993

Gunnar Eliasson

Contents

1. Summary

A hierarchical model of private demand in Sweden is formulated and partially estimated. Two consumer demand systems, the classical Linear Expenditure System (LES) and a recently proposed model based on a "flexible functional form", have been estimated in this study of Swedish consumption from 1931 to 1990. Each model has comparative advantages related to substitution. The LES does not "consider" substitution in a proper sense, as does the MLGL-model discussed as an alternative. The LES, originally proposed by Stone (1954) is, however, still going strong and an appropriate choice when applied to the broadest aggregates. At the highest level of aggregation satiation has not been reached and substitution is not so important. The sensitivity of the demand for each broad category to variations of *total* consumer expenditures is the most important aspect. The lower the level of the consumption tree, the more reasonable the opposite hypothesis.

This project is part of a large IUI inquiry into the nature of economic information processing in the Swedish economy. A special ambition has been to develop an econometric method that allow us to capture the demand for transport and communication services in total demand, with particular emphasis on telecommunication services, and its possible substitute, postal services. The empirical application focuses on these demand categories. The two communication services (postal and tele communications) are found to be weak substitutes. They are also price inelastic, postal services more so than telecommunication. The expenditure elasticity of telecommunication services is however greater than one, while postal services is found to be expenditure inelastic with an elasticity of less than 0.7.

2. Introduction

This study has four objectives. The *first* objective is to continue a tradition of the Industrial Institute for Economic and Social Research (IUI) and update earlier models of Swedish household's demand for goods and services. The reestimation is based on the Linear Expenditure System (LES) used in earlier studies, although restricted to the eight broadest categories of goods and services that was studied by Ragnar Benzel in 1957, and by Anders Klevmarken and Carl Johan Dahlman in 1971. The *second* objective has been to make a related special study of the Swedish private households' *aggregate demand for telecommunication*. In order to perform this task, the Minflex Laurent Generalized Leontief (MLGL) model, a specification of consumer demand systems proposed in the mid-eighties by William A. Barnett and Yul W. Lee, has been tried. The *third* objective has been to update the nonlinear expenditure system of the Micro-Macro model (MOSES), and set it up for special studies of the demand for various categories of information and of the importance of communications services in the economy. The *fourth* objective is to report some experience gained from using the MLGL-model on smooth as well as on rough time series data.

Price and expenditure elasticities based on the chosen specifications of consumer demand systems will be calculated, so that the combined effect of e.g. a decrease in aggregate private consumption due to a more restrictive fiscal policy or a recession, on the demand for a certain consumer category such as telecom, may be estimated. Elasticity measures can be more easily calculated if they are based on more "pragmatic" demand models, e.g. a log-linear relationship between quantities, prices and expenditures. However, the advantage gained by using a "regular" demand system, like the Linear Expenditure System, is that these models are explicitly derived from the well developed neoclassical theory on households. Each empirical application can be checked for consistency with economic theory. To use systems of demand functions may possibly have the disadvantage of making the analyst feel like having a strait-jacket on, to quote Leif Johansen (Johansen, 1981).

3. Data

The demand for telecommunications services is an integrated part of the households' total private consumption and is published as a separate category by Statistics Sweden (in the National Accounts). By using earlier research reports from IUI, covering the period 1931–1968, and the annual national accounts data from the late sixties and onwards, data on the Swedish households' use of telephone services can be traced back sixty years as one category among a (more or less) standardized item in a "purpose-oriented" taxonomy of the consumer demand categories (see appendix). These series have been linked together from the most recent observation and backwards with one overlapping year. This linking has been applied *both* to reported *values*, because of level-differences in overlapping years among various sources, and certainly also to *volumes* that are calculated in the price-level of different base-years.

4. Consumer demand systems, a short background

A short summary of the basic elements of the neoclassical theory of the household, will hopefully be of some value as a refreshment and as a background to the discussion of the properties of different demand systems, notably the Linear Expenditure System and some more recent alternatives, with utility functions based on "flexible functional forms".

4.1 Some basic elements of neoclassical consumer theory

One basic assumption of neoclassical consumer theory is that the representative consumer has a clear preference structure with reference to the set of goods and services that is available to her or him. In a standard neoclassical context, the consumer is also assumed to be completely informed about the price structure on the market and to have a budget at his/her disposal to spend on private consumption. The consumer is free, capable and willing to comprehensively survey the market in order to select that particular collection of goods and services that maximizes his/her utility, given preferences, prices and a personal budget. Consumption is the result of an explicit optimization process.

Formally, the set of goods and services ("commodities") that might be bought in a period, called the "commodity space" C, is assumed to be a closed, convex set in the non-negative orthant of an n-dimensional Euclidean space. A weak preference relation is assumed to exist among the elements of C. This property is based on two axioms.

A1. A complete preordering

For *any* commodity vectors **x** and **q** in C, either
x is preferred to **q** ($\mathbf{x} \succeq \mathbf{q}$) or
q is preferred to **x** ($\mathbf{q} \succeq \mathbf{x}$)

A2. Continuity

The preference set, $P_x = \{ \mathbf{q} \in C \mid \mathbf{q} \succeq \mathbf{x} \}$ and
the nonpreference set, $N_x = \{ \mathbf{q} \in C \mid \mathbf{x} \succeq \mathbf{q} \}$
should both be closed sets for any $\mathbf{x} \in C$.

If A1 and A2 both apply, there exists a continuous, real valued function U(.), defined on C, for which

$$U(\mathbf{x}) \geq U(\mathbf{q}), \text{ if and only if } \mathbf{x} \succeq \mathbf{q} . \qquad (1)$$

The real-valued function U(.) is called an ordinal, direct utility function; *ordinal* because U(.) transforms the preference relations to an ordinal scale and *direct* because the utility is directly expressed as a function of the demanded volumes of the n commodities. The utility function is normally assumed to be at least twice differentiable.

$$g = \frac{\partial U(x)}{\partial x} > 0$$
$$H = \frac{\partial^2 U(x)}{\partial x^2} \text{ such that } q'Hq \prec 0, \text{ all } q \in C \,. \tag{2}$$

All elements of the gradient **g** of U(.) are assumed to be positive, because all commodities are assumed to be normal "goods" in the sense that a greater quantity is always preferred to a smaller one.

U(.) *is assumed to be a concave* function, hence its matrix of second order partial derivatives **H**, the Hessian, must be negative definite. This assumption is equivalent to the requirement that each *preference set* ***P*** in **C** shall be (strictly) *convex*, i.e,

$$\mathbf{P_q} = \{ \mathbf{x} \in \mathbf{C} \mid \mathrm{U}(\mathbf{x}) \geq \mathbf{q} \}, \text{ convex } \forall \mathbf{q} \in \mathrm{C}. \tag{3}$$

If $\mathbf{x_1}$ and $\mathbf{x_2}$ are both preferred to **q**, then it is natural to assume that the linear combination $\mathbf{x} = \alpha\, \mathbf{x_1} + (1 - \alpha)\, \mathbf{x_2}$ must also be preferred to **q** for any $\alpha \in (0,1)$. However, in a negative definite Hessian matrix **H** of U(.), the diagonal elements must be negative. These elements represent the rate of change of the marginal utility of each commodity. So, the marginal utility of any commodity is always positive but at a decreasing rate with an increase in volume (diminishing marginal utility).

The neoclassical consumer will now maximize U(.), subject to a budget constraint

$$\max_{\mathbf{x}} \ \mathrm{U}(\mathbf{x}), \text{ subject to } \mathbf{p}'\,\mathbf{x} \leq \mathrm{y}, \ \mathbf{x} \in \mathbf{C}, \tag{4}$$

where **p** is the price vector and y the consumer's budget.

The solution to this usually nonlinear optimization problem must satisfy the following n+1 equations, the first order conditions of a constrained maximum

$$\begin{aligned} y - p'x &= 0 \\ \frac{\partial U\,(x)}{\partial x} - \lambda p &= 0\;. \end{aligned} \tag{5}$$

If the utility function is regular, as assumed, with a negative definit matrix of second order partial derivatives, *a solution exists*,

$$\begin{aligned} x^* &= x^*(p,\ y) \\ \lambda^* &= \lambda^*(p,\ y)\;. \end{aligned} \tag{6}$$

The first n equations of (6), $\mathbf{x}^*$, represent the consumers' optimal demand. The demand functions are uniquely determined, implicit functions of the prices and the income, dependent on the particular utility function U(.). The demand functions are homogenous of degree zero with respect to the prices and the income (no money illusion), and have continuous first order (at least) partial derivatives with respect to the prices, **p**, and the expenditure, y.

The optimal Lagrange multiplier, λ^*, may be interpreted to represent the optimal increase of the consumer's utility, associated with a marginal increase of the budget. If the optimal demand, $\mathbf{x}^*(\mathbf{p},y)$, is substituted into the utility function, $U(\mathbf{x}^*(\mathbf{p},y))$, the utility will indirectly become a function of prices and expenditures, $U^*(\mathbf{p},y)$. Hence, $U^*(\mathbf{p},y)$ is referred to as an indirect utility function. As first shown by R. Roy in 1942, the relation between optimal demand of a commodity and the indirect utility function is,

$$x^*_i = -\frac{\partial U^*/\partial p_i}{\partial U^*/\partial y},\quad i = 1,\ \ldots\ ,\ n \tag{7}$$

(Roy, 1942, and Intriligator, 1971).

4.2 Sequential decision making in a hierarchical structure

Consumption data are traditionally organized in a hierarchical structure by Statistics Sweden (SCB). Are the consumers' decisions also hierarchically organized? This may be the case but it can certainly not be taken for granted. The decision to buy one commodity at the lowest level of the consumption tree may, in principle, be influenced by the prices of all other "low-level" commodities. Also if a flat but broad, low-level interdependence would dominate real decisions, a detailed analysis of consumer behaviour by means of *any* (high dimensional) consumer demand system would be difficult for several reasons.

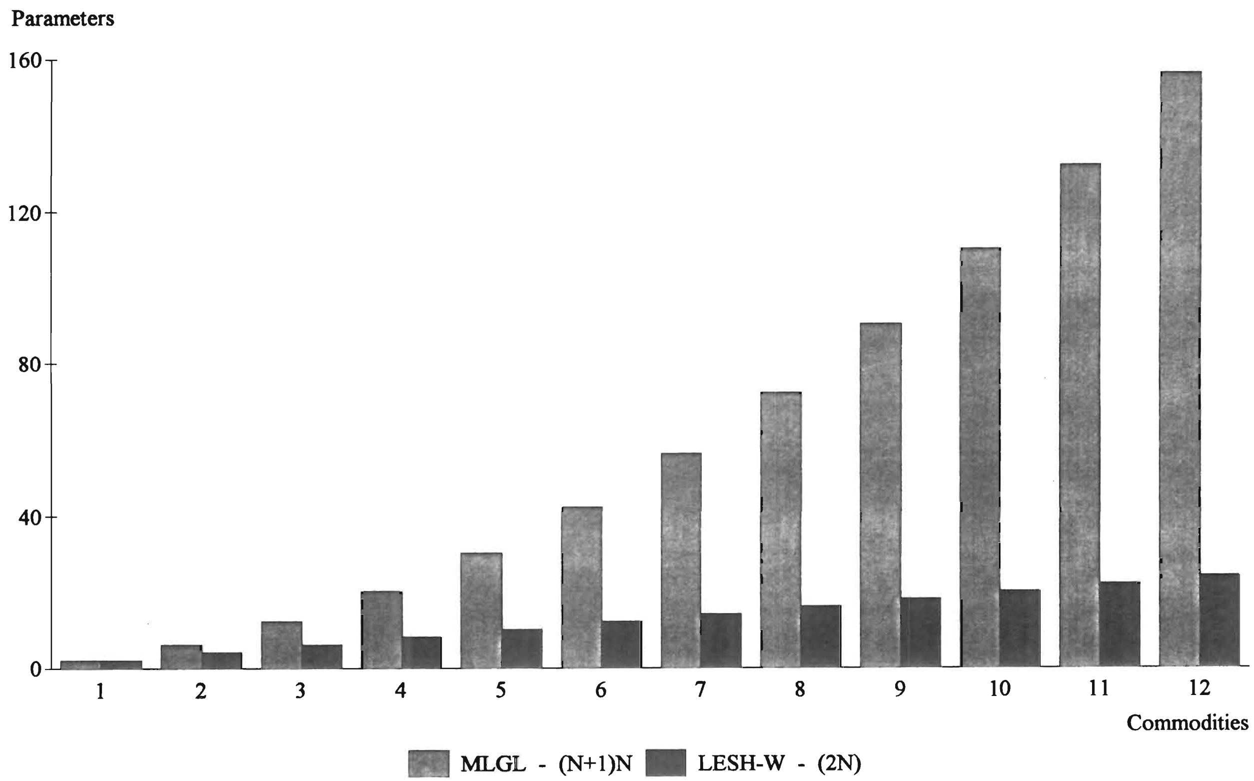
4.2.1 The curse of dimensionality

The main drawback of high-dimensional consumer demand systems is that the number of parameters increase with the number of separate commodity groups. The rate of increase varies however. The *Linear Expenditure System* has the advantage of being *parsimonious* in terms of parameters, as compared to more recently proposed demand-system specifications based on flexible functional forms, like the Translog model (TL), the Generalized-Leontief model (GL) or the Minflex Laurent versions of these two models that will be discussed in more detail below. The number of *LES-parameters increase* only *linearly* with the number of simultaneously optimized commodity groups, while the corresponding increase is *proportional to the number of commodities raised to the power of two for the other models*. Observations made during sixty years have been available for this study, a comparatively long period. Normally, the number of available and useful observations are less than that, due to events like World War II, considerable changes in economic policy, technology etc.

