THE INDUSTRIAL INSTITUTE FOR ECONOMIC AND SOCIAL RESEARCH

TWO MODELS OF AN OPEN ECONOMY

Two Models
of An Open Economy

Industriens Utredningsinstitut

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FOREWORD

In recent years IUI has concentrated much of its research efforts on the development and use of econometric models of the Swedish economy. Although full scale economy-wide models are difficult to estimate and implement, they are essential for analyzing many relevant policy problems. One result of IUIs research in this area is ISAC – a disequilibrium model in the "Keynesian" tradition – which is documented in this book. The analytical focus of this model is on the testing of theory and the evaluation of policy. The model has been used in evaluating energy policy, fiscal programs, and local government control, as reported in earlier IUI publications. This project has been headed by Professor Bengt-Christer Ysander.

In constructing ISAC and applying it to energy analysis, IUI has collaborated with EFI at the Stockholm School of Economics, particularly with Professor Lars Bergman. His parallel research with the equilibrium model – ELIAS – which is also documented in this volume, represents another, more "classical" tradition in macro model building.

The common purpose of the two models is to capture the various mechanisms by which disturbances and shifts in world markets induce structural adjustment in the small and open Swedish economy. The models are two different analytical tools that have been applied to the same type of problems, and a comparison between them should therefore prove both methodologically and empirically interesting.

Stockholm in January 1986

Gunnar Eliasson

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by Bengt-Christer Ysander

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1. A "Model" History of the Swedish Economy

The postwar development of the Swedish economy is reflected in the history of its medium-term macroeconomic models. The 50s and 60s were a period of fast growth and relative price stability. Swedish exports were boosted first by reconstruction needs in war-ravished Europe and later by the trade liberalization which, part of the time, was reinforced by an undervaluation of the Swedish crown. The stability of the international monetary system and the seemingly well established pattern of expanding world trade induced the government to take for granted continued industrial competitiveness and the possibility of full employment policies. Medium-term policies were focused on distributing the gains of industrial productivity growth and international trade through a fastly expanding public welfare system.

The main task of the government's medium-term surveys was to check on the consistency of allocative schemes and to provide a framework for the long-term public expenditure plans, which – in the 60s – were becoming increasingly elaborate by means of various PPBS[1]-procedures. When the first medium-term macroeconomic model was developed by the Treasury in the late 60s, its aim was mainly to enhance computational convenience and further ensuring internal consistency in constructing balanced growth scenarios for the medium-term surveys (Åberg 1971). A static multisectoral model was used to compute desired equilibrium values at some future target date and to describe – or prescribe – balanced growth paths from the present to the target. The model only dealt with developments in real terms with consistent aggregate price and wage structures being computed afterwards. Although becoming successively more detailed and elaborate in terms of sector disaggregation etc., the methodology of Treasury models remained unchanged throughout the 70s (Ministry of Finance 1976).

Meanwhile the focus of medium-term problems changed drastically during the 70s. The increased price and exchange rate instability and uncertainty, demonstrated, and partly caused by, the oil price hikes, gave in Sweden rise to dramatic swings in industrial production and investment activity during the latter part of the decade. The shifts in world market demand also revealed a seriously deteriorated position for many of the raw materials and investment goods on which Swedish exports had been traditionally based. Interest became focused on the need for structural adjustment in industry to eliminate a mounting balance of payment deficit. The rapid expansion of local government consumption and central government transfers, partly used as a means of "bridging" the employment problems during international recessions, had simultaneously created a domestic budget deficit problem, which, with mounting real interest rates, threatened to lead fiscal policy into a "debttrap".

These problems meant new challenges for macromodelling. A major task of medium-term models now had to be to take explicit account of the price uncertainty and to measure the propagation and impact of changes in world market prices

[1] *P*lanning – *P*rogramming – *B*udgeting – *S*ystems.

and in Swedish competitiveness. It thus became of strategic importance to integrate price and wage formation into the models and to be able to use the models for tracing the effects of price changes on industrial profitability and investment in different sectors.

To be able to analyze the mechanisms of structural change the models must furthermore be dynamic in the sense of i.a. taking explicit account of the economic inertia represented by a given vintage structure of industrial capital and the lagged response of consumer demand. Finally, to provide a basis for policy analysis, the models should make it possible to study how various kinds of price policies and stabilization regimes could be used to ease the necessary structural and financial adjustments.

The two models documented in this book represent the first attempts – started in the late 70s – to meet these challenges.[2] They are based on widely different modelling concepts but did nevertheless result from a cooperative effort, incorporate many common dynamic mechanisms, exploit the same data-base and share many common econometric estimates. They were also initially employed to analyze the same problems, namely the impact of oil price shocks on the Swedish economy and possible policy means to insure in advance against, and/or to ease the adjustment after, unexpected dramatic changes in world market prices. The results of that study have already been extensively documented (cf. Ysander 1983b,c).

The first model, ELIAS,[3] is a six sector multiperiod equilibrium model developed by Lars Bergman at the Stockholm School of Business as a dynamic counterpart to an earlier static model of his (Bergman 1982). Like the other model, ISAC,[4] it incorporates price and wage formation and explicit links with the world market, treats capital accumulation in terms of vintages and determines consumer demand by linear expenditure functions. Apart from these dynamic elements it does not however – unlike ISAC – recognize any other sources of market inertia or rigidities. The market for both products and factors are thus assumed to clear by price adjustment in each period – where however the "period" may be interpreted as extending over several years.

The ELIAS model, as well as separate submodels developed by Bergman for different sectors of energy use, has been extensively used for the analysis and evaluation of Swedish energy policy. Besides the studies resulting from the common project already mentioned above (cf. Bergman-Mäler 1981, 1983) the model was used for evaluating the probable impact of discontinuing nuclear power production in Sweden (Bergman 1981).

The second model, ISAC, is a 36 sector disequilibrium model, developed by Ysander, Nordström and Jansson at the Industrial Institute for Economic and Social Research (IUI) in Stockholm, building on an earlier static model at the institute resembling the above mentioned Treasury model. By incorporating va-

[2] For a discussion of other models, mainly developed for pedagogical purposes, cf. Lybeck et al. (1984).

[3] *E*nergy, *L*abor, *I*nvestment *A*llocation and *S*ubstitution.

[4] *I*ndustrial *S*tructure *A*nd *C*apital Growth.

rious kinds of rigidities in price and wage formation it allows for disequilibria both in foreign trade and in the factor markets. The behavior of local governments, who employ about a quarter of total labor in Sweden, is endogenously determined by way of an integrated submodel. A fairly detailed treatment of central government taxes and transfers makes it possible also to use the model for simulating various kinds of fiscal policies and stabilization regimes.

The ISAC model has been used for several different kinds of policy evaluation studies. The studies concerning energy policy have already been mentioned (cf. Ysander 1983a, Nordström-Ysander 1983 and Ysander-Nordström 1983). A medium-term macroeconomic forecast was carried out by model simulations in 1979 (Ysander-Jansson-Nordström 1979). The long-term interrelation between industrial structural change and government expansion was analyzed in a study in 1980 (Nordström-Ysander 1980). Finally the efficiency of fiscal policy and wage policy in controlling local governments has been analyzed by model simulations (Ysander-Nordström 1985).

The Swedish Treasury has also recently raised its ambitions in regard to medium-term models. Partly based on the experience of the ISAC model and with the aid of two of its authors, an aggregate dynamic model, incorporating for the first time price and wage formation, was developed within the Treasury during 1982 and used for the 1984 medium-term survey. (Cf. Nordström 1982, Ministry of Finance 1984).

2. International Trade and Structural Change

A common starting point for the modelling work is the fact that Sweden is a small open economy which in both models is interpreted to mean i.a. that the supply of imports is perfectly elastic and that developments in the world economy can be treated as exogenously given.

A major task for the macromodels is to represent as accurately as possible the mechanisms by which price and demand changes in the world market are transmitted to the Swedish economy and there induce structural adjustment.

The transmission is modelled by export- and import-functions for the tradable goods sector of Swedish industry. In both models the level of export and import for each type of commodity depends on relative price and on the demand in world market and domestic market, respectively. Econometric estimates of price and demand elasticities are used, directly in ISAC and modified by theoretical considerations in ELIAS.

The prices of Swedish producers can thus deviate from those of foreign competitors. In Bergman's competitive equilibrium model this is explained by the assumption of "country-specific" commodities (cf. Armington 1969), while ISAC assumes price-setting producers in monopolistic competition adjusting their profit margins with regard both to the competitors' prices and to capacity utilization.

This specification of export and import functions must, however, be regarded as very rough approximations. There are reasons to expect that in some cases it is the relative change rather than the level that will be a function of relative price development. The elasticity can moreover be expected to vary depending both on specific competitive conditions limiting the range of price competition and on general business conditions. The market reactions to price increases and price decreases are not necessarily symmetrical in amplitude and time-structure. A particular form of specification bias is also implied by the omission of all transaction and marketing costs other than price.

In the macromodels these behavior patterns are aggregated over different commodities and different time-phases which means i.a. that sight is lost of the changes in commodity composition within each aggregate. This may in itself make it difficult to interpret the price elasticities. We may be comparing aggregates of different compositions – e.g. domestic sales versus imports of a certain composite good or exports versus world market trade – which means that the measured changes in relative price may in part not be due to different price development for individual goods but simply reflect the different composition of the aggregates. Changes in composition may moreover call forth changes in the aggregate rate of price change even without a changing rate in any individual line of goods. There is e.g. some evidence indicating that Swedish firms react to a profit squeeze by concentrating their marketing efforts on those less contested markets and commodity lines where they can raise prices without losing too many customers. The fact that the estimated relative price elasticities are rather low in magnitude – between 1 and 2 – may therefore partly be explained by marketing efforts being concentrated to price-insensitive subgroups within each aggregate.

The price elasticities used may thus be biased particularly in measuring effects of general relative price changes. However, sensitivity analysis seems to indicate that for a reasonable range of elasticity values the demand growth will still exert a dominant influence (Nordström 1982).

Changes in world market growth and/or price can be expected to affect Swedish industry in several stages. The immediate impact on sales will first be reflected in changing factor earnings – wages and profits. It will be further buffered by a changed rate of scrapping of old production capacity, and in ISAC – with producers in monopolistic competition – also by modified price-setting in foreign and domestic markets, respectively. By changing price and profit expectations the transmitted signals will in the next stage also influence investment behavior and by that the long-term resource allocation and industrial structure.

3. The Dynamics of Capital Accumulation

The time-lags and inertia in structural adjustment are in both models represented by distinguishing the various vintages of productive capital. Capital is thus modelled as being "putty-clay". In the ex ante production functions there are no restrictions on input substitution. The shares for capital, labor and energy can be freely selected in order to minimize cost in terms of expected input prices. Once installed however the capacity becomes "clay", i.e. the capital ratio is fixed and no further substitutions between major input aggregates are possible. At any given point of time the total production capacity of an industrial sector will thus be made up of a long line of different vintages, whose diverse input ratios "embody" the price expectations current at the time each vintage capital was invested. As a consequence the ex post functions will exhibit decreasing returns to scale in labor even when the ex ante production functions are linearly homogeneous.

The vintage structure of capital explains both why structural adjustment to world market changes is slow and why complete specialization is avoided even in competitive equilibrium models like ELIAS.

A long-term deterioration in the international competitive position of a particular branch of Swedish industry will only gradually be reflected in a shrinking of the productive capacity of that industry. The structural adjustment will come about by an increase in the scrapping of old plants and a decrease in the investment in new capacity. The funds and real resources thus released will successively be transferred to other and move profitable lines of production. In the same manner increased capital costs, caused e.g. by raised real interest rates, will only gradually become fully reflected in changed capital ratios.

The structural adjustment will in general proceed somewhat slower in ISAC than in the competitive equilibrium world of ELIAS, since "sticky" pricing and an evening out of utilization between plants – due to technological reasons and employment considerations – will delay the weeding out of plants earning a zero or negative quasi-rent. On the other hand, the decline in sales and profit will for the same reason be correspondingly greater and will react both on prices and investment and thus reinforce adjustment. Altogether adjustments in ISAC will, however, be somewhat slower and carried out on a lower profit level than what would be the case in a perfectly competitive world.

Since both models assume a fixed exchange rate regime, a deterioration of the terms-of-trade by e.g. import price increases not matched by price increases in the export markets, will not be accommodated by a changing exchange rate. Adjustment to the diminished real national income must instead come about through lowered real wages and various kinds of import substitutions. Tendencies of rising deficits in the balance of payments will in the ELIAS model be automatically countered by a rise in domestic saving, while in ISAC this would normally require government intervention. In both models the change in import prices would modify the pattern of input prices and thus influence both the choice of technology and the allocation of investment resources. Changed profit expectations and capacity utilization will also affect investment decisions in ISAC, where these decisions are represented by investment functions, while in ELIAS the total gross saving ratio is exogenously given.

4. The Energy Economy

The experience of the oil price hikes in the 70s made it particularly interesting to model and measure the effects on the Swedish economy of changes in the international energy markets. As already mentioned the concern about the impact of oil price changes was in fact one of the major reasons for starting the modelling work.

A principal ambition in both models has therefore been to represent as accurately as possible the structure of energy demand both by households and by industry. In modelling industrial energy demand technological choices are represented in a way that can be interpreted as reflecting a multistage decision-making. In ELIAS nested CES-functions are used. The first choice is between a capital-labor composite and an energy composite. In the second stage the substitution possibilities within the composite commodities are exploited, capital versus labor and electricity versus fuels, respectively. Once these ex ante choices are made, the input shares are regarded as fixed for that vintage.

In ISAC an ex ante choice is made regarding the input shares for capital, labor, electricity, fuels and other intermediate goods. Within the fuels aggregate, however, ex post substitution can be made between oil, coal and domestic fuels. The rationale behind this assumption is that the choice of fuel often has only limited effects on the rest of the installed production technology.

This relatively detailed specification of industrial energy demand combined with the vintage capital approach makes it possible to study the technological adjustment necessitated by e.g. sudden oil price increases. One of the main results of the energy policy studies mentioned earlier was to show that a major part of the macroeconomic impact of oil price hikes on the Swedish economy is due to various kinds of economic inflexibility. The ELIAS simulations were particularly concerned with the technological inflexibilities and with ways of designing and investing more flexibility in regard to energy use into the economy. In the study employing the ISAC model which incorporates a good deal of price and market inflexibilities as well as a wide range of government policy instruments, interest was more focused on designing and safeguarding flexible policies to compensate for the rigidities of the markets (Ysander 1983b,c).

5. Classical versus Keynesian Modelling

We have so far mainly dealt with the aims and features common to the two models – the way price signals and shifts in the world markets are transmitted through foreign trade into Sweden's small open economy and there gradually transform capital structure, production technology and energy use through the successive replacement of old vintages.

Of at least equal importance and interest, however, are the conceptual differences in approach between the models. They, in fact, seem well suited to exemplify the methodological dissimilarities between the two traditions in macromodelling often, somewhat inaccurately and simplistically, identified as "classical" and "Keynesian".[5]

[5] For a stringent but more narrow definition of classical and Keynesian macromodels, cf. e.g. Sargent (1979).

The label "classical" is here meant to refer not so much to the common foundations of neoclassical analysis as to some characteristic premises and priorities shared both by "old" and "new" classical economists. Models in this tradition are usually built on the assumption of a competitive equilibrium with immediate market clearing, and priority treatment is often given to the supply side of the economy, with demand, and particularly public demand and policy, dealt with in a more summary fashion. In the unavoidable trade-off between on the one hand theoretical consistency and coherence and on the other hand empirical verification and realism, model builders in this tradition tend moreover to favor the former.

The "Keynesian" modellers usually lean the other way and therefore tend to incorporate various forms of marked rigidities and limits to competition into their macromodels, even at the risk of having to accept "adhocery" and divergencies from the postulates of rational behavior. The recognition of market failures is moreover seen as implying the need for government intervention and demand management, requiring a more detailed specification of demand structure and policy instruments.

In terms of this oversimplified dichotomy ELIAS should undoubtedly be called a classical model while ISAC seems well qualified for the Keynesian label.

ELIAS is a competitive equilibrium model, assuming immediate market clearing. It extends the earlier analysis of multisectoral growth by Leif Johansen (1960) mainly by "opening" the model and explicitly formulating the linkage to the world markets and by taking account of the dynamics of capital accumulation by using a vintage representation of technological choice. The emphasis is on analyzing supply side developments although household demand is represented by a linear expenditure system, common with ISAC. Government demand is taken as an exogenously given aggregate and no taxes or any other policy instruments are explicitly specified. The specification of the model is to a large extent directly derived from neoclassical economic theory. To achieve this some empirical accuracy must be sacrificed by substituting calibrations and "guesstimates" for econometric estimates. It is also in line with the ambition of theoretical transparency to keep the model relatively small and compact. The specification has moreover been formulated so as to make it possible to use rational expectations and some simulation experiments have indeed been based on this assumption.

In ISAC, belonging to the tradition of Keynesian disequilibrium models (cf. Barker 1976), the modellers have gone far in the opposite direction, trying to incorporate various kinds of adjustment obstacles – besides immalleable vintage capital also sticky wages and prices, cash-flow restrictions on investment financing and the inertia and lags observed in both private and local government consumption. Government demand and policy instruments are specified in some detail with local government budgets being endogenously determined. The model is "extra-Keynesian" in the sense that it has been designed to make it possible to take into consideration also rigidities in local governments and various possible constraints on central government policy. These ambitions together with the desire to be able to exploit available statistics and survey data on individual industrial branches have made the model big and rather complex. This unavoidably entails the risk for inconsistencies and difficulties in tracing unambiguously the effects of variations in the exogenous variables. As far as possible the implementation of the model has been based on econometrical estimates.

These differences between the models will affect the analysis of structural change in important ways. With the equilibrium assumptions of the ELIAS model we can study in isolation the long-run impact of world market disturbances on Sweden's industrial structure with the immalleability of capital as the only factor of inertia and without getting the causal picture blurred and distorted by central government interventions and endogenous local government reactions. The picture will be clear but partial in the sense of neglecting interrelations between private and public sectors and focusing only on the industrial structure.

With the ISAC approach market disequilibria – surpluses or deficits of foreign exchange, public budgets, production capacity and labor – will be a normal feature of model projections and will have feed-back effects in the form of price modifications, rationing of supply and changing patterns of local government allocations. From a disequilibrium situation the model economy may finally – with the help of or despite economic policy measures – fetch up in a new equilibrium or steady-state growth, barring new disturbances. The adjustment path will in most cases affect the final structure in several important ways. To study this interdependence between short-run instability and long-term structural growth is indeed one of the main purposes of the ISAC model.

6. Market Behavior

The differences between the two models are best exemplified by looking at the functioning of the various markets.

In the *markets for tradable goods* flexible prices will guarantee equilibrium between demand and supply in ELIAS. Total demand is composed of export demand, investment demand, household demand and intermediate demand. The intermediate demand – apart from energy – is determined by input coefficients, which are the same for all vintages. The demand for investment goods is derived from the investment within the different sectors. Household demand is determined by the aggregate consumption propensity – assumed constant in ISAC and indirectly determined in ELIAS by the given gross saving ratio. The distribution of total household demand between different commodities is in both models determined by a linear expenditure system. Supply by the profit maximizing and price-taking firm will in the assumed competitive equilibrium be set at levels equating market price with marginal cost.

In ISAC the producers are instead price-setters in monopolistic competition. Their profit-maximization is moreover constrained by i.a. employment considerations, forcing them to try to keep an even rate of utilization between the plants in operation,[6] although successively scrapping units with a negative quasi-rent. The

[6] For a discussion of the modelling of constrained optimization in disequilibrium model, cf. i.a. Bureau-Miqueu-Norotte (1984).

producers may also discriminate in their price-setting between foreign sales and the home market. In both cases their pricing can be said to reflect an attempt to cover costs, computed as average variable cost plus planned depreciation and a target rate of return on installed capital. This "cost-price" is then modified to take account both of foreign competition and of variations in capacity utilization. As already exemplified above, the "sticky" prices and the evening out of capacity use will tend to slow down structural adjustment, while the possibility of price discrimination will sometimes mean that home consumers subsidize foreign sales by inflated domestic prices. Production is assumed to adjust to actual sales with proportionate variations in stock-keeping. Disequilibria will thus occur in the form of over- and underutilization of capital. A slow adjustment of capacity will take place, since investments are determined by the rate of utilization as well as by past profit performance.

Supply in the *labor market* is exogenously given in both models. Demand is derived from the levels of current production in the private and public sectors.

The mechanism for wage determination is, however, very different in the two models. In ELIAS the general wage level is determined as an equilibrium price, guaranteeing full employment. In order to take account of labor heterogeneity, however, an exogenous wage structure is imposed by multiplying the general wage with sector-specific coefficients.

Wage-setting in ISAC is instead modelled as the outcome of a negotiating process, where wage earners try to get compensated for both inflation and productivity gains but where the final result will be modified by current market conditions, i.e. unemployment. Long-run wage adjustment will thus be reinforced by the change in total employment, resulting from the current bargaining. For public employees a one-year lag in wage settlement, relative to the private sector, has been usual and has been assumed to continue in the future. Apart from this time-lag in compensation, there are no sector specific wage differentials in ISAC.

There are no explicit financial markets in either model. The rate of exchange is in both models assumed to be exogenously determined. The treatment of *capital transactions,* i.e. investments is however different.

In ELIAS the gross saving ratio in the economy is assumed to be given. Since the size of the balance of payment deficit or surplus is further assumed to be set by government at some target value, total domestic saving or investment will be determined along with domestic production. Investment resources are then allocated between the different production sectors in proportion to the expected relative profitability of new capacity within each sector. In principle the market rate of interest is determined as the "cut-off" rate which makes total investment demand equal to the given supply of investment resources. In practice, however, a slightly more roundabout way is employed to ensure also that rates of return will tend to equalize between sectors despite the absence of returns to scale in the ex ante production functions. The capital market implicit in this modelling can perhaps be described in terms of a given "loanable fund" with the market rate of interest determined by the "marginal efficiency of capital".

In ISAC the rate of interest is instead assumed to be determined by conditions in the international financial markets. The small open economy with its heavy dependence on international finance is supposedly unable under a fixed exchange rate regime to deviate substantially and persistently from international standards of yield. This assumption could be interpreted as implying a central bank policy of monetary accommodation aimed at keeping the public's demand for money satisfied at the given rate of interest. The internationally determined rate of return requirement is further transformed in the model to a particular rate for each branch of industry by taking account of differences in depreciation rate, tax treatment and solidity.

Investments in the different sectors are in ISAC functions of capacity utilization and past "excess" profits. These excess profits are measured relative to the user cost of capital, with the rate of return requirement as a major component. The required rate of return will influence the firm's pricing and investment and by that also its saving, but will not affect the saving ratio for households. The main burden of adjusting total domestic saving to avoid surpluses or deficits in external payments will thus fall on the public budgets, particularly the state budget.

7. Government Behavior and Stabilization Regimes

In deterministic equilibrium models, where instant market clearing is insured by flexible prices, stabilization and incomes policies are almost by definition superfluous. There is consequently no need for a detailed specification of public budgets in order to pinpoint the various available policy instruments or to study the possible destabilizing effects of endogenous local government action. It is thus consistent with the modelling ambition in ELIAS to treat public spending as an exogenous aggregate and to define taxing only implicitly by the assumed constant ratio of gross saving in the economy.

The opposite is true of the disequilibrium model, ISAC. The interrelation between stabilization problems and structural change is here in the focus of interest. In order to be able to study these interrelations a detailed account of public allocations, of taxes and transfers and a brief accounting of their distributive effects are built into the model. By the use of a submodel, LOGOS, (cf. Ysander 1985), the behavior of local governments is moreover endogenously determined.[7]

[7] The local government sector in Sweden is, by international standards, big, financially independent and expansive. It employs more people than the manufacturing industry and thus exerts a dominant influence on wages and labor market conditions. During the 70s the overall net effect of variations in local government activity appears to have been procyclical and destabilizing. The LOGOS model is a ten-equation system determining five kinds of service expenditures, two kinds of transfers, investments, borrowing and, as a residual, the local income tax rate (cf. Ysander (forthcoming)). Its incorporation in the macromodel has made it possible to explain i.a. the inefficiency of grants policy in changing the local government share of total consumption and the possibility that local government may by itself generate cyclical movements in wages and prices and reinforce disturbances from abroad (Ysander-Nordström 1985).

One of the main purposes with the ISAC model has been to use it for policy evaluation. Apart from the studies of energy policies, already mentioned, the main emphasis has been on medium-term stabilization regimes, ranging from various fiscal policies and control measures directed at local governments to wage controls and other income policies. In Keynesian terms one could say that the exogenous interest rate in ISAC combined with inelastic household saving and sticky wages make fiscal policy needed. At the same time adaptive expectations, demand-sensitive investment functions and the absence of financial crowding-out usually ensure that fiscal measures will have a short-run impact on employment before this effect is overtaken by a wage-propelled inflation.[8]

8. Comparing Models

Having examined the various characteristics of the two models we will try to sum up the discussion by comparing the models in terms of realism, relevance, resilience and robustness.

Econometric methods are supposed to help us in determining to what extent a model explains real life events satisfactorily. Unfortunately we are seldom able – even when we try – to use econometric tests also to choose between alternative models – alternative simplifications of reality.

Some of the conceptual differences between ELIAS and ISAC belong to the core of current macroeconomic controversy. The attempts made so far to test econometrically key assumptions and parameters do not seem to have yielded very conclusive or generally convincing results. One cannot escape the impression that the macroeconomic argument is often not just about the right answers but in equal measure about the right choice of questions. We argue about *realism* even when we really disagree about *relevance.*

The different aims of our two models exemplify the range of relevant questions. The interest in ELIAS is mainly focused on analyzing as clearly as possible the mechanisms of long-term structural adjustment, while largely abstracting from the short-term aberrations and stabilization problems on the demand side. ISAC instead aims at providing quantitative answers to these short-term questions and investigates how the management of short-run problems can affect the process of long-term structural change. The choice of question may depend on your time perspective or special interests but can also be contingent on whether you think short-run problems matter and can be meaningfully treated by public policy.

[8] In practice the stabilization policy experiments are usually set up in the following manner. The multiplier effects of variation in policy instruments on certain chosen target indicators are measured for a predetermined medium-term period in relation to a reference growth path. The results are then tested to see if the dynamic structure can be viewed as approximately linear in a local neighborhood of the reference path. If this is possible it is a straightforward matter to compute "optimal policy packages" or the trade-off between policy instruments in terms of the target indicators. (For a discussion of linearized model analysis and the application of control methods, cf. e.g. Kuh-Nease 1982 and Chow 1982).

The problem of *resilience* is important both in equilibrium and disequilibrium models. With resilience we here mean the ability of an economic system to absorb temporary outside shocks and adjust back to a stable growth path close to the one initially given without intervention in the form of policy changes. Intuitively one tends to presume that models with a Walrasian market equilibrium like ELIAS will have a high degree of resilience. The flexible prices will buffer the shocks and there will be no risks that the feed-back of disequilibria will further reinforce the deviation from the original growth path. In the case of ELIAS the intuition seems true as far as can be judged from the simulations carried out. For ISAC it is more difficult to know what to expect. Experiments show, however, that feed-back mechanisms in the labor and capital markets will tend in time to restore full employment of both people and capacity. There is no corresponding mechanism though for the balance of payment.

Another important aspect of models concerns *robustness,* i.e. their structural stability as related to the parameters of the models. To what extent are the constant parameters really constant and, if not, how sensitive is the model performance to parameter changes?

A number of sensitivity tests for parameter changes has been carried out for ISAC. They show i.a. that even though the model performance changes continuously and not very dramatically with variations in single key parameters, quite large deviations may occur if several parameters are allowed to vary simultaneously. The problem with these kinds of test is of course that we have no reliable way of deciding in advance which are the parameters most important to the dynamics of the system. Methods have been developed for measuring the relative importance of parameters in terms of a linear approximation of non-linear models (cf. e.g. Kuh-Nease 1982). These methods however still leave us with the question how valid the approximation is outside the chosen reference path.

Another aspect of the problem concerns the reliability of econometric estimates, when applied deterministically in new economic situations (cf. Lucas 1976). How far do we go wrong by neglecting not only random variations but also adaptive changes by the economic agents reacting to new conditions? These are obviously important questions, particularly for models like ISAC, which depend to a great extent on econometric estimates. The fact that both external conditions and economic policy in Sweden have changed drastically since the period of estimation increases the uneasiness. We have so far no way of resolving this dilemma. We can only hope that errors of this kind will not be able to change the sign and direction of the results. A reasonable conjecture could perhaps be that the implications are less important for short-run behavior than for long-run performance (cf. Brandsma-Hughes Hallet 1984). Whatever the truth of this much work remains before we can be sure of having achieved reasonably reliable and robust models.

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Elias – A Model of Multisectoral Economic Growth in a Small Open Economy

by Lars Bergman

Contents

1 Introduction*

In the following a computable model of medium-term multisectoral economic growth in a small, open economy is presented. The model is based on basic notions of general equilibrium theory; supply and demand factors interact and product and factor prices are assumed to be flexible enough to ensure equilibrium on all markets. However, the supply side is considerably more elaborated than the demand side. Thus, in each production sector different vintages of capital are distinguished and the substitutability of capital, labor, and different kinds of energy is modeled in some detail. Factors such as the formation of disposable incomes and the determination of savings, on the other hand, are treated in a rather crude way. On the supply side the substitutability of energy, labor and capital, as well as the sectoral allocation of energy, labor, and investments, is treated in some detail. Accordingly, the model has been christened ELIAS, *E*nergy, *L*abor, *I*nvestment *A*llocation and *S*ubstitution.

The general modeling approach is to emphasize the interdependence of goods and factor markets, and the role of relative prices in the medium-term resource allocation process. Moreover, with two exceptions[1] the structural equations of the model are derived from explicit production and preference functions together with optimization behavior assumptions rather than from econometric analyses of the past behavior of the economy. To a large extent the specification of the model, as well as the estimation of parameter values, is directly based on neoclassical economic theory. Accordingly, the present model primarily is a tool for quantitative analysis within the conceptual framework suggested by general equilibrium theory and the theory of economic growth, and in its present form it is not primarily intended to be a forecasting model.

The origin of the modeling effort presented here is the work of Johansen in the late 1950s (see Johansen 1960). However, Johansen's original so-called MSG-model essentially was a general equilibrium model of a closed economy; net exports entered as exogenously determined final demand components and complementary imports were proportional to domestic production. This property has been retained in later Norwegian versions of the MSG-model. (See for instance Longva et al. 1980.)

The first Swedish MSG-model, developed by Restad (1976), incorporated a current account constraint. On the basis of that constraint and fixed import shares

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[1] These exceptions are the determination of the gross savings ratio and the sectoral allocation of investments.

in the domestic production sectors and a given composition of total exports, the "export requirements" were calculated. The compatibility of these "export requirements" and world market demand conditions was then analyzed outside the model. In a later Swedish MSG-model, developed by the author (see Bergman 1978), explicit price-dependent export and import functions were incorporated. Thus, both the supply side and the demand side of the "rest of the world" were taken into account.

However, although these models are models of small, open economies, neither of them incorporates foreign trade in a theoretically satisfactory way. According to the conventional notion of a "small, open economy", the prices of traded goods should be determined by and be equal to world market prices. In the long run factor prices, and the structure of the production system, in the small economy have to adjust in order to maintain equilibrium between world market prices and domestic production costs. This feature of multisectoral economic growth in open economies is, to a varying extent, lacking in the MSG-models mentioned above. This study is an attempt to incorporate foreign trade in the MSG-model in a way which is consistent with the notion of a "small, open economy"[2] as well as with observations on the actual specialization pattern in the economy.

It is well-known that a multisectoral growth model of an open economy with two homogeneous primary factor of production, linearly homogeneous production functions and parametric world market prices, i.e. a model based on the standard assumptions of the Heckscher-Ohlin model, fails to generate development paths similar to actually observed development paths in real-world economies. Thus, while there are, at most, two producing and exporting sectors in the trade-exposed part of the model economy in equilibrium (or, in general, m producing sectors if there are m homogeneous factors of production; see Samuelson 1953), real-world economies exhibit a rather incomplete specialization pattern in the production system. Moreover, in the model economy traded goods of a given type are either domestically produced and exported or imported, while there is a considerable amount of intra-industry trade even within rather disaggregated real-world production sectors.

Of course, there is a number of ways in which the basic assumptions of the Heckscher-Ohlin model can be changed in order to achieve equilibrium allocations with several producing sectors in the trade-exposed part of the economy as well as with intra-industry trade. The approach adopted here is to stay within the general equilibrium framework, but add two factors of significant importance in a medium-term perspective: The immobility of the existing capital stock, and the heterogeneity of products of the same type but with different country of origin.[3] The "small open economy" assumptions are retained in the sense that the model-economy is assumed to face a perfectly elastic supply of imports. Moreover, it is assumed that the development of production less export in the rest of the world

[2] For a different approach to the same end, see Norman-Wergeland (1977).
[3] By "type" is simply meant number in some commodity classification system such as the SITC.

can be treated as an exogenous magnitude in spite of the fact that the import of the home country is the export of the rest of the world.

Explicit recognition of the immobility of the existing capital stock affects the assumptions about supply conditions in the Heckscher-Ohlin model; a distinction has to be made between production functions ex ante and ex post. If the ex ante supply conditions are represented by a linearly homogeneous neoclassical production function in capital and labor, and capital becomes an immobile resource once it is invested, the ex post production function will exhibit decreasing returns to scale in the only variable factor, labor. Accordingly, unit production costs will be a function of the level of output, and a given production sector can always maintain international competitiveness by a suitable change in the scale of operation. Consequently, complete specialization is avoided.

Explicit distinction between products of the same type but with different country of origin, together with the assumption that these products are not perfect substitutes,[4] implies that consumers at home, as well as abroad, will demand both imported and domestically produced units of products of a given type. Consequently, there can be both import and export of each type of tradable goods in equilibrium.

This approach to the treatment of the intra-industry trade phenomenon was initially proposed by Armington (1969) and has later on been employed in several numerical general equilibrium models.[5] It should perhaps be noted that the "Armington assumption" implies that each trading sector in each country produces a unique good. Thus, even if the production functions are linearly homogeneous in the variable inputs, the relation mentioned above between the number of homogeneous factors of production and the number of producing sectors in equilibrium does not hold when the Armington assumption is adopted.

The purpose of this report is to describe the model and discuss its implementation on actual data. In Section 2 a one-sector version of the model is presented. There are two reasons for beginning the presentation by considering an aggregated version of the model. One is, of course, that it is easier for the reader to get an overview of the model when it is presented in a condensed form. The second reason is that the mechanisms generating economic growth in the model economy can be demonstrated without complicating the picture by sectoral disaggregation. Thus, when the complete multisectoral model is presented in Section 3, the exposition can be focused on the treatment of intermediate inputs (particularly energy), the allocation of household expenditures between types of consumer goods, and the allocation of gross investments between production sectors. The procedure used for solving the model is briefly described in Section 4, and in Section 5, finally, some problems related to the practical implementation and use of the model are discussed.

[4] The less than perfect substitutability of aggregated commodity groups with different countries of origin can be shown to be a result of aggregation over individual products which do have perfect substitutes produced in other countries.

[5] See for instance Dervis et al. (1979) for a brief survey. For some econometric evidence, see Frenger (1980).

2 A One-Sector Version of the Model

To begin with we consider a case where only one type of output is produced in the domestic production system. It is convenient to structure the exposition with respect to the three markets dealt with in the model: the commodity market, the labor market, and the capital market. For each one of these we consider the determinants of supply and demand, and specify the equilibrium conditions.

2.1 The Commodity Market

On the supply side there are to sources: import and domestic production. The "home" country is assumed to be small enough to face a completely elastic supply of imports. Thus, any demanded quantity of imports in period t, $M^D(t)$, is supplied at the price $P^M(t) = V(t)P^{WI}(t)$, where V(t) is the exchange rate in period t and $P^{WI}(t)$ is the world market price, in foreign currency units, of the imported commodity in that period. To normalize the price system, V(t) is set equal to unity.