4.2.2 The quality of data

The National Accounts is the main source of information and a valuable source of time series data covering many important quantitative aspects of private consumption. However, as a "rule-of-thumb" and according to SCB officials, the quality of the data often decreases with the level of aggregation.

The number of parameters of the Linear Expenditure System and the Minflex Laurent Generalized Leontief model as a function of the number of commodities

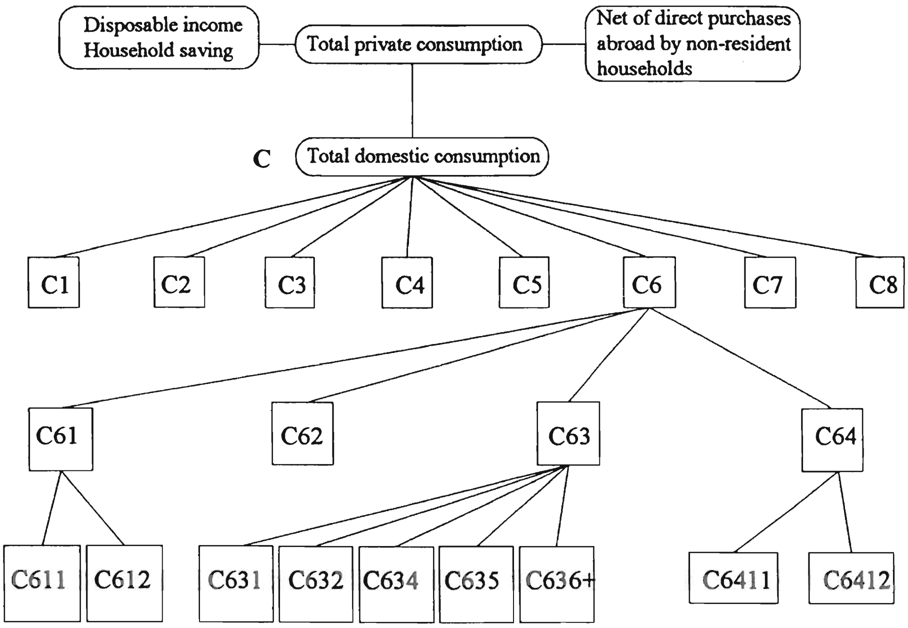


The contamination of errors becomes more significant in low-level series, and normally shows up in the rate of change of the variable that may behave in a less credible way during some periods. "Errors" may occur as regular measurement errors, but may also be caused by changes in statistical methodology from one period to another. During the process of aggregation, the errors will contaminate the broader aggregates, but will also "cancel each other out" to some extent if they are randomly distributed with mean zero. So, the broader the aggregate, the more reliable the relative performance of its development over time, but the greater also the distance to the real consumer's real decision to spend.

4.2.3 Hierarchical modelling, the only feasible alternative

The Swedish National Accounts now publish data concerning 99 "function defined" categories of goods and services demanded by private consumers. It is obviously not possible to estimate any single demand system of that dimension, given the information available versus the number of parameters to estimate. However, it is possible to estimate demand functions as components of some demand system for each one of these commodities, provided that a hierarchical approach is taken. This approach is correct if consumers first decide about how they would like to allocate the total amount of money available for private consumption among the broadest categories, given information about the aggregate price level of each group. After that decision, they go on and gradually make more detailed decisions along each one of the various branches of the consumption tree, only considering the price structure within each group.

As pointed out by Angus Deaton, whether or not the "tree-budgeting process" describes the actual behaviour, it is a most useful abstraction for the analyst, because an unmanageable optimization problem can now be handled by the break-down to a number of smaller sub-systems. When decisions are taken in a certain node, information from all parallel nodes are left out of consideration. Although, it is quite possible to imagine that information is passed from a lower level to a higher in order to update the "price-index"


Disposable income
Household saving
Total private consumption
Net of direct purchases abroad by non-resident households
C
Total domestic consumption
C1
C2
C3
C4
C5
C6
C7
C8
C61
C62
C63
C64
C611
C612
C631
C632
C634
C635
C636+
C6411
C6412

information of that level, which may trigger a reallocation. This may in turn cause a further feed-back to the parent node above etc. The information processing would be similar to a budget-dialogue in an organization.

4.2.4 A uniform choice of demand system or not?

Given the necessity to estimate separate demand (sub)systems on different levels of a hierarchic data structure, the next question arises: Should the specific system selected on the top level also be used uniformly on lower levels, or is it possible and even appropriate to select other systems for the lower levels? It has been argued by e.g. Lawrence Lau in the late seventies, that "uniformity is desirable because all commodities should be treated symmetrically" (Lau, 1977). The uniform approach has also been used earlier by e.g. Angus Deaton in a comprehensive study of consumer demand in post-war Britain in the seventies (Deaton, 1975).

5. The Linear Expenditure System (LES)

The Linear Expenditure System is not the first system of demand equations, but it is the first that was explicitly derived from a specific utility function. Already in 1942, Leser formulated a demand system that would later be known as the indirect addilog system, when it was rediscovered and theoretically confirmed to be a solution to a constrained utility maximization problem by Houthakker in 1960 (Deaton, 1975). The linear utility function, from which the LES is derived, was discussed in 1947–1948 by L.R. Klein and H. Rubin, by P.A. Samuelson and one year later by R.C. Geary. The first empirical application of the basic version of the Linear Expenditure System (eq. 10, below) was performed by J.R.N. Stone in 1954, analysing inter-war consumer demand in the U.K.

5.1 The basic form of the LES

The consumer is now assumed to maximize the following utility function,

$$U_t = U_t^0 + \sum_{i=1}^{n} \beta_i \log(q_{it} - c_{it}) \tag{8}$$

subject to the budget constraint,

$$\sum_{i=1}^{n} q_{it} \, p_{it} = y_t \, . \tag{9}$$

The desired demand for commodity i in period t is q_{it}, while c_{it} is a parameter deviced to represent the "necessary consumption" of this commodity. The price of a commodity at time t is represented by p_{it} and a budget by y_t .

The implied demand functions derived from the constrained maximization of (7) are,

$$q_{it} = c_{it} + \frac{\beta_i}{p_{it}} (y_t - \sum_{k=1}^{n} p_{kt} \, c_{kt}), \quad i = 1, ..., n \, . \tag{10}$$

The utility function (7) has a negative definit Hessian, if and only if all β-elements are positive and if the "uncommitted" income (also referred to as "the supernumerary income") is nonnegative

$$(y_t - \sum_{k=1}^{n} p_{kt} \, c_{kt}) \geq 0, \quad t = 1, ..., T \, . \tag{11}$$

From the demand system (10) it follows that the β-parameters may be interpreted as marginal expenditure propensities.

5.2 Inertia by formed habits incorporated in the LES

Like earlier Swedish analysts, when estimating the linear expenditure system, I have adopted the tradition introduced by Pollak and Wales, of incorporating into the LES a model of inertia in consumption caused by formed habits

(Pollak and Wales, 1969). In this tradition, the "necessary" or "committed" consumption of a commodity is modelled as a function of the consumption level of that commodity during the previous year. This introduces, in a simple but credible way, dynamics in the consumption model. The influence on present decisions from habits formed by past experience is in all probability a real factor. The simplest possible model is the following one,

$$c_{it} = \alpha_i \; q_{it-1} \; . \tag{12}$$

If this equation is substituted into (10), the LESH-pq model follows, with the expenditures on commodity i as dependent variable,

$$p_{it}q_{it} = \alpha_i \, p_{it}q_{it-1} + \beta_i \, (1 - \sum_{k=1}^{n} \alpha_k \, p_{kt}q_{kt-1}) + \epsilon_{it} \; . \tag{13}$$

If (11) is divided by the expenditures, y, we obtain the *mathematically equivalent* LESH-w version of this model, a system of expenditure share equations,

$$w_{it} = \alpha_i \, \frac{p_{it}q_{it-1}}{y_t} + \beta_i \, (1 - \sum_{k=1}^{n} \alpha_k \, \frac{p_{kt}q_{kt-1}}{y_t}) + \epsilon_{it} \; , \tag{14}$$

with expenditure shares $w_{it} = q_{it} \, p_{it} \, / \, y_t \, , \;\; i = 1, \ldots , n \; .$

In order to reduce heteroscedasticity, equation (14) is preferable to (13). According to (12), the α-parameters relate to the reservation level of commodity i, while the βs still are marginal expenditure propensities. The parameters are assumed to be constants or at least independent of q_{it}.

5.3 Estimation procedure and parameter estimates

Each estimation procedure must consider the following conditions. The budget restriction is always in effect and the expenditure shares sum to one (most easily seen from (14)). As a result,

$$\sum_{k=1}^{n} \varepsilon_{kt} = 0, \qquad t = 1, ...,T \, . \tag{15}$$

When estimating the LES model, the matrix of second order moments,

$$\hat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \hat{e}_t \, \hat{e}_t' \, , \tag{16}$$

would become singular, if the residual vector $\hat{\mathbf{e}}_t$ consisted of elements of all n equations that are linearly dependent. In order to avoid this result, one equation is left out of the estimation process and the estimate of $\boldsymbol{\Omega}$, now a square matrix of dimension n-1, will hopefully be non-singular.

5.3.1 Iterative least squares, the original ("Stone Age") method

British economists and econometricians were pioneers in using electronic computing in applied economic analysis. As early as in 1949, the Department of Applied Economics in Cambridge got access to the second operational electronic computer in the world, which was built after ENIAC in the USA. The electronic computing capacity in the early fifties, was, as a rule, too limited to estimate all parameters of a "large-scale", nonlinear economic model simultaneously, but sufficient to admit "lean production in applied econometrics. The advantage with limited computing capacity is that it puts more strain on the analyst to actively search for efficient algorithms, sometimes based on particular features of the problem to solve.

As can be seen from equations (10), (13) and (14), the demand equations of the linear expenditure system are bilinear in the parameters, $\{\boldsymbol{\beta}, \mathbf{c}\}$ and $\{\boldsymbol{\alpha}, \boldsymbol{\beta}\}$ respectively, which is a mild form of non-linearity. The linearity of the LES refers to the utility function, not to the demand system. Richard Stone managed to estimate model (10) by first fixing the elements of $\mathbf{c}$ and estimate $\boldsymbol{\beta}$ by ordinary least squares. That is possible, since the model is linear in $\boldsymbol{\beta}$ with fixed $\mathbf{c}$. In a following step, the estimated $\boldsymbol{\beta}$-parameters were assumed to be fixed, and the $\mathbf{c}$-elements estimated by OLS. This process was continued iteratively, until convergence was reached according to some criterium, unfortunately a slow process (Deaton, 1975).

One *advantage* gained by using least squares as an estimation method is the *robustness*. No explicit assumptions about the particular form of density function behind the stochastic process realized as residuals will influence the estimates. The following assumptions are sufficient,

$$E(\epsilon_{it})=0$$

$$E(\epsilon_{it},\epsilon_{js}) = \begin{cases} \sigma_{ij} & if \quad s=t \\ 0 & if \quad s\neq t. \end{cases} \tag{17}$$

The errors are assumed to have mean zero and a constant covariance among simultaneous error terms. Otherwise, the error terms are assumed to be uncorrelated. When using an iterative ordinary least squares method of Stone's design, the σ_{ij} are implicitly assumed to build a diagonal matrix of moments, or more specifically a unit matrix (Deaton, 1975).

Dahlman and Klevmarken relaxed this restrictive assumption and estimated the moment matrix. The n-1 dimensional estimate of Ω was used to apply generalized least squares iteratively in an IUI study of the private consumption in Sweden from the early seventies (Dahlman and Klevmarken, 1971).

5.3.2 Maximum-likelihood (ML), now available in nonlinear modelling

If the residuals ϵ_{it} can be assumed to be "drawn" from a particular probability distribution, a likelihood function can be established and maximized over the parameter space, given the sample of observed values, according to principles established by R. A. Fischer in the early 1920s. The relevance of Fisher's concept to economic research was realized by among others the Cowles Commission for Research in Economics, but time was not yet ripe. Now it is, triggered by the development of the electronic computing capacity over the last two decades. ML-methods have become generally accessible as useful, general methods in estimating nonlinear economic models (Cramer, 1986).

If the distributional assumptions are correct, the ML-method provides estimators with statistical properties that are now known to be valuable, at least in large samples. If, on the other hand, the ML-estimator would be based on an inapplicable choice of distribution function, ML-estimates *may* be biased and misleading. In that case, the analyst should apply some nonlinear least-squares method, or some other robust technique. He could certainly also reformulate the likelihood function, after having reconsidered the choice of underlying density function.

Maximum likelihood requires by construction an explicit choice of density function. In practice, the error terms are assumed to be independent and identically distributed random variables generated by a multivariate normal probability density function,

$$\epsilon_t \sim i.i.d. \quad N(0,\Omega^0) \ . \tag{18}$$

This assumption may be a matter of convenience more than of actual knowledge. In the context of demand analysis, the normality assumption is probably credible, at least on the higher levels of aggregation. I have decided to follow this tradition.

Let me in the rest of this paragraph *change notation* to some extent and address to the maximum likelihood techniques in more general terms. The LESH-w model (14) or some other similar model, given by a system of g = (n-1) expenditure equations can be represented as,

$$w_t = \varnothing(x_t \ , \ \beta) + \varepsilon_t \ , \tag{19}$$

where $\boldsymbol{\phi}(\mathbf{x}_t, \boldsymbol{\beta})$ is the particular deterministic model as a real valued function of the exogenous variables, $\mathbf{x}_t$, and the p-dimensional parameter vector, $\boldsymbol{\beta}$. The most probable value of $\boldsymbol{\beta}$, given $\mathbf{x}_t$ and (17), is the parameter vector that maximizes the likelihood function (in log-form)

$$\log L = C - \frac{T}{2} \log |\Omega| - \frac{1}{2} \sum_{t=1}^{T} e_t(\beta)' \, \Omega^{-1} \, e_t(\beta) \,, \tag{20}$$

where residuals are defined as

$$e_t = w_t - \phi(x_t, \beta) \,, \tag{21}$$

and Ω represents the matrix of second order moments (15).

The first order conditions for a maximum of the likelihood function with respect to the parameter vector β is given by

$$0 = f(\beta) = \sum_{t=1}^{T} \frac{\partial \phi}{\partial \beta}(x_t, \hat{\beta})' \hat{\Omega}_{\beta}^{-1} \, e_t(\hat{\beta}) \,. \tag{22}$$

If β is a p-dimensional parameter vector, the *score vector* **f**(β) is also p-dimensional.

The information matrix, associated with parameter vector β, is

$$\Psi_{(p \times p)} = E\, f(\beta) f(\beta)' \,, \tag{23}$$

and the asymptotic covariance matrix is given by its inverse

$$V(\beta) = \Psi^{-1} \,. \tag{24}$$

Full information maximum likelihood, FIML, would require that the g(g+1)/2 elements of the symmetric inverse moment matrix Ω^{-1}, were also estimated simultaneously with β. The equation system (21) would then be augmented by another g(g+1)/2 equations and the information matrix would expand accordingly into a block diagonal matrix (Cramer, 1986).

5.3.3 An iterative ML-procedure used

I now return to the original problem and notation used before the previous paragraph. I prefer to estimate the parameters $\{\alpha, \beta\}$ simultaneously but not explicitly include the elements of Ω^{-1}. By not using a regular FIML procedure, I must stick to the iterative tradition in the following sense.

I started to estimate the LESH-w system (14) by using nonlinear least squares, assuming Ω^{-1} to be a unit matrix. This procedure converged nicely, given any theoretically feasible start values of $\{\alpha, \beta\}$. The estimated parameters $\{\alpha_{ls}, \beta_{ls}\}$, were used to compute Ω_{ls}^{-1}.

In the second step, the parameters $\{\alpha_{ls}, \beta_{ls}\}$ were used as start values and the matrix Ω^{-1} was set equal to Ω_{ls}^{-1}, in the search for the parameters $\{\alpha, \beta\}$ that minimized the Euclidean norm of the vector of score functions (22), $||f(\alpha, \beta)||_2$, productively assisted by the NAG-subroutine E04FDF. This routine is deviced for unconstrained minimization of a sum of squares function without any requirement of user supplied analytical derivatives and hence easy to use.

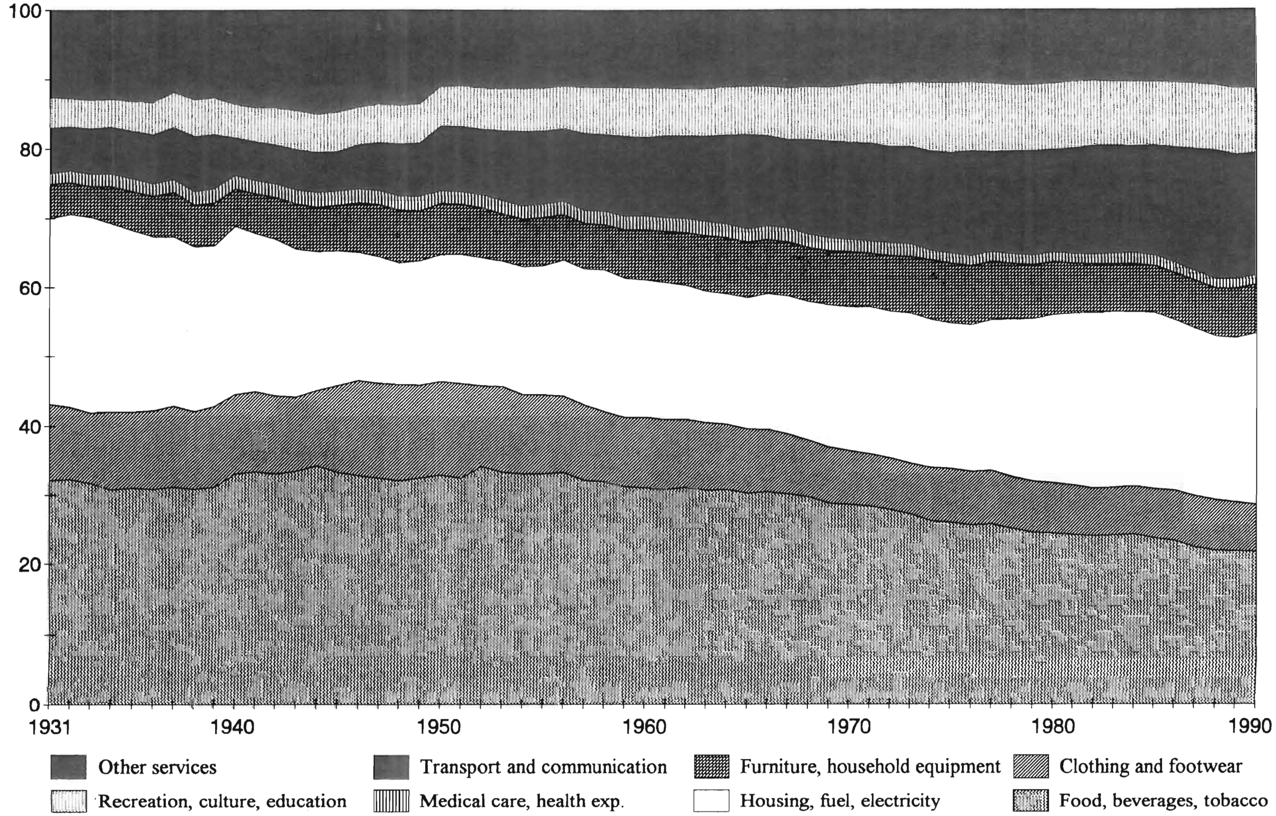
In the third step, the parameters $\{\alpha_{ml}, \beta_{ml}\}$ were used as start values and the matrix Ω^{-1} was set equal to Ω^{-1}_{ml} in a new search for a minimum value of $||f(\alpha, \beta)||_2$. The third step was repeated and $\{\alpha_{ml}, \beta_{ml}\}$ converged rapidly to a limit, $\{\underline{\alpha}_{ml}, \underline{\beta}_{ml}\}$.

5.3.4 The estimated LESH-w parameters

The ml-estimates $\{\underline{\alpha}_{ml}, \underline{\beta}_{ml}\}$ of the LESH-w model applied to the main eight consumption groups, C1–C8, obtained according to the procedure presented in 5.3.3 are tabulated with asymptotic standard errors in parenthesis. The asymptotic standard errors, computed as square roots of the diagonal elements of $V(\beta)$, are all comfortably low and all elements of $\{\underline{\alpha}_{ml}, \underline{\beta}_{ml}\}$ are significantly greater than zero and consistent with regularity conditions of the utility function. When estimating model (14), the last equation was excluded. The marginal expenditure propensity related to financial services, hotel & restaurants etc, C8, was computed residually,

$$\beta_8 = 1 - \sum_{k=1}^{7} \beta_k \ . \tag{25}$$

Expenditure shares of private domestic consumption



LESH-w, parameter estimates
period, 1931-1990

α_1 =	.975394	β_1 =	.228219
	(.009057)		(.025772)
α_2 =	.935412	β_2 =	.194242
	(.019062)		(.013765)
α_3 =	1.004544	β_3 =	.089659
	(.005221)		(.014679)
α_4 =	.954934	β_4 =	.113795
	(.015350)		(.008414)
α_5 =	.989033	β_5 =	.018392
	(.010787)		(.001832)
α_6 =	.984855	β_6 =	.142009
	(.012457)		(.017208)
α_7 =	1.008866	β_7 =	.051899
	(.008946)		(.008510)
α_8 =	.948517	β_8 =	.161785
	(.019029)		(n.a.)

The sample period is 1931 to 1990. The 60 years covered by the sample include World War II, the Korea inflation, OPEC I and II, a number of devaluations of the SEK as well as considerable changes of technology and economic structure. It is a positive surprise to see how well the LESH-w model performs during all these years.

LESH-w, Measures of association, R2

Total R2= .9713

C1, R2=	.9696
C2, R2=	.9697
C3, R2=	.9723
C4, R2=	.9714
C5, R2=	.9713
C6, R2=	.9749
C7, R2=	.9756
C8, R2=	.9644

Following earlier studies, the measures of association are computed as,

$$R^2_{all} = 1 - \frac{\sum_i \sum_t (w_{it} - \hat{w}_{it})^2}{\sum_i \sum_t (w_{it} - \bar{w}_{it})^2}$$

$$R^2_{C_i} = 1 - \frac{\sum_t (w_{it} - \hat{w}_{it})^2}{\sum_t (w_{it} - \bar{w}_{it})^2} \quad . \tag{26}$$

5.4 Price and expenditure elasticities

The short run (instantaneous) price elasticities of the LESH model(s) are defined as follows and tabulated referring to the last observation ,

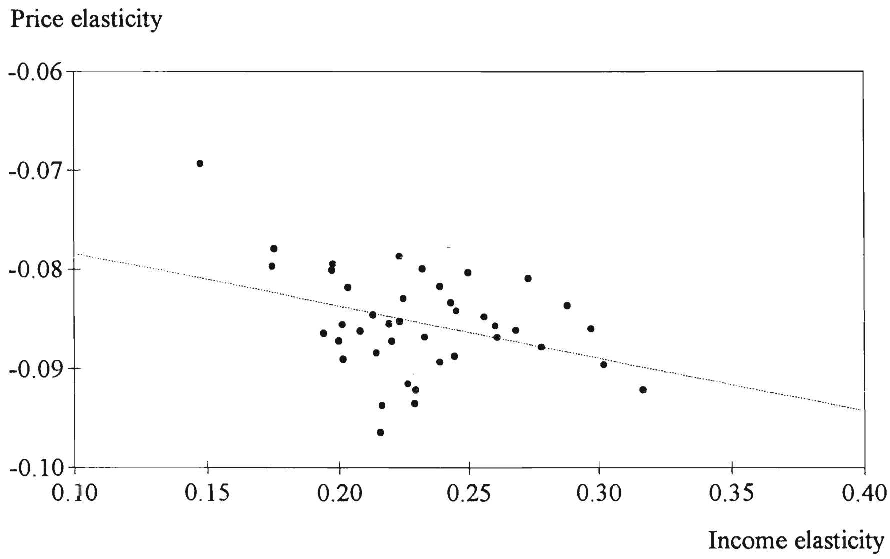
$$e_{ii0} = \frac{p_{it}}{q_{it}}\frac{\partial q_{it}}{\partial p_{it}} = -1 + (1-\beta_i)\alpha_i \frac{q_{it-1}}{q_{it}}, \quad i=1, ..., n$$

$$e_{ik0} = \frac{p_{kt}}{q_{it}}\frac{\partial q_{it}}{\partial p_{kt}} = -\alpha_k \beta_i \frac{p_{kt}}{p_{it}}\frac{q_{kt-1}}{q_{it}}, \quad i \neq k \quad . \tag{27}$$