The domestic producers face an ex ante production function, $f(\tilde{K}, \tilde{L}, t)$, which is linearly homogeneous in capital, K, and labor, L, and has the usual neoclassical properties.[6] It is assumed that once an investment is carried out, the invested capital becomes an immobile resource, subject to depreciation at a constant annual rate. Thus, assuming that the ex ante production function is a Cobb-Douglas function, the technological constraints on the production system can be characterized by set of ex post production functions

$$X_\upsilon(t) = K_\upsilon(t)^\alpha L_\upsilon(t)^{1-\alpha} e^{\lambda\upsilon}; \qquad \upsilon = 0,1, \ldots, t-1 \tag{1}$$

where $K_0(0)$ is given as an initial condition and K(t) is exogenously given by

$$K_\upsilon(t) = (1-\delta)^{t-\upsilon-1} I(\upsilon); \qquad \upsilon = 0,1, \ldots, t-1; \quad t>0$$

and where δ is the exogenously given rate of depreciation and $I(\upsilon)$ is gross investment in period υ. The exogenously given rate of embodied technical progress is represented by the shift parameter λ in the ex ante production function.

Assuming optimization behavior on the part of the producers, technological constraints and producer behavior can be summarized by a set of profit functions, one for each vintage and time period. These profit functions can be written

$$\Pi_\upsilon(P, W; t) = K_\upsilon(t) e^{\lambda\upsilon} (1-\alpha)^{\frac{1-\alpha}{\alpha}} P^{\frac{1}{\alpha}} W^{\frac{\alpha-1}{\alpha}}.$$

[6] The "~" over the variables indicate their ex ante nature.

By Hotelling's lemma (see for instance Varian 1978) the supply from an individual vintage of production units is given by the partial derivative of the profit function with respect to the price of output. Thus, it holds that

$$X_\upsilon(t) = K_\upsilon(t)e^{\lambda\upsilon}\left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{\frac{\alpha-1}{\alpha}}; \tag{2}$$

and the total supply of domestically produced goods is

$$X(t) = \sum_{\upsilon=0}^{t-1} X_\upsilon(t); \tag{3}$$

It is assumed that imports qualitatively differ from domestically produced commodities, and that there is a constant elasticity of substitution, μ, between the two sources of supply in all domestic uses. Thus, domestic users demand as CES-composite, Y, of imports and domestically produced commodities, defined by a time-independent "production" function

$$Y = \left\{d_d(X-Z)^{\frac{\mu-1}{\mu}} + d_m M^{\frac{\mu-1}{\mu}}\right\}^{\frac{\mu}{\mu-1}};$$

where Z is export of the domestically produced commodity.

The price index, $P^D(t)$, of the composite good is defined by the unit cost function corresponding to the "production" function defining the composite good. Accordingly

$$P^D = \psi(P, P^M) = \left\{d_d^\mu P^{1-\mu} + d_m^\mu PM^{1-\mu}\right\}^{\frac{1}{1-\mu}}; \tag{4}$$

Thus, the domestic producers receive P(t) for their products and the importers receive $P^M(t)$, while the domestic consumers pay $P^D(t)$ for the composite good they consume.

The demand for the composite good is the sum of consumption and investment demand, i.e. the sum of C(t) and I(t). By Shephard's lemma (see for instance Varian 1978) the demand for domestically produced goods per unit of composite goods demanded is given by the partial derivative of $\psi(P, P^M)$ with respect to P. Thus, the equilibrium condition for the market for domestically produced goods can be written

$$X(t) = \frac{\partial\psi(P(t), P^M(t))}{\partial P}\{C(t) + I(t)\} + Z(t);$$

or

$$X(t) = d_d^\mu\left(\frac{P^D(t)}{P(t)}\right)^\mu \{C(t) + I(t)\} + Z(t); \tag{5}$$

where Z(t) is the export demand for domestically produced goods.

In a parallel way the equilibrium condition for imported goods becomes

$$M(t) = d_m^\mu\left(\frac{P^D(t)}{P^M(t)}\right)^\mu \{C(t) + I(t)\}; \tag{6}$$

Division of eq. 5 by eq. 6 yields another expression for import demand (using the definition $P^M(t) = V(t)P^{WI}(t)$)

$$M(t) = \left(\frac{d_m}{d_d}\right)^{\mu} \left(\frac{P(t)}{V(t)P^{WI}(t)}\right)^{\mu} \{X(t) - Z(t)\};$$

Assuming that the import to the "rest of the world" from the "home" country can be described by an import function of this kind, and also assuming that the "home" country is small enough to make $X(t) - Z(t) \approx X(t)$ in the rest of the world, the export function of the home country becomes

$$Z(t) = Z^0 \left(\frac{P(t)}{V(t)P^{WE}(t)}\right)^{\varepsilon} e^{\sigma t}; \qquad (7)$$

where Z^0 is a constant, ε the elasticity of substitution between domestically produced and imported goods in the "rest of the world", P^{WE} is the price of goods and σ is the rate of growth of production in the rest of the world.[7]

The demand for composite goods for investment purposes is derived from an assumption that a fixed fraction, s, of total factor income is saved, and that the sum of domestic savings and the current account deficit is spent on fixed investment. Thus it holds that

$$sP(t)X(t) - V(t)D(t) = P^D(t)I(t); \qquad (8)$$

The demand for composite goods for consumption purposes is determined by the budget constraint of the consuming sector and the savings behavior assumption introduced above. If the budget constraint of the consuming sector is explicitly incorporated in the model, an equilibrium solution will imply current account equilibrium by Walras' law. Alternatively, a current account constraint can be incorporated as a way of implicitly defining the budget constraint of the consuming sector. The latter approach is adopted here, and thus $P^D(t)C(t)$ is implicitly determined by

$$P(t)Z(t) = P^M(t)M(t) + V(t)D(t); \qquad (9)$$

where $V(t)D(t)$ is the exogenously given current account surplus.

2.2 The Labor Market

The supply of labor, L(t), is completely inelastic and determined outside the model. The allocation of the labor force over vintages is, however, determined within the model, and in such a way that the total demand for labor equals the supply. Thus the labor market part of the model can be described by means of one single equation; the equilibrium condition for the labor market. By Hotelling's lemma this condition can be written

[7] P^{WE} might differ from P^{WI} because the former corresponds to a fob price while the latter is a cif price.

$$L(t) = \sum_{\upsilon=0}^{t-1} - \frac{\partial \Pi_\upsilon(P(t), W(t); t)}{\partial W};$$

or

$$L(t) = \sum_{\upsilon=0}^{t-1} K(t) e^{\lambda \upsilon} \left(\frac{W(t)}{(1-\alpha) P(t)} \right)^{-\frac{1}{\alpha}}; \tag{10}$$

2.3 The Capital Market

In this part of the model the real rate of interest is determined by the process in which new vintages of production units are created. In the one-sector version of the model the creation of new vintages of production units is essentially equivalent to the choice of capital intensity in the new vintage. This is because the gross savings ratio is exogenously given. Thus, the overall rate of capital formation is not related to the real rate of interest.

The choice of capital intensity in new plants depends, among other things, on the expected future prices of output and labor. Price expectations are represented by the following functions

$$\tilde{P}(t) = \zeta^P P(t) + (1 - \zeta^P) P^F(t); \tag{11}$$

$$\tilde{W}(t) = \zeta^W W(t) + (1 - \zeta^W) W^F(t); \tag{12}$$

where $\tilde{P}(t)$ and $\tilde{W}(t)$ are the prices which producers expect to be the representative, or average, prices during the life-time of the production unit under construction.

The parameters ζ^P and ζ^W are simply tools for the specification of different expectation formation mechanisms. Thus, when ζ^P and ζ^W are equal to unity, price expectations are "static". The variables $P^F(t)$ and $W^F(t)$ are exogenously determined. They can either be regarded as weighted averages of previous prices and wages, or as some kind of official price and wage forecast. If $\zeta^P = \zeta^W = 0$, and $P^F(t)$ and $W^F(t)$ are given appropriate values, a case with "rational" expectations can be simulated.

New production units are designed on the basis of the ex ante production function, or, equivalently, the ex ante unit cost function. The equilibrium real rate of interest, R(t), should be such that the expected unit production cost in production units under construction should be equal to the expected price of output. Thus, denoting the ex ante unit cost function by $\varkappa(W, Q; t)$, the real rate of interest is determined by the equilibrium condition

$$\tilde{P}(t) = \varkappa(\tilde{W}(t), \tilde{Q}(t); t);$$

or, with $\varkappa(\cdot)$ written in explicit form,

$$\tilde{P}(t) = \frac{1}{e^{\lambda t}} \alpha^{-\alpha} (1-\alpha)^{\alpha-1} \tilde{Q}(t)^{\alpha} \tilde{W}(t)^{1-\alpha}; \tag{13}$$

where $\tilde{Q}(t)$ the "user cost of capital", is defined by

$$\tilde{Q}(t) = \tilde{P}^D(t)(\delta + R(t)); \tag{14}$$

2.4 A Summing-up

The equations of the one-sector version of the model are displayed in Table 2.1. As can be seen in the table, the complete model can be described by means of 11 equations in 11 unknowns.

As has already been mentioned, the basic elements of the model are contained in the one-sector version. By disaggregating the model into several sectors, however, it becomes a tool for analysis of structural changes, hidden in the growth process generated by the one-sector model. In order to extend the model presented in this section into a multisectoral model, basically three additional features of the economic system have to be incorporated: interindustry transactions, allocation of consumer expenditures between categories of consumer goods, and allocation of investible funds between investing sectors. The difference between the one-sector model discussed so far and the multisectoral model presented in the next section is that the latter includes mechanisms which can handle these types of phenomena.

Table 2.1 *The Equations of the One-sector Version of the Model*

Supply of Domestically Produced Goods

$$X(t) = \sum_{\upsilon=0}^{t-1} X_\upsilon(t) = \sum_{\upsilon=0}^{t-1} K_\upsilon(t) e^{\lambda\upsilon} \left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{\frac{\alpha-1}{\alpha}};$$

Demands for Imports

$$M(t) = d_m^\mu \left(\frac{P^D(t)}{P^M(t)}\right)^\mu \{C(t) + I(t)\};$$

Export Demand for Domestically Produced Goods

$$Z(t) = Z^0 \left(\frac{P(t)}{V(t) P^{WE}(t)}\right)^{-\varepsilon} e^{\sigma t};$$

Current Account Constraint

$$P(t)Z(t) = P^M(t)M(t) + V(t)D(t);$$

Savings – investment equilibrium

$$sP(t)X(t) - V(t)D(t) = P^D(t)I(t);$$

Product Market Equilibrium

$$X(t) = d_d^\mu \left(\frac{P^D(t)}{P(t)}\right)^\mu \{C(t) + I(t)\} + Z(t);$$

Labor Market Equilibrium

$$L(t) = \sum_{\upsilon=0}^{t-1} K_\upsilon(t) e^{\lambda\upsilon} \left(\frac{W(t)}{(1-\alpha)P(t)}\right)^{-\frac{1}{\alpha}};$$

Capital Market Equilibrium

$$\tilde{P}(t) = e^{-\lambda t}\alpha^{-\alpha}(1-\alpha)^{\alpha-1} \{\tilde{P}^D(t)(\delta + R(t))\}^\alpha \tilde{W}(t)^{1-\alpha};$$

Definitions

$$P^M(t) = V(t) P^{WI}(t);$$

$$P^D(t) = \{d_d^\mu P(t)^{1-\mu} + d_m^\mu P^M(t)^{1-\mu}\}^{\frac{1}{1-\mu}};$$

$$\tilde{P}(t) = \zeta^P P(t) + (1-\zeta^P) P^F(t);$$

$$\tilde{W}(t) = \zeta^W W(t) + (1-\zeta^W) W^F(t);$$

Endogenous Variables

$P(t)$, $P^D(t)$, $W(t)$, $R(t)$, $X(t)$, $C(t)$, $I(t)$, $Z(t)$,

$M(t)$, $\tilde{P}(t)$, $\tilde{P}^D(t)$, $\tilde{W}(t)$.

Exogenous Variables

$L(t)$, $D(t)$, $V(t)$, $P^{WI}(t)$, $P^{WE}(t)$, $P^F(t)$, $W^F(t)$.

3 The Multisectoral Model

Basically the exposition in this section is structured in the same way as in Section 2. That is, supply and demand factors as well as equilibrium conditions for each one of the commodity, labor, and capital markets are dealt with in separate subsections. Before that, however, the necessary background for the exposition is given in three introductory subsections. Thus, a general overview of the model is given in 3.1 and in 3.2 the goods and sectors in the model economy are defined. In subsection 3.3 the assumptions about technology are discussed, and the unit cost and profit functions utilized later on in the exposition are derived. Then the commodity, labor and capital markets are dealt with in subsections 3.4, 3.5 and 3.6 respectively. In subsection 3.7 the complete model is summarized. All symbols used in the exposition are defined in an appendix to this section.

3.1 General Characteristics

The model is a multisector, multiperiod model of economic growth in a small, open economy, i.e., an economy facing a perfectly elastic supply of imports at given world market prices. It is assumed that capital once invested in a specific sector cannot be reallocated to some other sector. Moreover, technical change is assumed to be embodied. Consequently there is a number of "vintages" of capital in each production sector at a given point in time.

The total supply of labor is given, and so is the gross savings ratio. For each period the model endogenously allocates the available labor force over sectors and vintages, and total gross investments over sectors. Investments in period t give productive capacity from period t + 1 and on. The allocation mechanisms are derived from optimization behavior assumptions. Thus, producers are assumed to maximize profits subject to technological constraints and the single aggregated household sector is assumed to maximize a utility function subject to a budget constraint.

Technological constraints and the utility functions are exogenously given and explicitly specified. (The choice of specification is discussed in Section 5). The public sector is treated as a production sector in which the use of inputs is determined on the basis of cost minimization considerations. The demand for public services is exogenously given.

There are no financial assets in the model, and the exchange rate is exogenously given. The budget constraint of the household sector is implicitly determined by a constraint on the current account. Thus, the tax and transfer system is essentially implicit in the model. All product markets and the labor market are treated as if they were competitive, and generally relative prices are assumed to be flexible enough to clear all markets. However, in another version of the model, rigidities in the adjustment of real wages are introduced.

3.2 Goods and Sectors

The model describes an economy with $n+3$ production sectors producing $n+3$ goods. There is no joint production, and consequently each good is produced in one sector only. Accordingly, there is no real distinction between domestically produced goods and the domestic production sectors. However, when it is natural to think in terms of "sectors" the index j will be used, while index i will be used when it is natural to think in terms of "goods".

The production sectors are numbered from 0 to $n+2$, 0 being the fuels production sector and 1 the electricity sector, while $n+1$ is the housing sector and $n+2$ the public sector. Foreign trade is ruled out in the last two sectors, as is exports from sector 0. There is also a "book-keeping" sector, $n+3$, in which different goods are aggregated into one single capital good.

Thus, the economy produces $n+3$ goods of which n, at least in principle, can be exported. Imports are classified by means of commodity index running from 0 to n. In addition to these $n+3$ goods, two types of complementary imports are used within the economy. These complementary imports are inputs used in the fuels and electricity producing sectors, respectively, and which cannot be produced within the country. Examples of such inputs are crude oil used in domestic refineries in a country without oil resources, or enriched uranium used in domestic nuclear power plants in a country without uranium enrichment capacity.

3.3 Technology and Producer Behavior

The basic assumption about technology is that there exists a constant returns to scale ex ante production function in each sector, i.e. a relation between planned output and the planned use of inputs. This relation is specified in accordance with

$$\tilde{X}_j = \min\left\{f_j(\tilde{K}_j, \tilde{L}_j, \tilde{X}_{0j}, \tilde{X}_{1j}; \upsilon), \frac{\tilde{X}_{2j}}{a_{2j}}, \ldots, \frac{\tilde{X}_{nj}}{a_{nj}}\right\}; \qquad j = 0, 1, \ldots, n+2$$

where X_j is output, K_j the use of capital, L_j the use of labor, X_{ij} the use of intermediate input i in sector j and ~ indicates the ex ante nature of the variables. The coefficients $a_{2j}, \ldots, a_{nj}$ indicate the minimum input of the corresponding good per unit of output in sector j. The variable υ is a time index, indicating that the ex ante production function shifts over time.

On the basis of the ex ante production function efficiency requires that

$$\tilde{X}_j = f_j(\tilde{K}_j, \tilde{L}_j, \tilde{X}_{0j}, \tilde{X}_{1j}; \upsilon); \qquad j = 0, 1, \ldots, n+2$$

$$\tilde{X}_{ij} = a_{ij}\tilde{X}_j; \qquad i = 2, 3, \ldots, n; \qquad j = 0, 1, \ldots, n+2$$

In parametric form the functions $f_j(\cdot)$ are nested CES – Cobb Douglas functions. Thus,

$$\tilde{X}_j = A_j \{a_j \tilde{F}_j^{\varrho_j} + b_j \tilde{H}_j^{\varrho_j}\}^{\frac{1}{\varrho_j}};$$

$$\tilde{F}_j = \tilde{K}_j^{\alpha_j} \tilde{L}_j^{1-\alpha_j} e^{\lambda_j \upsilon};$$

$$H_j = \{c_j \tilde{X}_{0j}^{\gamma_j} + d_j \tilde{X}_{1j}^{\gamma_j}\}^{\frac{1}{\gamma_j}} e^{\lambda_j^* \upsilon};$$

where $(1-\varrho_j)^{-1}$ and $(1-\gamma_j)^{-1}$ are elasticities of substitution, and λ_j and λ_j^* are exogenously given rates of technological progress.

Whereas the ex ante production function is a planning concept, the ex post production function is a relation between actual output and the actual use of inputs. It can be derived from the ex ante production function by assuming that the capital stock and the energy input coefficients are given once a production unit has been taken into operation. Moreover, it is assumed that the once installed capital equipment depreciates at a given, sector specific rate. This means that the ex post production function in period t for vintage υ in sector j can be written

$$X_{\upsilon j}(t) = \min \left\{ g_{\upsilon j}(L_{\upsilon j}(t), t), \frac{X_{\upsilon 0j}(t)}{a_{\upsilon 0j}}, \frac{X_{\upsilon 1j}(t)}{a_{\upsilon 1j}}, \frac{X_{\upsilon 2j}(t)}{a_{\upsilon 2j}}, \ldots, \frac{X_{\upsilon nj}(t)}{a_{\upsilon nj}} \right\};$$

$$j = 0, 1, \ldots, n+2$$

where $X_{\upsilon j}(t)$ and $L_{\upsilon j}(t)$ are production and employment, respectively, in vintage υ in sector j, and $a_{\upsilon 0j}$ and $a_{\upsilon 1j}$ are minimum energy input coefficients. The production function $g_{\upsilon j}(\cdot)$ is directly related to the ex ante production function by

$$g_{\upsilon j}(L_{\upsilon j}, t) = f_j(K_j, L_j, X_{0j}, X_{1j}, \upsilon);$$

subject to

$$K = I_j(\upsilon)\,(1-\delta_j)^{t-\upsilon-1}; \qquad t \geq \upsilon + 1$$

$$X_{0j} = a_{\upsilon 0j} X_{\upsilon j};$$

$$X_{1j} = a_{\upsilon 1j} X_{\upsilon j};$$

where $I_j(\upsilon)$ is the gross investment in sector j in period υ. Using the symbols of the ex ante production function, the function $g_{\upsilon j}(\cdot)$ thus becomes

$$g_{\upsilon j}(L_{\upsilon j}, t) = \left[\frac{a_j \left[A_j ((1-\delta_j)^{t-\upsilon-1} I_j(\upsilon))^{\alpha_j} e^{\lambda_j \upsilon} \right]^{\varrho_j}}{1 - b_j A_j^{\varrho_j} \left[(c_j a_{\upsilon 0j}^{\gamma_j} + d_j a_{\upsilon 1j}^{\gamma_j})^{\frac{1}{\gamma_j}} e^{\lambda_j^* \upsilon} \right]^{\varrho_j}} \right]^{\frac{1}{\varrho_j}} L_{\upsilon j}^{(1-\alpha_j)} \equiv A_{\upsilon j}(t) L_{\upsilon j}^{1-\alpha_j};$$

Efficiency requires that in each time period, sector and vintage, it holds that

$$X_{\upsilon j}(t) = A_{\upsilon j}(t) L_{\upsilon j}(t)^{1-\alpha_j}; \qquad j = 0, 1, \ldots, n+2$$

$$\upsilon = 0, 1, \ldots, t-1$$

$$X_{vij}(t)=\begin{cases}a_{vij}X_{vj}(t) & \text{when } i=0,1\\ a_{ij}X_{vj}(t) & \text{when } i=2,3,\dots,n\end{cases}\qquad \begin{matrix}j=0,1,\dots,n+2\\ v=0,1,\dots,t-1\end{matrix}$$

Observe that energy input coefficients differ across vintages in a given sector, while that is not the case for the non-energy intermediate coefficients. Note also that the input of labor per unit of output is dependent on the level of production. Thus, while the ex ante technology exhibits constant returns to scale, the ex post technology exhibits decreasing returns to scale.

With these equations the description of technology is complete. However, as profit maximization behavior on the part of the producers is assumed throughout, it is more convenient to use a dual representation of technology and producer behavior. That is, to describe the ex ante technology by means of an ex ante unit cost function and the ex post technology by means of profit functions for each vintage of production units. This means that input demand and output supply functions can be determined without explicitly making use of necessary conditions for profit maximum. However, before the derivation of the cost and profit functions, the input and output prices facing the producers have to be defined.

In accordance with the assumption that the modeled economy is "small", it is assumed that there is a perfectly elastic supply of imports at world market prices P_i^{WI} (and P_i^C for complementary imports). As these prices should be regarded as c.i.f. prices, tariffs and indirect taxes have to be added in order to get the market prices of imports. Moreover, as P_i^{WI} is a price in foreign currency units, it has to be multiplied by an exchange rate. Thus, the relation between c.i.f. import prices in foreign currency, P_i^{WI}, and domestic market prices of imports, P_i^M, is given by:

$$P_i^M(t)=\left(1+\frac{\Delta\varphi}{1+\varphi_i^0}\right)V(t)P_i^{WI}(t);\qquad i=0,1,\dots,n\ ^{8}\qquad (15)$$

The variable V(t) is the exogenously given exchange rate. Moreover, $V(t)P_i^{WI}(t)$, where i is a major import good, is taken to be the numeraire of the price system, and is accordingly set equal to unity. This means that the other $P_i^{WI}(t)$s represent world market prices relative to the world market price of the numeraire good.

It is assumed that imported and domestically produced goods with the same

[8] Let M_i^* be imports at constant cif prices, and define $\varphi_i=\varphi_i^0+\Delta\varphi_i$, where φ_i is base year tariffs and indirect taxes on imports, while $\Delta\varphi_i$ reflects changes in that parameter thereafter. Imports at constant market (or purchaser's) prices can then be written $M_i=(1+\varphi_i^0)M_i^*$. At a given point in time the domestic market value of imports can be written

$$(1+\varphi_i^0+\Delta\varphi_t)VP^{WI}M_i^*=VP^{WI}\left[(1+\Phi_i^0)M_i^*\right]+\frac{\Delta\varphi_i}{1+\varphi_i^0}VP_i^{WI}\left[(1+\varphi)M_i^*\right]=$$

$$=\left(1+\frac{\Delta\varphi_i}{1+\varphi_i^0}\right)VP_i^{WI}M_i\equiv P_i^M M_i.$$

But as $M_i^*=M_i/(1+\varphi_i^0)$ this becomes

$$P_i^m M_i=\left(\frac{1+\varphi_i^0+\Delta\varphi_i}{1+\varphi_i^0}\right)VP_i^{WI}M_i\quad\text{or}\quad P_i^M=\left(1+\frac{\Delta\varphi_i}{1+\varphi_i^0}VP_i^{WI}\right).$$

classification, say i, are not perfect but relatively close substitutes. Thus, equally classified goods from the two sources of supply are aggregated into a composite good in accordance with some "production" function representing their substitutability. This function is assumed to be a CES-function and to apply to all domestic users of the goods in question.

Under these conditions the unit cost of composite good i is solely a function of the price of imported units of good i, $P_i^M(t)$, and the price of domestically produced units of good i, $P_i(t)$. Thus, if the "price" of a composite good, $P_i^D(t)$, is defined as the unit cost of that good, we get

$$P_i^D(t) = \psi_i(P_i(t), P_i^M(t)) \equiv \{d_{di}^{\mu_i} P_i(t)^{1-\mu_i} + d_{mi}^{\mu_i} P_i^M(t)^{1-\mu_i}\}^{\frac{1}{1-\mu_i}}; \quad i = 0, 1, \ldots, n \quad (16)$$

On the basis of Shephard's lemma (see Varian 1978) the demand for domestically produced goods of type i per unit of composite good i demanded is given by the partial derivative of $\psi_i(P_i, P_i^M)$ with respect to P_i. The demand for imported goods of type i is determined in a parallel way.

As there is only one type of labor in the model economy and a competitive labor market is assumed, there should only be one wage rate. However, in order to make it possible to roughly take labor heterogeneity into account, an exogenous wage structure is imposed. Thus the sectoral wage rates are defined by

$$W_j(t) = \omega_j W(t); \quad j = 0, 1, \ldots, n+2 \quad (17)$$

where $W_j(t)$ is the sectoral wage rate, $W(t)$ an index of the general wage rate and ω_j a sector specific parameter.

Energy prices might differ across sectors as a result of sector specific energy taxes. Thus, it holds that

$$P_{ij}(t) = (1 + \tau_i(t) + \xi_{ij}(t)) P_i^D; \quad i = 0, 1; \quad j = 0, 1, \ldots, n+2 \quad (18)$$

where $\tau_i(t)$ is a general tax on energy of type i and $\xi_{ij}(t)$ is a specific tax on the use of that kind of energy in sector j.

Decisions concerning actual production are based on current market prices, i.e., $P_{0j}^D(t)$, $P_{1j}^D(t)$, $P_2^D(t), \ldots, P_n^D(t)$, $W_j(t)$, and the ex post production functions, while decisions concerning the design of new plants are based on expected prices and the ex ante production functions. Expected prices of outputs and inputs respectively, are determined by

$$\tilde{P}_i(t) = \zeta_i^0 P_i(t) + (1 - \zeta_i^0) P_i^{OF}(t); \quad i = 0, 1, \ldots, n+2 \quad (19)$$

$$\tilde{P}_i^D(t) = \zeta_i^I P_i^D(t) + (1 - \zeta^I) P_i^{DF}(t); \quad i = 0, 1, \ldots, n \quad (20)$$

$$\tilde{P}_i(t) = \zeta_i^C V(t) P_i^C(t) + (1 - \zeta_i^C) P_i^{CF}(t); \quad i = 0, 1 \quad (21)$$

$$\tilde{W}_j(t) = \zeta^W W_j(t) + (1 - \zeta^W) W_j^F(t); \quad j = 0, 1, \ldots, n+2 \quad (22)$$

where $p_i^{hF}(t)$, h = 0, I, C, W, are exogenous variables. By an appropriate choice of values of these variables, and setting $\zeta_i^h = 1$, h = 0, I, C, W, a case with rational expectations can be stimulated, while $\zeta_i^h = 1, h = 0, I, C, W$, implies static expectations. Throughout expectations about indirect taxes and tariffs as well as about the exchange rate are assumed to be static.

The user cost of capital is determined by

$$\tilde{Q}_j(t) = \tilde{P}_{n+3}(t)\ (\delta_j + R(t)); \qquad j = 0, 1, \ldots, n+2 \tag{23}$$

where

$$\tilde{P}_{n+3}(t) = \sum_{i=2}^{n} \tilde{P}_i^D(t)\, a_{i,\, n+3}; \tag{24}$$

where the coefficients $a_{i,\, n+3}$ add up to unity, and where $R(t)$ is the real rate of interest.

The profit function for a production unit of vintage υ in sector j can now be defined as the solution to the problem

$$\Pi_{\upsilon j}(P^*_{\upsilon j}, W_j; t) = \max_{X_{\upsilon j},\, L_{\upsilon j}} \{P^*_{\upsilon j} X_{\upsilon j} - W_j L_{\upsilon j} | X_{\upsilon j} = A_{\upsilon j}(t)\, L_{\upsilon j}^{1-\alpha_j}\}$$

$$j = 0, 1, \ldots, n+2; \quad \upsilon = 0, 1, \ldots, t-1$$

where,[9] in a given time period t,

$$P^*_{\upsilon j}(t) = (1 - \Theta_j) P_j(t) - \sum_{i=0}^{1} P_{ij}(t)\ a_{\upsilon ij} - \sum_{i=2}^{n} P_i^D(t)\, a_{ij} - V(t)\, P_j^C(t)\, b_{jj}; \tag{25}$$

$$j = 0, 1, \ldots, n+2$$
$$\upsilon = 0, 1, \ldots, t-1$$

In explicit form the profit function becomes

$$\Pi_{\upsilon j}(P^*_{\upsilon j}, W_j; t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \alpha_j (1 - \alpha_j)^{\frac{1-\alpha_j}{\alpha_j}} P^{*\,\frac{1}{\alpha_j}}_{\upsilon j} W_j^{\frac{\alpha_j - 1}{\alpha_j}}; \qquad j = 0, 1, \ldots, n+2 \tag{26}$$

The ex ante unit cost function represents the minimum cost of inputs per unit of output at given prices, subject to the ex ante production function in per unit form. As the ex ante technology exhibits constant returns to scale, the ex ante unit cost is independent of the expected level of production, and thus a function of expected input prices only. According to the ex ante production function, non-energy intermediate inputs are used in fixed proportions, while the proportions of other inputs can be varied. Thus, disregarding time indices the ex ante unit cost $\varkappa_j$ is given by the function

$$\varkappa_j = \varkappa_j^*(\tilde{W}_j, \tilde{Q}_j, \tilde{P}_{0j}, \tilde{P}_{1j}; \upsilon) + \sum_{i=2}^{n} \tilde{P}_i^D a_{ij} + \tilde{V}\tilde{P}_j^C b_{jj} + \theta \tilde{P}_j; \qquad j = 0, 1, \ldots, n+2 \tag{27}$$

[9] Provided $P^*_{\upsilon j}(t) > \sum_{i=0}^{1} P_{ij}(t)\, a_{\upsilon ij}$. Otherwise $P^*_{\upsilon j}(t) = 0$.

where the net unit cost function $\varkappa_j^*(\cdot)$ represents the minimum cost of labor, capital, fuels and electricity per unit of output. The other terms represent the cost of non-energy intermediate inputs, complementary imports and an ad valorem output tax, respectively.

Thus, the net unit cost function $\varkappa_j^*(\cdot)$ is the solution to the problem

$$\text{Min } \frac{1}{\tilde{X}_j} \{W_j\tilde{L}_j + Q_j\tilde{K}_j + P_{0j}\tilde{X}_{0j} + P_{lj}\tilde{X}_{lj}\};$$

subject to

$$\tilde{X}_j = A_j \left\{a_j\tilde{F}_j^{\varrho_j} + b_j\tilde{H}_j^{\varrho_j}\right\}^{\frac{1}{\varrho_j}};$$

$$\tilde{F}_j = \tilde{K}_j^{\alpha_j}\tilde{L}_j^{1-\alpha_j}e^{\lambda_j \upsilon};$$

$$\tilde{H}_j = \left\{c_j\tilde{X}_{0j}^{\gamma_j} + d_j\tilde{X}_{lj}^{\gamma_j}\right\}^{\frac{1}{\gamma_j}} e^{\lambda_j^* \upsilon};$$

for $j = 0, 1, \ldots, n+2$.

In explicit form the net unit cost function thus can be written

$$\varkappa_j^*(W_j, Q_j, P_{0j}, P_{lj}; \upsilon) = A_j^{-1}\left\{a_j^{\frac{1}{1-\varrho_j}}\left[e^{-\lambda\upsilon}\alpha_j^{-\alpha_j}(1-\alpha_j)^{\alpha_j-1}Q_j^{\alpha_j}W_j^{1-\alpha_j}\right]^{\frac{\varrho_j}{\varrho_j-1}} + \right.$$
$$\left. + b_j^{\frac{1}{1-\varrho_j}}\left[e^{-\lambda^*\upsilon}\left(c_j^{\frac{1}{1-\gamma_j}}P_{0j}^{\frac{\gamma_j}{\gamma_j-1}} + d_j^{\frac{1}{1-\gamma_j}}P_{lj}^{\frac{\lambda_j}{\lambda_j-1}}\right)^{\frac{\gamma_j-1}{\gamma_j}}\right]^{\frac{\varrho_j}{\varrho_j-1}}\right\}^{\frac{\varrho_j-1}{\varrho_j}} \qquad j = 0, 1, \ldots, n+2 \tag{28}$$

Having now defined the prices, cost and profit functions of the model economy, the derivation of the complete model is quite straightforward. Output supply and input demand functions can be directly derived from the eqs. (26) and (28), which are thus the main building blocks of the model.

3.4 The Commodity Markets

3.4.1 *Supply of Domestically Produced Goods*

By Hotelling's lemma the supply of domestically produced goods from production units of vintage υ in sector j is given by

$$\frac{\partial \Pi_{\upsilon j}(P_{\upsilon j}^*(t), W_j(t); t)}{\partial P_{\upsilon j}^*} = X_{\upsilon j}(t); \qquad \begin{array}{l} j = 0, 1, \ldots, n+2 \\ \upsilon = 0, 1, \ldots, t-1 \end{array}$$

Thus, on the basis of this equation and eq. (26), total supply of goods produced in sector j is given by

$$X_j(t) = \sum_{\upsilon=0}^{t-1} A_{\upsilon j}(t)^{\frac{1}{\alpha_j}}\left[\frac{W_j(t)}{(1-\alpha_j)P_{\upsilon j}^*(t)}\right]^{\frac{\alpha_j-1}{\alpha_j}}; \qquad j = 0, 1, \ldots, n+2 \tag{29}$$

There are four types of demand for goods produced in the home country: intermediate, consumption, investment and export demand. The first three types of demand are derived from the demand for composite goods, while export demand is a direct demand for domestically produced goods. Investment demand is implicitly determined by the savings-investment equilibrium condition, while the other types of demand have to be explicitly specified.

3.4.2 *Intermediate Demand for Composite Goods*

By the technology assumptions the demand for intermediate inputs is given by

$$X_{ij}(t) = \begin{cases} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij} X_{\upsilon j}(t) & \text{when } i = 0, 1 \\ a_{ij} X_j(t) & \text{when } i = 2, 3, \ldots, n \end{cases} \qquad j = 0, 1, \ldots, n+3 \tag{30}$$

3.4.3 *Household Demand for Composite Goods*

There is only one aggregated household sector in the model. It is assumed that the household sector maximizes a utility function of the form

$$u(t) = \sum_{s=1}^{k} \beta_s \log(Q_s(t) - Q_s^0); \quad \sum_{s=1}^{k} \beta_s = 1$$

where $Q_s(t)$ is the consumption of a consumer commodity group, defined as a convex combination of the composite goods $i = 0, 1, \ldots, n$ and the domestically produced good $n+1$. The constraint on the parameters β_s implies that the resulting linear expenditure system will satisfy the budget constraint. The resulting household demand equations for composite goods can be written

$$C_i(t) = \sum_{s=1}^{k} t_{is} \left\{ Q_s^0 + \frac{\beta_s}{P_s^Q(t)} \left(E(t) - \sum_{s=1}^{k} P_s^Q(t) Q_s^0 \right) \right\}; \qquad i = 0, 1, \ldots, n+1 \tag{31}$$

where $E(t)$ is total household consumption expenditures, and

$$P_s^Q(t) = \sum_{i=0}^{1} P_{ic}(t) t_{is} + \sum_{i=2}^{n} P_i^D(t) t_{is} + P_{n+1}(t) t_{n+1,s}; \qquad s = 1, 2, \ldots, k \tag{32}$$

$$P_{ic}(t) = (1 + \tau_i(t) + \xi_{ic}(t) P_i^D(t); \qquad i = 0, 1 \tag{33}$$

and where $\sum_{i=0}^{n+1} t_{is} = 1$.

3.4.4 *Import and Export Demand*

On the basis of Shephard's lemma and the definition of the price of composite good i (see eq. (16)), the demand for competitive imports follows directly from the demand for composite goods. Thus, import demand can be written

$$M_i = \frac{\partial \psi_i(P_i(t),\, P_i^M(t))}{\partial P_i^M} \left\{ \sum_{j=0}^{n+3} X_{ij}(t) + C_i(t) \right\};$$

or, in explicit form,

$$M_i(t) = d_{mi}^{\mu_i} \left[\frac{P_i^D(t)}{P_i^M(t)} \right]^{\mu_i} \left\{ \sum_{j=0}^{n+3} X_{ij}(t) + C_i(t) \right\}; \qquad i = 0, 1, \ldots, n \tag{34}$$

Observe that $M_i(t)$ is competitive import at constant purchaser's prices. In accordance with the discussion on p. 39, the corresponding import at constant c.i.f. prices is $M_i(t)/(1+\varphi_i^0)$, where φ_i^0 represent base year tariffs and indirect taxes on imported goods of type i.

The demand for complementary imports follows from the technology assumptions, and can be written:

$$M_j^c(t) = b_{jj} X_j(t); \qquad j = 0, 1 \tag{35}$$

Export demand can be regarded as the import demand of the rest of the world. By assuming that there is a constant elasticity of substitution, ε_i, between goods produced in the rest of the world and goods with the same classification produced in the home country, the export demand equations can be written:

$$Z_i(t) = Z_i^0 \left[\frac{P_i(t)}{V(t)\, P_i^{WE}(t)} \right]^{\varepsilon_i} e^{\sigma_i t}; \qquad i = 1, 2, \ldots, 4 \tag{36}$$

where σ_i is the exogenously given rate of growth of the production of commodity group i in the rest of the world.

3.4.5 *Equilibrium Conditions*

The derivation of the equilibrium conditions is straightforward. Thus, by Shephard's lemma and the definition of the price of composite good i the equilibrium conditions for the markets for domestically produced goods become

$$X_i(t) = \frac{\partial \psi_i(P_i(t),\, P_i^M(t))}{\partial P_i} \left\{ \sum_{j=0}^{n+3} X_{ij}(t) + C_i(t) \right\} + Z_i(t); \qquad i = 0, 1, \ldots, n \tag{37}$$

or, in explicit form,

$$X_i(t) = d_{di}^{\mu_i} \left(\frac{P_i^D(t)}{P_i(t)} \right)^{\mu_i} \left\{ \sum_{j=0}^{n+3} X_{ij}(t) + C_i(t) \right\} + Z_i(t); \qquad i = 0, 1, \ldots, n \tag{38}$$

$$X_i(t) = C_i(t); \qquad i = n+1,\, n+2 \tag{39}$$

$$X_{n+3}(t) = I(t); \tag{40}$$

Observe that $C_{n+2}(t)$ is the exogenously determined public consumption.

The budget constraint of the household sector is indirectly defined by a current account constraint. Thus, it is assumed that an "invisible government" instantaneously adjusts lump sum taxes so that household consumption expenditure is kept at a level compatible with a target surplus (or deficit), V(t)D(t), on the current account at the given level of public expenditures. Thus, each solution to the model will satisfy the condition

$$\sum_{i=1}^{n} P_i(t) Z_i(t) - \sum_{i=0}^{n} P_i^M(t) M_i(t) - \sum_{j=0}^{1} V(t) P_j^C(t) M_j^C(t) = V(t)D(t); \tag{41}$$

It is assumed that a given fraction, s, of the gross national product at market prices, $Y_m(t)$, is saved. Consequently the savings-investments equilibrium condition becomes

$$sY_m(t) - V(t)D(t) = P_{n+3}^D(t) I(t); \tag{42}$$

where I(t) is total gross investment and

$$Y_m(t) = E(t) + P_{n+2}(t)C_{n+2}(t) + P_{n+3}(t) I(t) + \sum_{i=1}^{n} P_i(t) Z_i(t) -$$

$$- \sum_{i=0}^{n} P_i^M(t) \frac{M_i(t)}{1+\varphi_i^0} + \sum_{j=0}^{1} V(t) P_j^C(t) M_j^C(t); \tag{43}$$

and where the price of capital goods is given by

$$P_{n+3} = \sum_{i=2}^{n} P_i^D(t)\, a_{i,\, n+3}; \tag{44}$$

Substitution of (41) and (43) in (42) yields the following expression for the savings-investment equilibrium condition

$$\frac{s}{1-s}\{E(t) + P_{n+2}(t)\, C_{n+2}(t)\} - V(t)D(t) = P_{n+3}I(t); \tag{45}$$

3.5 The Labor Market

By Hotelling's lemma the demand for labor by production units of vintage υ, sector j is given by

$$\frac{\partial \pi_{\upsilon j}(P_{\upsilon j}^*(t),\, W_j(t),\, t)}{\partial W_j} = - L_{\upsilon j}(t); \qquad \begin{array}{l} j = 0, 1, \ldots, n+2 \\ \upsilon = 0, 1, \ldots, t-1 \end{array}$$

or in explicit form

$$L_{\upsilon j}(t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \left(\frac{W_j(t)}{(1-\alpha_j) P_{\upsilon j}^*(t)} \right)^{-\frac{1}{\alpha_j}}; \qquad \begin{array}{l} j = 0, 1, \ldots, n+2 \\ \upsilon = 0, 1, \ldots, t-1 \end{array} \tag{46}$$

The equilibrium condition for the labor market becomes

$$L(t) = \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} L_{\upsilon j}(t); \tag{47}$$

where L(t) is the exogenously given supply of labor.

One step towards a full representation of the economy's properties in the short run is to relax the assumption about fully flexible real wage rates. In an alternative specification of the model, real wages are assumed to be flexible only upwards, i.e. a minimum wage rate $\bar{W}(t)$ is incorporated. The labor market equilibrium condition (47) is then replaced by the condition

$$W_j(t) = \begin{cases} \omega_j \bar{W}(t) \text{ when } \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} -\frac{\partial \pi_{\upsilon j}(P^*_{\upsilon j}(t), \omega_j \bar{W}(t); t)}{\partial W_j} \leq L(t) \\ \omega_j W(t) \text{ when } \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} -\frac{\partial \pi_{\upsilon j}(P^*_{\upsilon j}(t), \omega_j \bar{W}(t); t)}{\partial W_j} > L(t) \end{cases} \quad j=0,1,\ldots,n+2 \qquad (47')$$

where thus $\bar{W}(t)$ is the exogenously given index of the minimum real wage level, and W(t) is an index of the market clearing real wage level.

3.6 The Capital Market

In order to make the model complete, it remains to allocate gross investments over sectors, determine the technological coefficients in production units which are to be taken into operation in period t + 1, and to determine the real rate of interest. These three aspects of a solution to the model obviously are closely interconnected, and the purpose of this subsection is to discuss in some detail how the links between investment allocation, choice of technology and the real rate of interest are specified in the model. However, it should be stressed that even though the discussion is entirely based on theoretical considerations, the equations in this part of the model are not derived from neoclassical economic theory in the same direct way as the equations in other parts of the model. It should also be stressed that the main role of the equations discussed here is to provide the links between the various periods. That is, the equations (15)–(47) make up a complete model of resource allocation in period t, while the equations dealt with in this subsection "create" the new vintages of production units to be taken into operation in period t + 1.

In equilibrium the expected rate of return on marginal investments should be equal to the real rate of interest in all sectors. Thus, defining

$$\tilde{P}^*_j(t) = (1 - \tilde{\Theta}_j)\tilde{P}_j(t) - \sum_{i=2}^{n} \tilde{P}^D_i(t) a_{ij} - \tilde{V}(t)\tilde{P}^C_j(t) b_{jj}; \qquad j = 0, 1, \ldots, n+2 \qquad (48)$$

total investments should be allocated between the sectors in such a way that

$$\sum_{j=0}^{n+2} I_j(t) = I(t); \qquad (49)$$

where $I_j(t)$ is gross investments in sector j in period t, and the sectoral rates of return, $R_j(t)$, satisfy the conditions

$$P_j^*(t) = \varkappa_j^*(\tilde{W}_j(t), \tilde{Q}_j(t), \tilde{P}_{0j}(t), \tilde{P}_{1j}(t)); \qquad j = 0, 1, \ldots, n+2 \tag{50}$$

where $\tilde{Q}_j(t)$ is defined in accordance with

$$\tilde{Q}_j(t) = \tilde{P}_{n+3}(\delta_j R_j(t)); \qquad j = 0, 1, \ldots, n+2 \tag{51}$$

and the sectoral investments are determined by[10]

$$I_j(t) = \begin{cases} \delta_j X_j(t) \dfrac{\partial \varkappa_j^*(\cdot)}{\partial Q_j} \left(\dfrac{R_j(t)}{r(t)}\right)^{\varrho_j} & \text{when } R_j(t) \geq r(t) \\ 0 & \text{when } R_j(t) < r(t) \end{cases} \qquad j = 0, 1, \ldots, n+2 \tag{52}$$

where r(t) can be interpreted as a market rate of interest, and $\varkappa_j^*(\cdot)$ is evaluated at the prices $\tilde{W}_j(t)$, $\tilde{Q}_j(t)$, $\tilde{P}_{0j}(t)$, $\tilde{P}_{1j}(t)$, i.e. the capital intensity of new production units in a given sector j reflects the expected internal rate of return on investments in that sector. This implies that in each sector the producers take the expected rate of return on investments in that sector as a measure of the real rate of interest.

With this specification there is an inverse relation between $I_j(t)$ and $P_j(t+1)$. Thus, whenever current prices affect the price expectations, there is a tendency towards equalization of the expected internal rates of return on investments across sectors. This means that in the long run the choice of technology in new production units will reflect the opportunity cost of capital, while it will be largely unaffected by current market interest rates.

On the basis of Shephard's lemma, the energy input coefficients in production units of vintage t is determined by

$$a_{tij} = \frac{\partial \varkappa_j^*(\tilde{W}_j(t), \tilde{Q}_j(t), \tilde{P}_{0j}(t), \tilde{P}_{1j}(t))}{\partial P_{ij}}; \qquad i = 0, 1; \quad j = 0, 1, \ldots, n+2 \tag{53}$$

With this a solution to the model for period t provides all the data necessary for solving the model for period t + 1, i.e. the ex post production function units of vintage t are specified.

3.7 A Summing-up

The model presented here is designed for analysis of the adjustment of the economy to changes in exogenous conditions such as world market prices or energy prices. In the initial year there is only one vintage of capital in each production sector, but then a new vintage is created in each period. Thus the size of the model increases with the number of periods even though it is solved for only one period at a time.

In Table 3.1 below, the complete model is summarized. A complete list of the symbols used is given in the appendix to this chapter.

[10] Alternatively sectoral investments can be exogenously determined. That can be a reasonable approach particularly in the public sector.

Table 3.1 *The Equations of the Model*

Supply of domestically produced goods

$$X_{\upsilon j}(t) = \frac{\partial \pi_{\upsilon j}(\cdot)}{\partial P^{*}_{\upsilon j}}; \qquad j = 0, 1, \ldots, n+2 \quad \upsilon = 0, 1, \ldots, t-1$$

$$X_j(t) = \sum_{\upsilon=0}^{t-1} X_{\upsilon j}(t); \qquad j = 0, 1, \ldots, n+2$$

Intermediate Demand

$$X_{ij}(t) = \begin{cases} \sum_{\upsilon=0}^{t-1} a_{\upsilon ij} X_{\upsilon j}(t) & \text{when } i = 0, 1 \\ a_{ij} X_j(t) & \text{when } i = 2, 3, \ldots, n \end{cases} \qquad j = 0, 1, \ldots, n+3$$

Household Demand

$$C_i(t) = \sum_{s=1}^{k} t_{is} \left\{ Q_s^0 + \frac{\beta_s}{P_s^Q(t)} \left(E(t) - \sum_{s=1}^{k} P_s^Q(t) Q_s^0 \right) \right\}; \qquad i = 0, 1, \ldots, n+1$$

Export Demand

$$Z_i(t) = Z_i^0 \left(\frac{P_i(t)}{V(t) P^{WE}(t)} \right)^{\varepsilon_i} e^{\sigma_i t}; \qquad i = 1, 2, \ldots, n$$

Import Demand

$$M_i(t) = \frac{\partial \psi_i(\cdot)}{\partial P_i^M} \left\{ \sum_{j=0}^{n+3} X_{ij}(t) + C_i(t) \right\}; \qquad i = 0, 1, \ldots, n$$

$$M_j^c(t) = b_{jj} X_j(t); \qquad j = 0, 1$$

Product Market Equilibrium

$$X_i(t) = \frac{\partial \psi_i(\cdot)}{\partial P_i} \left\{ \sum_{j=0}^{n+3} X_{ij}(t) + C_i(t) \right\} + Z_i(t); \qquad i = 0, 1, \ldots, n$$

$$X_i(t) = C_i(t); \qquad i = n+1, n+2$$

$$X_{n+3}(t) = (t);$$

Savings Investment Equilibrium

$$\frac{s}{1-s} \{E(t) + P_{n+2}(t) C_{n+2}(t)\} - V(t) D(t) = P_{n+3}(t) I(t);$$

Labor Market Equilibrium

$$L(t) = \sum_{\upsilon=0}^{t-1} \sum_{j=0}^{n+2} - \frac{\partial \pi_{\upsilon j}(\cdot)}{\partial W_j};$$

Current Account Constraint

$$\sum_{i=1}^{k} P_i Z_i - \sum_{i=0}^{n} V(t) P_i^{WI}(t) \frac{M_i(t)}{1+\varphi_i^0} - \sum_{j=0}^{1} V(t) P_j^C(t) M_j^C(t) = V(t) D(t);$$

Energy Input Coefficients in New Plants

$$a_{tij} = \frac{\partial \varkappa_j^{*}(\cdot)}{\partial P_{ij}}; \qquad i = 0, 1; \quad j = 0, 1, \ldots, n+2$$

Investments in New Plants

$$\tilde{P}_j(t) = \varkappa_j^*(\tilde{W}_j(t),\ \tilde{Q}_j(t),\ \tilde{P}_{0j}(t),\ \tilde{P}_{lj}(t)) + \sum_{i=2}^{k} \tilde{P}_i^D(t)a_{ij} + \tilde{V}(t)\tilde{P}_j^C(t)b_{jj} + \Theta_j\tilde{P}_j(t); \qquad j = 0, 1, \ldots, n+2$$

$$I_j(t) = \begin{cases} \delta_j X_j(t)\, \dfrac{\partial \varkappa_j(\cdot)}{\partial Q_j} \left(\dfrac{R_j(t)}{r(t)}\right)^{\varrho_j} & \text{if } R_j(t) \geq r(t) \\ 0 & \text{if } R_j(t) < r(t) \end{cases} \qquad j = 0, 1, \ldots, n+2$$

$$I(t) = \sum_{j=0}^{n+2} I_j(t);$$

Definitions

$$\psi_i(P_i,\ P_i^M) = \left\{d_{di}^{\mu_i} P_i^{1-\mu_i} + d_{mi}^{\mu_i} P_i^{M^{1-\mu_i}}\right\}^{\frac{1}{1-\mu_i}}; \qquad i = 0, 1, \ldots, n$$

$$\pi_{\upsilon j}(P_{\upsilon j}^*,\ W_j;\ t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \alpha_j (1-\alpha_j)^{\frac{1-\alpha_j}{\alpha_j}} P_{\upsilon j}^{*\frac{1}{\alpha_j}} W_j^{\frac{\alpha-1}{\alpha_j}}; \qquad \begin{array}{l} j = 0, 1, \ldots, n+2 \\ \upsilon = 0, 1, \ldots, t-1 \end{array}$$

$$\varkappa_j^*(W_j,\ Q_j,\ P_{0j},\ P_{lj};\ \upsilon) = A_j^{-1}\left[a_j^{\frac{1}{1-\varrho_j}} \left(e^{-\lambda_j \upsilon} \alpha_j^{-\alpha_j} (1-\alpha_j)^{\alpha_j - 1} Q_j^{\alpha_j} W_j^{1-\alpha_j}\right)^{\frac{\varrho_j}{\varrho_j - 1}} + \right.$$

$$\left. + b_j^{\frac{1}{1-\varrho_j}} \left(e^{-\lambda_j^* \upsilon} \left(c_j^{\frac{1}{1-\gamma_j}} P_{0j}^{\frac{\gamma_j}{\gamma_j - 1}} + d_j^{\frac{1}{1-\gamma_j}} P_{lj}^{\frac{\gamma_j}{\gamma_j - 1}}\right)^{\frac{\gamma_j - 1}{\gamma_j}}\right)^{\frac{\varrho_j}{\varrho_j - 1}}\right]^{\frac{\varrho_j - 1}{\varrho_j}}; \qquad j = 0, 1, \ldots, n+2$$

$$P_{\upsilon j}^*(t) = (1-\Theta_j)P_j(t) - \sum_{i=0}^{1} P_{ij}(t)a_{\upsilon ij} - \sum_{i=2}^{n} P_i^D(t)\ a_{ij} - V(t)P_j^C(t)b_{jj}; \qquad j = 0, 1, \ldots, n+2$$

$$P_i^M(t) = \left(1 + \frac{\Delta\varphi_i}{1+\varphi_i^0}\right) V(t)P_i^{WI}(t); \qquad i = 0, 1, \ldots, n$$

$$P_i^D(t) = \psi_i(P_i(t),\ P_i^M(t)); \qquad i = 0, 1, \ldots, n$$

$$P_{ij}(t) = (1 + \tau_i(t) + \xi_{ij}(t)P_i^D(t)); \qquad i = 0, 1, \ldots, n+2,\ c$$

$$P_s^Q(t) = \sum_{i=0}^{1} P_{ic}(t)t_{is} + \sum_{i=2}^{n} P_i^D(t)t_{is} + P_{n+1}(t)t_{n+1,\,s}; \qquad s = 1, 2, \ldots, k$$

$$P_{n+3}(t) = \sum_{i=2}^{n} P_i^D(t)\ a_{i,\,n+3};$$

$$W_j(t) = \omega_j W(t);$$

$$\tilde{P}_i(t) = \zeta_i^0 P_i(t) + (1-\zeta_i^0)P_i^{OF}(t); \qquad i = 0, 1, \ldots, n+2$$

$$\tilde{P}_i^D(t) = \zeta_i^I P_i^D(t) + (1-\zeta_i^I)P_i^{DF}(t); \qquad i = 0, 1, \ldots, n$$

$$\tilde{P}_j^C(t) = \zeta_j^C P_j^C(t) + (1-\zeta_j^C)P_j^{CF}(t); \qquad j = 0, 1$$

$$\tilde{W}_j(t) = \zeta^W W_j(t) + (1-\zeta^W)\ W_j^F(t); \qquad j = 0, 1, \ldots, n+2$$

$$\tilde{Q}_j(t) = \tilde{P}_{n+3}(\delta_j + R_j(t)); \qquad j = 0, 1, \ldots, n+2$$

Exogenous variables

$L(t)$, $C_{n+2}(t)$, $D(t)$, $V(t)$, $P_i^{WI}(t)$, $P_i^{WE}(t)$, $P_j^C(t)$, $P_i^{OF}(t)$, $P_i^{DF}(t)$, $P_i^{CF}(t)$, $W_j^F(t)$, $\tau_i(t)$, $\xi_{ij}(t)$.

Appendix. List of Symbols

A. *Endogenous variables in period t*

$X_{\upsilon j}(t)$	gross output in sector j = 0, 1, . . ., n + 2, vintage υ = 0, 1, . . ., t − 1 in period t.
$X_j(t)$	total output in sector j = 0, 1, . . ., n + 2 in period t.
$X_{n+3}(t)$	output of investment goods in period t.
$X_{ij}(t)$	use of commodity i = 0, 1, . . ., n + 2 in sector j = 0, 1, . . ., n + 3, in period t.
$I_j(t)$	gross investments in sector j = 0, 1, . . ., n + 2 in period t.
$I(t)$	total investments in period t.
$C_j(t)$	household consumption of commodity i = 0, 1, . . ., n + 1 in period t.
$E(t)$	total household consumption expenditures in period t.
$Z_i(t)$	export of production sector output i = 1, 2, . . ., n in period t.
$M_i(t)$	import of goods competing with production sector output i = 0, 1, . . ., n, in period t.
$M_j^C(t)$	complementary imports used in sector j = 0, 1 in period t.
$P^*_{\upsilon j}(t)$	value added per unit of gross output in sector j = 0, 1, . . ., n + 2, vintage υ = 0, 1, . . ., t − 1 in period t.
$P_i(t)$	price of production sector output i = 0, 1, . . ., n + 3 in period t.
$P_{ij}(t)$	user price of energy of type i = 0, 1 in sector j = 0, 1, . . ., n + 2 in period t.
$P_i^D(t)$	user price of commodity i = 0, 1, . . ., n in period t.
$P_{ic}(t)$	user price of energy of type i = 0, 1 in the household sector in period t.
$P_s^Q(t)$	price of consumer commodity group s = 1, 2, . . ., k in period t.
$W_j(t)$	wage rate in sector j = 0, 1, . . ., n + 2 in sector t.
$W(t)$	index of the level of wages in the economy as a whole in period t.
$r(t)$	the market rate of interest in period t.
$\tilde{P}_j(t)$	expected future price of production sector output j = 0, 1, . . ., n + 2 in period t.
$\tilde{P}_{ij}(t)$	expected future user price of energy of type i = 0, 1 in sector j = 0, 1, . . ., n + 2 in period t.
$\tilde{P}_i^D(t)$	expected future user price of commodity i = 0, 1, . . ., n in period t.
$\tilde{P}_j^C(t)$	expected future price of complementary imports in sector j = 0, 1 in period t.
$\tilde{W}_j(t)$	expected future wage rate in sector j = 0, 1, . . ., n + 2 in period t.
$R_j(t)$	expected rate of profit on investments in sector j = 0, 1, . . ., n + 2 in period t.
a_{tij}	expected, and actual, input of energy of type i = 0, 1, per unit of output in plants designed for sector j = 0, 1, . . ., n + 2 in period t.

B. *Exogenous variables in period t*

$L(t)$	supply of labor in period t.
$C_{n+2}(t)$	public consumption in period t.
$D(t)$	surplus on current account in period t, expressed in foreign exchange.
$V(t)$	exchange rate (units of domestic currency per unit of foreign currency) in period t.

$P_i^{WI}(t)$ c.i.f. price level in period t, expressed in foreign currency, of imported goods competing with domestically produced commodity $i = 0, 1, \ldots, n$.

$P_i^{WE}(t)$ f.o.b. price level in period t, expressed in foreign currency, on foreign markets where domestic producers of commodity $i = 1, 2, \ldots, n$ compete.

$P_j^{C}(t)$ c.i.f. price level in period t, expressed in foreign currency, of complementary imports used as inputs in sector $j = 0, 1$.

$P_i^{OF}(t)$ predicted future price of production sector output i in period t.

$P_i^{DF}(t)$ predicted future user price of commodity group i in period t.

$P_j^{CF}(t)$ predicted future price of complementary imports in sector $j = 0, 1$ in period t.

$W_j^{F}(t)$ predicted future wage rate in sector j in period t.

$\tau_i(t)$ ad valorem tax on energy of type $i = 0$ in period t.

$\xi_{ij}(t)$ ad valorem tax on energy of type $i = 0, 1$ used in sector $j = 0, 1, \ldots, n+2$ in period t.

$\xi_{ic}(t)$ ad valorem tax on energy of type $i = 0, 1$, used in the household sector in period t.

C. *Parameters*[11]

s gross savings ratio for the economy as whole.

$a_{\upsilon ij}$ input of energy $i = 0, 1$ per unit of output in vintage $\upsilon = 0, 1, \ldots, t-1$ in sector $j = 0, 1, \ldots, n+2$.

a_{ij} input of commodity $i = 2, 3, \ldots, n$ per unit of output in sector $j = 0, 1, \ldots, n+2$.

b_{jj} input of complementary imports per unit of output in sector $j = 0, 1$.

t_{is} relative weight of commodity $i = 0, 1, \ldots, n+1$ in consumer commodity group $s = 1, 2, \ldots, k$.

ω_j index of the relative wage rate in sector $j = 0, 1, \ldots, n+2$.

Θ_i ad valorem indirect tax on commodity $i = 0, 1, \ldots, n+2$.

$\Delta\varphi_i + \varphi_i^0$ custom duty and indirect tax on the c.i.f. value of imports of commodities competing with commodity group $i = 0, 1, \ldots, m$.

δ_j annual rate of depreciation of capital in sector $j = 0, 1, \ldots, n+2$.

σ_i annual rate of change of production of commodity group $i = 1, 2, \ldots, n$ in the rest of the world.

μ_i, ε_i elasticity of substitution between domestically produced and imported units of commodity $i = 0, 1, \ldots, n$ at home and abroad, respectively.

β_s^Q expenditure allocation parameter in the household demand function for consumer commodity group $s = 1, 2, \ldots, k$.

d_{di}, d_{mi} distribution parameter in the CES-function representing the trade-off between imported and domestically produced goods of type $i = 0, 1, \ldots, n$.

ϱ_j the elasticity of investments in sector $j = 0, 1, \ldots, n+2$ with respect to the expected excess profit ratio.

ζ_i^0 the relative weight of the current price of production sector output $i = 0, 1, \ldots, n+2$ in the formation of expectations about that price.

[11] The difference between "exogenous variables" and "parameters" is that the former may have different values in different periods, while the latter are constant over time.

ζ_i^I	the relative weight of the current price of commodity $i = 0, 1, \ldots, n$ in the formation of expectations about that price.
ζ_j^C	the relative weight of the current price of complementary imports to sector $j = 0, 1$ in the formation of expectations about that price.
ζ^W	the relative weight of the current wage rates in the formation of wage expectations.
Z_i^0, Q_s^0, A_j	constants in the export, household demand and ex ante production functions, respectively.

4 The Solution Procedure

The purpose of this section is to give a brief overview of the solution algorithm developed by A. Por for this model. The discussion here does not deal with the mathematical techniques used in various parts of the algorithm. The aim is only to indicate the main steps in the solution procedure, and to give an economic interpretation of that procedure. Before turning to the main topic of this section, however, a few properties of the model should be pointed out, and some of the equations have to be written in a different form.

In accordance with eq. (25) it holds that

$$P^{*}_{\upsilon j}(t) = (1 - \Theta_j) P_j(t) - \sum_{i=0}^{1} P_{ij}(t) a_{\upsilon ij} - \sum_{i=2}^{n} P^{D}_{i}(t) a_{ij} - V(t) P^{C}_{j}(t) \bar{b}_{jj}; \quad \upsilon = 0, 1, \ldots, t-1 \quad j = 0, 1, \ldots, n+2$$

By a slight rearrangement of the terms this can be written so as to define a new variable, $P^{*}_{j}(t)$. That is

$$P^{*}_{j}(t) = P^{*}_{\upsilon j}(t) + \sum_{i=0}^{1} P_{ij}(t) a_{\upsilon ij} = (1 - \Theta_j) P_j(t) - \sum_{i=2}^{n} P^{D}_{i}(t) a_{ij} - V(t) P^{C}_{j}(t) \bar{b}_{jj}; \quad \upsilon = 0, 1, \ldots, t-1 \quad j = 0, 1, \ldots, n+2$$

By denoting the energy costs per unit of output $e_{\upsilon j}(t)$, i.e.

$$e_{\upsilon j}(t) = \sum_{i=0}^{1} P_{ij}(t) a_{\upsilon ij}; \qquad \upsilon = 0, 1, \ldots, t-1 \quad j = 0, 1, \ldots, n+2$$

the value added per unit of output in vintage υ in sector j can be written

$$P^{*}_{\upsilon j}(t) = P^{*}_{j}(t) - e_{\upsilon j}(t);$$

Thus the value added per unit of output can be subdivided into two parts, where one, the new variable $P^{*}_{j}(t)$, applies to all vintages in sector j and the other, the unit energy costs, is vintage specific. Moreover, when the prices of intermediate inputs are given, the values of $e_{\upsilon j}(t)$ are given as well. This means that total output in sector j can be expressed as a function of P^{*}_{j} and the wage rate W_j.

The variable $P^{*}_{j}(t)$ represents the difference between the producer price of the output in sector j, and the unit cost of non-energy intermediate inputs. Clearly it can happen that $e_{\upsilon j}(t)$ exceeds $P^{*}_{j}(t)$. Since negative $P^{*}_{\upsilon j}(t)$ would imply zero production under conditions of profit maximization behavior, $P^{*}_{\upsilon j}(t)$ has to be determined in accordance with

$$P^{*}_{\upsilon j}(t) = \begin{cases} P^{*}_{j}(t) - e_{\upsilon j}(t) & \text{if } P^{*}_{j}(t) > e_{\upsilon j}(t) \\ 0 & \text{if } P^{*}_{j}(t) \leq e_{\upsilon j}(t) \end{cases} \tag{54}$$

If some $P^*_{\upsilon j}(t) = 0$ in equilibrium, the corresponding $X_{\upsilon j}(t)$ is also zero, which can be easily seen when the suppy function of production units of vintage υ in sector j is written in the following explicit form.

$$X_{\upsilon j}(t) = A_{\upsilon j}(t)^{\frac{1}{\alpha_j}} \left\{ \frac{(1-\alpha_j)P^*_{\upsilon j}(t)}{W_j(t)} \right\}^{\frac{1-\alpha_j}{\alpha_j}};$$

Assuming that the prices $P_0(t), \ldots, P_{n+2}$ and the wage rate $W(t)$ are given, and using eq. (54), this supply function can be written

$$X_{\upsilon j}(t) = f_{\upsilon j}(P^*_j(t)) \qquad \begin{array}{l} \upsilon = 0, 1, \ldots, t-1 \\ j = 0, 1, \ldots, n+2 \end{array} \tag{55}$$

where $f_{\upsilon j}$ denotes a nonlinear function of the variable $P^*_j(t)$. Observe that when the domestic producer prices $P_0, \ldots, P_{n+2}$ are given, the domestic user prices $P^D_0(t), \ldots, P^D_n(t)$ are given as well. (See eq. (16)).

On the basis of eq. (31) it holds that

$$C_i(t) = E(t)\left(\sum_{s=1}^{k} \frac{t_{is}\beta_s}{P^Q_s(t)}\right) + \sum_{s=1}^{k} t_{is}Q^0_s - \sum_{s=1}^{k} \frac{t_{is}\beta_s}{P^Q_s(t)}\left(\sum_{s=1}^{k} P^Q_s(t)Q^0_s\right); \tag{56}$$

Thus, when the prices are given, household demand for commodity group i is a linear function of household expenditure $E(t)$.

Moreover, when prices are given, the "input" of domestically produced goods per unit of composite goods demanded is also given for all types of composite goods. Thus we can define

$$d_i(t) = d_{di}^{\mu_i}\left(\frac{P^D_i(t)}{P_i(t)}\right)^{\mu_i};$$

and treat $d_i(t)$ as a constant as long as the prices are kept unchanged. The equilibrium condition for the markets for domestically produced goods can now be written (setting $d_i(t) = 1$ for $i = n+1, n+2$)

$$X_i(t) = d_i(t)\left\{\sum_{j=0}^{n+3} X_{ij}(t) + C_i(t)\right\} + Z_i(t); \qquad i = 0, 1, \ldots, n+2$$

and on the basis of eq. (30) this becomes

$$X_i(t) = d_i(t)\left\{\sum_{j=0}^{n+3}\sum_{\upsilon=0}^{t-1} a_{\upsilon ij}X_{\upsilon j}(t) + C_i(t)\right\} + Z_i(t); \qquad i = 0, 1 \tag{57}$$

for the energy sectors and

$$X_i(t) = d_i(t)\left\{\sum_{j=0}^{n+3} a_{ij}X_j(t) + C_i(t)\right\} + Z_i(t); \qquad i = 2, 3, \ldots, n+2 \tag{58}$$

for the other production sectors.

As the total production in a given sector is the sum of the production in the individual vintages of production units, substitution of eq. (56) in eq. (57) yields, after simple reorganization of the terms,

$$\sum_{\upsilon=0}^{t-1} f_{\upsilon j}(P_j^*(t)) - d_i(t)\left\{\sum_{j=0}^{1}\sum_{\upsilon=0}^{t-1} a_{\upsilon ij} f_{\upsilon j}(P_j^*(t))\right\} =$$

$$= d_i(t)\left\{\sum_{j=2}^{n+3}\sum_{\upsilon=0}^{t-1} a_{\upsilon ij} f_{\upsilon j}(P_j^*(t)) + C_i(t)\right\} + Z_i(t); \qquad i = 0, 1 \qquad (59)$$

Note that when prices are given, the left-hand side is a non-linear function of $P_0^*(t)$ and $P_1^*(t)$, while the right-hand side is linearly dependent on E(t) ((through $C_i(t)$ by eq. (56)) and non-linearly dependent on $P_2^*(t), P_3^*(t), \ldots, P_{n+2}^*(t)$.

On the basis of eqs (35), (39), (40), (41) and (45) it holds that

$$\frac{s}{1-s}\{E(t) + P_{n+2}(t)X_{n+2}\} = \sum_{i=1}^{n} P_i(t)Z_i(t) -$$

$$- \sum_{i=0}^{n} P_i^M(t)M - \sum_{j=0}^{1} V(t)P_j^C(t)b_{jj}X_j(t) + P_{n+3}(t)X_{n+3}(t); \qquad (60)$$

When prices are given $Z_i(t)$ can be treated as constants (by eq. (36)) while $M_i(t)$ will be linearly dependent on $X_i(t)$ (by eqs. (34), (36) and (37)). Thus, with all prices and the wage rate given eq. (60) represents a linear dependency among the variables $X_0(t), X_1(t), \ldots, X_{n+3}(t)$ and E(t).

The model is solved for one period at a time, and for each period the solution procedure can be subdivided into two blocks. The first block deals with the commodity and labor markets, while the second, which does not affect the first, deals with the capital market. For the first block the solution procedure involves three main steps. In the following each one of these steps will be briefly described.

Step I

In this step an initial guess about the producer prices and the wage rate is made. These values are denoted $\hat{P}_0(t), \hat{P}_1(t), \ldots, \hat{P}_{n+2}(t)$ and $\hat{W}(t)$. When $t = 0$ these values are exogenously given, while the solution for the previous period is taken as the initial guess for $t > 0$. Using these prices, and on the basis of eq. (16), values for the domestic user prices, denoted $\hat{P}_0^D(t), \hat{P}_1^D(t), \ldots, \hat{P}_n^D(t)$, are computed, while a value for the price of capital goods, $P_{n+3}(t)$, is computed in accordance with eq. (44). Then it is possible to establish an initial guess $\hat{P}_0^*(t), \hat{P}_1^*(t), \ldots, \hat{P}_{n+2}^*(t)$ on the basis of eqs. (25) and (54). When these values are determined it is possible to represent $X_0(t)$ and $X_1(t)$ as linear functions of E(t).

Step II

II.1.: *The values for $W(t)$, $P_0(t), \ldots, P_{n+3}(t)$, $P_0^*(t)$, $P_1^*(t), \ldots, P_{n+2}^*(t)$ are kept fixed*

As $X_0(t)$ and $X_1(t)$ are now (with prices kept fixed) represented as linear functions of $E(t)$, values for the variables $X_2(t)$, $X_3(t)$, ..., $X_{n+3}(t)$ and $E(t)$ can be determined by a linear equation system derived from eqs. (41), (58) and (60). The computed output levels are denoted by $\hat{X}_2(t)$, $\hat{X}_3(t)$, ..., $\hat{X}_{n+3}(t)$.

II.2: *The values for $W(t)$, $P_0(t)$, $P_1(t)$, ..., $P_{n+3}(t)$ $P_0^*(t)$, $P_1^*(t)$ are kept fixed*

On the basis of step II.1, a set of values $\hat{\hat{P}}_2^*(t)$, $\hat{\hat{P}}_3^*(t)$, ..., $\hat{\hat{P}}_{n+2}^*(t)$ are determined by eq. (55), i.e.

$$\hat{X}_i(t) = \sum_{\upsilon=0}^{t-1} f_{\upsilon i}(\hat{\hat{P}}_i^*(t)); \qquad i = 2, 3, \ldots, n+2$$

II.3: *The values for $W(t)$, $P_0(t)$, $P_0^*(t)$, $P_1^*(t)$ are kept fixed*

Using the values $\hat{\hat{P}}_2^*(t)$, $\hat{\hat{P}}_3^*(t)$, ..., $\hat{\hat{P}}_{n+2}^*(t)$ thus computed and the equality (derived from eqs. (16) and (25) and the definition of $P_1^*(t)$)

$$P_j^*(t) = (1 - \Theta_j) P_j(t) - \sum_{i=2}^{n} \psi_i(P_i(t),\, P_i^M(t))\, a_{ij} - V(t)\, P_j^C(t)\, b_{jj};$$

new values for the producer prices can be computed. These values are denoted $\hat{\hat{P}}_2(t)$, $\hat{\hat{P}}_3(t)$, ..., $\hat{\hat{P}}_{n+2}(t)$. If these are sufficiently close to the previous values the procedure continues. Otherwise, it returns to step II.1, i.e. on the basis of $\hat{\hat{P}}_2^*(t)$, $\hat{\hat{P}}_3^*(t)$, ..., $\hat{\hat{P}}_{n+2}^*(t)$ new values for $X_2(t)$, $X_3(t)$, ..., $X_{n+2}(t)$ are computed.