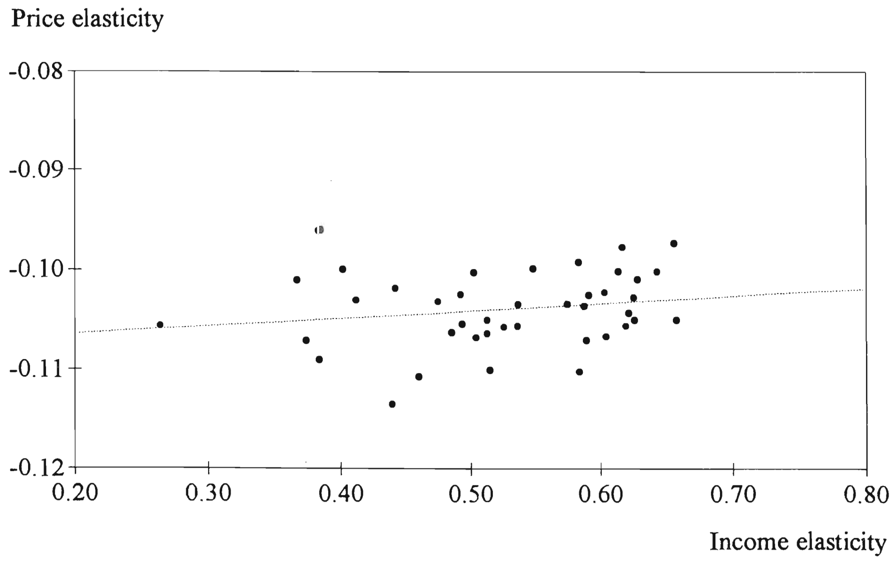
LESH-W, *Instantaneous* cross & own price elasticities in 1990, due to the income-effect of a price change

	C1	C2	C3	C4	C5	C6	C7	C8
C1	-.2452	-.0677	-.2669	-.0718	-.0168	-.1951	-.1023	-.1192
C2	-.5782	-.2360	-.6956	-.1871	-.0438	-.5086	-.2667	-.3108
C3	-.0743	-.0227	-.0842	-.0240	-.0056	-.0654	-.0343	-.0399
C4	-.3327	-.1015	-.4003	-.1503	-.0252	-.2927	-.1535	-.1788
C5	-.2316	-.0706	-.2786	-.0749	-.0262	-.2037	-.1068	-.1245
C6	-.1634	-.0498	-.1966	-.0529	-.0124	-.1558	-.0754	-.0878
C7	-.1155	-.0352	-.1390	-.0374	-.0087	-.1016	-.0438	-.0621
C8	-.2980	-.0909	-.3585	-.0964	-.0226	-.2621	-.1374	-.2118

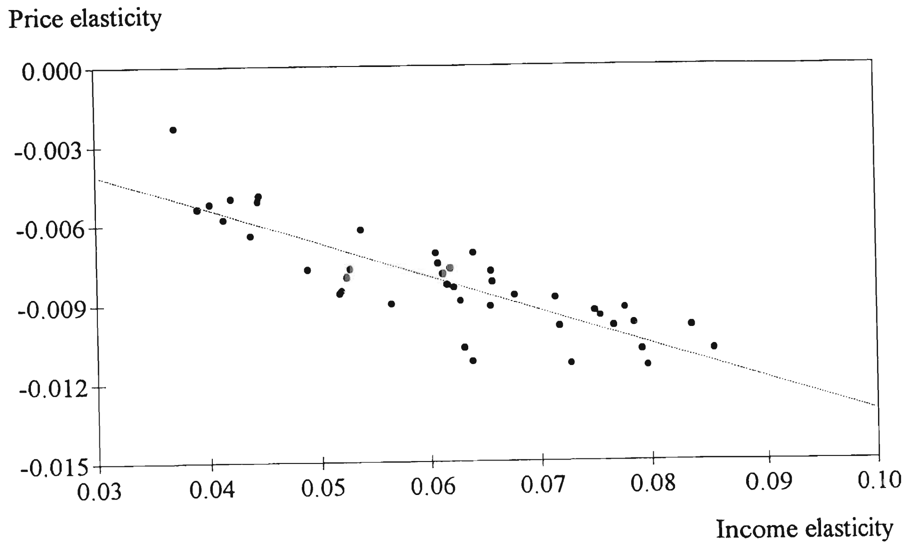
Food, beverages and tobacco,
adjusted elasticities 1950 - 1990, LESH-W



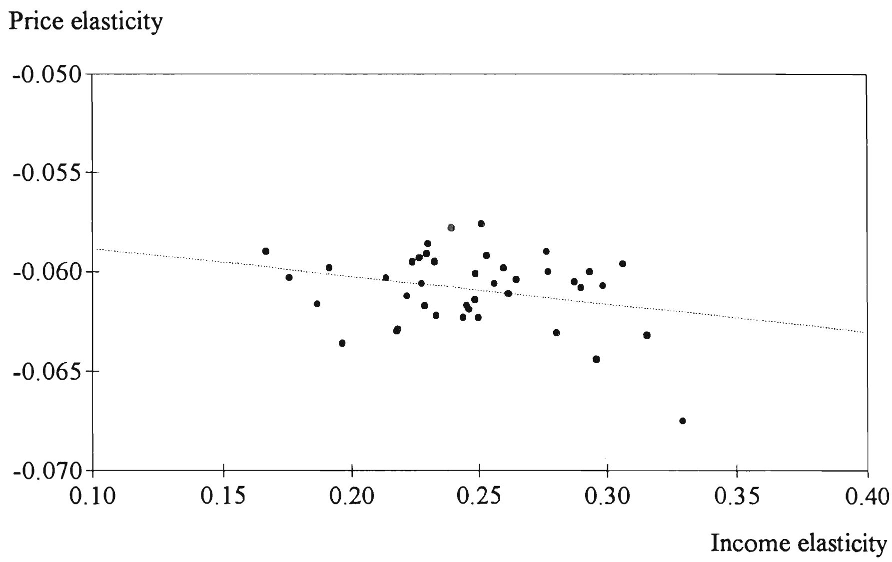
Clothing and footwear,
adjusted elasticities 1950 - 1990, LESH-W



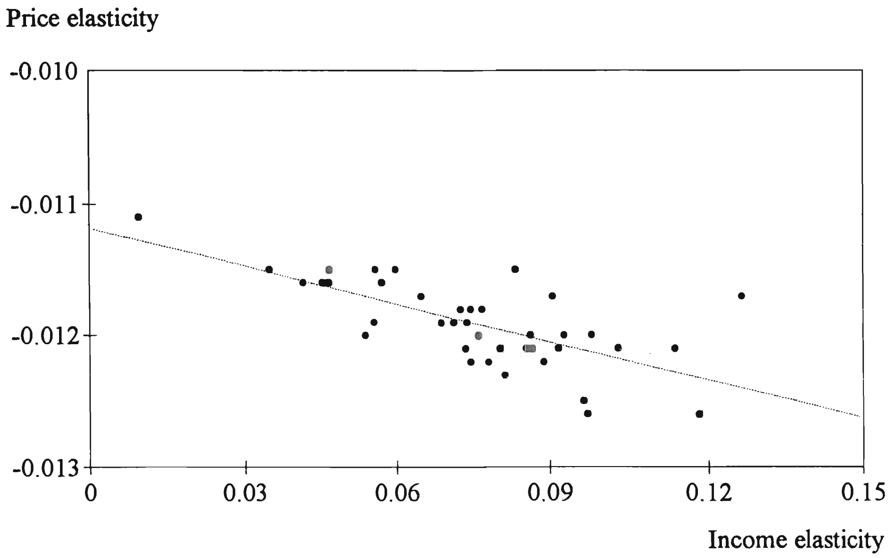
Housing, fuel and electric energy,
adjusted elasticities 1950 - 1990, LESH-W



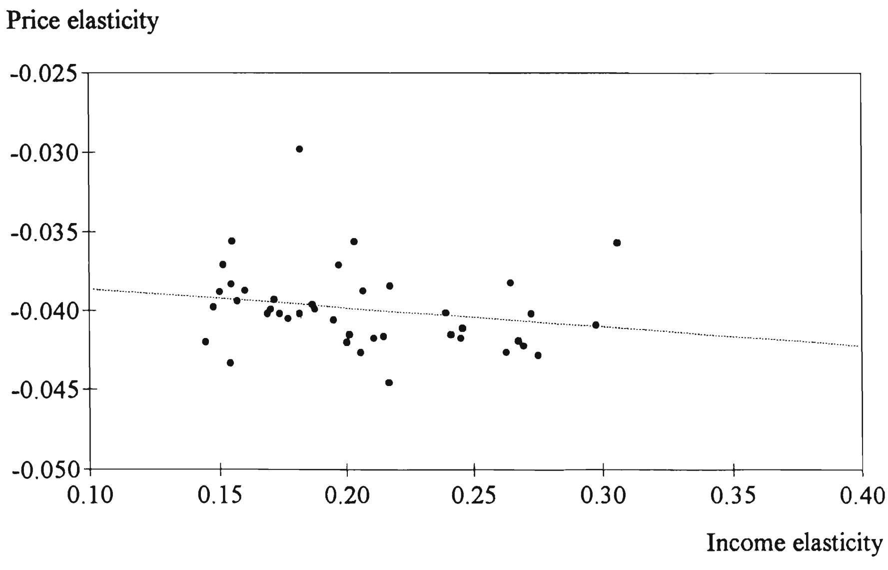
Furniture, furnishings, household equipment and opr.,
adjusted elasticities 1950 - 1990, LESH-W



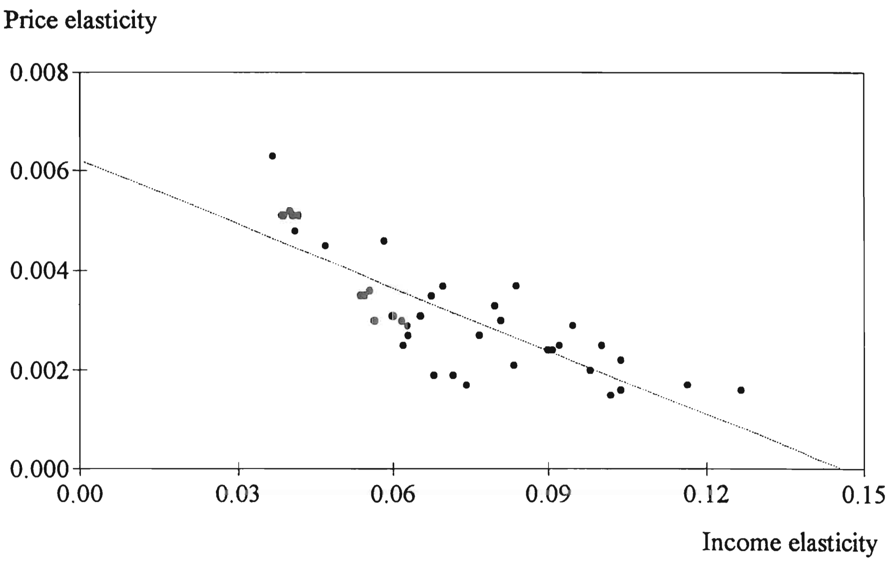
Medical care and health expenses,
adjusted elasticities 1950 - 1990, LESH-W



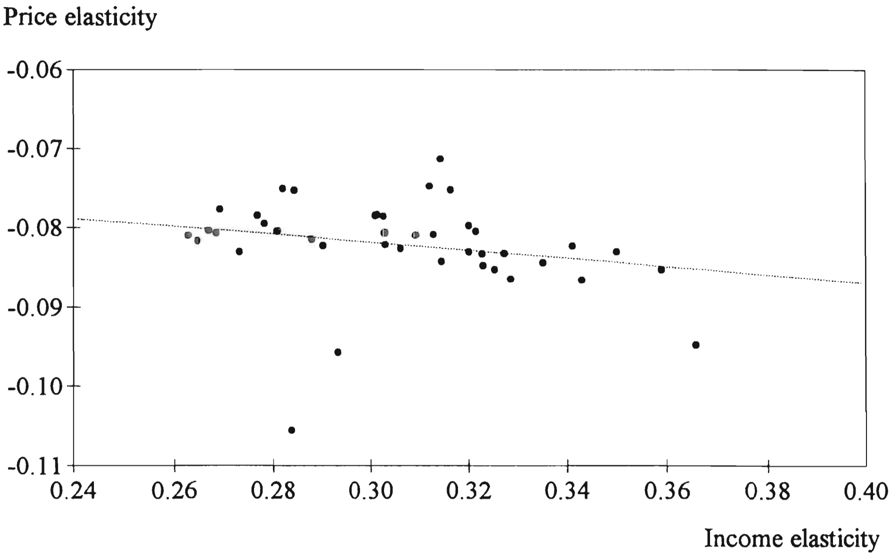
Transport and communication,
adjusted elasticities 1950 - 1990, LESH-W



Recreation, entertainment, education and cultural services,
adjusted elasticities 1950 - 1990, LESH-W



Financial services, hotels & restaurants, other goods & services,
adjusted elasticities 1950 - 1990, LESH-W



The LESH model's short run expenditure elasticity concerning commodity i is,

$$E_{i0} = \frac{y_t}{q_{it}} \frac{\partial q_{it}}{\partial y_t} = \frac{\beta_i}{w_{it}}, \quad i = 1, \ldots, n \,. \tag{28}$$

The simple dynamics of the LESH model allows net "long-run" price and income elasticities to be defined. Long-run refers to the period after the price or expenditure changes (Dahlman and Klevmarken, 1971).

$$e_{ii1} = \frac{p_{it-1}}{q_{it}} \frac{\partial q_{it}}{\partial p_{it-1}} = e_{ii0} + \sum_{k=1}^{n} e_{ik0}\, e_{ki0}, \quad i = 1, \ldots, n$$

$$E_{i1} = \frac{y_{t-1}}{q_{it}} \frac{\partial q_{it}}{\partial yt-1} = E_{i0} + \sum_{k=1}^{n} e_{ik0}\, E_{k0}, \quad i = 1, \ldots, n \,. \tag{29}$$

In the perceding diagrams, pairs of "long-run" own price elasticity measures, e_{ii1}, and expenditure elasticities, E_{i1}, are plotted concerning all main consumption categories during the period 1950–1990. With one exception, a negative relation seems to exist between the long-run own price elasticity and the long-run expenditure elasticity which on this high level is equal to the income elasticity if the saving ratio is fixed.