II.4.: *The values for $W(t)$, $P_0(t)$, $P_1(t)$ are kept fixed*

Taking the values of $P_2^*(t)$, $P_3^*(t)$, ..., $P_{n+2}^*(t)$ and $E(t)$ thus computed as given, eq. (60) represents a nonlinear equation system with two equations in the two unknowns $P_0^*(t)$ and $P_1^*(t)$. By solving this system, i.e. clearing the energy markets, new iterative values, $P_0^*(t)$ and $P_1^*(t)$, for these variables are obtained. If these values are close to the previous ones, the procedure continues. Otherwise, it returns to step II.1, where thus the new values for $P_0^*(t)$ and $P_1^*(t)$ change the linear dependency between, on the one hand, $X_0(t)$ and $X_1(t)$, and, on the other hand, $E(t)$. If the values for $P_0^*(t)$, ..., $P_1^*(t)$, ..., $P_{n+2}^*(t)$ converge the steps II.1–II.4 means that a set of market clearing prices for the non-energy markets, at given wage rates and energy prices, has been found. However, the "net prices" for energy, i.e. the values for $P_0^*(t)$ and $P_1^*(t)$, which bring about the supply of energy which is demanded at the prices ruling at this stage of the procedure, might be inconsistent with the ruling producer prices of energy, i.e. $P_0(t)$ and $P_1(t)$.

II.5.: *The value for $W(t)$ is kept fixed*

The consistency of the values of $P_0^*(t)$ and $P_1^*(t)$ determined in step II.4, and the initial values $\hat{P}_0(t)$ and $\hat{P}_1(t)$ are checked by means of the equality

$$P_j^*(t) = (1 - \Theta_j)P_j(t) - \sum_{i=2}^{n} \psi_i(P_i(t),\ P_i^M(t)),\ a_{ij} - V(t)P_j^C(t)\ b_{jj}; \qquad j = 0,\ 1$$

and the values for $P_2(t)$, $P_3(t)$, ..., $P_n(t)$ computed in step II.3. If the equation is approximately satisfied, the procedure continues. Otherwise, it returns to step II.1, using the values for $P_0(t)$ and $P_1(t)$ which satisfy the above equation at the values $P_0^*(t)$ and $P_1^*(t)$ computed in II.4. When step II.5 finally is completed, a market clearing price system, at given wage rates, has been found.

Step III

With given wage rates and commodity prices, the demand for labor is given by eq. (46) added over vintages and sectors. If the demand for labor thus computed is sufficiently close to the supply of labor the procedure stops. Otherwise, W(t) is adjusted (the sign of the revision of W(t) being the same as the sign of the excess demand for labor) and the procedure is repeated from step II.1. When step III is completed a solution for the resource allocation in period t (except the sectoral allocation of investments) is obtained, i.e. the first block in the solution procedure is completed.

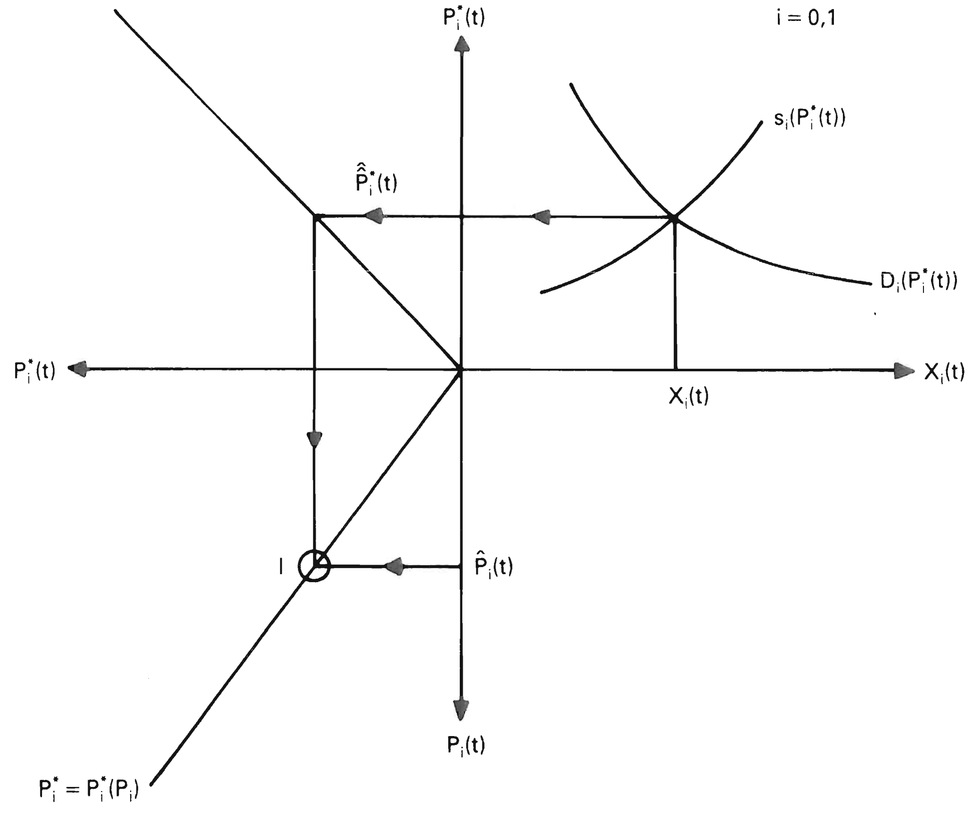
Before turning to the second block in the solution procedure, a few additional remarks about the steps II.1–II.5 in the first block should be made. The first point to note about these steps is that the two energy sectors are treated in a different way than the other production sectors. The reason for this is that the different vintages in a sector do not differ in terms of the use of non-energy intermediate inputs. Thus, provided prices, the production in the energy sectors and total household expenditures are given, the commodity market equations for sectors 2, 3 ..., n + 3 can be treated as an input-output model, i.e., a system of linear equations with equal number of equations and unknowns. In the corresponding equations for the energy sectors, however, all vintages in all sectors appear explicitly. Accordingly, the energy markets cannot be cleared unless the allocation of total production over vintages is simultaneously determined.

With this background it is easy to describe step II in the solution procedure in economic terms. In II.1 initial prices of energy are fixed. These prices can be regarded as prices facing the user of energy. Then in II.2 and II.3 market clearing prices for non-energy goods are determined; by an iterative process equality between the prices accepted by the producers and the prices facing the consumers is established, and throughout this process the markets for non-energy goods are cleared. When step II.3 is completed, the supply and demand functions for energy are also determined. The situation is illustrated by Figure 4.1 below where both the supply of and demand for fuels and electricity are expressed as functions of $P_i^*(t)$,[12] $i = 0$, 1, respectively.

In step II.4, then, the market clearing values of $P_i^*(t)$, $i = 0$, 1, are determined

[12] If Figure 4.1 is assumed to apply for $i = 0$, it presupposes that a solution for $i = 1$ already has been obtained.

Figure 4.1 *Market Clearing in an Energy Market*

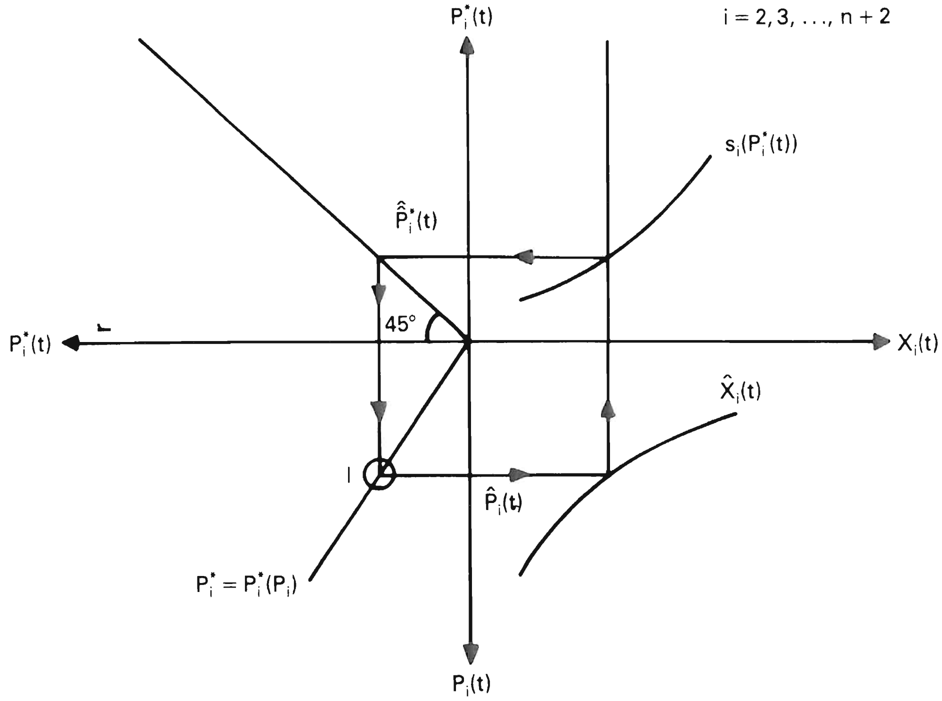


and in II.5 these values are compared with those obtained on the basis of the previous energy prices, i.e. $\hat{P}_0(t)$ and $\hat{P}_1(t)$. In Figure 4.1 the two sets of estimates of $P_0^*(t)$ and $P_1^*(t)$ coincide as they should in a final solution to the model.

Generally speaking, the solution procedure implies that producers always satisfy the demand which is established at the prevailing prices. However, once they realize that the market clearing output levels lead to marginal production costs, which differ from the prices, the prices are revised in such a way that equality between price and marginal cost is attained. But when prices change demand changes as well and the equality between price and marginal cost may again not be realized. The process continues until all commodity markets are cleared at prices which are compatible with profit maximization in all production units. At an intermediate step in the process, however, the prices needed to induce producers to supply the demanded quantity differ from the prices the consumers are faced with. Figure 4.2 illustrates how a solution for the non-energy sectors is attained, i.e., the steps II.2 and II.3.

Step III in the solution procedure does not have to be described in detail. It can be regarded as a Walrasian tâtonnement process in which the excess demand for

Figure 4.2 *Market Clearing in a Non-energy Market*



labor at a given wage rate is computed, and then the wage rate is gradually adjusted until the excess demand is eliminated. In this process the sign of the wage revision is the same as the sign of the excess demand.

The solution procedure for the second block, the allocation of investible funds over investing sectors, is much less complicated than the one described above. Once a solution for the first block is obtained, the variables $R_j(t)$ and $\partial\varkappa_j^*(\cdot)/\partial Q$ can be computed. Since I(t) is determined in the first block, the second block can be represented by the single equilibrium condition

$$I(t) = \sum_{j=0}^{n+2} I_j(t);$$

where $I_j(t)$ is a function of the market interest rate, r(t). However, $I_j(t)$ is a discontinuous function of r(t). Thus it may happen that no solution exists for the sectoral investment rule given by eq. (52). In order to obtain a solution in all cases, the rule is somewhat augmented in the solution procedure. Thus, if no solution can be obtained on the basis of eq. (52), the set of sectors competing for investible funds, i.e., the sectors for which $R_j(t)$ is greater than or equal to the market rate of interest at the point of discontinuity, is augmented with the sector with the highest $R_j(t)$ of those not in that set. This procedure is repeated until a solution is obtained.

5 Implementation, Parameter Estimation, and Model Behavior

5.1 General Remarks

When building a model of resource allocation in a national economy, the modeler is faced with several trade-off problems. One of these is that attempts to give a relatively detailed representation of various resource allocation mechanisms tend to involve variables which are difficult, or even impossible, to observe. For instance, explicit incorporation of the notion of "putty-clay" technology facilitates a realistic representation of factor substitution and technical change in a medium-term perspective, but requires an explicit representation of the ex ante technology and price expectation formation process, i.e. phenomena which are very difficult to observe. Conversely, models of resource allocation can be entirely based on observable data only if quite dramatic simplifications of the resource allocation mechanisms are accepted.

Obviously the model presented here is based on a number of simplifying assumptions. Yet it represents an attempt to model the process of factor substitutions and technical change in a relatively detailed and, hopefully, realistic way. The price of this realism is the incorporation of a number of variables which generally are difficult or impossible to observe. In addition the model contains variables for which only one or a few observations can be obtained. This applies, for instance, to the intersectoral deliveries of goods and services. Consequently most parameters in the model cannot be estimated by means of standard econometric techniques.

A substitute technique is the so-called "reference equilibrium method", which has been employed in the implementation of the model presented in this report. This approch to parameter estimation *starts* from the assumption that the model is a true representation of the economy. It is also, implicitly, assumed that the variables dealt with in the model can be measured correctly. If these assumptions are accepted, a statistical description of the state of the economy at one single point in time can be taken as a solution to the model.

With this point of departure the parameter estimation procedure becomes simple and straightforward. To begin with, all the parameters of the model are expressed as functions of the exogenous and endogenous variables, and then these functions are evaluated at the observed values of the model's variables at a particular point in time.

In formal terms the model is written

$$f_i(x, y, \alpha) = 0; \qquad i = 1, \ldots, m$$

where the vector x denotes the endogenous variables, the vector y the exogenous variables and the vector α the parameters. At a given point in time the endogenous and the exogenous variables attain the values $\bar{x}$ and $\bar{y}$ respectively, and the model becomes a system of equations in which the vector α contains all the unknowns, i.e.

$$f_i(\bar{x}, \bar{y}, \alpha) = 0; \qquad i = 1, 2, \ldots, m.$$

In simple cases, such as when the parameters of an input-output model are to be estimated, one consistent set of observations on exogenous and endogenous variables is sufficient for unique determination of all the parameters of the model. Generally, however, extraneous information about functional forms and some parameter values is needed as well.

When this procedure is adopted model validation obviously becomes very difficult. The statistical significance of individual parameter estimates cannot be ascertained in the "usual" way, and lack of data often prevents a full ex post comparison between model predictions and the actual development of the economy. In addition, models of the type presented here often are used for simulation of the effects of policies and other changes in exogenous conditions which have not been experienced before. Thus, one may wonder about the proper interpretation of results obtained from a model, in which the parameters have been estimated by means of the reference equilibrium method.

The easiest answer to this question, of course, is to maintain that the principal aim of the study is methodological, and, consequently, that the numerical results should be regarded only as illustrations of a method. This position no doubt implies the minimum commitment for the author, and may in some cases be the only reasonable one, but it also implies some waste of the resources put into model development. After all, a considerable amount of data *is* available, and efficient use of these data might turn the model into a useful tool for quantitative analysis of resource allocation problems in a real economy.

The model presented in this report has been implemented on Swedish data. The implementation represents a serious effort to make efficient use of available data. The resulting numerical model, which is briefly described in the following subsection, is intended to be used for quantitative analysis of resource allocation problems related to Swedish energy policy. The nature and purpose of the quantitative analysis in which the model will be used is best described by a quotation from Leif Johansen:[13] "The data and the quantitative analysis do serve the purpose of illustrating the method and the model. But, at the same time, if I were required to make decisions and take actions in connection with relationships covered by this study, I would (in the absence of more reliable results, and without doing more work) rely to a great extent on the data and the results presented in the following sections. Thus, the quantitative analysis does not *solely* serve the purpose of illustrating a method. I do believe that the numerical results also give a rough description of some important economic relationships in reality".

[13] See Johansen (1960), p. 3.

5.2 Functional Forms and Parameter Values

As mentioned above, the model has been implemented on the basis of Swedish data. The main data source is an input-output table for 1979, aggregated up to seven production sectors. The sector classification, which can be seen in Table 5.1, is chosen in order to be useful for analyses of the economic impact of national energy policies and alternative assumptions about world market oil prices.

Table 5.1 *Production Sector Definitions*

	Sector	SNI
0	Fossil fuels production	353, 354
1	Electricity production	4
2	Mainly import competing industries	11, 13, 31, 32, 33, 3412, 3419, 342, 355, 361, 362
3	Mainly exporting energy intensive industries	12, 2, 3411, 351, 37101 37102
4	Other mainly exporting industries	352, 356, 3699, 37103, 372, 38, 39
5	Sheltered industries and service production	3691, 3692, 5, 6, 7 8, 9 (priv.)
6	The public sector	
7	The capital goods sector (book keeping sector)	

The aim has been to identify the smallest possible number of sectors which is compatible with a meaningful analysis of the issues under study. There are several reasons to keep the number of sectors as small as possible. One is simply that the costs for solving and storing the model are quite sensitive to its size. Another, and more important reason, is that no complete input-output table for 1979 is available. Thus, the estimates of the intersectoral flows of non-energy[14] goods and services have to be based on the 1975 input-output data.[15] As the input-output relations between large aggregates of sectors tend to be more stable than the corresponding relations between more disaggregated sectors, a relatively high level of aggregation accordingly was preferable in this case. A third reason is that the possibilities of getting good estimates of world market prices, technical change, etc. for a large number of sectors from published sources are quite limited, and if such assumptions cannot be differentiated between individual sectors, there is no point in a far-reaching disaggregation of the production system.

[14] Estimates of the 1979 energy flows are, however, available.

[15] For 1975 a complete input-output table is available. However, as only one year had elapsed since the 1973/74 oil price increases, it is quite likely that the energy input coefficients which can be observed in the 1975 input-output table do not represent equilibrium values. To some extent the same problem applies to the 1979 data, but in view of the errors in the national accounts up to 1978 that have been discovered, the 1979 data was regarded as the best choice.

Tables 5.2–5.4 summarize some base-year data about the production sectors, and in Table 5.5 the ten aggregates of household consumption goods dealt with in the model are defined.

Table 5.2 *Base Year Sectoral Employment, Energy Use, and Value Added Shares*

Sector	L_j/L	$X_{0j}/(X_0 + M_0)$	$X_{1j}/(X_1 + M_1)$	Y_j/Y
0	0.0009	0.0251	0.0025	0.0223
1	0.0095	0.0814	0.0249	0.0255
2	0.1226	0.0595	0.0620	0.1721
3	0.0472	0.1006	0.1557	0.0612
4	0.1303	0.0470	0.0862	0.1562
5	0.4258	0.4213	0.5346	0.3514
6	0.2637	0.0309	0.1138	0.2113
7	–	–	–	–
Σ	1.0000	0.7658[a]	0.9795[a]	1.0000

[a] The difference between this value and unity is made up of the shares of household consumption and exports.

Table 5.3 *Base Year Input Coefficients for Capital, Labor, Fuels, and Electricity*

Sector	K_j/X_j	L_j/X_j	X_{0j}/X_j	X_{1j}/X_j
0	0.2174	0.0003	0.0512	0.0023
1	6.3451	0.0031	0.1850	0.0255
2	0.7361	0.0047	0.0159	0.0075
3	1.5517	0.0044	0.0649	0.0452
4	0.6229	0.0052	0.0132	0.0109
5	2.6974	0.0093	0.0639	0.0366
6	1.5461	0.0118	0.0096	0.0159
7	–	–	–	–

Table 5.4 *Base Year Shares of Export and Import, and Export and Import Shares in Individual Production Sectors*

Sector	Z_i/Z	M_i/M	Z_i/X_i	$M_i/(X_i + M_i)$
0	0	0.1946	0	0.6060
1	0.0030	0.0059	0.0210	0.0494
2	0.1530	0.2454	0.1277	0.2024
3	0.2283	0.1056	0.4597	0.2086
4	0.4963	0.3745	0.4342	0.2888
5	0.1194	0.0740	0.0566	0.0417
6	–	–	–	–
Σ	1.0000	1.0000		

All import measures inclusive of complementary imports.

Table 5.5 *Definition of Household Consumption Goods*

	Consumption good	SNA[a]
1.	Food	1.1
2.	Beverages and tobacco	1.2–1.4
3.	Clothing	2, (8.1.2)
4.	Gasoline	6.2.2
5.	Medical products and personal care	(4.5.1), 5.1, 8.1.1, (8.1.2)
6.	Gross rents, fuel and power	3.1.1, 3.2
7.	Transport	(6.1), (6.2.1), 6.2.3, 6.3
8.	Recreation	5.2, (6.1), (6.2.1), 7.1, 8.2.1, (8.2.2)
9.	Furniture and other furnishing	4.1, 4.2.1, (4.3.1), 4.3.2, (4.4.1), (4.5.1)
10.	Other	(4.3.1), (4.4.1), (4.5.1) 4.5.2, 4.6, 5.3, 6.4, 7.2–7.4, (8.1.2), (8.2.2) 8.2.3–8.3.2, 8.5, 8.6

[a] SNA numbers in ( ) means "part of".

Given the specification of the model in terms of the general characteristics of supply and demand functions, there is still some freedom to choose functional forms for the ex ante production functions, the household utility function and the "production functions" defining the composite goods in the home country and in the rest of the world. In the following the choices actually made will be briefly discussed. It should, however, be pointed out that other choices can be made without major revisions of the solution algorithm.

Clearly the knowledge about ex ante production functions is very limited. This would suggest that a flexible functional form such as the translog production function or a generalized Leontief cost function should be used for the representation of the ex ante technology. However, in view of the difficulties to obtain relevant price-data for the estimation of these functions, they were not too attractive. Instead it seemed resonable to choose a functional form with a small number of parameters, and in which each one of the parameters has a simple economic interpretation. In other words, if one is forced to use a lot of "guesstimates" the number of guesses should be kept at a minimum and concern economically meaningful magnitudes.

From these points of view the nested CES-Cobb-Douglas function (see p. 38) was attractive, but not the only possible choice. The distribution parameters (α_j) in the Cobb-Douglas part of the function were estimated by means of income distribution data for the base-year, i.e. for 1979, and the input-output coefficients (a_{ij}) are estimated by means of the ratios of intersectoral flows (X_{ij}) and gross output (X_j) that year. The ex ante elasticity of substitution between the capital-labor composite and the fuels electricity composite was set equal to 0.75 in all sectors except the energy sectors. This figure is compatible with the results presented in Pindyck (1980),[16] but of course subject to significant uncertainty. Lacking better information, finally, the same values were assumed for the elasticity of substitution between fuels and electricity. A selection of the adopted parameter values are displayed in Table 5.6.

The choice of a linear expenditure system for the representation of household demand is motivated by the fact that such systems have been estimated on Swedish data. The linear expenditure system used here is estimated by J. Dargay and A. Lundin (1981). It differs from other systems in that it treats fuels as a separate household consumption good (see Table 5.5), which is an advantage in the types of studies the present model will be used. However, as all linear expenditure systems it does not take substitution effects into account, which clearly is a disadvantage.

The "production functions" defining the composite goods consumed in the home country and the composite goods consumed in the rest of the world were all specified as CES-functions. The CES-specification is convenient since it leads to import and export functions which are relatively easy to estimate. However, the assumption that the same "production function" applies to all domestic users of good i is generally not plausible. If goods produced in different countries actually are qualitatively different, the substitutability between imported and domestically produced goods of the same "type" should, in general, differ between domestic users. In particular, for industrial users the substitutability should reflect the properties of technology, while it should reflect the properties of preferences for users in the household sector.[17]

However, whereas the model would remain fundamentally the same if different composite goods were defined for different domestic users, the estimation problems would increase significantly. In view of these problems the simplest possible specification was chosen. Thus, all domestic users of a given composite good, say i, are assumed to use the same type of composite good, i.e., use the same "production function" to define the composite good.

The numerical values of the parameters of the import and export functions have been chosen partly on the basis of econometric evidence, partly on the basis of theoretical considerations. Thus, one source of information is Hamilton (1980), another is Restad (1981). However, neither of these sources, or other possible sources, use a sector classification which can be aggregated into the one used in this study. Moreover, many of the estimated export price elasticities have absolute

[16] On the basis of pooled time-series data from ten countries, Pindyck estimated, among other things, the elasticity of substitution between capital and labor (σ_{KL}), between capital and energy (σ_{KE}) and between labor and energy (σ_{LE}) for the industrial sector. His results should be regarded as estimates of the long-run elasticities of substitution, and can thus be used to characterize the properties of the ex ante technology. With our assumptions included in paranthesis, Pindyck's results were the following:

σ_{KL}: 0.77–0.82(1.00), σ_{KE}: 0.61–0.86(0.75), σ_{LE}: 0.93–0.97(0.75).

[17] Available econometric evidence does *not* support the hypothesis that the same "production function" can be used to define the composite good i for all sectors. See Frenger (1980).

values which are so low that they are hard to accept on theoretical or even common sense grounds. Consequently, a considerable amount of judgement and "fingerspitzgefühl" has gone into the estimates of ε_i and μ_i displayed in Table 5.6.

Table 5.6 *Estimated Values of Some Key Parameters*

Sector	α_j	$(1-\varrho_j)^{-1}$	$(1-\gamma)^{-1}$	ε_j	μ_j
0	0.8362	0.25	0.25	0.0	0.5
1	0.7555	0.25	0.25	−1.0	0.5
2	0.3770	0.75	0.75	−2.0	4.0
3	0.3076	0.75	0.75	−5.0	0.5
4	0.2053	0.75	0.75	−4.0	2.0
5	0.3600	0.75	0.75	−2.0	0.5
6	0.0436	0.75	0.75	–	–
7	–	–	–	–	–

It should be noted that the export functions and the current account constraint are specified in such a way that the "home country" has some autonomy in the pricing of its exports. Whether or not a significant deviation between domestic production cost, i.e., the variable $P_i(t)$, and the corresponding world market price, i.e., the variable $V(t)P_i^{WE}(t)$, would exist in equilibrium depends on the absolute values of ε_i as well as on the properties of the supply functions. As deviations between domestic production costs and world market prices are not consistent with the notion of "a small open economy", some constellations of parameter values would necessitate a respecification of the foreign trade part of the model. Otherwise the model would indicate terms of trade gains of the optimum tariff type from domestic energy taxation. Such a respecification would then be carried out along the following lines.

The total output in sector j is assumed to consist of an aggregate of a large number of goods. The price $P_j(t)$ is taken to be the price index of this aggregate. Some of the goods in the aggregate are exported at given world market prices. The price $P_j^{WE}(t)$ is taken to be the price index of the aggregate of goods exported from sector j. Using the same symbols as before, the value of the total output from sector j can be written in two identical ways in accordance with

$$P_j(t)X_j(t) = V(t)P_j^{WE}(t)Z_j(t) + P_j^N(t)\ \{N_j(t)\};$$

where thus $P_j^N(t)$ is the price index and $N_j(t)$ the quantity of the nonexported part of the output from sector j.

The current account constraint should now be written

$$\sum_{i=1}^{n} P_i^{WE}(t)Z_i(t) - \sum_{i=0}^{n} P_i^{WI}(t)\frac{M_i(t)}{1+\varphi_i^0} - \sum_{j=0}^{n} P_j^C(t)M_j^C(t) = D(t);$$

while the unit cost of the composite goods used within the country should be defined

$P_i^D(t) = \psi_i(P_i^N(t), P_i^M(t));$ $\quad i = 0, 1, \ldots, n$

rather than

$P_i^D(t) = \psi_i(P_i(t), P_i^M(t));$ $\quad i = 0, 1, \ldots, n$

Thus, the model can be made roughly consistent with the usual "small economy" assumptions also in the case where the absolute values of the parameters ε_i are rather small. However, unless the energy tax variables are given quite high values, "optimum tariff effects" do not seem to be a problem at the parameter values displayed in Table 5.6.

5.3 Some Numerical Results

To conclude the description of the ELIAS-model, a few results from model simulations are presented. These results primarily serve the purpose of indicating the functioning of the model. Thus, there is no point in carrying out an extensive discussion about underlying assumptions about exogenous variables; it is sufficient to say that the "Base case" presented below is based on the assumptions and results of the most recent long-term economic survey published by the Ministry of Economic Affairs.

Table 5.7 *The Calculated Impact of Increasing Labor Supply*
Annual percentage growth rates 1979–91

	Base case	Labor Plus case
Private consumption[a]	1.3	2.5
Public consumption[ab]	0.9	0.9
Gross investments[a]	2.9	3.6
Exports[a]	4.0	4.9
Imports[a]	1.7	2.4
GDP[a]	2.2	3.0

[a] In constant 1979 prices.
[b] Exogenously determined.

To begin with two "comparative dynamics" experiments are presented. Thus, in Table 5.7 the impact of an increasing supply of labor is displayed. In the Base case the labor force is assumed to decline by 0.2 percent per annum, while it is assumed to grow by 0.8 percent per annum in the Labor Plus case.

The next "comparative dynamics" experiment concerns the role of expectation formation and the functioning of the vintage capital and putty-clay properties of the technology. Thus, in the Base case static expectations were assumed, i.e., the producers/investors were assumed to expect current relative prices to prevail in the future as well. In the Oil Price Foresight case, on the other hand, the producers/

investors are assumed to correctly foresee an annual 2 percent increase in the relative price of imported oil. As a result, total oil consumption declines by 0.1 percent per annum in the Oil Price Foresight case, while it increases by 1.1 percent per annum in the Base case.

In Table 5.8 below the development of sectoral oil input coefficients in these two cases is displayed. A distinction is made between the average sectoral oil input

Table 5.8 *The Calculated Development of Sectoral Oil Input Coefficients 1979–91 under Various Assumptions about Oil Price Expectation Formation*

	Base case		Oil Price Foresight case	
	Average sectoral oil input coefficient	Oil input coefficient in new plants, %	Average sectoral oil input coefficient	Oil input coefficient in new plants, %
a. *Sector 2: Mainly import competing industries*				
1979	0.0159	98	0.0159	75
1982	0.0158	86	0.0142	75
1985	0.0149	84	0.0129	76
1988	0.0140	84	0.0118	78
1991	0.0132	84	0.0108	81
	−1.5 p.a.		−3.2 p.a.	
b. *Sector 3: Mainly exporting energy intensive industries*				
1979	0.0650	100	0.0650	76
1982	0.0650	94	0.0610	79
1985	0.0636	92	0.0570	81
1988	0.0618	91	0.0535	83
1991	0.0599	90	0.0505	84
	−0.7 p.a.		−2.1 p.a.	
c. *Sector 4: Other mainly exporting industries*				
1979	0.0132	100	0.0132	77
1982	0.0132	90	0.0116	80
1985	0.0125	89	0.0104	84
1988	0.0118	88	0.0095	86
1991	0.0110	89	0.0087	87
	−1.5 p.a.		−3.4 p.a.	
d. *Sector 5. Sheltered industries and service production*				
1979	0.0639	100	0.0639	77
1982	0.0639	94	0.0603	78
1985	0.0630	93	0.0571	80
1988	0.0618	94	0.0542	83
1991	0.0606	92	0.0517	84
	−0.4 p.a.		−1.8 p.a.	

coefficients and the oil input coefficients in new plants, i.e. plants designed in the period in question and taken into operation the following period. The latter set of coefficients are expressed as percentages of the average sectoral oil input coefficient.

It should be noted that the assumptions made in the Oil Price Foresight case lead to approximately the same, but relatively low, oil input coefficients in new plants in all periods, while these coefficients tend to decline over time in the Base case where expected oil prices gradually increase. Also it should be noted that the development of average oil input coefficients over time partly reflects the properties of the new technology, and partly reflects the amount of gross investments in the sector in question.

In Table 5.9, finally, some comparative static experiments are presented. The results are presented as elasticities computed around the 1991 Base case equilibrium values.

Table 5.9 *Computed Elasticities of Selected Macro-economic Variables with Respect to Oil and Export Price Changes*

	Elasticity with respect to changes in		
	Oil prices		
	Import price change	Domestic oil tax change	Export price in sector 4[c]
Private consumption[a]	−0.17	0.01	0.10
Gross investments[a]	−0.16	−0.01	0.13
Exports[a]	0.24	0	−0.04
Imports[a]	−0.11	0	0.22
GDP[a]	0.01	0	0
National income[b]	−0.19	0.01	0.24
Terms of trade	−0.37	0	0.24
Oil consumption	−0.04	−0.03	0.01

[a] Constant 1979 prices.
[b] Evaluated at equilibrium factor prices.
[c] The variable $P_4^{WE}(t)$.

As can be seen in the table the Armington assumption in conjunction with the exogenously given current account deficit (or surplus) leads to very strong terms-of-trade effects of world market price changes. The significant increase in exports which is necessary to restore equilibrium in the case of a world market oil price increase, suggests that rigidities in the adjustment process might cause additional income losses. It should also be noted that, due to the technology assumptions, the short-run price elasticity of the demand for oil is very low.

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ISAC – A Model of Stabilization and Structural Change in a Small Open Economy

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Contents

1 Introduction

In the following we present a model for the Swedish economy called ISAC – *I*ndustrial *S*tructure *A*nd *C*apital Growth. The principal aim of our modelling efforts has been to form an instrument suitable for the medium and long-term study of stability and growth. The model also reflects our concern to arrive at empirically estimable relations and functions, and to make possible projections over a longer time span in order to analyze some crucial policy problems concerning, e.g. the balance of payment, energy policy, and the control of local government spending. The model has so far been used mainly for policy studies dealing with the time periods 1980–85 and 1980–2000, respectively.

1.1 Inertia and Adjustment in a Small Open Economy

In a small open economy like Sweden much interest is focused on the way in which the economy adjusts to changes in trade volume and prices on the world markets. There are many ways in which this adjustment process can be modelled. In a pure neoclassical model of competitive equilibrium the emphasis would be on the final results of the assumed price adjustments, disregarding the various sources of inertia and the effects of intermittent disequilibria. The assumed adjustment mechanisms would then usually not be directly estimable from the dynamic performance of real life economies, and the model would be best suited for comparative static studies of medium and long-term tendencies.

In ISAC we have gone in an opposite direction, trying as far as possible to incorporate various kinds of adjustment obstacles, like the sticky wages and prices of markets characterized by monopolistic or oligopolistic competition, the immalleability of vintage capital, the cash-flow restrictions on investment financing, and the inertia and lags observed in both private and local government consumption. We recognize regulated prices like the rate of exchange and cost components mainly determined from abroad like the rate of return requirement. Market disequilibria – surpluses or deficits of foreign exchange and of production capacity and labor – will therefore be a normal feature of model projections and will have feedback effects in the form both of price modifications and a rationing of supply. From a disequilibrium situation the model economy may finally – with the help of or despite economic policy measures – fetch up in a new equilibrium or steady-state growth barring new disturbances. Even then, however, the adjustment path will in most cases affect the final equilibrium in important ways. To study this interdependence between short-term instability and long-term growth is indeed one of the main purposes of the ISAC-model.

The ambition to incorporate a realistic description of the "imperfect" adjustment mechanisms is a feature which the ISAC-model has in common with i.a. the

LIFT-model of the University of Maryland and the MGM-model of the Cambridge Growth project (Barker 1974, 1976). The development of ISAC has indeed run parallel with the development of the MGM-model and has been influenced and aided by the experience earned from the Cambridge Growth project.