6. The Minflex Laurent Generalized Leontief model (MLGL)

The LES represents a simple, elegant and useful model of consumer demand. Its simplicity is one of its most attractive features, but the simplicity is obtained by a restrictive "view" of the consumer's possibilities to substitute among commodities as a response to changing relative prices. That restriction is not harmful if the LES is used as an instrument to analyse broad aggregates, like the eight main groups in this study. If we walk down along some branch of the consumer tree, we may have good reasons to expect that substitution effects become more important. The utility function on these levels must support equations that provide credible measures of substitution, price, and expenditure elasticities.

6.1 Constant elasticities of substitution and flexible functional forms

In the early 1960s, Hirofumi Uzawa formulated and proved the following equivalence. Assume that there are n factors of production, {1, ..., n} that can be completely partitioned into S subsets $\{N_1, ..., N_S\}$, so that $N_s \cap N_t = \emptyset$, $s \neq t$. Corresponding to this partition, the volumes and the prices of the production factors can be partitioned to subvectors $x^{(1)}, ..., x^{(S)}$ and $p^{(1)}, ..., p^{(S)}$ respectively.

Each member of the class of production functions defined as,

$$f(x) = \prod_{s=1}^{S} f^{(s)}(x^{(s)})^{\rho_s}, \text{ where}$$

$$f^{(s)}(x^{(s)}) = \left(\sum_{i \in N_s} \alpha_i x_i^{-\beta_s} \right)^{-\frac{1}{\beta_s}}$$

$$\alpha_i > 0, \quad i=1, \dots, n \qquad (30)$$

$$-1 < \beta_s < \infty, \; \beta_s \neq 0$$

$$\rho_s > 0, \; \sum_{s=1}^{S} \rho_s = 1 ,$$

has constant partial elasticities of substitution, σ_{ij}, with respect to each pair of production factors according to,

$$\sigma_{ij} = \begin{cases} 1, & \text{if } i \in N_s, j \in N_t, t \neq s \\ \sigma_s, & \text{if } i,j \in N_S \end{cases} \qquad (31)$$

$$\sigma_s = \frac{1}{1 + \beta_s} \neq 1 .$$

Uzawa also showed that *all* linearly homogenous, strictly quasi-concave and three times continuously differentiable production functions are included in the class defined by (30).

The partial elasticities of substitution are defined by Uzawa in terms of the unit cost function, λ(p), i.e. the cost to produce one unit of output by means of the optimal mix of available factors of production,

$$\sigma_{ij} = \frac{\lambda \dfrac{\partial^2 \lambda}{\partial p_i \, \partial p_j}}{\dfrac{\partial \lambda}{\partial p_i} \, \dfrac{\partial \lambda}{\partial p_j}} \, , \tag{32}$$

where

$$\lambda(p) = \sum_{i=1}^{n} p_i \, x_i^*(p) \, . \tag{33}$$

In a broad and important class of production functions with a globally regular behaviour, the associated partial elasticities of substitution will take constant, but *not arbitrary* values (Uzawa, 1962). Further, it follows from (30), that least restricted in terms of freedom to adapt to an arbitrary elasticity of substitution, is the well known CES-function defined for two factors of production, originally proposed by Arrow, Chenery, Minhas and Solow in an article published the year before Uzawa's note (Arrow, et. al., 1961). The more factors of production, the tighter the restrictions on the partial substitution elasticities.

Since the regularity conditions of the neoclassical theory of firms are parallel to those of the neoclassical theory of households, the arguments as above also go for the relation between the curvature of utility functions and partial elasticities of substitution among pairs of consumer goods and services.

6.2 The TL-, GL- and MLGL-models briefly compared

The restrictions discussed above, inspired Erwin W. Diewert to propose a "Generalized Leontief Production Function" (GL) in 1971, and Laurits Christensen, Lawrence R. Lau and Dale W. Jorgenson to propose "Transcendental Logarithmic Production Frontiers" (TL) two years after. These two are the main exponents of what is now known as "flexible functional forms", *deviced to permit inferences without prior constraints about elasticities.*

However, when the monotonicity and curvature properties of the TL- and GL-utility functions, (35) and (36) respectively, were inspected at data points on some distance from the sample mean of the real or simulated dataset used to estimate the model's parameters, the utility function occasionally ceased to be concave or strictly quasiconcave. The conditions under which the TL- and GL-models tend to offend against the regularity conditions are different. If the AUES are low and the consumer's preferences not too influenced by the income level, the GL-demand functions will as a rule be supported by a quasiconcave utility function. If the same condition applies to the preferences (near homothetic) but the elasticities of substitution are close to one, the TL-utility function will as a rule be regular. When preferences change significantly when the income level changes and the AUES are neither close to zero nor to one, both models tend to have a very limited region where their utility functions are regular.

The Minflex Laurent Generalized Leontief model, the MLGL-model, was proposed by William A. Barnett, Yul W. Lee and M. Wolfe in the mid-eighties. They proposed a specification of the utility function and related demand equations that, without becoming irregular, would be able to "adapt to" an arbitrary set of Allen-Uzawa elasticities of substitution at *any point in a region* containing at least the convex hull of the dataset used to estimate the parameters of the model (Barnett and Lee, 1985a and b).

Let us first see how the MLGL-model relates to its predecessors among the demand models based on "flexible functional forms". Consider the following definitions of variables,

p_i price of group i
q_i volume of good i
$y = p'q$, total expenditure on $1,\ldots,n$; current prices

$$w_i = \frac{p_i q_i}{y} \quad i=1,\ldots,n,$$
$$v_i = \frac{p_i}{y} \quad i=1,\ldots,n, \qquad (34)$$
$$u_i = \sqrt{v_i} \quad i=1,\ldots,n,$$
$$\bar{u}_i = \frac{1}{u_i} \quad i=1,\ldots,n,$$

and the following three utility functions defined in prices and expenditures, i.e. indirect utility functions,

the Trans-log (TL)

$$\ln V(v) = \sum_i \alpha_i \ln v_i + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln v_i \ln v_j , \qquad (35)$$

the Generalized Leontief (GL)

$$\bar{V}(v) = a_0 + a' u + u' A u , \qquad (36)$$

and the Minflex Laurent Generalized Leontief (MLGL)

$$\bar{V}(v) = a_0 + 2 a'u + \sum_{k=1}^{n} a_{kk} v_k + \sum_{j=1,\neq i}^{n} \sum_{k=1,\neq i}^{n} a_{jk}^2 u_j u_k - \sum_{j=1,\neq i}^{n} \sum_{k=1,\neq i}^{n} b_{jk}^2 \bar{u}_j \bar{u}_k . \qquad (37)$$

Each one of these models can provide a second order local approximation to a regular but unknown utility function. A more elaborate discussion of the flexibility criterion is provided by (Diewert, 1971) and (Barnett, 1985a and b). For all models, the matrix (or matrices) of second order parameters are restricted to be symmetric. There is also a need for parameter normalization, because the expenditure share equations emerging from each one of them are homogenous of degree zero in the parameters. An example related to the MLGL-model is given by equation (40). Otherwise there are no explicit restrictions on the parameters of the TL- or the GL-models. In contrast to that, *all parameters of the MLGL-model are a priori restricted to be nonnegative* (Barnett, 1985b, p. 1 426). Possibly, if the second order parameters are specified to be squared as in (37), they could in principle be estimated without a nonnegativity restriction. Anyhow, the TL- and GL-models possess more unconstrained coefficients than the MLGL-model. On the other hand, given a specified number of commodities, the MLGL-model has a greater number of parameters as compared to the other ones. That is not an advantage to the statistical efficiency when estimating the parameters of the MLGL-model, but it contributes to its flexibility properties.

Why then introduce this model without paying attention to the Occham razor principle? The TL- and GL-models are approximations of the indirect utility function based on a Taylor-series expansion. The MLGL-model on the other hand is based on a Laurent-series expansion, normally used to handle singularities in (complex) analytical functions, where they are defined to be series of the type,

$$\sum_{k=-\infty}^{k=+\infty} a_k (z - a)^k . \qquad (38)$$

In the utility function approximation (37), the last term corresponds to exponent k= -1 referring to (38). The main argument in favour of using a Laurent series type of approximation, is that the remainder term is the sum of two terms that always move in opposite directions, which is favourable as compared to a Taylor series expansion. The advantage to be expected is that (37) will keep desirable curvature properties over a larger region than will (35) and (36). The particular advantage pointed out by Barnett and Lee is that the regular region of the MLGL-model increases when income increases. With some recent exceptions, this has normally been the case in the region of the most recent observations. The MLGL-model can be expected to behave well when used in forecasting or simulation experiments (Barnett and Lee, 1985b).

The MLGL-model implies the following set of demand equations, expressed as expenditure shares and with error terms added.

$$w_i = \frac{a_i u_i + a_{ii} v_i + \sum_{j=1,\neq i}^{n} a_{ij}^2 u_i u_j + \sum_{j=1,\neq i}^{n} b_{ij}^2 \bar{u}_i \bar{u}_j}{a' u + \sum_{k=1}^{n} a_{kk} v_k + \sum_{j=1,\neq i}^{n} \sum_{k=1,\neq i}^{n} a_{jk}^2 u_j u_k + \sum_{j=1,\neq i}^{n} \sum_{k=1,\neq i}^{n} b_{jk}^2 \bar{u}_j \bar{u}_k} + \epsilon_i \,. \qquad (39)$$

Equation (39) is homogenous of degree zero in the parameters. The following normalization is used in the process of estimation,

$$\sum_{i=1}^{n} a_{ii} + 2 \sum_{i=1}^{n} a_{ii} + \sum_{j=1,\neq i}^{n} \sum_{k=1,\neq i}^{n} a_{ij}^2 - \sum_{j=1,\neq i}^{n} \sum_{k=1,\neq i}^{n} b_{ij}^2 = 1 \,. \qquad (40)$$

The $\mathbf{A}=[\mathbf{a}_{ij}]$ and $\mathbf{B}=[\mathbf{b}_{ij}]$ matrices are assumed to be symmetric and the diagonal elements of **B** restricted to zero. At least all first order parameters are a priori restricted to be nonnegative. Among the second order parameters there is an optional restriction, the "minflex condition",

$$a_{ij}\, b_{ij} = 0, \quad i \neq j \,. \qquad (41)$$

This restriction refers to a minimum amount of parametric freedom needed by the model to be fully flexible.

6.3 Estimation of the MLGL-model

Also the MLGL-model has been estimated with a maximum likelihood method. The same standard assumptions concerning the error distribution was adopted as in the previous estimation of the LES, i.e.

$$\epsilon_i \sim i.i.d. \; N(0,\Omega^0)$$

$$E(\epsilon_{it})=0$$

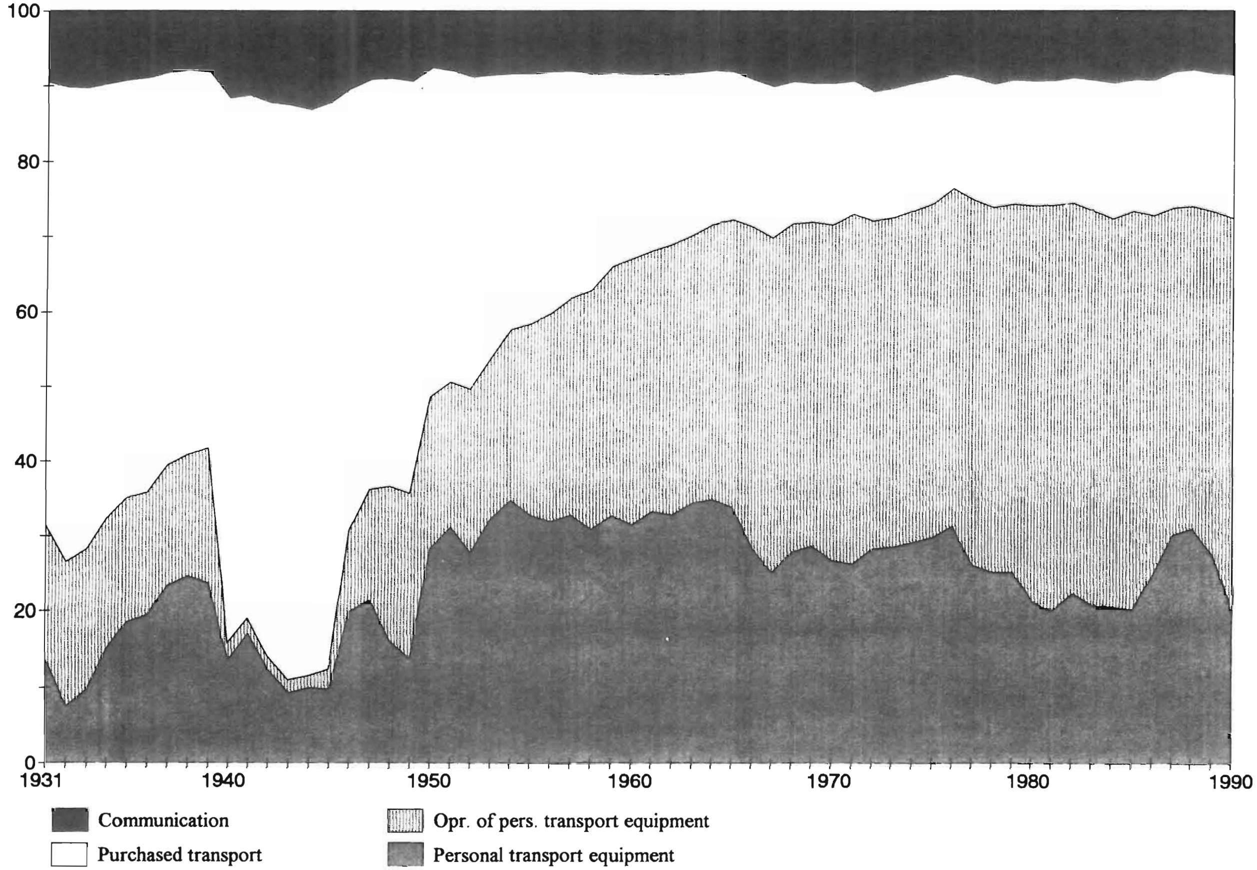
$$E(\epsilon_{it},\epsilon_{js}) = \begin{cases} \sigma_{ij} & if \;\; s=t \\ 0 & if \;\; s \neq t \end{cases}$$

The MLGL-model applied to the transport and communication level, with four service categories, below referred to as MLGL-4, has twenty parameters given the restrictions discussed above. In order of dimensionality of the parameter space, this is comparable to sixteen parameters in the LES application to eight commodity groups. The MLGL-2 model applied to communications, with postal service and telecommunication, has only six parameters. Despite this fact, the MLGL-models turned out to be far more labour intense and CPU demanding, as compared to the LES, mainly because of the a priori restriction that at least all first order parameters must be nonnegative. In order to comply with that requirement, I decided to present the first order conditions of maximizing the likelihood function (20) to another routine in the NAG-library, E04UPF, that is designed to minimize an arbitrary smooth sum of squares function subject to constraints. These may include simple bounds on the parameters, linear constraints and/or smooth nonlinear constraints. However, the routine requires that its user not only supplies program-code concerning the score functions but also concerning their first order partial derivatives. Also the nonlinear constraining function, i.e. equation (40), and its derivatives must be supplied, although a minor problem. In the MLGL-2 case this means six score functions and 36 derivatives, but in the MLGL-4 case there are 20 score functions and 400 derivatives, now with contributions from three equations.

6.3.1 MLGL-4 parameters, transport and communication

In a four commodity version of the MLGL-model there are twenty parameters. The estimates obtained are as follows in the table.

Expenditure shares, transport and communication



MLGL-4, parameters – transport and communication. Sample period: 1960–1990

a1 = 1.00000000	a11 = 1.53690260 (***)	a22 = 0.01396012 (***)
a2 = 0.00000000	a12 = 0.000000000	a23 = 1.15836590 (***)
a3 = 0.71016405 (***)	a13 = 0.000000000	a24 = 0.00000000
a4 = 0.68330769 (***)	a14 = 0.000000000	a33 = 0.04187396 (***)

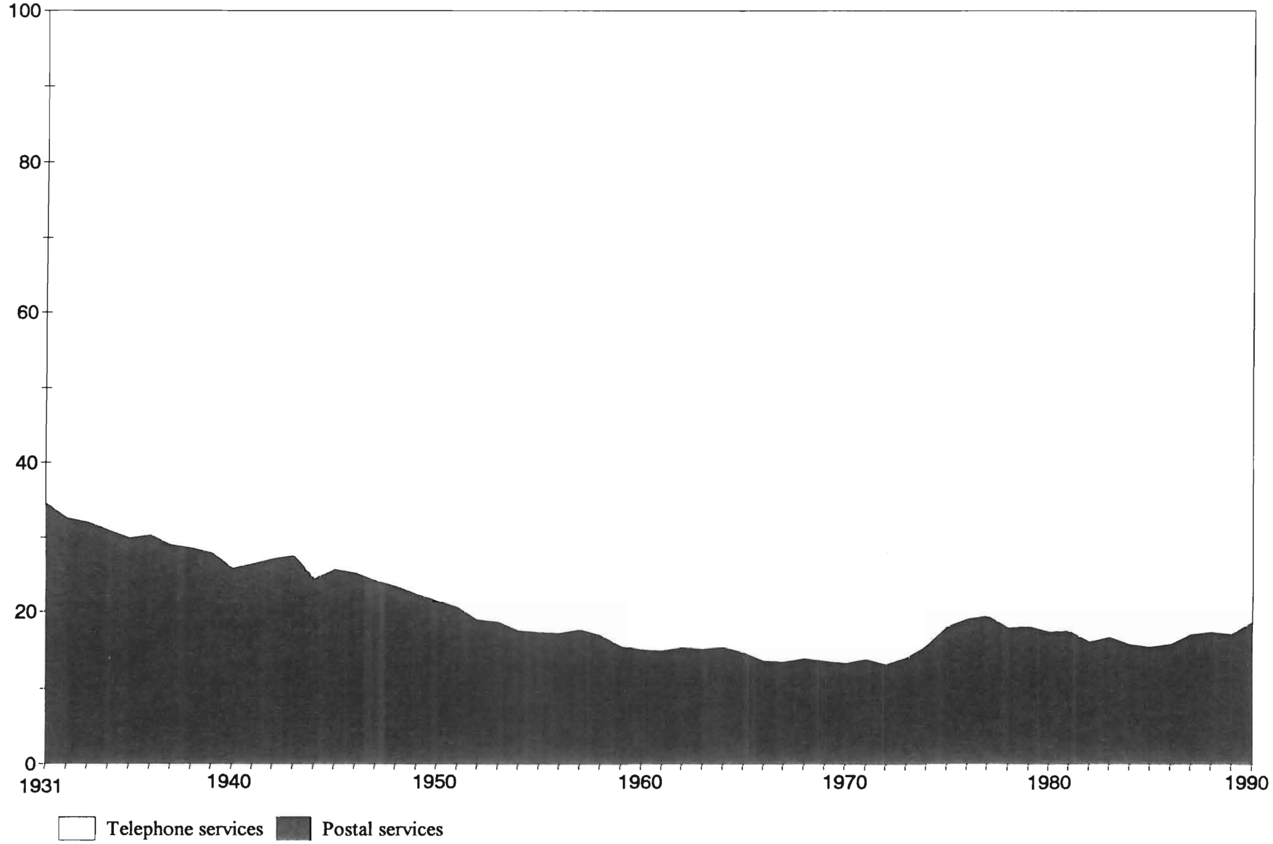
a34 = 0.00000000	b14 = 0.01394199 (0.72204)
a44 = 0.01396170 (***)	b23 = 0.03619080 (0.01427)
b12 = 1.43301800 (0.01580)	b24 = 0.72847446 (0.01377)
b13 = 0.34562565 (0.06338)	b34 = 0.51132322 (0.01797)

The asymptotic standard errors are out of range, except for the b-parameters. The parameters are subject to a non negativity restriction. No conclusions can be drawn directly from inspection of parameter values. The economic implications require computation of elasticity measures. One "technical" observation is that the normalizing restriction (40) is very "weak". Also with this restriction in effect, the expenditure shares are still very close to be homogeneous of degree zero in the parameters. This may explain the large values of most asymptotic standard errors.

6.3.2 MLGL-2 parameters, postal services and telecommunication

In a two commodity version of the MLGL-model, there are only six parameters. The estimates obtained are as follows in the table.

Expenditure shares, communication



MLGL-2, parameters – post and telecommunication. Sample period 1931–1990

a1 = 0.01384737 (***)	a12 = 0.03060796 (***)
a2 = 0.30026396 (***)	a22 = 0.00000000 (***)
a11 = 0.36992652 (***)	b12 = 0.00000004 (0.03303)

6.4 Substitution, price and expenditure elasticities

The expenditure elasticity of the MLGL-model is computed as,

$$E_{i0} = \left(\frac{\bar{A}_i'}{\bar{A}_i} - \frac{D'}{D} \right) y \ , \tag{42}$$

where the definitions of variables in (34) are supplemented as follows,

$$A_i = a_i u_i + a_{ii} v_i + \sum_{j=1, \neq i}^{n} a_{ij}^2 u_i u_j + \sum_{j=1, \neq i}^{n} b_{ij}^2 \bar{u}_i \bar{u}_j \ , \tag{43}$$

$$\bar{A}_i = \frac{A_i}{v_i} \ , \tag{44}$$

$$\bar{A}_i' = \frac{\partial \bar{A}_i}{\partial y} \ , \tag{45}$$

$$D = \sum_i A_i \ , \tag{46}$$

$$D' = \frac{\partial D}{\partial y} \ . \tag{47}$$

The price elasticities are given by

$$e_{ij} = w_{ij} \, (\sigma_{ij} - E_{i0}) \, , \tag{48}$$

where σ_{ij} is the partial elasticity of substitution (AUES) computed as,

$$\begin{aligned}
\sigma_{ij} = \{ \; & \frac{a_i \mu_i w_j}{w_i} + \frac{a_j \mu_j w_i}{w_j} + \frac{(a_i \mu_i A_i + a_j \mu_j A_j)(\sum_{k \in \Lambda_{ij}} A_k)}{w_i w_j \, (\sum_k A_k)^2} \\
& - \sum_{k \in \Lambda_{ij}} a_k \mu_k + \frac{a_{ij}^2 u_i u_j}{w_i w_j} - [\, 2 - 3(\frac{w_i}{w_j} + \frac{w_j}{w_i}) \\
& + \frac{\sum_{k \in \Lambda_{ij}} A_k^2}{w_i w_j \, (\sum_k A_k)^2} - 2(\frac{1}{w_i} + \frac{1}{w_j}) \frac{(\sum_{k \in \Lambda_{ij}} A_k)}{\sum_k A_k}] b_{ij}^2 \, \bar{u}_i \bar{u}_j \\
& - \frac{4 \, (A_i - \sum_{k,\, k \neq i} A_k)}{w_i \sum_k A_k} \sum_{k \in \Lambda_{ij}} b_{ik}^2 \bar{u}_i \bar{u}_k \\
& - \frac{4 \, (A_j - \sum_{k,\, k \neq i} A_k)}{w_j \sum_k A_k} \sum_{k \in \Lambda_{jj}} b_{jk}^2 \bar{u}_j \bar{u}_k \; \} \, / \, 2 \, D \\
\textit{where} \;\; & \Lambda_{ij} = \{ \, k \mid k \neq i, \, k \neq j \, \}
\end{aligned} \tag{49}$$

and the own AUES as,

$$\begin{aligned}
\sigma_{ii} = \{ \; & - \sum_{j,\, j \neq i} a_j u_j - \frac{a_i \mu_i}{w_i^2 (\sum_{j,\, j \neq i} A_j)^2} (\sum_{j,\, j \neq i} A_j)^2 \,) - \frac{u_i}{w_i^2} \sum_{j,\, j \neq i} a_{ij}^2 u_j \\
& - [\, \frac{3 \, (\sum_j A_j^2)}{w_i^2 (\sum_j A_j)^2} - \frac{2 \sum_{j,\, j \neq i} A_j}{w_i \sum_j A_j} + \frac{3 \, (\sum_{(j,k) \in \Lambda_i} \sum A_j A_k \,)}{w_i^2 \, (\sum_j A_j \,)^2}] \\
& \times \bar{u}_i \sum_{j,\, j \neq i} b_{ij}^2 \bar{u}_j - 4 \sum_{(j,k) \in \Lambda_i} \sum b_{jk}^2 \, \bar{u}_j \bar{u}_k \; \} \, / \, 2 \, D
\end{aligned} \tag{50}$$

where $\Lambda_i = \{ \, (j,k) \mid j \neq k, \, j \neq i, \, k \neq i \, \}$.