The history of the ISAC-model can, to a certain extent, be said to reflect the developments of the Swedish economy during the 70s. The model work was started in the middle of the 70s. The aim was to construct a static planning model for the whole economy of the kind then used for medium-term planning in many Treasuries around Europe. Starting out from the experience of fast growth and price stability in the 60s it was deemed sufficient and appropriate to limit the modelling ambitions to the description or prescription of balanced growth paths from the present to a future target date. The emphasis was on developments in real terms with consistent price structures being computed afterwards. This first version of the model was used for a medium-term survey, carried out in 1976 by the Industrial Institute for Social and Economic Research (IUI) in Stockholm.[1]

The increased price instability and price uncertainty engineered in part by the 1973 oil crisis were in Sweden followed by dramatic swings in industrial investment activity during the rest of the decade. In the second phase of model development priority was therefore given to the integration of price formation into the model computation and the introduction of investment functions for the various industrial branches. This second version was i.a. employed in a second IUI survey, published in 1979.[2]

At the end of the 70s the economic interest in Sweden was increasingly focused on the determinants of our international competitiveness and on the need for structural adjustment in industry in order to eliminate our mounting trade deficit. Thus, there were compelling reasons in the final phase of model development for introducing more explicit mechanisms for price and wage setting, for making the capital structure in industry more explicit with the aid of a vintage capital approach, and for incorporating a special submodel for local government spending. To make possible an explicit analysis of various means of reducing our oil dependence a relatively detailed treatment of energy consumption was also incorporated into the model.

During the early years of the 80s the model was used for various kinds of policy analysis within IUI. Structural change and strategic choices concerning the growth of public service were analyzed with the help of model simulations over the period 1980–2000 (Nordström-Ysander 1980). The adjustment problems caused by the oil price hikes and alternative ways of "insuring" against similar future macroeconomic shocks have been the subject of a series of studies making use of the detailed energy specification of the model (Ysander 1983a, b, 1984). The incorporation of a

[1] For a detailed account of this version see "IUI:s långtidsbedömning 1976. Bilagor." (1977), in particular Chapters 1–3 by U. Jakobsson, G. Normann and L. Dahlberg, respectively.

[2] Cf. Eliasson-Carlsson-Ysander (1979). The second version is described by the present authors in Ysander et al. (1979).

submodel of local government behavior (the LOGOS model) has made it possible to make simulation studies of the interaction between local government and the rest of the economy and to analyze the cyclical impact of local government budgets (Ysander-Nordström 1985). Since 1981 the model is also at the disposal of the Swedish Treasury, where it has been used for complementing medium-term surveys and as a starting point for developing new, aggregated, model versions (cf. Nordström 1982).

1.2 Market Behavior

The markets explicitly treated in the ISAC model are throughout characterized by a monopolistic or oligopolistic competition. The nonregulated prices are set by the sellers, not by any neutral "market" mechanism. This certainly does not imply the absence of any adjustment towards market equilibrium. But sticky prices and wages can make the adjustment considerably slower and more indirect.

On the *markets for tradable goods* prices develop as weighted averages of changes in cost, in world market prices and in capacity utilization, where different weights can be used for domestic and foreign sales, respectively. This can be interpreted as saying that basically the firms are trying to cover costs, where costs are computed as average variable cost plus planned depreciation and a target rate of return on installed capital.[3] This "cost price" is then modified to take account both of the foreign competition and of variations in capacity utilization.

There are no long-run returns to scale in industrial production but capacity within each branch is distributed between vintages, typically differing in technology and in productivity. In a world of perfect competition only those vintages would be used which can earn a quasi-rent at given world market prices. The way we have modelled the Swedish economy is quite different. Typically, a Swedish industrial branch is dominated by a small number of big firms – each comprising several capital vintages, both in different plants and within the same plant, which moreover often represent different technological stages of the production processes. We therefore assume that, due to technological reasons and to considerations of regional employment responsibilities, the firms will at each time have the same capacity utilization in each vintage. In the short run this may imply accepting current losses in some vintages. Since the firms are assumed also to vary the relative scrapping of vintage capacity in inverse proportion to profitability, vintage use will, nevertheless, in the long run adjust to differences in quasi-rent. The short-run impact on the firm of, say, a change in world market prices will thus be to a certain extent cushioned both by the "sticky" pricing and the evening out of capacity use – making the compensatory change in trade volume correspondingly greater. Since changing trade and capacity use will react back both on pricing and – together with profits – on investments, the adjustment will be reinforced. The

[3] For a discussion of pricing with target rate of return and of the empirical evidence for its use cf. Eckstein-Fromm (1968).

adjustment will, however, still be somewhat slower and carried out on a lower profit level than what would be the case in a perfectly competitive world.

The response to the firms' price-setting will on domestic markets be determined by intra-firm demand, by governmental demand, and by consumers' final demand as expressed in a system of linear expenditure equations. On the foreign markets, the response, described by a set of export functions, will be determined by the relative price development and by the world market trade expansion.

For nontradable goods the prices are assumed to be completely determined by cost, including a target rate of return on capital.

Wage-setting on the *labor market* is modelled by a Phillips curve type of approach, making wages in the private sector depend on past profits, productivity development and inflation, and current unemployment. This outcome can be interpreted in a manner somewhat analogous to that used for the price-setting in the product market. The wage earners will try to get compensated for both inflation and productivity gains but the final result will, however, be modified by current market conditions, i.e., unemployment. Long-run wage adjustment will thus be reinforced by the change in total employment resulting from the bargaining.

For public employees a one-year lag in wage settlement has been usual and has been assumed in the model to continue also in the future. The sensitivity of wages to employment conditions is obviously of strategic importance for the functioning and the policy implications of the model. In the model version documented here the empirical estimates of this sensitivity may well be biased by the choice of open unemployment as indicator of market conditions and by our assumption of an aggregated labor market with exogenously given supply.

A consequence of assuming immediate adjustment of supply to demand at the set prices in the product markets, i.e., no unplanned stock changes, is that disequilibria will occur also in another factor market in the form of *over- and underutilization of capital.* As already told above, a slow adjustment of capacity will, however, take place since investments are determined by the degree of utilization as well as by past profit performance.

This kind of investment function is obviously in line with the assumption of monopolistic competition in the product markets. The influence of past profits can be said to reflect both profit expectations and the cash flow restrictions on investment finance. A similar mixture of financial conditions and utilization rates will also determine local government investment in the various areas of service production.

There are no explicit *financial markets* in the model. Instead both the rate of exchange and the required rate of return on capital are assumed to be exogenously determined. The rate of exchange is treated as an instrument of government policy. Although some causes of exchange disequilibria may be weakened by self-correcting adjustments elsewhere in the model, there is thus no feedback mechanism leading automatically to exchange adjustment in the absence of government intervention.

A similar – and related – need of government intervention arises in the capital and the money markets. The required rate of return in the small open economy is assumed to be determined abroad by conditions in the international financial markets. This internationally determined rate of return is then transformed to a particular rate for each branch of industry by taking account of differences in depreciation rate, tax treatment and solidity. The required rate of return will influence the firm's pricing and investment and by that also its saving, but will not affect the saving ratio for households.[4] The main burden of adjusting total domestic saving to avoid surpluses or deficits in external payments will thus fall on the public budgets, particularly the state budget.

1.3 The Impact of World Markets

What happens in the world markets is of decisive importance for a small open economy like Sweden. The inter-relations between Swedish markets and foreign markets are in the ISAC-model described by export- and import-functions with the relative price (domestic/world) and market expansion as determinants. The measurement and interpretation of the involved elasticities do, however, involve some rather knotty problems.

That most estimated relative price elasticities both for exports and imports are rather low – in the estimates we use between 1.5–2.0 on the average – is well in line with our basic assumption of monopolistic competition determining foreign markets as well as the domestic ones. Unfortunately, we cannot be sure that the aggregate elasticities we measure reflect market condition for the commodities concerned. To be able to discern the various biases that might be involved in the estimates it might be worthwhile to start by recalling some main possible reasons or characteristics of monopolistic competition, explaining the existence of price differences between similar products.

One such characteristic may be simply *product differentiation.* The products of different firms – and countries – are then perceived as being different, even though the difference may only exist "in the eyes of the perceiver".

A second characteristic can involve differences concerning non price elements of transaction cost, i.e., *differences in market strategies.* The search cost of the buyer will, e.g., vary with advertising efforts.[5] Contractual forms and credit conditions will affect both cost and risk for the buyer, etc.

A third possible reason for the emergence of a monopolistic competition structure is the existence of entry costs and of various forms of *restrictions on free competition.*

[4] Having failed to arrive at any reliable estimate of the aggregate consumption function – partly due to methodological shifts in the official database – we have in this model version kept the aggregate saving ratio for households constant at the 1977 level. The choice of level as such is important only by its implications for the net balance of the state budget.

[5] Limited information and a randomized search may indeed by itself explain why the individual firm faces a sloping demand curve.

It seems intuitively reasonable to assume that all these three characteristics are to a certain extent relevant on most markets for international trade. If true, this means that unbiased estimates of elasticities should require not only that all transaction costs are taken into account but, even more important, that data are disaggregated by product, firm and market.

The realities of official statistics, however, are far removed from these utopian ideals. That our measurement of elasticities involves a *specification bias* by not including other transaction costs than price, is so obvious that we usually do not bother to reflect on it. Of more immediate concern is usually *the aggregation bias,* that comes from measuring price on aggregates of commodities and markets, that are different and moreover shifting in composition. That we may be comparing aggregates of different compositions – e.g. domestic sales versus imports of a certain composite good or exports versus world market trade – means that the measured changes in relative price may in part not be due to different price development for individual goods but simply reflect the different composition of the aggregates. Changes in composition will moreover call forth changes in the aggregate rate of price change even without any changing rate in any individual line of goods. A registered increase of the export price index for a certain aggregate commodity may, e.g., reflect the fact that the exporters – reacting, say, on sinking profits and trade – are falling back on a defensive market strategy, retiring to "safe" markets, where earlier market penetration allows them to raise prices faster while consciously staying away from too competitive markets and goods.

For these reasons aggregation will usually lead to a misrepresentation of behavioral responses. It means that we implicitly assume a two-stage maximization, where the buyer first reacts on an aggregate price, determining an aggregate volume of the composite commodity and then in the next stage "distributes" this composite demand volume optimally between the individual goods with respect to their respective prices. It is well known that such a two-stage maximization is in general only possible when the composite commodity from the buyer's point of view can be expressed as a linear homogeneous function of the individual commodities, and that even then "the correct" aggregate price index will be identical with the usual weighted price index only in the trivial case where individual price developments are identical, i.e., when the individual goods are perfect substitutes.[6]

The aggregation bias obviously makes interpretation of the elasticities involved very uncertain, makes the long-run stability of measured relations questionable and the assumption of a constant elasticity – used in this model also – particularly suspect.

The Swedish official statistics, however, carry the aggregation one step further by aggregating in the usual way the imported and domestically produced quantities

[6] For a thorough treatment of aggregation and decentralization cf. i.a. Blackaby-Primont-Russel (1975) and (1978), particularly Chapters 5–9. A detailed discussion of two-stage maximization as part of the aggregation problem is presented in Bliss (1975), Chapter 7.

of the same industrial category of goods, and using this aggregate in estimating, e.g., input-output relations. Although the inconsistency or bias becomes particularly glaring in this case since different prices are explicitly quoted for the two parts that are treated as perfect substitutes, it really only means a marginal strengthening of the general aggregation bias. Since there is no base in official data for a radically different approach, we have here chosen to follow the official statistics and accept this further approximation in estimating demand elasticities. All we can generally say is that the probability for a serious bias arising out of this particular kind of aggregation should decrease with the disaggregation of industrial sectors.

Most recent estimates of price elasticities in foreign trade in West-European countries are of the same low order as the ones we have used. Whether this should increase our confidence in the estimates is, however, questionable, since they may well contain common biases. If we accept the estimates as true, they imply that demand and/or supply of tradable goods is rather inflexible both for us and our trading partners – that there are rigidities in the economies concerned which may be due to monopolistic competition or to more general causes, e.g. obstacles to mobility in the factor markets.

Low price elasticities do, however, have some obvious and important consequences for economic policy. For one thing they make currency devaluation – even without compensating wage change – a more sluggish instrument for improving the trade balance and for expanding employment. They can also make a selective taxation or indeed tariff on imported goods a more efficient way of improving the external balance than, say, general export subsidies or indeed devaluation of the currency. In a full employment situation, e.g., taxing an imported good with a highly elastic domestic demand involves a limited loss of consumers' surplus to be compared to the resource cost and terms-of-trade loss involved in trying to force a corresponding general expansion of exports. Import taxation might still be an efficient way of solving trade deficit problems if instead of having a very elastic total demand, you have a highly elastic domestic supply – especially in a situation with less than full capacity utilization.[7]

1.4 Government Behavior

In an open economy with a large public sector like Sweden, government behavior can affect the adjustment of the economy in important ways. It may serve as a source of inertia or as an inbuilt stabilizer in the traditional sense. It may on the

[7] One should perhaps at this point add three cautionary notes. The possible efficiency of import taxation – remarked upon by many economists around Europe – is of course conditional on no retaliation measures being taken by the trading partners and no losses of dynamic efficiency resulting from the decrease in specialization. Secondly, the possible relative efficiency of selective import taxation has nothing to do with the optimal tariff argument, since it does not involve any change in terms of trade. Thirdly, the possible advantage tends to be grossly exaggerated by those who are calculating in fixed prices and thus neglecting all losses of consumers' surplus.

other hand give rise to new oscillatory movements and it is, finally, expected to provide the means to facilitate the necessary adjustment through economic policy.

In analyzing government behavior it is, however, necessary to distinguish between local and central government. Local government in Sweden has up till now not been independently involved in income distribution and stabilization policy except to a very limited extent. Economic policy has largely been left as a responsibility for central government, whose budget is dominated by transfer payments. Local government has traditionally been the main provider of public services.

Local government

The restructuring of Swedish economy in postwar years has been rapid. While agricultural employment has been drastically reduced, a matching increase has occurred in the service sector, particularly in the public services, which have doubled their share of GNP and trebled their employment share. The major part of this expansion – in education, medical care and social welfare – took place within the local governments which now employ almost a quarter of the labor force and channel almost a third of the national income through their budgets. Local governments have in this way grown into the same order of size as manufacturing industry, which meanwhile has kept its share both of GNP and of employment relatively unchanged. There is also a substantial degree of decentralization and freedom from central government control within the c. 300 different local government units. There are 277 municipals and 24 counties.

There are several potential ways for central government to control local government. However, when it comes to actual effective control of total expenditures or investments – or the consumption share of local government – postwar history makes it indeed very doubtful if local governments have really been more controlled or more needful of the wishes of central decisionmakers than the big manufacturing corporations. Although about a fourth of local government financing comes from central government grants, until recently there was no attempt to use grant policy for controlling total local government expenditures.

Against this background the assumption – common to almost all national policy models – that local government expenditures can be treated as a central policy parameter, seems particularly unwarranted. The treatment of local government expenditures as an exogenous parameter can obviously lead to misrepresentations of economic policy problems in several important ways. It overestimates both the effectiveness of existing policy means and the degree of inbuilt stability and inertia guaranteed by the existence of the public sector. At the same time it neglects the importance of swings in local government activity, generated independently and/or reinforcing disturbances from elsewhere in the economy, and makes it impossible to study explicitly the problems of central control of local government expenditures.

For these reasons local government actions are treated endogenously in the ISAC-model by way of a special aggregate submodel.

The submodel is represented by a 10-equation system explaining production expenditures (1–5), transfer payments to households (6–7), investments (8), loans (9) and – residually – taxes (10). The production technology used by local governments is throughout assumed to be of the Leontief type with fixed coefficients, so that production volumes can be directly transformed into required inputs from the business sector.

The submodel is derived from an additive quadratic goal function maximized under a budget constraint and can be generally described as a model of budgeting behavior for local government decisionmakers. The goal function includes consideration not only of service and transfer targets but also of private disposable income, of capacity utilization and of the net real wealth of local government.

The explanation of service production derived from this kind of quadratic goal function can be interpreted as reflecting a certain kind of budgetary procedure. To begin with, demographic factors, the need and cost of capacity expansion and private income developments determine the maximum amount of money that the decisionmakers in local government would like to spend on a certain category of service. These maximal claims must then somehow be cut down to fit into the available tax income, defined by a two-year lag in disbursement of the tax yield. The cutting ratio for a certain category – determining how much of the total reduction required that will have to fall on this category – will vary directly with the price of the service and inversely with the rate of decline of the marginal utility for the service. This "two-stage" budgetary procedure can be viewed as corresponding to the two-level hierarchy of budgeting and of political representation used in Swedish local governments.

Local government transfers have been classified into two categories of housing subsidies to households – subsidies to rent and subsidies to other current housing expenditures, i.e., water, electricity, heating, etc. Local government is in both cases modelled as trying to maximize the housing standard that can be bought for a given amount of money. How big the total transfer expenditure will be, given a certain subsidy level, is then determined by housing demand – or more exactly by the local governments' expectations of housing demand.

The corresponding investments will be determined by capacity utilization, liquidity and credit market conditions. The two latter factors will also decide to what extent the investments should be loan financed. Finally, current tax rate will be residually determined by the budget restraint.

Because of the two-year lag in final dispersement of taxes to the local governments and because many of the expenditure decisions must be based on expectations formed on the experience from the preceding year, there may also be unplanned changes of liquidity. These lags together with the interaction between local government expenditures and wage determination in the total economy can give rise to cyclical swings in local government expenditure.

Central government

A relatively disaggregated treatment of the central government budget is a necessary prerequisite for the intended use of the ISAC-model for policy analyses. The central government budget expenditures are slightly bigger in total amount than those of the local government but are in contrast to these dominated by transfer payments.

Central government consumption is treated as exogenous and is divided up into six categories. The corresponding investments are determined by a simple accelerator principle. The production technology, like that of local government, is assumed to be characterized by fixed input-output ratios.

There are altogether ten forms of central government transfer payments distinguished in the model: industrial subsidies, local government grants and eight types of transfers to households, ranging from various pensions to sickness and unemployment benefits. The size of these transfers is exogenously determined by existing laws or by government decisions. The same is true for the four types of transfers *to* central government which apart from the national income tax also include social security contributions and other payroll taxes. The final incidence of payroll taxes is hard to measure and still open to controversy. We have, however, throughout made the simplifying assumption that payroll taxes are entirely rolled back onto the wage earners.

Besides the budget items there are other policy parameters for central government, e.g., the exchange rate, tax rate limits for local government and "wage policy". With "wage policy" we mean the possibility under certain conditions to shift the "Phillips curve". This can be interpreted in terms of a labor market policy – changing mobility or matching on the market – or as an adjustment of the marginal tax structure, making it easier to co-ordinate wage settlements for different wage groups without inflationary effects.

1.5 Income Formation and the Level of Aggregation

It would have been desirable to be able to use the model also for analyzing income distribution problems, particularly the interaction between income distribution on the one hand and allocation and stabilization policies on the other. Unfortunately this has proved possible to do only to a very limited extent.

Many different kinds of income are distinguished in the treatment of income formation in the model but only a few types of income recipients. Transfers that help to make up disposable income for the households mirror the transfers in the government sector which we have already dealt with above. As for factor incomes a first distinction is made between wages and entrepreneurial income.

When it comes to income recipients, however, there are only two categories, pensioners and wage earners, although the latter can be split into entrepreneurs and wage earners. The unemployed are grouped together with the employed. As long as we cannot make a corresponding separation of the expenditure system for private consumption – and the available data do not allow this as yet – the value for distributional analysis of a further disaggregation of income recipients is anyhow limited.

The expenditure system for private consumption – the same for all consumers – distinguishes between 14 different commodities. Production within the business sector is more disaggregated, recognizing altogether 23 different sectors. Within each sector there are five different types of factors used, i.e., besides intermediate inputs, capital, labor, fuel and electricity.

Production capital is, however, further disaggregated through the vintage approach, with each vintage being not only ear-marked for a specific branch but also reflecting the relative price expectations in the investment period and thus being potentially different from the rest of the vintages. The difference between different vintages is further enhanced by the fact that a major part of technical development is treated as embodied, which also means incidentally that a major part of productivity changes in manufacturing is explained endogenously in the model by investments and structural change.

Compared to production capital, aggregation has been carried much further with respect to labor. Although there are different wage levels in different manufacturing branches and a lagged development of wage for public employees, labor supply is treated in the model as one homogeneous aggregate. If capital can be said to be treated as "putty-clay", the treatment of labor could be characterized as "putty-putty". Not only are there no human capital distinctions between different kinds of labor, but there is also no branch or vintage specific labor. In keeping with our ambition to model the various kinds of inertia or adjustment obstacles, it would undoubtedly have been desirable to have a regionally and branchwise segmented labor market with explicit mobility obstacles.

The choice of a suitable level of aggregation is generally difficult and all the more so when, as in our case, you want to be able to do model projections into a distant future. For the use of models over long time periods there are both obvious advantages and disadvantages of aggregation.

The advantages are well-known. Aggregate models are easier to handle and easier to understand. They require less detailed future projections which in turn may make them appear, as more generally valid. From the estimation point of view, there is also the hope that some random or specification errors may get swamped in the aggregation.

The disadvantages are equally self-evident. First of all, there are all the usual statistical problems connected with aggregation. By aggregating relations over different micro units and thereby also hiding their interaction, the derived aggregate relations will be less autonomous, the parameters therefore less stabile and you may be facing increasing problems of simultaneity between your variables. On top of that you will have the kind of aggregation bias in estimating behavioral reactions which we already discussed above. Many of these aggregation problems become, almost definitionally, more serious as you extend the projections in time.

Equally important is the economic argument against aggregation in long-term studies, which says that by carrying aggregation too far you may altogether miss the possibility of analyzing structural changes that should be a primary concern for any long-term economic study. It is after all mostly in the relation between sectors and more specifically in the capital structures that you have enough inertia and long enough lead-times to be able to make meaningful statements about long-term effects.

Our choice of aggregation levels in the ISAC-model has been pragmatically guided both by the availability of data and by our focus of interest. We chose a more disaggregated treatment of manufacturing because of its importance for foreign trade and international specialization which has been one of our primary concerns. The government sector, and particularly the local government sector, was likewise dealt with in rather great detail because of our belief that the rapid growth of the government budgets represents a major structural change of importance also for the functioning of the rest of the economy.

1.6 Short-run and Long-run Dynamics

Already the short introductory comments above may have raised the following question in the reader's mind. Is the model mainly meant to describe the short and medium-term functioning of the economy or should it be primarily regarded as an instrument for the analysis of long-term growth and growth policy?

There are undoubtedly many short-run dynamic aspects of the model, that seem to indicate that short-term policy analyses should be its main application. It is meant to be solved year by year, it contains a great number of lag structures, all the usual types of Keynesian multiplier and accelerator loops, a large number of stabilization policy instruments and also some dynamic mechanisms like the Phillips curve and local government behavior, which are central to any explanation of cyclical movements in prices and production.

On the other hand, the emphasis on structural change, the rather detailed treatment of capital formation and capital structure in manufacturing and of long-term energy substitution possibilities, with concommittant explanations of endogenous productivity developments would rather seem to point to an interpretation in terms of long-term growth.

Since the model is neither a full-fledged business cycle model but still close enough to play havoc with any simple and straightforward growth pattern, the reader might well ask what this hybrid model could really be suited for. As we will be returning to this question of model use later on in the last section, a short comment may be sufficient at this point.

Let us start by granting that the model by no means is competent to handle all short-term policy effects. Already the absence of financial markets makes its use as a full-fledged business cycle model somewhat suspect. It is undoubtedly more convincing as an instrument to study the long-term effects of longt-term policy, a permanent change, say, in exchange regime, in the given rate of return requirement, in average profit levels or in household saving habits.

There are, however, two kinds of studies which particularly require a mixture of short and long-run features in the models used. One is concerned with studying long-term effects of short-term policies, i.e., the way short-run stability characteristics of the economy and the chosen stabilization policy affect long-term growth patterns and possibilities.

The second deals with the contrary problem: the effects on short-run stability and stabilization possibilities of long-term policies. How can, e.g., long-term choices of, say, taxing structure, consumption shares for households, local and central government, respectively, or government price/interest regulations or guarantees, affect the way the economy reacts later to disturbances from the world markets and the measures required and available for stabilization policies? The ISAC-model has, to a large extent, been particularly tailored for this type of studies of the policy interrelations between short and long run.

2 An Outline of the Model

The full version of ISAC contains hundreds of variables and relationships. The large size of the model is an inevitable outcome of our attempts to model explicitly the structure of i.a. the industrial sector. This task requires disaggregation not only between branches but also, we believe, within branches to capture the economy's sluggish accommodation to changing relative prices through dated capital formation.

Since we have also wanted to use the model for policy analysis, the submodels for the household sector as well as for the local government sector have been given a detailed design to allow for a fairly realistic treatment of different possible economic political measures.

Although the disaggregation contributes to the size of the model, what makes it complex is rather the relations between variables specified in the model. The purpose of this chapter is to describe some of these relations in an aggregated model setting. Section 2.1 will give an overview of the model structure while some important interactions, the wage-price formation and the structure of production capacity in manufacturing branches respectively, are presented in more detail in section 2.2. In the last section a somewhat simplified one-sector version of the model is presented.

2.1 A Key Map of the Model

Figure 2.1 is an attempt to illustrate the overall structure of ISAC. The figure shows main variables and submodels and the most important relations between them. The block diagram is built around a sector balance for industry which assures that supply equals demand. The exogenous determinants of the development of the model economy can be divided into two sets. The first set of variables is external to the economy in the sense that neither the development of the economy nor the economic political decision-making is assumed to exert any influence on them. These variables, marked by single-squares in Figure 2.1, are world markets (prices and volumes), disembodied technical change, labor supply and, finally, a rate of interest that is assumed to be imposed on the economy from abroad.

The other set of exogenous variables is the policy instruments, indicated by rhombs. They are the exchange rate, central government consumption and various kinds of fiscal parameters affecting the household sector through taxes and transfers as well as the local government sector through i.a. the grant system.

Finally there is a policy instrument that can influence the wage rate albeit not completely control it.

Given these two sets of exogenous variables every item of the sector balance will

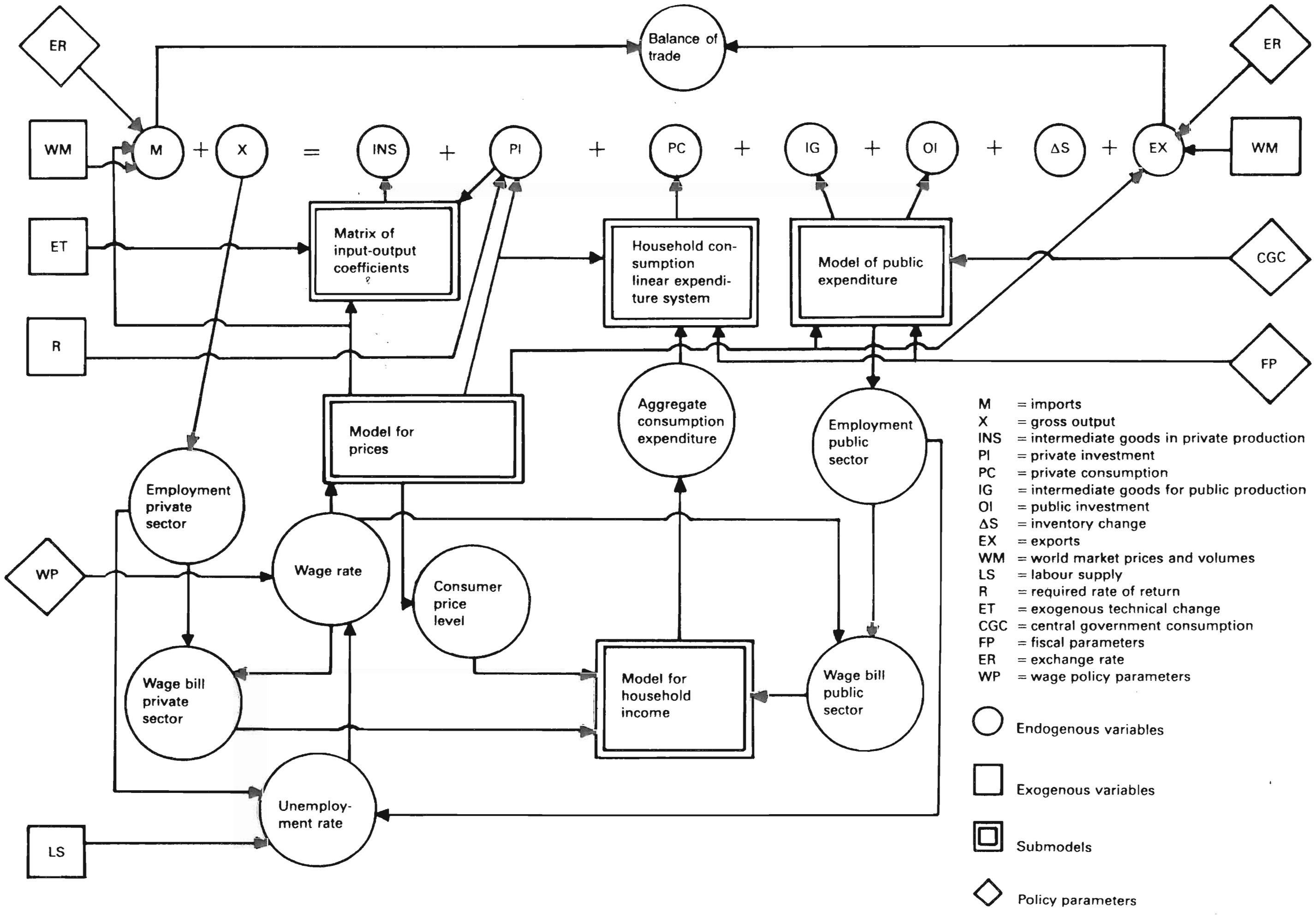


Figure 2.1 *Main Structure of the ISAC-Model*

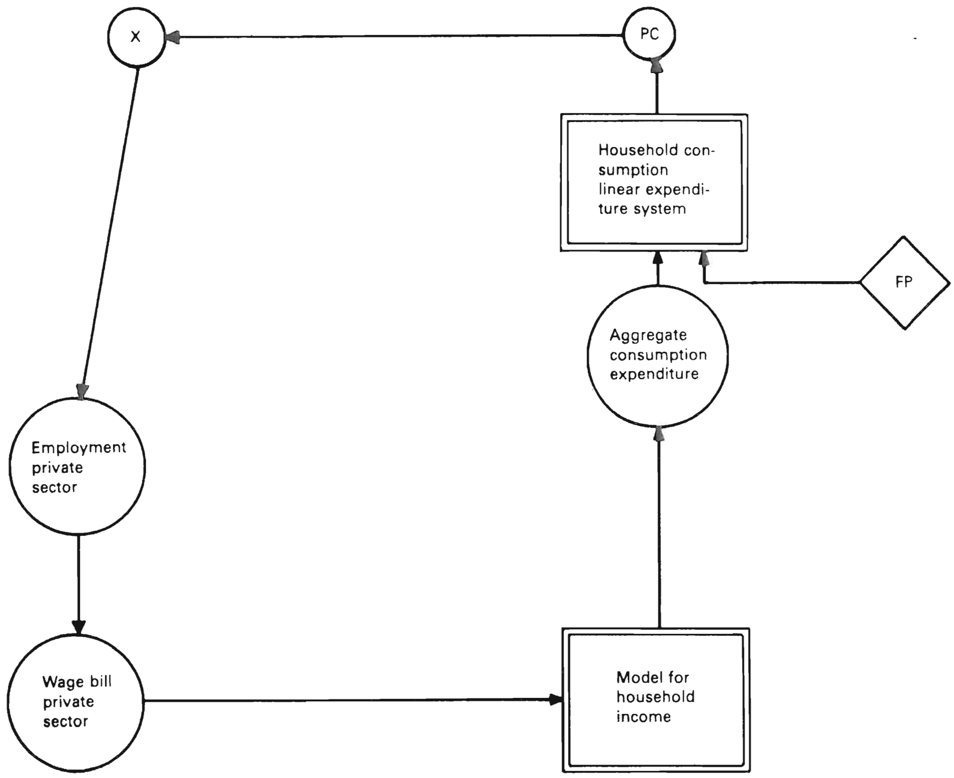
be determined in the way that is outlined below. The demand for intermediary goods in industry (INS) is of course given by current production (X) and the i/o-matrix. Since the present model version only allows for substitution of energy, capital and labor in production processes and no exogenous trends are attached to commodity input coefficients the i/o-matrix is almost constant over time.

Investments in industry (PI) are determined by investment functions taking account of influences from profit expectations as well as current capacity utilization. Through the vintage mechanism the volume of investment will affect average capital- and labor-productivity in industry. Productivity growth also depends on the rate of scrapping of old vintages which is assumed proportional to quasi-rents earned in each vintage. So far the vintage approach is only implemented for branches in the manufacturing sector while capital in other branches is treated as homogeneous in each branch. Also investment functions are specified and estimated only for manufacturing branches and otherwise set either as an exogenous trend or related to production in some simple way.

Private consumption is determined by a rather detailed specification of income and expenditure in the household sector. The main source of gross income is wages and salaries from industry, thus providing the model with a multiplier link between the activity level in the economy and private consumption expenditures as shown in Figure 2.2. Starting, as in the figure, with some exogenous change in fiscal parameters (FP), e.g. a reduced income tax rate, household income will increase. Since the savings ratio is a constant fraction of disposable income most of the tax reduction will result in consumption expenditure thus increasing demand for commodities. Some of these will be imported but domestic production and employment will increase creating more wage income and so on. The size of multiplier is also affected by wage inflation as a consequence of the higher activity level in the economy. This mechanism, which is not shown i Figure 2.2, will on the one hand positively affect total wage income through higher wage rates. On the other hand it will also limit the real multiplier effect through higher inflation, and increased imports and reduced exports following the rise in the domestic price level. The net real effect of these secondary mechanisms after 2–3 years may be positive or negative depending i.a. on price elasticities in foreign trade and the sensitivity of wages to increased pressure on the labor market. Other important sources of household income are wages, salaries and transfers from the public sector. After deduction of various taxes households are left with disposable income. The savings ratio is assumed fixed and real consumption expenditure is distributed between fourteen consumption categories by a linear expenditure system. The feed-back mechanism between the production system and consumption demand through the household sector is important to explain the model's short-term response to an exogenous disturbance, either in the form of e.g. a change in world market growth or in the form of some fiscal measure towards the household sector.

Public sector demand for intermediary and investment goods (IG, OI) is partly a policy variable (central government), partly endogenous (local government). The local government model, as it is presently implemented, tends to produce fairly

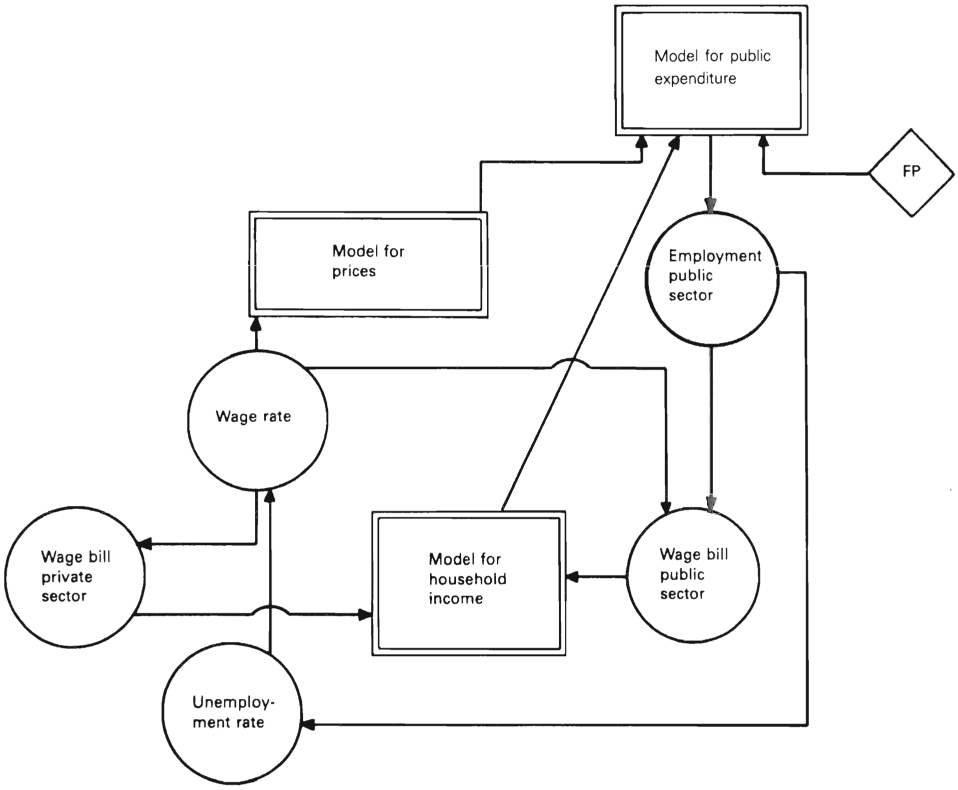
Figure 2.2 *The Production – Consumption Loop*



strong oscillations in the economy through its interactions with the labor market on the one hand and the household sector via the endogenous local tax rate on the other. These links are shown in Figure 2.3.

Suppose central governments increase their categorical grants to local governments, indicated by a change in fiscal parameters (FP) in the figure. The immediate impact of this measure will be to decrease local governments' net costs of production and hence induce them to step up real expenditures. The grants will also improve their financial situation making tax increases unnecessary. Since no other fiscal measures are assumed, i.e. the original grants' increase is not financed by central government taxes, the activity level in the economy will increase and so will employment. The resulting wage inflation will partly "finance" the local government expenditure increase by eroding households' income and by worsening the external balance. But the wage increase will also lead to increased net costs of production for local governments and slow down their expansion. However, because of the two year lag in the disbursement of centrally collected local tax-income, local governments' financial situation will again improve at a time when wages are moderate as a consequence of weak demand. Thus with a two year lag the inflated household income will feed back to local governments' acting as a

Figure 2.3 *The Local Government Consumption – Labor Market Loop*



stimulus to increased expenditures.[1]

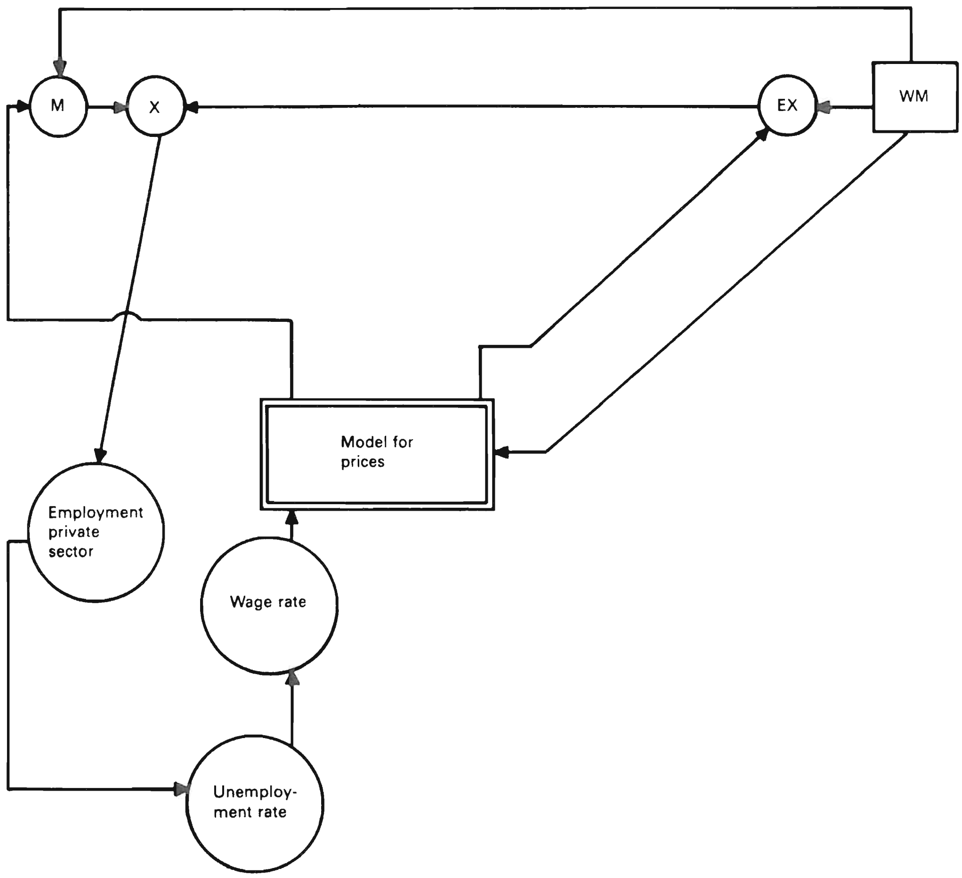
Changes in stocks (ΔS) are modelled in a very simple fashion with total stocks in the economy proportional to production in "stockholding" branches. Mostly, however, the model is run with exogenous stock investments.

Finally, the sector balance for industry includes *imports and exports* (M, EX). These are, of course, of great importance considering the large export- and import-shares, especially for the manufacturing branches, in the Swedish economy. As discussed in Chapter 1 we assume that price differentials between imported and domestically produced goods can persist over long periods, and also that Swedish exporters are not necessarily price-takers on the world market. The implication of these assumptions is that imports and exports will depend on relative prices and that domestic producers can price themselves out of domestic as well as foreign markets as was the case in the middle of the seventies.

Used together with the wage equation, the foreign trade functions introduce a

[1] Simulation experiments carried out so far with the local government submodel, reported i.a. in Ysander-Nordström (1985) are still on an explorative level and the stability tests discussed in Chapter 5 are accomplished with exogenous local governments.

Figure 2.4 *The Foreign Trade – Labor Market Loop*



mechanism that tends to dampen the effects of world market disturbances. The immediate impact of a general world market price increase, for example, will be higher domestic inflation through imports and a pressure on the labor market due to improved relative prices and hence increased net demand from abroad. This will however create both inflation expectations and wage drift on the labor market pushing up wages. The higher wages will not only remove pressures on the labor market but also reduce the initial surplus on foreign account. These mechanisms, indicated in Figure 2.4, are further discussed in Chapter 5.

2.2 Price and Wage Formation

The previous section gave a summary description of the determination of industry's sector balance in the model. Obviously prices and the equations determining price formation play an important role for the solution of the model.

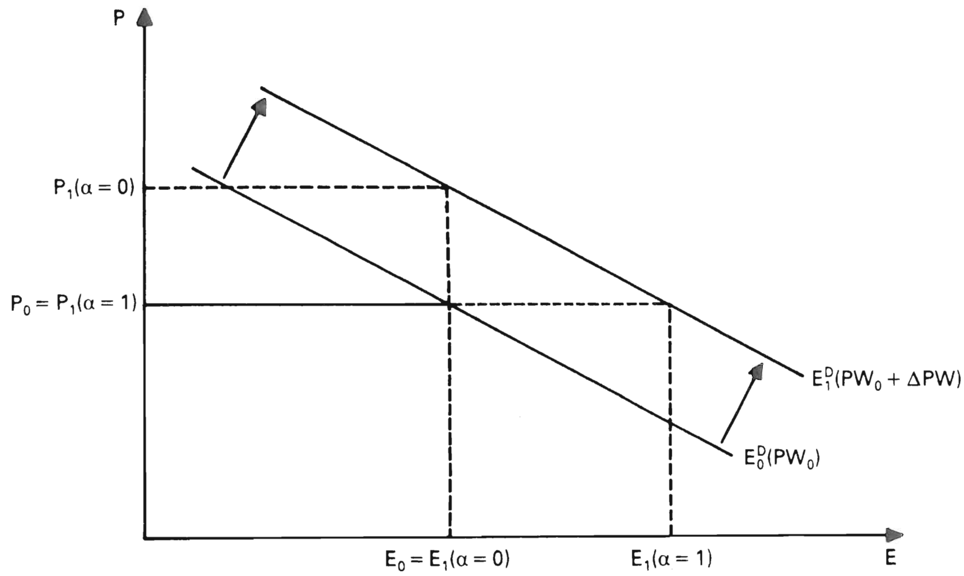
As mentioned in Chapter 1 prices are based on average rather than marginal costs. Unit cost is taken to include "normal" profits, i.e. the mark-up over average operating costs is equal to the average capital cost share. Theoretically this kind of pricing can be underpinned by assuming market imperfections and adjustment costs. It also seems to be well in accordance with observed behavior.

For a small country, however, with large export shares it seems natural that producers not just pass on their costs to the world market without regard to competitors' prices. The price equations in ISAC also allow for this and prices (P) are accordingly specified as a geometric average of unit cost (C) and world market price (PW) in Swedish currency:

$$\dot{P} = \alpha \cdot \dot{C} + (1 - \alpha) \cdot \dot{PW};$$

where dotted variables are growth rates. The size of the parameter α is of crucial importance. Obviously α equal to unity implies pure mark-up pricing. On the other extreme, with α equal to zero, producers are assumed always to follow world market prices. The implications of the two extreme points of the interval for α are shown in Figure 2.5. Initially domestic prices, costs and world market prices are assumed to coincide ($P_0 = PW_0 = C$). Facing the demand curve E_0^D for Swedish products, exports will amount to E_0. Now we introduce a shift in world market prices, ΔPW.

Figure 2.5 *Export Effects of Different Pricing Parameters*



Equations:
$P \sim C^{\alpha} PW^{1-\alpha}$
$E^D \sim (P/PW)^{-\beta}$

Suppose that $\alpha = 1$, i.e. that pure mark-up pricing is used, and that the cost curve does not shift. Then the Swedish export price will still be equal to P_0 making it possible to gain market shares in face of the new export demand curve E_1^D as a result of competitors' price increase, ΔPW. Exports will rise to $E_1(\alpha = 1)$ in the figure. On the other hand, if α is zero, exports will not change at all despite the world price increase, since domestic producers will simply follow the world market pricing. With constant costs this will of course increase profits.

This does not correspond to the common "small country assumption" with purely price-taking domestic producers facing a totally elastic demand. Since we here assume that foreign demand for Swedish products has a limited price elasticity, producers setting their price equal to the average world market price will only retain their market share and may not be able to sell all they wish at that price.

In the discussion above costs were for simplicity assumed to be unaffected by the world price increase. Of course price increases abroad will influence domestic inflation. One way in which world inflation will be felt is through imported goods. Another is through price increases by domestic producers if they take the opportunity to raise their profit margins.

However, the model economy also includes an internal source of inflation. Wages are determined by an expectations augmented Phillips curve where the overall wage rate growth is explained by last year's consumer price growth, the inclusion of which can be thought of as either an expectational or a compensatory element, the current rate of unemployment, profit levels and finally productivity growth.[2]

Theoretically Phillips curves can be specified in a great many ways. Two features of this particular specification should be noted since they are important to the model behavior. The first one is the rather unsophisticated formation of expectations that is assumed. It may in fact seem more natural to view the one-year lagged consumer price variable as a compensatory mechanism for past inflation, especially since the parameter was estimated to be close to unity. The second noteworthy characteristic of the chosen wage equation is the absence of lags in the unemployment variable.[3] These two properties make wages react very quick and strong to inflationary pressures making e.g. the domestic consequences of external inflationary shocks fade away in a rather short period.

2.3 A One-sector Version of the Model

As a further help in getting acquainted with the main features of the model we will here present a condensed one-sector version of the model, i.e. a model economy with only one type of private commodity and one kind of public service.

The equations of the simplified model will be presented in a slightly different order from that later used for the presentation of the full model in Chapter 3. The

[2] Estimated parameter values are given in Chapter 4.
[3] For a discussion of possible lag structure cf. Jansson (1982).

name of the variables are however the same with minor exceptions. Large roman letters are variables in the model with a bar indicating exogenous variables. Small greek and roman letters stand for parameters, the former singling out economic-political parameters. Throughout the presentation current time index is suppressed unless necessary for interpreting the relations. Lagged variables are denoted by subscript "−1" etc.

2.3.1 *The Product Market*

The model is technically built around a commodity balance in fixed prices:

$$M + X = H + E; \tag{1}$$

where

M = import
X = domestic production
H = domestic demand
E = export.

The implication of relation (1) is that any quantity of commodities that is demanded will also be supplied. With exports and imports being separately determined as functions of domestic and foreign prices (cf. 2.3.4 below) the model thus allows for over- and under-utilization of existing production capacity.

Relation (1) of course also holds in current prices thus defining the price level on the domestic market in terms of import price and domestic producer price. The latter is for simplicity assumed equal to the export price in the one-sector model:

$$P \cdot H = PE \cdot (X - E) + \theta \cdot \overline{PW} \cdot M; \tag{2}$$

where

P = domestic market price
PE = export price (domestic producer price)
θ = exchange rate
$\overline{PW}$ = world market price in foreign currency

Final domestic demand can be decomposed:

$$H = \overline{Q^p_{IM}} \cdot X + \overline{Q^o_{IM}} \cdot OC + PC + INV + \overline{DS};$$

where

$\overline{Q^p_{IM}} \cdot X$ = private sector demand for intermediate goods
$\overline{Q^o_{IM}} \cdot OC$ = public sector demand for intermediate goods
PC = private consumption demand
INV = gross-investment demand
$\overline{DS}$ = demand for increase in stocks

Q_j^s = input-output coefficients; s = p, o (private, public sector), j = C, L, IM (capital, labor, intermediate goods).

Although public consumption is given as an endogenous variable in the condensed model, only part of it – local government consumption – is endogenous in the full model. The same holds for investments where investment functions exist only for the manufacturing branches. Investments in other branches in the private sector as well as in the public sector are either exogenous or determined by a simple investment quota.

2.3.2 *Production Technology*

The production technology in the private sector is described by a vintage approach to capital formation. Ex ante input coefficients for labor and capital are given as functions of input prices:

$$Q_j^{t^*} = F_j^{t^*}(W, P_C); \qquad j = C, L \tag{3, 4}$$

where

Q_j^t = input coefficient for capital (j = C) and labor (j = L) respectively in vintage t* at the date of installment
W = wage rate
P_C = cost of capital calculated as $P(\bar{R} + d)$ where $\bar{R}$ is the exogenously required rate of return and d the depreciation rate.

Once installed the capital-output ratio of the vintage is fixed, while labor productivity is subject to an exogenous increase, *l*, every year throughout all vintages:

$$Q_L^{t^*,t} = Q_L^{t^*}/(1 + l)^{(t - t^*)}; \tag{5}$$

while

$$Q_C^{t^*,t} = Q_C^{t^*}; \tag{6}$$

Since labor productivity will differ between vintages average productivity depends on both the pace with which new technology is implemented, i.e., gross investments, and, in the full model, the rate of scrapping of old plants in response to either rising unit labor costs or unfavorable world market prices depressing quasi-rents. Provided that the utilization ratio is equal for all vintages the aggregate input coefficients can be calculated by weighting with capacity:

$$Q_L^p = \sum_{t^* = t_0}^{t} XCAP^{t^*,t} \cdot Q_j^{t^*,t}/XCAP; \qquad j = C, L \tag{7, 8}$$

where t_0 is the date of installment of the oldest vintage. $XCAP^{t^*,t}$ and XCAP are given below by (11) and (12) respectively.

2.3.3 *Production Capacity Growth and Depreciation*

New capacity with new technology is added through gross investments. Capacity utilization and expected profits determine the amount of investments in the private sector, with the capacity variable dominating the short-run behavior:

$$INV = F_{inv}(EP_{-1}, UR_{-1}); \tag{9}$$

where EP is an "excess profit" variable with a five year lag-structure relating the rate of profit to the exogenous required rate of return $\bar{R}$. UR is a capacity utilization index. The excess profit will be a function of labor cost, capital cost and product price:

$$EP = F_{EP}(Q_L^p, W, Q_C^p, P, \bar{R}, d);$$

Assuming the best technology in new plants given by (3, 4) the addition to capacity will be:

$$XCAP^{t^*} = INV/Q_C^{t^*}; \tag{10}$$

To calculate the net increase in production capacity depreciation must also be considered. In the full model the rate of scrapping of old plants (vintages) is assumed to depend on profitability. In the condensed model we assume the rate of depreciation to be constant over time as well as over vintages:

$$XCAP^{t^*, t} = (1-d)^{(t-t^*)} \cdot XCAP^{t^*}; \qquad t^* = t_0, \ldots, t \tag{11}$$

This gives total production capacity:

$$XCAP = \sum_{t^*=t_0}^{t} XCAP^{t^*, t}; \tag{12}$$

With actual production, X, determined by demand the utilization ratio, UR, must be allowed to fluctuate:

$$UR = X/XCAP; \tag{13}$$

2.3.4 *Foreign Trade*

The foreign trade sector plays an important part for the growth patterns generated by the model. Since exports as well as imports depend on domestic producers' prices relative to the world market prices, a domestic inflationary pressure will lead to losses of market shares and to a deteriorating balance on foreign account although with some delay:

$$E = c \cdot (PE/\theta \cdot \overline{PW})^{-e_1} \cdot (PE/\theta \cdot \overline{PW})_{-1}^{-e_2} \cdot \overline{WM}; \tag{14}$$

$$M = f \cdot (PE/\theta \cdot \overline{PW})^{g_1} \cdot (PE/\theta \cdot \overline{PW})_{-1}^{g_2} \cdot H^h \tag{15}$$

where

$\overline{WM}$ = volume of world trade

The balance on current account, CA, is given by the sum of the trade balance and net transfer payments (including interest payments), $\overline{TP}$, which for simplicity are assumed exogenous in the condensed model:

$$CA = PE \cdot E - \theta \cdot \overline{PW} \cdot M + \overline{TP} \tag{16}$$

2.3.5 *Pricing*

In setting their prices domestic producers are assumed to take into account the following three factors – their unit cost of production, the world market price (in domestic currency) and finally the utilization of production capacity:

$$PE = a\,(\overline{Q^{p}_{IM}} \cdot P + Q^{p}_{C} \cdot P \cdot (\bar{R} + d) + Q^{p}_{L} \cdot W)^{r} \cdot (\theta \cdot \overline{PW})^{1-r} \cdot UR^{u}_{-1} \tag{17}$$

(17) shows that producers will only partially pass on cost inflation to consumers at home and abroad. On the other hand if market demand is soaring, part of the high prices will be cashed into higher profits while the possible market gains will only be realized to some extent. The price equation also assumes that producers actively will try to use spare capacity by lowering their prices and that rising demand with capacity at normal levels will push up prices.

2.3.6 *Consumption Demand*

The volume of private consumption depends on the households' factor income from the production system and their transfer income from the public sector.

$Y = F_Y(W, L, P, \overline{TR})$;

where

Y = nominal gross income
L = total employment
$\overline{TR}$ = real transfer income.

The wage rate as well as the employment level, both of which are endogenous variables, will thus affect household disposable income and consumption:

$$PC = F_{pc}(Y, P, \tau, \bar{S}); \tag{18}$$

where

PC = private consumption in fixed prices
τ = tax parameters
$\bar{S}$ = savings ratio.

The tax parameters are important instruments in the policy simulations with the model.

Central government consumption (CGC) is exogenous while local consumption is determined by household income, production costs and demographic factors (DEM):

$$OC = F_{oc}(Y, P, W, \overline{DEM}, \overline{CGC}); \tag{19}$$

2.3.7 *Labor Market and Wage Determination*

Productivity is assumed to be independent of the utilization ratio. Thus, total employment is calculated as:

$$L = Q_L^p \cdot X + \overline{Q_L^o} \cdot OC; \tag{20}$$

Supply of labor, L_S, is exogenous, which gives the unemploment rate, as:

$$U = (\bar{L}_S - L)/\bar{L}_S; \tag{21}$$

The model incorporates a Phillips-like wage equation, where wage increases are determined by expected change in consumer prices, current unemployment, excess profits and productivity growth:

$$\Delta W = F_w(\Delta P_{-1}, U, EP_{-1}, \Delta Q_{L-1}^p) + \eta; \tag{22}$$

where Q_L^p is the inverse of labor productivity.

The autonomous wage inflation η can be used as a control variable in the simulations.

2.3.8 *The One-sector Model*

Summing up the condensed model gives us:

$$M + X = H + E \tag{1}$$

where $H = \overline{Q_{IM}^p} \cdot X + \overline{Q_{IM}^o} \cdot OC + PC + INV + \overline{DS}$

$$P \cdot H = PE \cdot (X - E) + \theta \cdot \overline{PW} \cdot M \tag{2}$$

$$Q_j^{t^*} = F_j^{t^*}(W, P(\bar{R} + d)); \qquad j = C, L \tag{3, 4}$$

$$Q_L^{t^*, t} = Q_L^{t^*}/(1 + l)^{(t - t^*)}; \qquad t^* = t_0, \ldots, t \tag{5}$$

$$Q_C^{t^*, t} = Q_C^{t^*}; \tag{6}$$

$$Q_j^p = \sum_{t^* = t_0}^{t} XCAP^{t^*, t} \cdot Q_j^{t^*, t}/XCAP; \qquad j = C, L \tag{7, 8}$$

$$INV = F_{inv}(Q_L^p, W, Q_C^p, P, \bar{R}, d, UR_{-1}); \tag{9}$$

$$XCAP^{t^*} = INV/Q_C^{t^*}; \tag{10}$$

$$XCAP^{t^*, t} = (1 - d)^{(t - t^*)} \cdot XCAP^{t^*}; \qquad t^* = t_0, \ldots, t \tag{11}$$

$$XCAP = \sum_{t^* = t_0}^{t} XCAP^{t^*, t}; \tag{12}$$

$$UR = X/XCAP; \tag{13}$$

$$E = c \cdot (PE/\theta \cdot \overline{PW})^{-e_1} \cdot (PE/\theta \cdot \overline{PW})_{-1}^{-e_2} \cdot \overline{WM}; \tag{14}$$

$$M = f \cdot (PE/\theta \cdot \overline{PW})^{g_1} \cdot (PE/\theta \cdot \overline{PW})_{-1}^{g_2} \cdot H^h; \tag{15}$$

$$CA = PE \cdot E - \theta \cdot \overline{PW} \cdot M + \overline{TP}; \tag{16}$$

$$PE = a(\overline{Q_{IM}^p} \cdot P + Q_C^p \cdot P \cdot (\bar{R} + d) + Q_L^p \cdot W)^r \cdot (\theta \cdot \overline{PW})^{1-r} \cdot UR_{-1}^u \tag{17}$$

$$PC = F_{pc}(W, L, P, \overline{TR}, \tau, \bar{S}) \tag{18}$$

$$OC = F_{oc}(Y, P, W, \overline{DEM}, \overline{CGC}) \tag{19}$$

$$L = Q_L^p \cdot X + \overline{Q_L^o} \cdot OC \tag{20}$$

$$U = (\bar{L}_S - L)/\bar{L}_S \tag{21}$$

$$\Delta W = F_w(\Delta P_{-1}, U, EP_{-1}, \Delta Q_{L-1}^p) + \eta \tag{22}$$

where $EP = F_{EP}(Q_L^p, W, Q_C^p, P, \bar{R}, d)$.

With these twenty-two relations the following endogenous variables are determined:

X = production
E = exports
M = imports
PC = private consumption
OC = public consumption
INV = investment
P = domestic market price
PE = export price (domestic producer price)

$Q_{C,L}^{t^*}$ = use of capital/labor per output in new vintages
$Q_{C,L}^{t^*,t}$ = use of capital/labor per output in vintage t* at time t
$Q_{C,L}^{t}$ = average use of capital labor per output at time t
$XCAP^{t^*}$ = production capacity in vintage t* at the date of installment
$XCAP^{t^*,t}$ = production capacity in vintage t* at time t
XCAP = production capacity at time t
UR = capacity utilization ratio
W = wage rate
CA = current account
L = total employment
U = unemployment ratio

3 The Equations of the Model

3.1 Definitions and Conventions

The following definitions and conventions have been used in this chapter:

Large romans = matrices
Small romans without subscripts = vectors
Small letters with subscripts = scalars
Superscripts = indices of categories, etc.
Subscripts = indices of branches, goods, etc.
"Roofed" letters = vectors turned into diagonal matrices
The transpose of a vector or matrix is denoted by ' , e.g. x'
(t − n) = n-year lag. When no time indication is given, time t is assumed
Dotted variables indicate growth rates, e.g. $\dot{x} = \frac{dx}{dt} \cdot \frac{1}{x}$

Parameters are mostly named by small greek letters or in the case of simple constants by roman a or b with appropriate subscripts and occasionally a superscript indicating explained variable or the like.

3.2 Commodity Balances in Fixed and Current Prices

The following two accounting identities state that total domestic demand (right-hand side) is equal to total domestic supply (left-hand side) for every commodity:[1]

$$m + x = A \cdot x + inv + pc + pu + ds + e;$$

$$\hat{p}^m \cdot m + \hat{p}^x \cdot x = \hat{p}^h \cdot (A \cdot x + inv + pc + pu + ds) + \hat{p}^e \cdot e;$$

3.3 The I/O-Matrix

The i/o matrix for the manufacturing industry is calculated from a vintage model of production technique for each of the 14 branches in the sector.[2]

Aggregated input coefficients in a new vintage at time t are given by a constant elastic function in last year's input prices:

$$Q^t_{m,i} = a_{m,i} \cdot b^t_{m,i} \cdot \prod_n p_{n,i}(t-1)^{\alpha_{m,n,i}}; \qquad i = 1, \ldots, 14$$

[1] A list of symbols is found in Appendix B.
[2] A detailed account of the vintage approach as applied to the iron and steel industry is given in Jansson (1983).

where

m, n = 1 for intermediate goods
2 for electricity
3 for fuels
4 for labor
5 for capital

The sum of the elasticities $\alpha_{m,n,i}$ over n is zero which makes the input share function homogeneous of degree zero, i.e. a proportional increase in all prices will not affect the choice of technique.

The input share equations can not be derived from cost minimization of a common production structure, so the constraints derived from the classical demand theory are not satisfied. The reason to leave the firm on theoretical grounds is purely pragmatic. The constant elastic form used makes it practically possible to foresee and track the impact of price changes on the technical development.

The $\alpha_{m,n,i}$ used are calculated from estimated translog functions (see Dargay 1983), at the observed sample average. Since the variations in the elasticities calculated over the observed period are moderate in spite of considerable changes in relative prices the constant elastic functions can be regarded as approximations to proper share functions.

Once installed the i/o-coefficients in a vintage are fixed except for the labor coefficient which is subject to an exogenous decrease reflecting improvements in organization of production, education of the labor force etc., i.e. $b_{m,i} \neq 1$ only for $m = 4$.

Assuming that i/o-coefficients in a vintage are independent of utilization ratios and that all vintages are used at the same intensity level the capacity distribution over vintages can be used to calculate overall i/o-coefficients in a branch. The capacity distribution in a branch is calculated from gross investments and scrapping of old vintages.

New capacity in branch i is given by gross investments and the chosen capital output coefficient

$$\bar{x}_i^t = g_i / Q_{5,i}^t;$$

Remaining capacity in old vintages depends on the rate of scrapping which is assumed inversely proportional to the quasi-rent earned by each vintage

$$d_i^v = \sum_{n \neq 5} Q_{n,i}^v p_{n,i}(t-1) / p_i^x(t-1)\, d_i; \qquad v = t_0, \ldots, t-1$$

Although this criterion will speed up the scrapping of old capacity in times of changing relative prices it may still allow vintages earning negative quasi-rents to be run for several years.

Given the rate of scrapping, current production capacity in old vintages in manufacturing branch i is following directly:

$$\bar{x}_i^v = (1 - d_i^v) \cdot x_i^v(t-1); \qquad v = t_0, \ldots, t-1$$

and the distribution of capacity over vintages is

$$\bar{\bar{x}}_i^v = \bar{x}_i^v / \bar{x}_i;$$

where $\bar{x}$ is total capacity:

$$\bar{x}_i = \sum_v \bar{\bar{x}}_i^v;$$

This will finally give us overall aggregated input coefficients for manufacturing branches as

$$Q_{n,i} = \sum_v Q_{n,i}^v \cdot \bar{\bar{x}}_i^v; \qquad \begin{array}{l} n = 1, \ldots, 5 \\ i = 1, \ldots, 14 \end{array}$$

The matrix of aggregate input coefficients for the manufacturing sector thus depends on the pace with which new capacity (and technology) is installed and old capacity phased out and thus becomes a slowly moving function of relative prices in the economy.

The aggregated i/o-coefficients, $Q_{n,i}$, with $n = 1, 2, 3$ are transformed to the standard 23 input goods format by matrix H giving the i/o-matrix A for the manufacturing sector:

$$A_{j,i}^m = \sum_n Q_{n,i} \cdot H_{j,n,i}; \qquad \begin{array}{l} n = 1, \ldots, 3 \\ i = 1, \ldots, 14 \\ j = 1, \ldots, 23 \end{array}$$

$Q_{4,i}$ and $Q_{5,i}$ are the inverse of labor and capital productivity respectively in each branch.

For the 9 branches outside the manufacturing sector there is no vintage approach adopted as yet in the model. For these branches capital stocks are assumed to be homogeneous and aggregate input shares are given by a constant elastic function in input prices. The disaggregated i/o-matrix for these 9 branches, A^o, is then compiled in the same way as A^m.

Finally A^m and A^o are combined to give the total 23×23 i/o-matrix $A = (A^m, A^o)$.

Investments, input shares and depreciations are all functions of predetermined variables only which means that they are not dependent on the model solution for year t, and therefore neither is A.

The assumptions that all vintages are used at the same intensity level and that i/o-ratios are independent of the utilization rate implies that the A matrix describes the input-output relations not only at full capacity production but also at any other actual production level. It also means that the utilization rate can be calculated at "branch level" for the whole business sector, disregarding the vintage structure in manufacturing branches:

$$ur_i = x_i / \bar{x}_i; \qquad i = 1, \ldots, 23$$

3.4 Energy Substitution[3]

Five energy categories are distinguished in the model: electricity, distant heating, oil, coal and domestic fuels. Energy consumption is accounted for in these five categories for each of the 23 business sectors, for the public sector and for the household sector.

The substitution between different energy forms in the business sector is assumed to take place in two stages. In the first stage a new production technique with five inputs is selected: electricity, fuel (incl. heating where appropriate), other intermediate goods, labor and capital as described in section 3.3.

In the second stage the choice between different kinds of fuels is made. It is assumed that ex post substitution is possible in all branches in this stage. The shares q^f for coal and domestic fuels of total fuel consumption depend on price according to the following relation:

$$q^f_{j,i} = a_{j,i} \cdot (b_{j,i} - p_{j,i}^{\mu_{j,i}}); \qquad i = 1, \ldots, 23 \quad j = 1, 2$$

where j = 1 for coal
2 for domestic fuels.

The p:s are up to five years lagged energy prices, including capital cost, relative to the price of oil-produced energy. Since the aggregate use of fuels is the sum of oil, coal and domestic fuels (all measured in Twh), the share of oil is determined as a residual. The above relation thus does not describe the choice of optimal fuel-shares at given prices, but rather the sluggish accommodation of energy demand to changing price relations.

The rationale behind the two-stage procedure for determination of energy demand in the business sector is the assumption that the choice of fuel has negligible effects on the rest of the installed production technique. This seems to be a good approximation in many cases since fuel is mostly used to produce heat and the heat production unit may be fairly separate from the rest of the production process.

3.5 Investments in the Business Sector

Investments in each branch of the manufacturing sector are determined by past profits and capacity utilization:

$$g_i = k_i(t-1) \cdot \left(\sum_{j=1}^{4} \gamma_{j,i} \cdot ep(t-j)_i + d_i \right) \cdot ur_i(t-1)^{a_i};$$

where the parameters $\gamma_{j,i}$ are all > 0, ep_i is an "excess profit" index relating realized gross operating surplus to the user cost of capital:

[3] A more thorough account of energy substitution in the model is given in Ysander (1983a, b).

$ep_i = (va_i - w_i l_i)/p_i^k \cdot k_i;$

where va_i is value-added and p_i^k is the cost of capital in branch i defined as:

$p_i^k = p_i^g(r_i(r_w));$

where r_i is the required rate of return in branch i, calculated as a function of the exogenous rate of interest r_w, and p^g the price of investment goods for branch i.

In branches outside the manufacturing sector investments are either exogenous or related to production in the branch.

To account for demand for different goods generated by investment activities the composition of gross investments is assumed to differ between branches but to be constant over time:

$inv = G \cdot g;$

where G is a matrix converting investments in investing branches into commodity demand directed towards producing branches.

3.6 Foreign Trade

The volume of export, e, and import, m, depends on market size and relative price for each commodity.

The general form of the export functions are:

$$e_i = a_i^e \cdot \prod_{j=0}^{1} [p_i^e(t-j)/\theta\, p_i^w(t-j)]^{\beta_{i,1+j}} \cdot wm_i;$$

where p^e and p^w are Swedish export price and world market price, respectively. The export price is expressed in Swedish currency while the world market price is expressed in foreign currency. θ is the exchange rate. The size of the world market is given by index wm for each good.

The import functions have a similar structure:

$$m_i = a_i^m \cdot \prod_{j=0}^{1} [\theta \cdot p_i^w(t-j)/p_i^{xh}(t-j)]^{\gamma_{i,1+j}} \cdot h_i^{\gamma_{i,3}}$$

Swedish producers' price of commodities sold on the domestic market, competing with imports, is p^{xh}. The variable h is total domestic demand.

As shown by the foreign trade equations, exports and imports depend on prices relative to the world market with a one-year lag. For many commodities lagged relative prices affect current export/import volume as much as current price relations.

3.7 Disposable Income in the Household Sector

The submodel for household income distinguishes between two kinds of individuals. "Pensioners" are people with most of their income from the social security system. The remainder is simply called "wage-earners" although it includes the total labor force, i.e. entrepreneurs as well as unemployed persons.

Individuals receive factor-income, capital-income and transfers from other sectors. After deduction of income- and payroll-taxes, and transfers to other sectors they are left with disposable income.

Factor income

The main part of factor income is gross wages and salaries, including payroll taxes and other collective fees. Factor income also includes part of the net surplus in producing sectors.

Gross wages and salaries are the product of wage/hour and the number of hours worked in different sectors including the public sector:

$$y_{11} = w' \cdot \ell^{w} + w^{s'} \cdot \ell^{s} + w^{\ell'} \cdot \ell^{\ell};$$

$$\ell = \ell^{w} + \ell^{e}$$

$$y_{12} = va' \cdot \ell^{e};$$

where ℓ^{w} and ℓ^{e} are number of hours worked in each branch by wage-earners and entrepreneurs respectively, and va is value added. According to national accounting conventions y_{12} also includes imputed income from owner-occupied houses.

Capital income

Capital income includes interest-payments calculated as a constant fraction of entrepreneurs' income:

$$y_{21} = a_{21} \cdot y_{12};$$

Other net capital income is calculated from financial assets, fa, which in turn are accumulated from total financial surpluses and deficits of the household sector:

$$y_{22} = a_{22} \cdot fa$$

Transfer income

This part of the submodel is fairly disaggregated and to a large extent exogenous except for inflation. There are six different types of transfer incomes:

1: National general pension (old age plus others)
2: Ditto local
3: National supplementary pensions (old age plus others)
4: Private (collective) pensions (old age)

5: Other transfer income (non-taxable)
6: Other transfer income (taxable)

The first four items are calculated as number of persons, np, times real income per capita, rp, and inflation (consumer price index):

$$y_{3i} = np_i \cdot rp_i \cdot cp; \qquad i = 1, \ldots, 4$$

Other transfer income, which mainly goes to wage earners, is divided according to whether it is taxable or not.

Non-taxable transfer income is set by an exogenous trend plus inflation:

$$y_{35} = a_{35} \cdot \exp(b_{35} \cdot t) \cdot cpi;$$

Taxable transfers are divided into sickness benefits, unemployment benefits and others. Sickness benefits are assumed to be proportional to wage sum while unemployed persons receive a constant fraction of average wage income per employee.

$$y_{36} = a_{36} \cdot y_{11} + a_{37} \cdot y_{11} \cdot u + a_{38} \cdot \exp(b_{36} \cdot t) \cdot cpi;$$

Transfer payments

Five types of transfer payments are distinguished:

1: National income tax
2: Local income tax
3: Social security contributions
4: Other payroll fees
5: Other transfer payments

National income tax is calculated from an aggregate progressive tax-function:

$$y_{41} = a_{41} \cdot skind \cdot n_b (besk/skind/n_b)^{b_{41}};$$

where the inflation compensation index "skind" is a one-year lagged consumer price index and n_b is number of assessed persons.

Taxable income, besk, is a fairly complicated function taking account of different kinds of deductions from gross income:

$$besk = F_b(y_{11}, \ldots, y_{36});$$

Local tax is calculated from taxable income using the local tax rate utd:[4]

$$y_{42} = utd \cdot besk;$$

Payroll taxes and fees are share of total gross wage:

[4] y_{42} is the local tax paid in year t. However, as spelled out in the local government model below it only reaches local authorities in year $(t-2)$.