6.4.1 Transport and communication

MLGL-4, transport and communication,
Elasticity measures *within* this group,
mean values 1960-1990 concerning

AUES, partial elasticities of substitution

	C61	C62	C63	C64
C61	-3.84628	1.16375	2.14738	2.58710
C62	1.16375	-1.37207	1.22415	1.39203
C63	2.14738	1.22415	-6.34214	2.58266
C64	2.58710	1.39203	2.36784	-14.58173

Cournot price elasticities

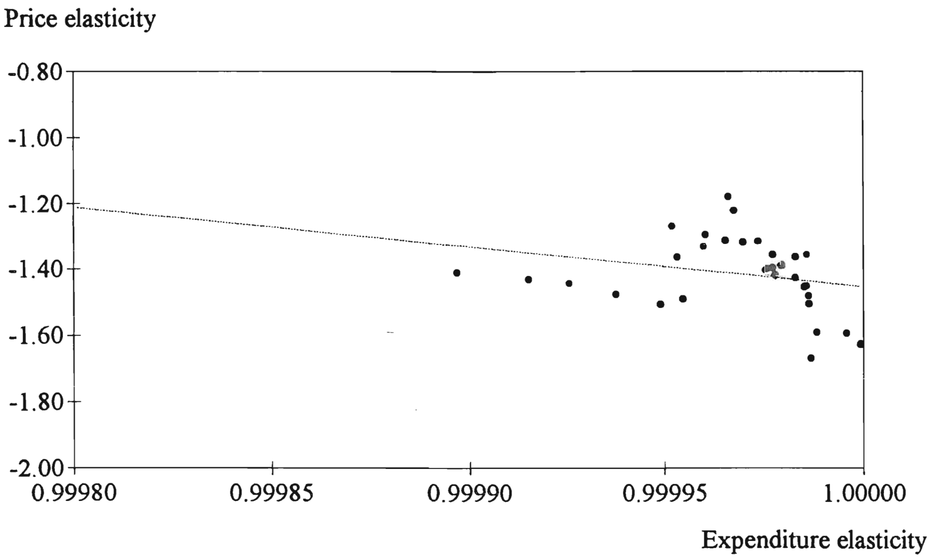
	C61	C62	C63	C64
C61	-1.25644	0.08247	0.20684	0.14490
C62	0.03724	-1.05726	0.03871	0.03523
C63	0.30011	0.10775	-1.32687	0.28913
C64	0.42206	0.18063	0.25016	-1.41369

Expenditure elasticities

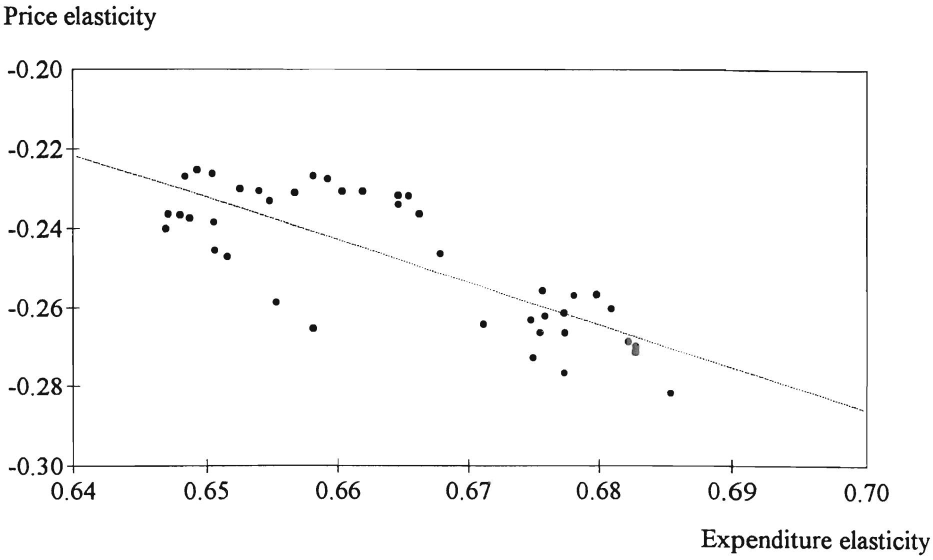
C61	C62	C63	C64
0.99998	1.00004	0.99998	0.99990

C61 – Personal transport equipment
C62 – Operation of personal transport equipment
C63 – Purchased transport services
C64 – Communication

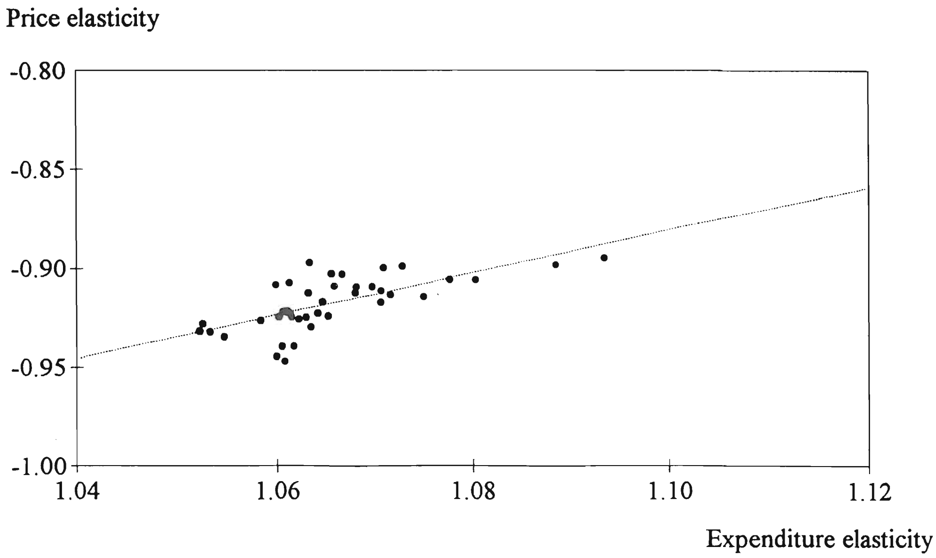
Communication, expenditure and own price elasticities within the transport and communication group
1950 - 1990, MLGL



Postal services, expenditure and own price elasticities within the communication group 1950 - 1990, MLGL



Telecommunication, expenditure and own price elasticities within the communication group 1950 - 1990, MLGL



6.4.2 Communication

MLGL-2, Postal services and Telecommunication
Elasticity measures *within* the communication group,
mean values 1931–1990 concerning

AUES, partial elasticities of substitution

	post	telecom
post	-0.73588	0.16935
tele	0.16935	-0.04394

Cournot price elasticities

	post	telecom
post	-0.26875	-0.39891
tele	-0.18408	-0.90046

Expenditure elasticities

post	telecom
0.66773	1.08461

7. The LES and the MLGL-model compared, some experience

The Linear Expenditure System was seminal to both theoretical research and empirical applications. According to my experience, there are several good reasons for the model's long lasting popularity. When applied appropriately, it is still very competitive. On the other hand, the appropriate selection of model depends on problem. If the main problem is to analyse substitution among goods and services on levels where substitution actually occurs, then models like the MLGL (or the Trans log version, MLTL) may be required to be able to make inferences as well as forecasts. Unfortunately, no free lunch is offered. These models are demanding in several ways.

8. Postal services and Telecommunication in total private consumption – some concluding remarks

As can be seen from the supplied graphical surveys, the importance of transport and communication to the household budget is twice as large now as compared to the state of sixty years ago. In this expansive group, communication has almost a fixed share. Communication is also more price elastic than any transport category. Communication is a clear substitute to personal transport equipment and purchased transport services.

In the communications group, the importance of Telecom increased constantly until 1970. However, the national accounts' statistics concerning price and volume of telecommunication covering the last two decades has to be checked more closely. With this reservation, the MLGL-2 model indicates that Telecom is more price elastic than Postal Service. Further, if the households decide to change their expenditures devoted to communication, Televerket and now also other telecom-operators will primarily gain or loose on that adjustment.

Appendix

Private consumption in Sweden as classified by Statistics Sweden (SCB)

Ändamål Function		
1000	Livsmedel, drycker, tobak	Food, beverages and tobacco
1100	Livsmedel	Food
1110	Bröd och spannmålsprodukter	Bread and cereals
1120	Kött	Meat
1130	Fisk	Fish
1140	Mjölk, ost och ägg	Milk, cheese and eggs
1150	Smör, margarin	Oils and fats
1160	Frukt och grönsaker utom potatis	Fruits and vegetables other than potatoes and similar tube
1170	Potatis, potatisprodukter	Potatoes, maniock and other tubers
1180	Socker	Sugar
1191	Kaffe, te, kakao	Coffee, tea, cocoa
1192	Konfektyrer m m	Confectionery etc.
1200	Alkoholfria drycker	Non-alcoholic beverages
1300	Alkoholhaltiga drycker	Alcoholic beverages
1310	Sprit, vin och starköl	Spirits, wine and export beer
1311	Sprit	Spirits
1312	Vin	Wine
1313	Starköl	Export beer
1320	Öl klass I och II	Beer n.e.c.
1400	Tobak	Tobacco
2000	Beklädnadsartiklar och skor	Clothing and footwear
2100	Beklädnadsartiklar inkl lagning	Clothing including repairs
2110	Kläder, tyger och garner	Clothing, fabrics and yarn
2120	Lagning av kläder	Repairs to clothing
2200	Skor inkl lagning	Footwear including repairs

2210	Skor	Footwear
2220	Lagning av skor	Repairs to footwear
3000	Bostad, bränsle och elström	Gross rent, fuel and power
3100	Bostad och vattenavgifter	Gross rents and water charges
3110	Hyra i flerfamiljshus	Gross rents
3120	Nyttjandevärdet av småhus	Imputed rents of owneroccupied dwellings
3130	Nyttjandevärdet av fritidshus	Imputed rents of secondary dwellings
3140	Hyresgästers reparationskostnader	Tenants repair costs
3200	Bränsle och elström	Fuel and power
3210	Elström	Electricity
3220	Gas	Gas
3230	Flytande bränslen	Liquid fuels
3240	Annat bränsle	Other fuels
3250	Fjärrvärme	Purchased heat
4000	Möbler, inrednings- och hushållsartiklar, förbrukningsartiklar för hushåll	Furniture, furnishings, household equipment and operation
4100	Möbler, mattor och reparationer	Furniture, carpets and repairs
4110	Möbler, mattor och belysning	Furniture, carpets and lamps
4120	Möbelreparationer	Repairs to furniture
4200	Hushållstextilier och andra inredningsartiklar inkl reparationer	Household textiles and other furnishings incl. repairs
4300	Spisar, mikrovågsugnar o d, kylskåp, tvättmaskiner och liknande större varaktiga hushållsinventarier inkl tillbehör och reparationer	Heating and cooking appliances, refrigerators, washing machines and similar major household appliances including fittings and repairs
4310	Spisar, mikrovågsugnar o d, kylskåp, tvättmaskiner och liknande större varaktiga hushållsinventarier inkl tillbehör	Heating and cooking appliances, refrigerators, washing machines and similar major household appliances including fittings
4320	Reparationer av hushållsapparater	Repairs to major household appliances
4400	Husgeråd inkl glas och porslin	Glassware, tableware and household utensils
4500	Hushållsartiklar och tjänster	Household operation except domestic services
4510	Hushållsartiklar	Non-durable household goods
4520	Köpta hushållstjänster (kemtvätt m m)	Household services excluding domestic service
4600	Lejda tjänster i hushåll	Domestic services
4610	Privat barnomsorg	Private welfare services, children
4620	Kommunal barnomsorg	Local government services, children
4630	Äldreomsorg	Welfare services elderly
5000	Hälso- och sjukvård	Medical care and health expenses
5100	Medicinska och farmaceutiska produkter	Medical and pharmaceutical products
5110	Läkemedel	Medicines
5120	Övriga apoteksvaror	Other products

5200	Glasögon, övrig terapeutisk utrustning	Therapeutic appliances and equipment
5300	Patientavgifter för sjukvård och tandvård	Fees paid to physicians, dentists and related practitioners
6000	Transport och samfärdsel	Transport and communication
6100	Personliga transportmedel	Personal transport equipment
6110	Nyinköp av bilar	Purchases of motor cars
6120	Övriga transportmedel	Other transport equipment
6121	Motorcykel och cykel	Motorcycles and bicycles
6122	Husvagn	Caravan
6200	Driftskostnader för personliga transportmedel	Operation of personal transport equipment
6210	Reparationer och tillbehör	Repair charges, parts and accessories
6220	Bensin, olja och smörjmedel	Gasoline, oils and greases
6230	Andra kostnader för bil	Other expenditures on cars
6231	Kontrollbesiktning	Compulsory tests of cars
6232	Körskolor	Driving lesson
6233	Garage	Garaging
6234	Parkering	Parking
6235	Billeasing	Car leasing
6300	Köpta transporttjänster	Purchased transport
6310	Järnväg	Railways
6320	Buss- och lokaltrafik	Bus and local traffic
6340	Taxi	Cabs
6350	Båt	Ships
6360	Flyg	Airlines
6370	Resebyråtjänster och utrikes charterflyg	Services of travel agencies and air charter
6380	Flyttning	Removal
6400	Post och tele	Communication
6411	Post	Postal services
6412	Tele	Telephone services
7000	Fritidssysselsättning, underhållning, undervisning och kulturella tjänster	Recreation, entertainment, education and cultural services
7100	Fritidsartiklar inkl reparationer	Equipment and accessories, including repairs
7110	Radio och TV	Radio and television
7120	Fotografisk utrustning, musikinstrument, båtar och andra större varaktiga fritidsartiklar	Photographic equipment, musical instruments, boats and other major durables
7121	Fotoutrustning m m	Photographic equipment
7122	Fritidsbåtar	Boats
7130	Andra fritidsartiklar	Other recreational goods
7140	Reparation av förstörelseart m m	Repairs to recreational goods etc
7141	Reparation, drift och underhåll av fritidsvaror	Parts and accessories for and repairs to recreational goods
7142	Hamntjänster	Port services
7200	Kulturella tjänster, tjänster med anknytning till fritidssysselsättning och underhållning exkl hotell-, restaurang- och kafétjänster	Entertainment, recreational and cultural services excluding hotels, restaurants and cafés
7211	Nöjen och fototjänster	Entertainment and photo services
7212	TV-licenser	Television licences
7213	Lotteri, toto och tips, bingo	Gambling, lotteries etc
7214	Veterinärtjänster	Veterinary services