$y_{43} = a_{43} \cdot y_{11};$
$y_{44} = a_{44} \cdot y_{11};$

Other transfer payments are calculated as:

$y_{45} = a_{45} \cdot \exp(b_{45} \cdot t) \cdot cpi;$

Disposable income

Summing up incomes and payments we get disposable income for the household sector as:

$$y_d = \sum_{j=1}^{6} \left[\sum_{i=1}^{3} y_{ij} - y_{4j} \right]$$

3.8 Private Consumption

Consumption expenditures per capita equal disposable income less savings. Savings are set as a constant fraction, s_r, of disposable income, and total population is n_s:

$cp = (1 - s_r) \cdot y_d/n_s;$

Total consumption expenditure per capita is distributed on 14 consumer goods, c_i, by a linear expenditure system:

$$p_i^c \cdot c_i = \gamma_i \cdot p_i^c \cdot c_i(t-1) + \beta_i \cdot \left(c_p - \sum_{k=1}^{14} \gamma_k p_k^c \cdot c_k(t-1) \right);$$

where $\sum_{j=1}^{14} \beta = 1$ and $i = 1, \ldots, 14$

The price vector p^c represents the prices of domestic absorption, p^h, converted to the consumer goods level by a constant matrix PK:

$p^c = PK \cdot p^h;$

Private consumption per capita is then transformed to total private consumption at the 23 commodity level:

$pc = PK \cdot c \cdot n_s;$

This gives the consumer price index as:

$$cpi = p^{h'} \cdot pc / \sum_{i=1}^{n} pc_i$$

3.9 Central Government

The development of central government consumption is exogenously determined. Seven different consumption purposes are distinguished.

1: National defense
2: Public order and safety
3: Education
4: Health
5: Social security and welfare services
6: Roads
7: Other services

The rate of growth of production, xs, and consumption, cs, for the various purposes is a constant proportion, gr, of an exogenously given common growth factor, g_0.

$$xs = g_o \cdot gr \cdot xs(t-1);$$

$$cs = (I - \hat{sa}) \cdot xs;$$

where the sa:s are shares of sales from production.

The need for intermediate goods is related to production in each central government sector. Together with exogenous investments this gives the central government demand from the 23 business branches:

$$fs = SG \cdot xs + sg' \cdot is;$$

where aggregate central government investment is called is. SG is a constant conversion matrix and sg a ditto vector.

3.10 Local Government

Production within local governments are split into five categories.

1. Education
2. Health
3. Social welfare
4. Roads (total expenditures)
5. Central administration, fire service, etc.

These expenditures are explained by linear expressions of the following form:

$$x\ell_i = a_{1i} \cdot z_{1i} + a_{2i} \cdot z_{2i} + a_{3i} \cdot z_{3i} + a_{4i} \cdot z_{4i} + a_{5i} \cdot z_{5i};$$

where z_{1i} are shift variables, z_{2i} and z_{3i} stand for investment consequences and capacity restrictions, while z_{4i} and z_{5i} reflect the impact of changes in real income, local tax rates and relative prices.[5]

As with central government, sales made by local authorities of goods and services at market prices are assumed to be a constant fraction of local production giving local consumption as:

$$c\ell = (I - \hat{\ell a}) \cdot x\ell;$$

Aggregate investments by local authorities are explained by a gradual adjustment to desired capital stock levels, with the rate of adjustment depending on capital good prices, interest rates, liquidity situation and real income development:

$$\Delta kp = a_{16} \cdot z_{16} + a_{26} \cdot z_{26} + a_{36} \cdot z_{36} + a_{46} \cdot z_{46} + a_{56} \cdot z_{56};$$

$$z_{16} = kp; \quad z_{26} = \Delta kp^*; \quad z_{36} = \Delta liq(t-1) \cdot z_{16};$$

where kp is actual and kp* desired real capital stock and Δliq is the change in local authorities' liquidity:

$$z_{46} = \varphi_6 \cdot kp^2 \cdot r \cdot \frac{besk}{besk(t-2)};$$

where φ_6 is the real relative price of capital goods (net of grants and user charges[6]), and r is the interest rate.

$$z_{56} = (1 - utd - av_0) \cdot besk \cdot z_{46} \cdot z_{16};$$

where av_0 is the share of nominal fees.

The depreciation is assumed to be a constant fraction of existing capital stock. Gross investments then become:

$$i\ell = \Delta kp + a_{66} \cdot z_{66};$$

where a_{66} is the depreciation rate and $z_{66} = kp(t-1)$.

Investments may alternatively be computed in a simplified manner as equal to desired capital stock changes plus reinvestments.

The local governments' expenditures on production and investments are converted into final demand of commodities from the business sector by LG and ℓg:

$$f\ell = LG \cdot x\ell + \ell g' \cdot i\ell;$$

Together with central government's demand this gives total public sector demand for commodities from the business sector:

$$pu = fs + f\ell;$$

Transfer payments, t, are split into two categories – subsidies to public utilities and direct transfers to the household sector. In the model the explanation of these payments is derived from the idea that the provision of housing space and of public

[5] All expressions explaining local authority behavior are derived from maximizing a quadratic goal-function under a budget restriction. An account of the model is given in Ysander-Nordström (1985).

[6] All prices in the local government submodel – LOGOS – are defined net in this sense.

utilities are arguments in the local governments' goal function, pursued indirectly by way of "price subsidies".

$t_i = a_{2i} \cdot \zeta_i + a_{3i} \cdot \mu_i + a_{4i} \cdot \gamma_i;$

$i = 1, 2$ 1 = public utilities' subsidies
2 = direct household (housing) subsidies,

where ζ_i represents the cost for the households relative to disposable income and μ_i and γ_i express the impact of developments in real income, local tax rate and relative prices.

The transfer payments can alternatively be treated in a simplified manner. The net amount of subsidies to public utilities are then approximated as a constant fraction of production in sector 18 (electricity, heat, etc.). Direct household transfers are set as a linear function of households' housing expenditures (z_{13}).

Local government expenditures ($e\ell$) are financed by taxes and state grants with liquidity changes acting as a buffer against planning failures. Given state grants (sb), net borrowing (Δdt), other net income (c), and interest payments (rdt) the local tax rate is determined residually by way of the budget restriction:

$$utd = \frac{1}{besk(t-2)} \cdot [-utd(t-2) \cdot (besk(t-2) - besk(t-4)) + e\ell - sb - \Delta dt - c + rdt];$$[7]

To simulate possible restrictions or inertia in local government political behavior, the model can alternatively be supplemented with a floor restriction on the tax rate complemented with a rule, prescribing that surpluses above a certain relative level are used to scale up current expenditure.

3.11 Stock Building

Total change of stocks, ds_a, in the economy is exogenous in the model. The commodity composition of stock investment goods, sc, is assumed constant over time. Inventory demand for commodities thus becomes

[7] When local governments are treated as exogenous and included in the policy arsenal the following specification of the total tax function is used instead.

$$y_{41} = t \cdot \left[a_{41} \cdot skind \cdot n_b \cdot (besk/skind/n_b)^{b_{41}} + utd \cdot besk\right];$$

where

y_{41} = total income tax
a_{41} = constant
skind = inflation compensation index
n_b = number of taxpayers
besk = taxable income
b_{41} = central government income tax elasticity
utd = local government tax rate
t = policy parameter.

$ds = sc \cdot ds_a;$

A simple stock-building model can be included when necessary. Still, only one aggregate inventory good is distinguished but demand for inventories is generated by four aggregate stockholding branches. The four branches are foresting, agriculture and fishing, producers of intermediate goods, producers of finished goods, and the wholesale and retail trade. Stocks are assumed proportional to production, y_i, in these aggregate branches which gives total stockholding as:

$$s_a = \sum_j s_j = \sum_j a_j \cdot y_j; \qquad j = 1, \ldots, 4$$

and

$$ds = sc \cdot ds_a = sc \cdot [s_a(t) - s_a(t-1)];$$

3.12 Prices

As is evident from the foreign trade equations, Swedish prices on foreign and domestic markets are allowed to differ from world market prices. These differences make Swedish producers lose or gain market shares. It is furthermore assumed that the export price, p^e, may differ from the price, p^{xh}, set for the domestic market.

$$p_i^e = a_{il} \cdot uc_i^{a_{i2}} \cdot (p_i^w \theta)^{a_{i3}} \cdot ur_i(t-1)^{a_{i4}}; \qquad i = 1, \ldots, 23$$

$$p_i^{xh} = b_{il} \cdot uc_i^{b_{i2}} \cdot (p_i^w \theta)^{b_{i3}} \cdot ur_i(t-1)^{b_{i4}}; \qquad i = 1, \ldots, 23$$

The unit cost of production, uc, is calculated as:

$$uc = A'p^h + vac;$$

with

$$vac_i = [w_i \cdot \ell_i + p_i^k \cdot k_i]/x_i;$$

that is, value added at normal profits.

The price equations above can be seen as a compromise between the common extreme assumptions of either a pure "cost plus" behavior or a pure price-taking in the foreign markets.

Given the Swedish producers' prices on the domestic market and import prices the implicit price index for commodities on the domestic market, p^h, is calculated as:

$$p_i^h = [(x_i - e_i) \cdot p_i^{xh} + m_i \cdot \theta \cdot p_i^w]/h_i;$$

where

$$h = x - e + m;$$

Adding export value and value of home market sales gives the unit price of domestic production:

$$p_i^x = \left[e_i \cdot p_i^e + (x_i - e_i) \cdot p_i^{xh}\right] / x_i;$$

For all business sectors gross profit, Π_i, will be determined residually:

$$\Pi_i = (p_i^x - A_i^{'} p^h) x_i - w_i \cdot \ell_i - i_i;$$

where A_i denotes column i of the matrix A and i_i denotes indirect taxes.

3.13 Employment

Total employment in the economy is derived from production and productivity in each sector.

For the business sector employment in branch i is given by:

$$\ell_i = x_i \cdot Q_{4,\, i}; \qquad i = 1, \ldots, 23$$

For the public sector labor input coefficients, q^s and q^ℓ, are exogenous although occasionally subject to a trend development:

$$\ell_i^s = xs_i \cdot q_i^s; \qquad i = 1, \ldots, 7$$

$$\ell_i^\ell = x\ell_i \cdot q_i^\ell; \qquad i = 1, \ldots, 7$$

Together with an exogenous labor supply, ℓ_s, these equations give the unemployment rate:

$$u = (\ell_s - x' \cdot Q_4 - xs' \cdot q^s - x\ell' \cdot q^\ell)/\ell_s;$$

where Q_4 is the vector of labor input coefficients in the business sector.

3.14 Wages

The changes in the wage rate is the same for all branches in the business sector and is determined by a kind of Phillips curve:

$$\dot{w}_0 = a_0 + a_1 \cdot cpi(t-1) + a_2 \cdot (u - u_0) + a_3 \cdot \left[\Pi^m(t-1) - \Pi_0\right] + a_4 \cdot \dot{Q}_4^m(t-1);$$

Thus the change in the wage rate is a function of current deviation of the unemployment rate from a "normal" or "frictional" unemployment rate (u_0), past inflation, cpi(t − 1), past deviation from a "normal" level of aggregate gross profit margin, Π_0, in the manufacturing sector and finally past change of aggregate labor productivity in the manufacturing sector (Q_4^m is aggregate labor input coefficient, i.e. the inverse of labor productivity). It is assumed that wage changes in the public sector is lagging one year behind the business sector.

Wage rate growth is thus given by:

$\dot{w}_i = \dot{w}_o$; $\quad$ $i = 1, \ldots, 23$

$\dot{w}_i^s = \dot{w}_o(t-1)$; $\quad$ $i = 1, \ldots, 7$

$\dot{w}_i^\ell = \dot{w}_o(t-1)$; $\quad$ $i = 1, \ldots, 7$

4 Implementation

This chapter deals with some aspects of the implementation of the model. The data base is fairly large and will not be given in full here. We will only report parameter values for some important equations. This is done in section 4.2. The first section, however, will describe the level of disaggregation of some main variables.

Since the model is large and complex the method used to solve it is of paramount importance. The solution algorithm is recounted in section 4.3.

4.1 Level of Aggregation

The fundamental level of aggregation in the model is that given by the distinction between 23 producing branches in *the business sector*. These are listed in Table 4.1 along with their SNI-classification numbers. Branches 4–17 (sometimes including branch 3) are referred to as the manufacturing sector. To give a hint of the relative size of the branches the last column also shows fixed price value added in percent of sector total in 1980.

The basic classification shown in Table 4.1 is also used for commodities and it is assumed that output of every branch is homogeneous at this level of aggregation.

Public consumption is disaggregated into thirteen real expenditure purposes – seven in the central government sector and six in the local government sector.[1] These thirteen categories, listed in Table 4.2, are also used to describe production within the public sector. It should be noted, however, that public enterprises are classified among business branches according to Swedish national accounting conventions.

Finally, *private consumption* is divided into fourteen categories, shown in Table 4.3, which are more suitable for analysis of demand than the basic 23 commodities division. The distribution of domestic private consumption expenditures between categories are determined by a linear expenditure system (cf. sections 3.8 and 4.2) and converted to demand for commodities.

[1] This is true only when local government expenditures are treated as exogenous. In the present version of the local government model two of the six categories are aggregated. For technical reasons the model code is specified with seven expenditure purposes in the local sector with one or several treated as "dummies".

Table 4.1 *Classification of Branches in the Business Sector*

Branch	SNI	Value added (percent)[a]
1. Agriculture, fishing	11, 13	3.0
2. Forestry	12	2.4
3. Mining and quarrying	2	0.8
4. Manufacture of food (sheltered)	3111–1, 3116–8	2.1
5. Ditto (exposed)	3113–15, 3119, 3121–1	0.9
6. Manufacture of beverages and tobacco	313–4	0.4
7. Textile, wearing apparel	32	1.2
8. Manufacture of wood, pulp and paper	33, 341	6.1
9. Printing and publishing industries	342	2.0
10. Manufacture of rubber products	355	0.3
11. Manufacture of industrial and other chemicals, and plastic products	351–2	2.3
12. Petroleum and coal refineries	353–4	0.2
13. Manufacture of non-metallic products (except products of petroleum and coal)	36	1.1
14. Basic metal industries	37	1.9
15. Manufacture of fabricated metal products, machinery and equipment, excl. shipbuilding	38 excl 3841	13.8
16. Shipbuilding	3841	0.6
17. Other manufacturing industries	39	0.2
18. Electricity, gas and water	4	3.3
19. Construction	5	9.7
20. Wholesale and retail trade	61–2	13.8
21. Transport, storage and communication	7	8.7
22. Letting of dwellings and use of owner-occupied dwellings	83101	10.4
23. Other private services	Rest of 6, 7, 8 and 9[b]	14.9
Total business sector	1–9	100.0

[a] Of total value added in the business sector at 1975 year's producer prices.
[b] Private part. Public authorities are active also in some business branches. These activities are, however, disregarded.

4.2 Values of Essential Parameters

This section presents parameter values of the main behavioral equations. Most of them, but not all, are estimated from historical data. Technical matters about estimation procedures will be omitted but references are given to more complete presentations.

The main set of parameters that are not properly estimated relates to the *price formation* equations, which give domestic producers' prices as functions of costs and world market prices. There is a distinction in the model between domestic producers' export prices (p^e) and import competing prices (p^{xh}):

$$p_i^e = a_{i1} uc_i^{a_{i2}} (\theta p_i^w)^{a_{i3}}; \qquad a_{i2} + a_{i3} = 1$$

$$p^{xh} = b_{i1} uc_i^{b_{i2}} (p_i^m)^{b_{i3}}; \qquad b_{i2} + b_{i3} = 1$$

Table 4.2 *Classification of Public Consumption*

	Classification according to Swedish national accounts	Central government consumption in 1979 (percent)[a]	Local government consumption in 1979 (percent)[a]
1. National defence	20	10.5	–
2. Public order and fire protection	13	4.0	0.9
3. Education	30	2.9	17.9
4. Health	40	1.2	23.2
5. Social security and welfare services	50	4.2	13.6
6. Roads	85	0.9	1.5
7. Other public services	6, 7, rest of 8, 9	8.2	11.0
Total public consumption	1–9	31.9	68.1

[a] Of total public consumption at 1975 year's prices.

Table 4.3 *Classification of Private Consumption*

	SNA	Percent[b]
1. Food	11, 7132, 7136	19.9
2. Beverages, tobacco	12–14	6.9
3. Clothing and footwear	20, 822(p)[a]	9.4
4. Cultural services	723–5, 73, 862	4.9
5. Personal care and effects	452, 512, 81	1.8
6. Gross rents and water charges	31	19.4
7. Private transport	611, 621(p), 623	5.5
8. Recreation	4114, 52, 612, 621(p), 71 excl. 71320 and 71360, 721–2, 821, 822(p)	7.4
9. Furniture	41 excl. 4114, 42–4, 451	6.9
10. Other consumption	Rest of 1–8	6.7
11. Electricity	321	2.0
12. Gas, fuels, steam	322–5	2.1
13. Gasoline	622	2.6
14. Purchased transports	63	2.6
Total private domestic consumption expenditure	1–8	98.1
Foreign travel, net	9	1.9
Total private consumption	1–9	100.0

[a] (p) denotes part of SNA-number.
[b] Of total private consumption expenditure 1980 in 1975 year's prices.

The sum of exponents for each branch (commodity) is in both cases restricted to unity giving domestic producers' prices as geometric average of production costs and world market prices.[2] With a_{i2} and b_{i2} equal to unity companies are assumed to compensate all domestic cost increases on the world market, thereby protecting their profit ratios even in the face of shrinking market shares – at home as well as abroad. On the other hand such price behavior also makes companies use reduced costs or increased world market prices (e.g. through currency devaluation) to improve their competitiveness with increased demand on domestic and foreign markets as a consequence. (Cf. section 2.2.) The reverse will of course follow if the a_{i2}:s and b_{i2}:s are close to zero.

As stated above we have not carried out any independent econometric analysis of price parameters. Intuitively it seems reasonable that the sensitivity to world market prices will vary considerable between branches, depending on type of commodity and on the degree of exposure to foreign competition. This conclusion is supported by Calmfors and Herin (1979) which so far is the only attempt to estimate price equations on Swedish data at a disaggregated level. Contrary to conventional wisdom, Calmfors and Herin estimated a rather low influence from world market prices and a correspondingly high sensitivity to domestic cost factors.

This was the case even for highly exposed branches with fairly homogeneous outputs such as paper and pulp, and iron and steel industries.

We have varied the price parameters in different applications of the model and have also sometimes let the price behavior differ between foreign and domestic markets (i.e. $a_{i2} \neq b_{i2}$ and $a_{i3} \neq b_{i3}$). The standard set of parameters is shown in Table 4.4, which gives the industrial branches in order of assumed increasing influence from world market prices on their own price setting behavior. As can be seen from the table, manufacturing branches (excl. petroleum refineries) are assumed to be equally sensitive to cost and world market price. It is, however, important to realize that the direct influence from foreign prices in the equations is not the only one. Domestic producers' prices will also to a varying degree react to world market prices through imported input goods which will affect production costs. The total foreign price influence will therefore always be greater than what is shown in Table 4.4.

For most branches Calmfors and Herin (1979) found no significant influence on price setting from capacity utilization. In the model simulations we have, nevertheless, frequently exploited a capacity variable in the price equations to speed up the adjustment to normal capacity utilization levels.

The *wage formation* is given by a kind of Phillips curve as decribed in section 3.14:

[2] The exchange rate (θ) times world market prices in foreign currency (p^w) may differ from import prices because of tariffs as well as for aggregational reasons. For the forecasting period, however, the rate of change of p_i^m is assumed to be equal to the rate of change of $\theta \cdot p_i^w$. The capacity utilization variable is omitted; cf. section 3.12.

$$\dot{w}_0 = a_0 + a_1 \cdot \dot{cpi}(t-1) + a_2 \cdot (u - u_0) + a_3 \cdot (\Pi^m(t-1) - \Pi_0) + a_4 \cdot \dot{Q}_4^m(t-1)$$

where dotted variables are growth rates.

The consumer price influence, $\dot{cpi}$, was first estimated not to differ significantly from unity and was therefore restricted in the final estimation, whose outcome is shown in Table 4.5. Estimation procedure and results are reported in detail in Jansson (1982). It should however be emphasized that the full compensation for past consumer price increases, as well as the rather high absolute value of the labor market coefficient, looks rather exceptional and that parameters in these kinds of wage equations are notoriously unstable.

Table 4.4 *Values of Parameters in Price Equations (standard set-up)*

Branch[a]	$a_{i2} = b_{i2}$	$a_{i3} = b_{i3}$
1, 18, 19, 22	1.0	0.0
2, 20, 21, 23	0.75	0.25
3–11, 13–17	0.5	0.5
12	0.0	1.0

[a] Branches are listed in Table 4.1.

Table 4.5 *Values of Parameters in Wage Equation*

Variable	Parameter
Constant	2.63
$\dot{cpi}$	1.0[a]
$(u-u_0)$	−4.14
$\Pi^m-\Pi_0$	0.56
$\dot{Q}_4^m$	−0.47

[a] Restricted.

As the wage inflation neutral "normal" unemployment (u_0) and manufacturing gross profit share (Π_0) in the wage equation, we have in most simulations used 2 percent and 27 percent respectively.

The *ex ante choice of production technique* in manufacturing branches is based upon a constant elastic function in relative input prices (cf. section 3.3):

$$Q^t_{m,i} = a_{m,n} b^t_{m,i} \cdot \prod_n p_{n,i}(t-1)^{\alpha_{m,n,i}}; \qquad i = 4, \ldots, 17 \quad m, n = 1, \ldots, 5$$

The $\alpha_{m,n,i}$:s are based on Dargay (1983) and are shown in Table 4.6. They are, however, modified in two ways. The first modification relates to the number of energy inputs, where the estimates only covered energy as a whole at the time when this part of the model was implemented. In order to, at least to some extent, account for possible substitution effects of changing relative prices for electricity and fuels, energy elasticities were split into parts. On the disaggregated branch level elasticities of substitution between electricity and fuels on the one hand and other inputs on the other hand were simply assumed to be equal in size. Moreover, in order to change as few estimated parameters as possible, cross price elasticities between electricity and fuels were set equal to zero. As estimates for different energy forms now are available on a disaggregated branch level these will be implemented in the next version of the model.

The second way in which we have modified the estimates arises from the ex ante – ex post distinction. The estimates are based on observed input coefficients for the whole production capacity in a branch. The response of these average coefficients to changing input prices is much weaker than the response in marginal additions to capacity. We assume that the influence from prices is increased by a factor three. All parameters in Table 4.6 are thus three times the estimated values. Finally, an exogenous trend is, in each branch, added to the labor coefficient with $b_{4,i}$ ranging from 0.98 to 0.99.

None of these modifications are of course satisfactory. However, given the very scant empirical basis for the disaggregated ex ante functions some kind of ad hoc modifications along these lines are unavoidable.

Lack of data at the time the present version of the model was made operational also seriously limited the implementation of the vintage structure of production capacity. In fact, the initial "distributions" in the different branches are implemented as an aggregate capital stock. This implies i.a. that the history of the system as given by the shape of the vintage-distribution is not taken account of.

Investments in manufacturing branches are explained by expected (past) profits and capacity utilization (cf. section 3.5):

$$g_i = k_i(t-1)\left[\sum_{j=1}^{4} \gamma_{j,i} \cdot ep(t-j)_i + d_i\right] ur_i(t-1)^{a_i};$$

The basis for the parameter values in Table 4.7 is found in Jansson (1983), who also recounts the assumptions behind this specification. The capacity utilization variable is, however, added afterwards with the a_i:s assumed equal to 3.0 for all branches in the standard version of the model. Compared to an outright accelerator this is a fairly modest influence. Together with the price equations this will provide the model with stabilizing responses to disturbances that affect capacity utilization. A higher rate of utilization will on the one hand lead to higher prices, thereby reducing demand, and to increased investments on the other, thus expanding capacity.

The distribution of *private real consumption* expenditures between consumer goods is explained by a linear expenditure system (cf section 3.8):

$$c_i = \left[\gamma_i p_i^c c_i(t-1) + \beta_i(c_p - \sum_{k=1}^{14} \gamma_k p_k^c c_k(t-1)\right]/p_i^c;$$

Table 4.6 *Values of Parameters in the Functions for ex ante Choice of Technique in Manufacturing Branches*

i	m/n	$\alpha_{m,n,i}$ 1	2	3	4	5
4	2	−0.090	−0.390	0.0	0.120	0.360
	3	−0.090	0.0	−0.390	0.120	0.360
	4	1.410	0.0	0.0	−1.620	0.210
	5	−0.240	0.060	0.060	0.540	−0.420
5	2	1.440	−1.410	0.0	0.450	−0.480
	3	1.440	0.0	−1.410	0.450	−0.480
	4	1.860	0.015	0.015	−1.950	0.060
	5	0.510	−0.045	−0.045	0.120	−0.540
6	2	1.500	−0.450	0.0	−0.960	−0.090
	3	1.500	0.0	−0.450	−0.960	−0.090
	4	1.770	−0.060	−0.060	−2.190	0.540
	5	−0.630	−0.015	−0.015	1.140	−0.480
7	2	3.570	−2.940	0.0	0.300	−0.930
	3	3.570	0.0	−2.940	0.300	−0.930
	4	1.290	0.015	0.015	−1.5690	0.270
	5	−0.090	−0.105	−0.105	1.080	−0.780
8	2	1.650	−1.230	0.0	−0.240	−0.180
	3	1.650	0.0	−1.230	−0.240	−0.180
	4	1.740	−0.030	−0.030	−1.830	0.150
	5	0.630	−0.030	−0.030	0.300	−0.870
9	2	0.900	−1.590	0.0	1.440	−0.750
	3	0.900	0.0	−1.590	1.440	−0.750
	4	1.050	0.015	0.015	−1.260	0.180
	5	0.660	−0.015	−0.015	0.570	−1.200
10	2	1.110	−1.530	0.0	0.450	−0.030
	3	1.110	0.0	−1.530	0.450	−0.030
	4	0.930	0.015	0.015	−1.290	0.330
	5	−0.270	0.0	0.0	0.810	−0.540
11	2	0.960	−0.780	0.0	−0.150	−0.030
	3	0.960	0.0	−0.780	−0.150	−0.030
	4	1.680	−0.015	−0.015	−1.620	−0.030
	5	0.810	−0.015	−0.015	−0.060	−1.720
13	2	1.650	−1.230	0.0	−0.240	−0.180
	3	1.650	0.0	−1.230	−0.240	−0.180
	4	1.740	−0.030	−0.030	1.830	0.150
	5	0.630	−0.030	−0.030	0.300	−0.870
15	2	2.130	−1.920	0.0	0.030	−0.240
	3	2.130	0.0	−1.920	0.030	−0.240
	4	1.650	0.0	0.0	−1.710	0.060
	5	0.510	−0.015	−0.015	0.240	−0.720
16	2	1.500	−1.680	0.0	0.330	−1.500
	3	1.500	0.0	−1.680	0.330	−0.150
	4	1.800	0.0	0.0	−1.950	0.150
	5	−0.030	−0.015	−0.015	0.510	−0.450

Note: m, n = 1 for intermediate goods; = 2 for electricity; = 3 for fuels; = 4 for labor; = 5 for capital. Branches are listed in Table 4.1.

Table 4.7 *Values of Parameters in Investment Functions*

Branch	$\gamma_{1,i}$	$\gamma_{2,i}$	$\gamma_{3,i}$	$\gamma_{4,i}$	$\sum_j \gamma_{j,i}$
4	0.0316	0.0000	0.0197	0.0000	0.0513
5	0.0000	0.0000	0.0322	0.0000	0.0322
6	0.0326	0.0739	0.0497	0.0285	0.1847
7	0.0470	0.0349	0.0104	0.0144	0.1067
8	0.0000	0.0101	0.0000	0.0175	0.0276
9	0.0838	0.0000	0.0163	0.0068	0.1069
10	0.0324	0.0124	0.0000	0.0000	0.0448
11	0.0119	0.0113	0.0000	0.0214	0.0446
13	0.0050	0.0291	0.0124	0.0000	0.0465
14	0.0132	0.0125	0.0433	0.0220	0.0910
15	0.0000	0.0201	0.0000	0.0235	0.0436
16	0.0753	0.0516	0.0000	0.0000	0.1269
17	0.0917	0.0000	0.0000	0.0000	0.0917

Note: Branches are listed in Table 4.1.

where $\sum_{j=1}^{14} \beta_j = 1$ and $i = 1, \ldots, 14$

The consumption goods are listed in Table 4.3. Parameter values, which are given in Table 4.8, are estimated in Dargay – Lundin (1981).

Foreign trade is explained by market size and relative prices (cf. section 3.6):

$$e_i = a_i^e \cdot \prod_{j=0}^{1} [p_i^e(t-j)/\theta \cdot p_i^w(t-j)]^{\beta_{i,1+j}} \cdot wm_i$$

and

$$m_i = a_i^m \cdot \prod_{j=0}^{1} [\theta \cdot p_i^w(t-j)/p_i^{xh}(t_i-j)]^{\gamma_{i,1+j}} \cdot h_i^{\gamma_{i,3}}$$

The export price elasticities are shown in Table 4.9. Only four export equations are based on estimated parameters (indicated by * in the table) but those branches nevertheless account for about two thirds of total export volume. Total elasticities range from −1.5 to −2.0 (except for export of chemicals) with lower absolute values assumed for export of services.

As shown in Table 4.10, two thirds of total import volume are determined by the use of estimated price- and market-elasticities. Generally, import is less sensitive to relative prices than exports with textile and machinery as notable exceptions. Since there is no indigenous production of coal and oil, imports are purely complementary but are affected by substitution with domestic energy.[3]

[3] Imports of branch 3 commodities is a mix of coal and other mineral raw material (excl. oil). For simplicity, however, all imports are treated as complementary.

Table 4.8 *Values of Parameters in Private Consumption Expenditure System*

Consumption	β	γ
1	0.1081	0.9405
2	0.1135	0.7998
3	0.0717	0.9000
4	0.0110	0.9797
5	0.0041	0.9498
6	0.0000	1.0100
7	0.3025	0.2502
8	0.1116	0.7600
9	0.0716	0.8301
10	0.0920	0.8898
11	0.0110	0.9306
12	0.0362	0.8700
13	0.0602	0.6697
14	0.0065	0.9500

Note: Branches are listed in Table 4.3.

Table 4.9 *Values of Parameters in Export Functions*

Branch	$\beta_{i,1}$	$\beta_{i,2}$	$\sum_j \beta_{i,j}$	Share of total export volume[b] in 1980 (percent)
1–6, 9, 10, 13, 14, 16, 17	−1.0	−1.0	−2.0	17.9
7*	−0.56	−0.92	−1.48	2.7
8*	−0.82	−1.34	−2.16	19.8
11*	−0.71	−0.39	−1.10	6.2
15*	−1.55	−0.51	−2.06	38.4
20, 21, 23	−1.3	0.0	−1.3	12.6
12, 18, 19, 22[a]	–	–	–	2.4

* Estimated parameters.
[a] Zero or exogenous exports.
[b] At 1975 year's prices.
Note: Branches are listed in Table 4.1.

Table 4.10 *Values of Parameters in Import Functions*

Branch	$\gamma_{i,1}$	$\gamma_{i,2}$	$\sum_j \gamma_{i,j}$	$\gamma_{i,3}$	Share of total import volume[b] in 1980 (percent)
8–10, 13, 16, 17	−0.5	−1.0	−1.5	1.5	8.0
4–6*	0.0	−0.53	−0.53	1.88	5.6
7*	−0.87	−1.63	−2.50	1.49	8.1
11*	−0.67	0.0	−0.67	1.27	10.3
14*	−0.69	−0.76	−1.45	1.32	5.8
15*	0.0	−2.71	−2.71	1.88	34.9
20, 21, 23	−1.3	0.0	−1.3	1.5	8.2
3, 12[a]	–	–	–	–	15.4
1, 2, 18, 19, 22[b]	–	–	–	–	3.7

* Estimated parameters.
[a] Imports of coal and oil is given directly by demand since no import competing domestic production exists.
[b] Zero or exogenous imports.
[c] Cif at 1975 year's prices.
Note: Branches are listed in Table 4.1.

4.3 The Solution Algorithm

Since the number of equations is large and there are many nonlinear relations between variables, there is no way to explicitly express the endogenous variables as functions of exogenous and predetermined variables. It is therefore not possible to solve the model without the use of an iterative procedure. The technique used in ISAC is the Gauss-Seidel algorithm. This algorithm is elementary, very simple to implement and has proved robust, efficient and cheap to use in terms of computer time. Through the iterative procedure it is also easy to add or detract equations without bothering about the order in which they are computed, which variables are endogenous, etc. This possibility greatly increases the flexibility of the model.

To show in a simplified way how the algorithm works let us represent the ISAC-model as a number of equations, one for each endogenous variable:

$$x_1 = g_1(x_1, x_2, \ldots, x_n, z)$$
$$x_2 = g_2(x_1, x_2, \ldots, x_n, z)$$
$$\vdots$$
$$x_n = g_n(x_1, x_2, \ldots, x_n, z);$$

where the x_i:s are endogenous variables and z is a vector containing all the exogenous and predetermined variables. The iteration starts with an initial guess of the x:s – normally the previous period's solution. Computing the right-hand side of the first equation gives a new value of x_1 which substitutes the old value in the subsequent calculations. The second equation gives a new value of x_2, which substitutes the old value, etc. This procedure is repeated through all n equations and is then started all over again from the first equation. It goes on until the process converges, i.e. until the computed values of the x:s on the left-hand side are close enough to the old ones on the right-hand side in the equation system above.

This method has proved to be very useful and robust for the type of models that ISAC represents. This is also the experience gained in connection with the MGM- and LIFT-models. For a more thorough discussion of the use of the Gauss-Seidel method for macro models, see Barker (1976).

5 Stability

Having treated in some detail the structure of ISAC we should now say something about the working and dynamic properties of the model. We emphasized already at the beginning that ISAC is better suited to analyze stability and stabilization problems than to deal with allocational or distributional policies because of its degree and form of aggregation. Without e.g. a sub-grouping of households and of private consumption, no serious incidence studies can be attempted. The situation is not quite so clearcut when it comes to long-term resource allocation. Structural change and productivity developments in manufacturing, shifting government shares in consumption and income formation are aspects of long-run economic adjustment which are explicitly recognized and analyzed in the model. The absence of financial markets, of an explanation of household saving and of regional dimensions still, however, makes it difficult to use the model for full-fledged studies of growth policies. What the model can be used for with advantage is studies of stability and of medium-term stabilization policies and their relation to growth and structure.

5.1 A Pragmatic Framework of Stability and Control

Anyone setting out to study the stability properties of a real-life dynamic system like the Swedish economy is bound to become frustrated at the very start in trying to assemble a suitable conceptual tool-box or framework. The stability concepts articulated and used in the mainstream of economic theory are so far removed from the dynamic problems of real economies as to be almost irrelevant to our purpose here. The systems treated in the literature are usually autonomous, i.e. time is not an essential variable. Stability definitions use empirically indeterminate concepts like "neighborhoods arbitrarily close to the origin" while any empirically useful stability concept should be concerned instead with the system staying within certain specified bounds. The systems usually discussed are moreover closed, i.e., do not explicitly contain exogenous variables like world market trade or stabilization policy instruments, etc.

We are thus forced to make a detour in order to define our concepts. To make this detour as short as possible, the presentation or definition of the five different stability notions used will be sketchy and direct without any side-glances on existing literature.

To simplify the discussion, let us assume that the ISAC-model could be represented by a system of first order difference equations:

$\Delta x_t = f(x_{t-1}, y_t, u_t, z, t);$

where

x = endogenous variables
y = exogenous variables
u = control variables ("policy instruments")
z = system parameters
t = time

If we specify a development over time for exogenous variables ($\bar{y}$) and policy instruments ($\bar{u}$), and an initial position for the economy (x_o) we can solve the system of equations for a *growth path* ($\bar{x}$).