7300	Böcker, tidningar och tidskrifter	Books, newspapers and magazines
7310	Böcker	Books
7320	Tidningar och tidskrifter	Newpapers and magazines
7330	Andra trycksaker	Other printed matter
7400	Undervisning	Education
7410	Kommunal musikskola	Music school
7420	Privat utbildning	Education
8000	Diverse varor och tjänster	Miscellaneous goods and services
8100	Varor och tjänster för personlig hygien	Personal care and effects
8110	Hår- och skönhetsvård	Services of barber and beauty shops etc.
8120	Toalettartiklar	Goods for personal care
8200	Övriga varor	Goods n.e.c.
8210	Smycken, ur	Jewellery, watches, rings and precious stones
8220	Andra personliga artiklar	Other personal goods
8230	Skriv- och ritartiklar samt kontorsmateriel	Writing and drawing equipment and supplies
8300	Utgifter för restaurang- och kafébesök samt hotellvistelse	Expenditure in restaurants, cafés and hotels
8310	Utgifter för restaurang- och kafébesök	Expenditure in restaurants and cafés
8320	Utgifter för hotelltjänster	Expenditure for hotels and similar lodging services
8500	Bank- och försäkringstjänster, andra finansiella tjänster	Financial services
8600	Andra tjänster, ej särskilt nämnda	Services n.e.c.
8611	Begravningskostnader	Undertaking
8612	Andra tjänster	Other services
Summa		Sum
	Hushållens konsumtion i utlandet	Direct purchases abroad by resident households
	Utländsk konsumtion i Sverige	Purchases by non-resident households
	Hushållens totala konsumtion	Total private final consumption expenditure by households
	Hushållens ideella organisationer	Private non-profit organizations
	Total privat konsumtion	Total private final consumption expenditure

Literature

Arrow, K.J., Chenery, H.B., Minhas, B.S. & Solow, R.M., 1961, Capital-Labour Substitution and Economic Efficiency, *Review of Economics and Statistics*, 43, No. 3.

Barnett, W.A., Lee, Y.W. & Wolfe, M., 1985a, The three dimensional global properties of the ML Translog and ML Generalized Leontief flexible functional forms, Discussion Paper, Dept. of Economics, University of Texas at Austin.

Barnett, W.A., & Lee, Y.W., 1985b, The global properties of the Minflex Laurent, Generalized Leontief and Translog flexible functional forms, *Econometrica*, 53, No. 6.

Barten, A.P., & Böhm V., 1982, *Consumer Theory*, in Arrow, K.J., & Intriligator, M.D., (eds.), Handbook of Mathematical Economics II, pp. 381–429, Amsterdam, North Holland.

Benzel, R., & Eklöf, K., 1957, *Den privata konsumtionen i Sverige*, Stockholm, Industriens Utredningsinstitut.

Christensen, L.R., Lau, L.J., & Jorgenson, D.W., 1973, Transcendental Logarithmic Production Frontiers, *Review of Economics and Statistics*, 55, No. 1.

Cramer, J.S., 1986, *Econometric applications of Maximum Likelihood methods*, Cambridge, Cambridge University Press.

Dahlman, C.J., & Klevmarken, N.A., 1971, *Den privata konsumtionen 1931–1975*, Stockholm, Industriens Utredningsinstitut.

Deaton, A., 1975, *Models and Projections of Demand in Post-War Britain*, London, Chapmand and Hall.

Diewert W. E. (1971), An application of the Shephard Duality Theorem:A Generalized Leontief Production Function, *Journal of Political Economy*, 79.

Diewert, W.E. & Wales, T.J., 1987, Flexible functional forms and global curvature conditions, *Econometrica*, 55, No. 1.

Geary, R.C., 1949–50, A note on "A constant utility index of the cost of living", *Review of Economic Studies*, 18.

Intriligator, M.D., 1971, *Mathematical Optimization and Economic Theory*, Englewood Cliffs, New Jersey, Prentice-Hall.

Johansen, L., 1981, *Suggestions towards freeing systems of demand from a strait jacket*, in Deaton, A., (ed.), Essays in the theory and measurement of consumer behaviour in honour of Sir Richard Stone, Cambridge, Cambridge University Press.

Klein, L.R. & Rubin, H., (1947–48), A constant utility index of the cost of living, *Review of Economic Studies*, 15

Klevmarken, N.A., 1981, *On the Complete Systems approach to Demand Analysis*, Stockholm, Industriens Utredningsinstitut.

Lau, L.J., 1977, Complete systems of consumer demand functions through duality, in Intriligator, M.C., (ed.), Frontiers of Quantitative Economics, Vol III, Amsterdam, North Holland.

Leser, C.E.V., 1941–42, Family budget data and price elasticities of demand, *Review of Economic Studies*, 9.

Pollak, R.A., & Wales, T.J., 1969, Estimation of the linear expenditure system, *Econometrica*, 37, No. 4.

Roy, R., 1942, *De l'utilité, contribution à la théorie des choix*, Paris, Hermann.

Samuelson, P.A., 1947–48, Some implications of linearity, *Review of Economic Studies*, 15.

Stone, J.R.N., 1954, Linear expenditure system and demand analysis: an application to the pattern of British demand, *Economic Journal*, 64.

Uzawa, H., 1962, Production functions with constant elasticities of substitution, *Review of Economic Studies*, 29.

IUI dissertations

Researchers who have worked on their degree at IUI and the title of their dissertation. (Doctor's degree in economics unless otherwise stated.)

1944–1954

Folke Kristensson (business administration). *Studier i svenska textila industriers struktur* (Studies in the Structure of Swedish Textile Industries). 1946.

Jan Wallander. *Flykten från skogsbygden* (The Exodus from the Forest Regions). 1948.

Erik Dahmén. *Svensk industriell företagarverksamhet* (Entrepreneurial Activity in Swedish Industry). 1950.

Roland Artle. *Svenskt distributionsväsende* (Swedish Distributive Trades). 1952.

Ragnar Bentzel. *Inkomstfördelningen i Sverige* (The Distribution of Income in Sweden). 1952.

1955–1965

Odd Gulbrandsen. *Strukturomvandlingen i jordbruket* (The Structural Transformation in Agriculture). 1957.

John Ekström (licentiate). *Den textila konsumtionen* (The Consumption of Textiles). 1958.

Göran Albinsson-Bruhner (licentiate). *Svensk verkstadsindustri. Struktur och utvecklingstendenser* (The Engineering Industries in Sweden. Structure and Tendencies of Development). 1961.

Lars Nabseth. *Löneökningars verkningar inom industrin. En studie av anpassningsprocessen inom företaget* (The Effects of Wage Increases in Industry. A Study of the Process of Adaptation within the Firm). 1961.

Erik Höök. *Den offentliga sektorns expansion. En studie av de offentliga civila utgifternas utveckling åren 1913–58* (The Expansion of the Public Sector – A Study of the Development of Public Civilian Expenditures in Sweden 1913–58). 1962.

Bengt G. Rundblad (sociology). *Arbetskraftens rörlighet. En studie av en lokal arbetsmarknad* (The Mobility of Labour. A Study of a Local Labour

Market). 1964.

1966–1976

Karl G. Jungenfelt. *Löneandelen och den ekonomiska utvecklingen. En empirisk-teoretisk studie* (Labour's Share and Economic Development). 1966.

Bengt Höglund (written at Lund University). *Modell och observationer. En studie av empirisk anknytning och aggregation för en linjär produktionsmodell* (A Study of Empirical Implementation and Aggregation for a Linear Model). 1966.

Gunnar Eliasson. *Kreditmarknaden och industrins investeringar* (Manufacturing Industry Finance and Short-Run Investment Behaviour). 1967.

Lars Kritz (licentiate, ethnogeography). *Godstransporternas utveckling i Sverige 1950–66 med utblick mot 1980* (Freight Transportation Trends in Sweden 1950–66 and the Outlook for the Seventies). 1968.

Lennart Ohlsson (licentiate). *Utrikeshandeln och den ekonomiska tillväxten i Sverige 1871–1966* (Foreign Trade and Economic Growth in Sweden 1871–1966). 1969.

Villy Bergström (licentiate, written at Uppsala University). *Den ekonomiska politiken i Sverige och dess verkningar* (Economic Policies in Sweden and their Results). 1969.

Lars Lundberg (licentiate, written at Umeå University). *Kapitalbildningen i Sverige 1861–1965* (Capital Formation in Sweden 1861–1965). 1969.

Lars Wohlin. *Skogsindustrins strukturomvandling och expansionsmöjligheter* (Forest-Based Industries: Structural Change and Growth Potentials). 1970.

John Skår. *Produksjon og produktivitet i detaljhandelen* (Production and Productivity in the Retail Sector). 1971.

Bengt Rydén. *Fusioner i svensk industri* (Mergers in Swedish Industry). 1971.

Anders Klevmarken (statistics). *Statistical Methods for the Analysis of Earnings Data with Special Application to Salaries in Swedish Industry*. 1972.

Rolf Rundfelt (business administration). *Reklamens kostnader och bestäm-*

ningsfaktorer (Advertising Costs in Sweden – Structure and Determinants). 1973.

Ulf Jakobsson och Göran Normann (partly written at Lund University). *Inkomstbeskattningen i den ekonomiska politiken* (Personal Income Taxation and Economic Policy). 1974.

Göran Eriksson. *Företagens tillväxt och finansiering* (Growth and Financing of the Firm). 1975.

Lennart Ohlsson. *Svensk verkstadsindustris internationella specialisering* (Patterns of Engineering Trade Specialization in Sweden). 1976.

Lars Kritz (ethnogeography). *Transportpolitiken och lastbilarna* (Transport Policy and the Lorries – A Study of the Effects of Regulation and Deregulation). 1976.

Lars Lundberg (written at Umeå University). *Handelshinder och handelspolitik* (Barriers to Trade and Trade Policy – Studies of the Effects on the Swedish Economy). 1976.

Siv Gustafsson. *Lönebildning och lönestruktur inom den statliga sektorn* (Determination and Structure of Salaries in the Government Sector of Sweden). 1976.

Johan Facht. *Emission Control Costs in Swedish Industry*. 1976.

1977–

Hans-Fredrik Samuelsson (business administration). *Utländska direkta investeringar i Sverige* (Foreign Direct Investments in Sweden – An Econometric Analysis). 1977.

Birgitta Swedenborg. *The Multinational Operations of Swedish Firms*. 1979.

Tomas Pousette (licentiate). *Efterfrågan på telefontjänster och telefoner* (Demand for Telephone Services and Telephones). 1976; *Teletjänster – priser och investeringar* (Telephone Services – Prices and Investments). 1979.

Gunnar Du Rietz. *Företagsetableringarna i Sverige under efterkrigstiden* (The Firm Establishments in Sweden in the Post-War Period). 1981.

Richard Murray. *Kommunernas roll i den offentliga sektorn* (The Role of the Local Governments in the Public Sector). 1981.

Jonas Agell (written at Uppsala University). *Tax Reforms and Asset Markets*. 1985.

Kenneth A. Hanson. *Adaptive Economics in Disequlibrium: Essays on Economic Dynamics*. 1986.

Nils Henrik Schager. *Unemployment, Vacancy Durations and Wage Increases: Applications of Markov Processes to Labour Market Dynamics*. 1987.

Joyce Dargay. *Factor Demand in Swedish Manufacturing: Econometric Analyses*. 1988.

Eva Christina Horwitz (licentiate, written partly at the Board of Commerce). *Analys av exportutvecklingen: Metoder och tillämpningar* (Analysis of Export Development: Methods and Applications). 1988.

Christina Hartler (licentiate). *Hushållens fastighetskapital och reala sparande*. (coauthor Jan Södersten.) 1989. *Agricultural Pricing and Growth*. 1991.

Thomas Lindh. *Essays on Expectations in Economic Theory*. 1992.

Eva M. Meyerson (sociology). *The Impact of Ownership Structure and Executive Team Composition on Firm Performance: The Resolution of a Leadership Paradox*. 1992.

Jonas Häckner. *Price and Quality: Essays on product differentiation*.1993.

Sten Nyberg. *Honesty, Vanity and Corporate Equity: Four microeconomic essays*. 1993.

Kent Rune Sjöholm (licentiate). *Hierarchical Modelling of Private Demand in Sweden*. 1993.

IUI Dissertation Series

1. *The Impact of Ownership Structure and Executive Team Composition on Firm Performance: The resolution of a leadership paradox.* Eva M. Meyerson. 1992.
2. *Price and Quality: Essays on product differentiation.* Jonas Häckner. 1993.
3. *Honesty, Vanity and Corporate Equity: Four microeconomic essays.* Sten Nyberg. 1993.
4. *Hierarchical Modelling of Private Demand in Sweden* (licentiate). Kent Rune Sjöholm. 1993.

ISSN 1102-6057
ISBN 91-7204-425-X