Let us now introduce our first "stability" concept by defining a *stable growth path* as one for which growth rates of all endogenous variables are "fairly constant" from time t_s and onwards:

$$\left|\Delta x_{i,\,t}/x_{i,\,t-1} - \Delta x_{i,\,s}/x_{i,\,s-1}\right| \leq \varepsilon_1; \qquad t,\, s \geq t_s$$

Our definition of stable growth should correspond to the use of that expression in common parlance but can not be applied strictly. It does not embrace all endogenous variables, e.g., the balance of payments which, being a difference between external income and expenditure, obviously may show large relative changes. The definition should be applied only to "basic" endogenous variables such as volumes of production and demand, price indices and the like.

As already stated the ISAC-model does not assure strict equilibrium solutions in the sense that excess demand may arise in some markets for shorter or longer periods. These possible excess demands, E_j, will concern currency (the balance of payment), labor and production capacity.

We can now define an *equilibrium growth path* where all excess demands are limited:

$$\left|E_{j,\,t}\right| \leq \varepsilon_2; \qquad t \geq t_s$$

Equilibrium growth in the ISAC-model implies stable growth as we have defined the concepts. This is the rationale for focusing our analysis on excess demands in the model simulations. Although the proposition is hard to prove strictly – because of the size and complexity of the model – it is intuitively reasonable and confirmed by extensive experimentation with the model. Assuming that values of exogenous variables and policy instruments do not exhibit large jumps it is, e.g., obvious that production capacity equilibrium is sufficient for stable growth in output. This is so because of the considerable inertia in capacity growth. Turning to the demand side of the model the composition of every category is fairly stable since no sudden relative price shifts between different goods are likely to occur given stable exogenous prices. Also equilibrium in the external balance assures no sudden shifts in domestic prices relative world market prices, and hence no drastic shifts in exports and imports. Finally, equilibrium in the labor market rules out sudden shifts in unit labor costs.

Although the model is not solved for equilibrium in some markets there are adjustment mechanisms that tend to make market disequilibria bounded and keep them from reinforcing each other. This mechanism together with suitable choice of policy instruments assures that an equilibrium growth path is always possible to find. This will accordingly also be stable as defined above.

Next we want a measure of what we will call the *resilience* of the system, i.e. the ability of the system, to absorb outside shocks by itself or without intervention in the form of policy changes. Given a stable groth path, x*, we introduce "disturbances" or major changes in some exogenous variables, while keeping everything else including the preset policy, unchanged. We call the system – or the specific growth path – resilient if it adjusts back to a stable growth path close to the original one:

$$|x_t(y_o + \Delta y_o) - x_t^*(y_o)| \leqslant \varepsilon_3; \qquad t \geqslant t_s$$

To quantify the concept of "degree of resilience" – and the same goes also for the other stability concepts introduced below – we obviously need to answer at least four kinds of questions. How big, relatively speaking, is the disturbance and where is it located in time and place? How long does it take – and at what amplitude of fluctuations – to bring back growth into a stable pattern? How big are the "stabilization" costs, measured in terms of some social "loss function"? How big is the following growth loss, if any, measured by that same function?

In the next step we assume that the resilience of the system is not sufficient, so that explicit policy intervention must be brought about to counteract the external disturbance.

We call the system *controllable* or *manageable* if there always exists a policy packet, ū', that will bring the system back close to the original undisturbed growth path:

$$|x_t(y_o + \Delta y_o, \bar{u}') - x_t^*(y_o, \bar{u})| \leqslant \varepsilon_4; \qquad t \geqslant t_s$$

Given a "loss function", defined in terms of welfare effects of growth paths as well as political costs of policy measures, optimal policies can be defined. No explicit loss function is however integrated in the model but far less sophisticated measures such as total private consumption, real wage rate, tax rates etc. are used to compare different policies. The reason is twofold. Firstly, the specification and estimation of a comprehensive social loss function is no easy task. Secondly, a strict optimization would require several iterations over possible policies and therefore significantly increase computer time.

So far we have taken the model structure, z, as given. If we take into account the possibility of changes in the system parameters we can define *structural stability*:

$$|x_t(z + \Delta z) - x_t^*(z)| \leqslant \varepsilon_5; \qquad t \geqslant t_s$$

that is small exogenous changes in z at a given time – i.e. "sensitivity tests" – will only produce small shifts in the growth path.

Finally, one may think of a system exhibiting a learning behavior in the sense that structural parameters are allowed to change as a consequence of shifts in exogenous variables. Such a learning property can be stabilizing as well as destabilizing. In the former case we will call the system *adaptive*, defining the concept as

$$\left|x_t(y_o + \Delta y_o, z_t') - x_t^*(y_o, z_t)\right| \leq \varepsilon_6; \qquad t \geq t_s$$

From the definitions it is obvious that resilience is a more restrictive concept of stability than adaptiveness. However the difference between the concepts can to some extent be attributed to the level of aggregation in the model.

To return to a stable growth path after some disturbance in world markets an adaptive parameter shift may be required for instance in export and import functions. In a more disaggregated model this parameter shift may correspond to redistributions of export- and import-shares between goods with parameters in the functional relationships unchanged. The latter disaggregated system would then be said to be resilient.

After this very cursory introduction of stability concepts let us try to illustrate them by some simple applications on the ISAC-model.

5.2 Stable Growth and Resilience

To ease an understanding of the stability behavior of the model, let us first recall some of its fundamental properties. There are three main types of markets where disequilibria or excess demand may occur. Production capacity may differ from the quantity actually produced, the unemployment rate may be higher or lower than the rate assumed to be caused by frictions in the labor market, and finally, foreign payments on current account may not balance, indicating an over- or undervalued currency.

For the first two of these disequilibria the model includes relations that tend to reduce deviations from "normal" values. If capacity falls short of demand, the reaction will be both to limit the increase in demand and to expand production capacity. To keep down the demand increase the price and wage equations are of great importance. The increased production will push up capacity utilization. This gives a direct upward pressure on supply prices. But there is also an indirect price effect. The increased demand will reduce unemployment below "normal" levels and wages will raise faster than otherwise.

Although producers are not assumed to use pure mark-up prices some of the wage inflation will be passed on to consumers. Increased demand will thus to some extent transform into higher prices. There will also be a tendency to expand capacity through investments, which will increase, ceteris paribus, when producers become short of capacity. Producers' prices will respond to demand increases affecting utilization ratios in the short run, but it may take several years before capacity, growing through higher investment, will catch up with production to reach "normal" utilization ratios.

The third disequilibrium, the current account, does not set directly in motion any balancing forces. The exchange rate is assumed fixed or rather exogenously determined and is included in the set of control variables used to steer the model (cf. section 5.3).

With this in mind we can go on to construct first of all a stable growth path, that can then serve as a reference path for the evaluation of system resilience when outside disturbances occur. To simplify the example the local government submodel has not been used, making all government consumption exogenously determined.

World market development and other exogenous variables for the period studied, 1980–2000, have been chosen from current Swedish projections, also used by us in other model simulations (cf. Ysander-Nordström 1985, for a detailed discussion of the assumptions). The model economy has then been steered through the eighties by use of tax and wage policy into a "turn-pike" 1990, characterized by narrowly bounded excess demand on the markets concerned as shown in Figure 5.1. New permanent values have then been set for the policy variables, so that the growth path during the nineties remains stable in this sense. (The steering technique will be further discussed in section 5.3).

This is indicated by Figure 5.2 where growth rates for some aggregated real and price variables are given. The short-run fluctuations in the beginning of the eighties are mainly due to the need to restore profit margins in industry. During the rest of the decade the slow growth of private consumption is explained by the need to restore the external balance. Having achieved that in 1990 (cf. Figure 5.1) taxes can be relaxed to let private consumption grow at a one percentage point faster rate than during the second half of the eighties. (Public consumption growth is assumed equal during the whole simulation period).

Generally, Figures 5.1–2 show that variations in growth rates of endogenous variables are fairly small if exogenous variables and policy instruments are given, and that there are no tendencies for the solution to "explode". More specifically, the figures indicate that possible excess demand can be kept within certain limits and that this implies a stable growth path during the same period. In the stability experiments discussed in the rest of the chapter we will therefore be content to show only excess demand diagrams.

In the first resilience test, we assume a 5 % jump upwards in world market prices in 1981/82 compared to the reference case. The higher price level is assumed to persist, implying equal growth rate during the rest of the simulation period. Public policy remains unchanged, since we want to study the "self-correcting" mechanisms in the system. The result is shown i Figure 5.3 as deviations from the reference case in the three "excess demand" markets. The immediate effect will be improved profits in export companies. However, there will also follow increased net demand from abroad since producers will partly take advantage of their improved relative cost position by capturing market shares. The current account will be improved by more than one half percent of GDP in 1982 and 1983 compared to the reference case. This development will, however, bring about reactions in the labor market and in the production system. As a consequence of increasing pressure from the demand side, wages and prices will shift upwards. The imported inflation in 1982 will add to inflation expectations and make wage inflation in 1983 even more severe. This price-wage spiral will reverse the process described above and will lead to unemployment and overcapacity in industry and so forth. As shown in Figure 5.3 the induced cycles will be gradually weakened and

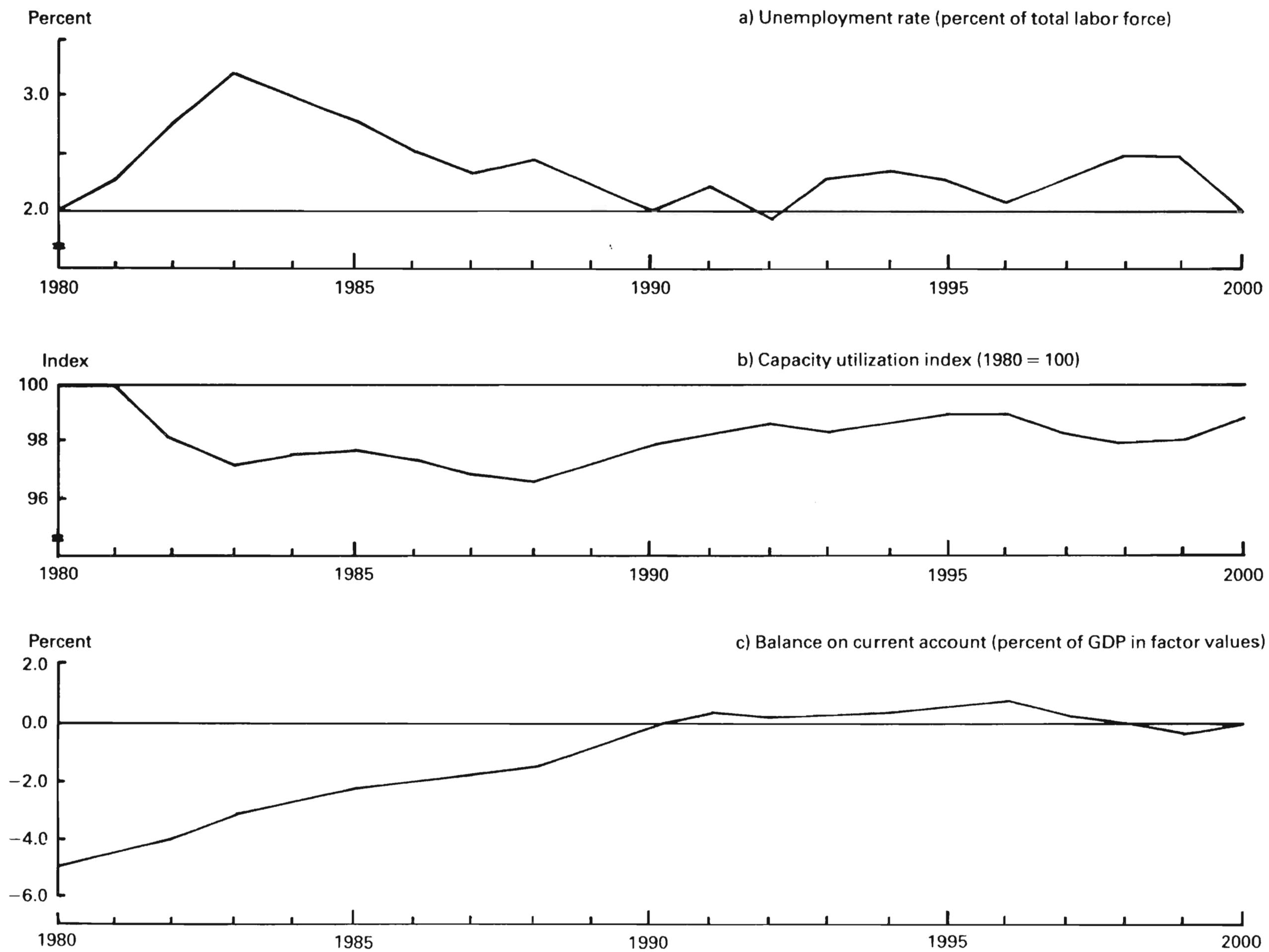


Figure 5.1 *The Reference Case*

Figure 5.2 *The Reference Case*
Percentage increase per year

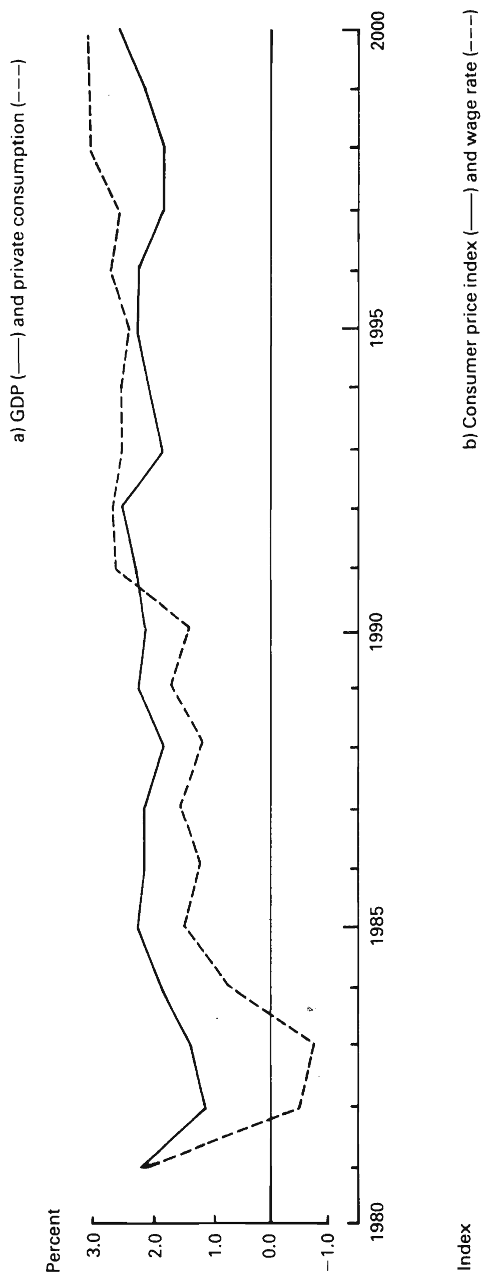
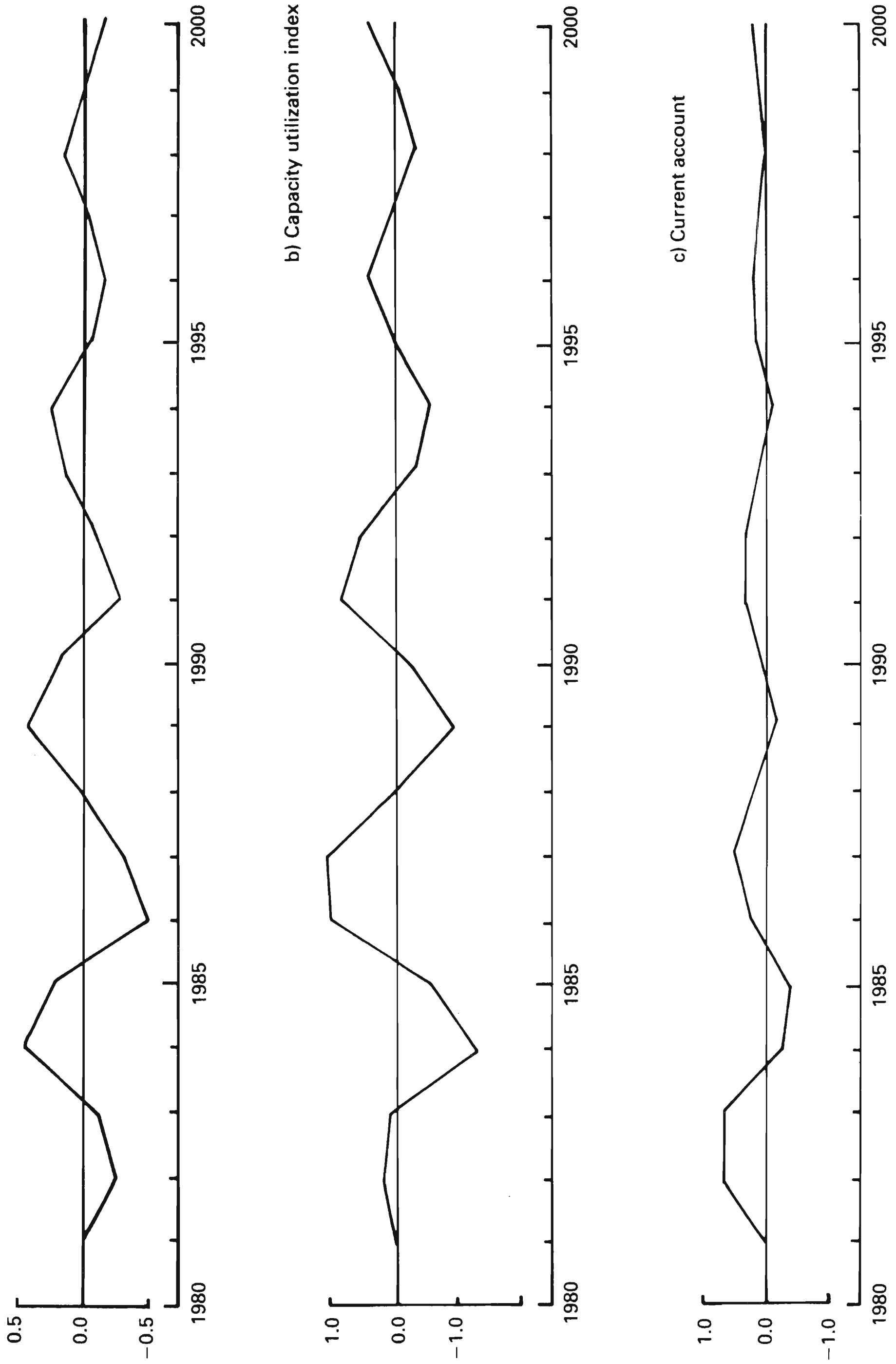


Figure 5.3 *Effects of Permanent 5 % World Price Increase 1982*
Difference in percentage points from Reference case



the economy will stabilize itself around its new long-run equilibrium growth path. In this particular case the only persistent effect is a 5 % rise in all price levels relative to the reference case.

In the second resilience test the economy is exposed to an overall 5 % increase of world market growth relative to the reference case in 1981/82. The story is told in Figure 5.4. In this case net export demand is affected directly without any improvement in relative prices. The same mechanisms as before will however produce reactions from the price side to hold back demand.

The fluctuations will fade away within a few years in this case but there will persist a surplus on current account even during the nineties. The model economy will accordingly not absorb this kind of exogenous disturbance and return to equilibrium growth. The reason for this is obvious since the exchange rate is exogenous and thus remains the same as in the reference case despite the significantly improved world trade. With floating exchange rates the current account surplus would have brought about currency appreciation.

The fact that not all excess demands vanish after a world trade disturbance does however not necessarily rule out the *stability* of the corresponding growth path as defined in section 5.1. This is indicated by Figure 5.5. Although effects of the disturbance on prices and quantities are registered for the whole simulation period they will fairly soon be confined to small deviations from the reference growth path.

In the simulations accounted for above we tested the resilience of the model for permanent shifts in world market variables. Figures 5.7–8 show the results of temporary disturbances of the same variables, i.e., the initial 5 % extra growth in 1982 will be followed by a 5 % fall two years later as compared to reference case growth rates. Although the excess demand cycles are clearly amplified in this case – especially when it comes to the world price disturbance – the oscillations are still dampened over time. However, Figure 5.6 also indicates that current parameters in the model may not be far from producing persisting excess demand cycles, at least in response to certain kind of external price disturbances. Finally, we may note that this time the system will resume equilibrium growth even in the face of a trade disturbance. The balance on current account given in Figure 5.7 will show no persistent surplus since the level world trade does not differ from the reference case as from 1984.

5.3 Controlling the Model

The interest in steering model solutions is due to the wish to compare the outcome of equilibrium growth paths under different external conditions. E.g., how will the space for consumption be affected by higher oil prices given that it is possible to preserve equilibrium growth?

As we have seen equilibrium growth is not assured in the model even in the long run, if parameter values are held constant. If however some of the model parameters are assumed free to change during a simulation it is always possible to attain

Figure 5.4 *Effects of Permanent 5% World Trade Increase 1982*
Difference in percentage points from Reference case

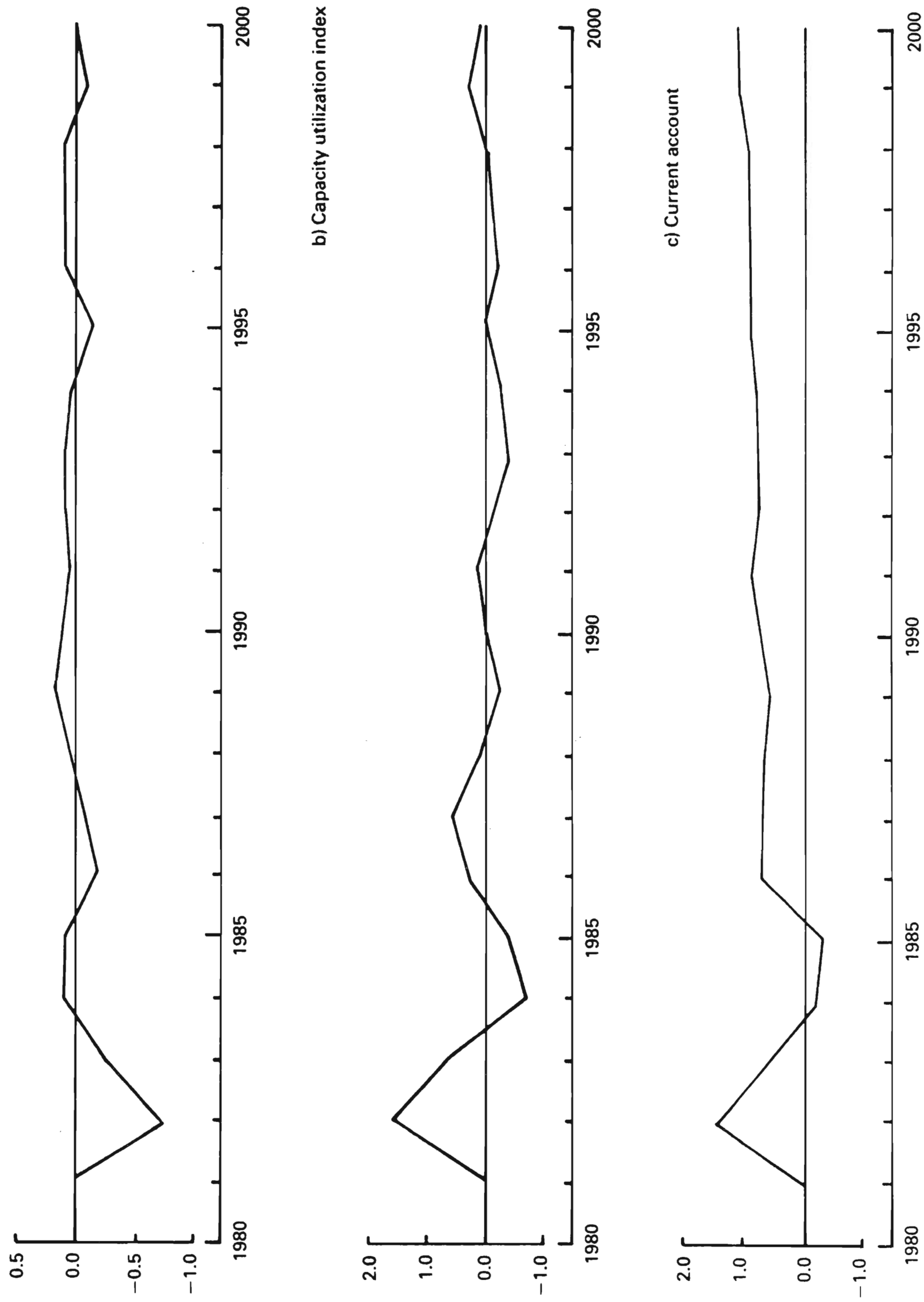


Figure 5.5 *Effects of Permanent 5 % World Trade Increase 1982*
Growth rate differences in percentage points from Reference case

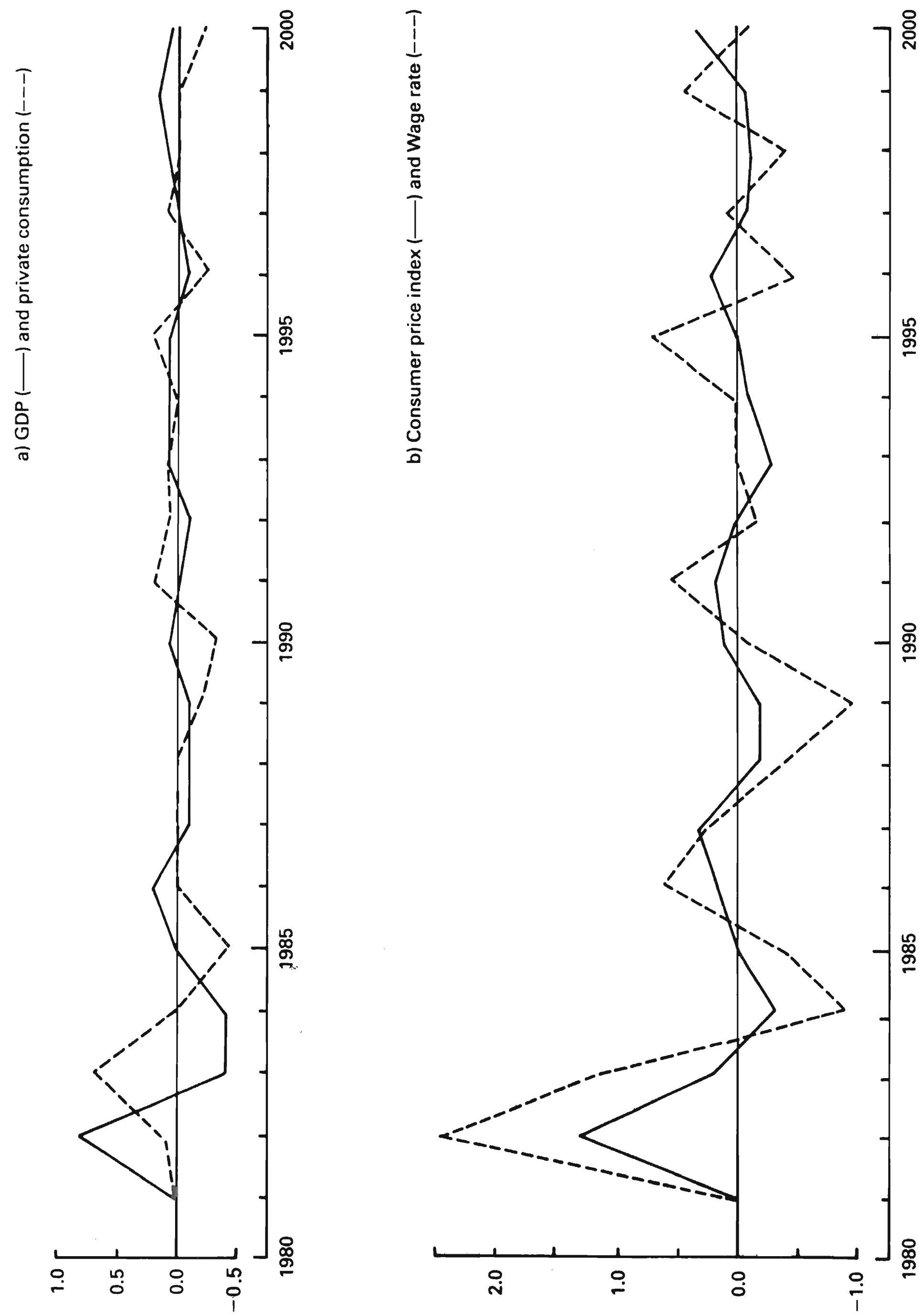


Figure 5.6 *Effects of Temporary 5% World Price Increase 1982–84*
Difference in percentage points from Reference case

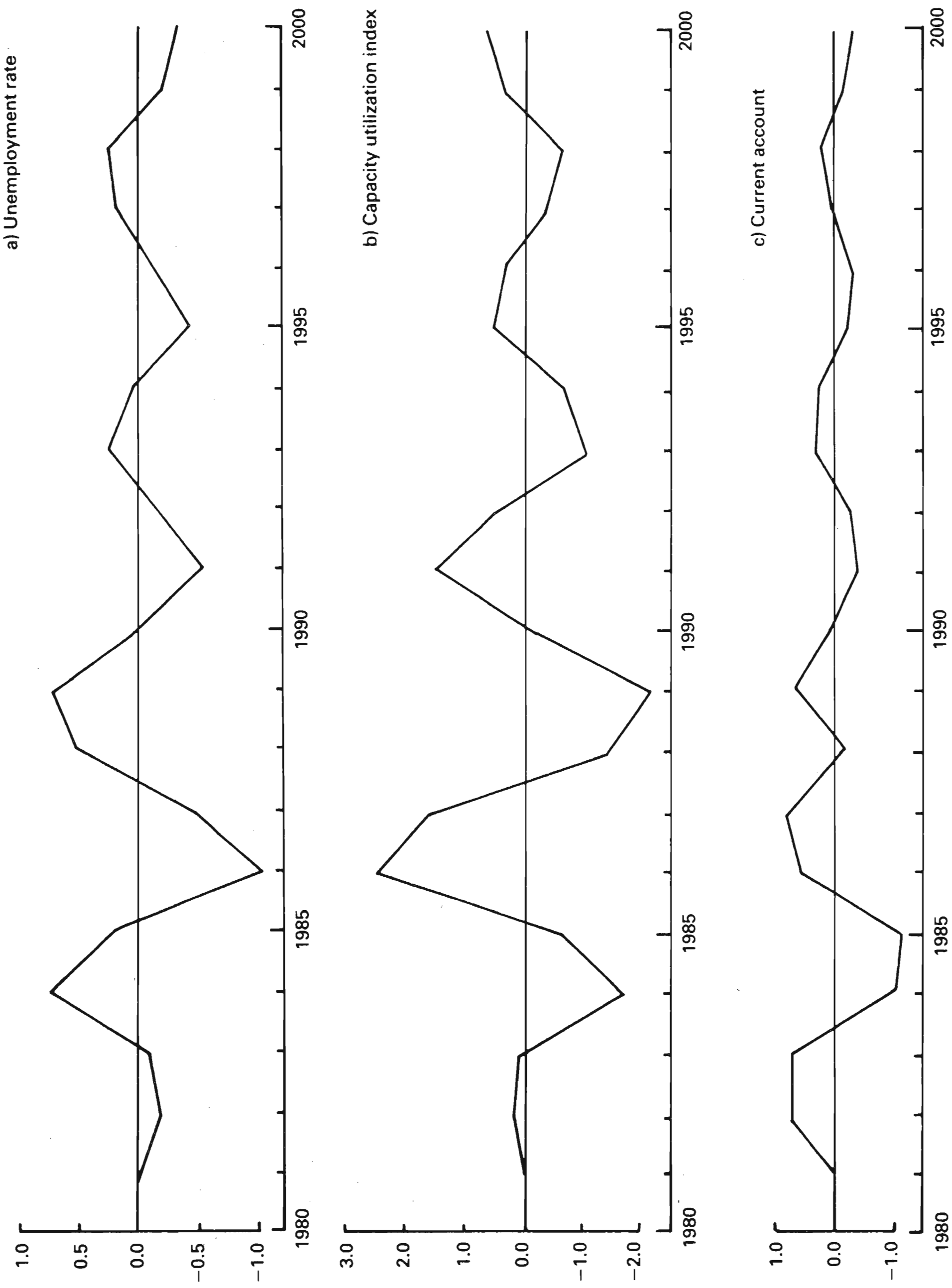


Figure 5.7 *Effects of Temporary 5 % World Trade Increase 1982–84*
Difference in percentage points from Reference case

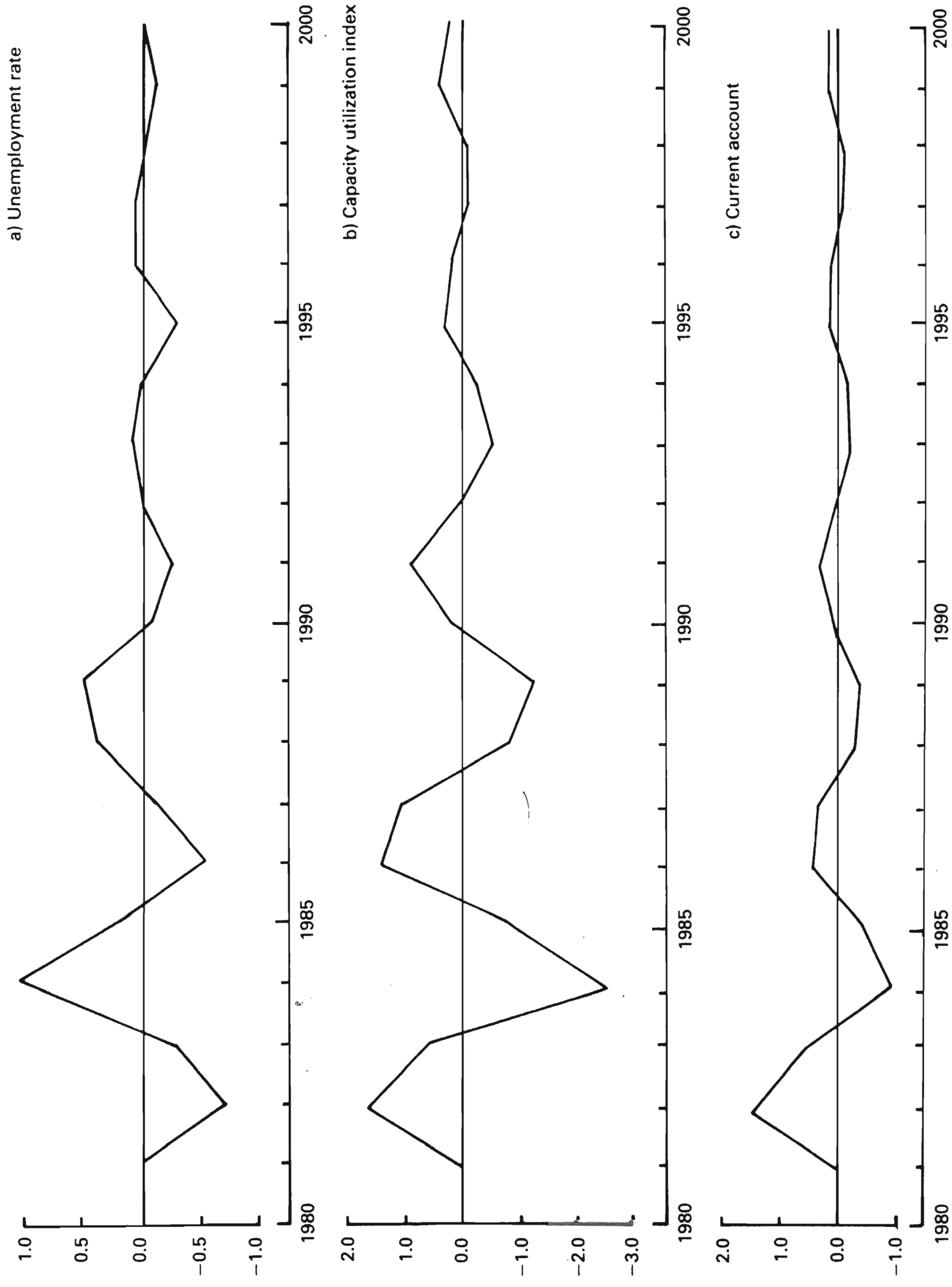
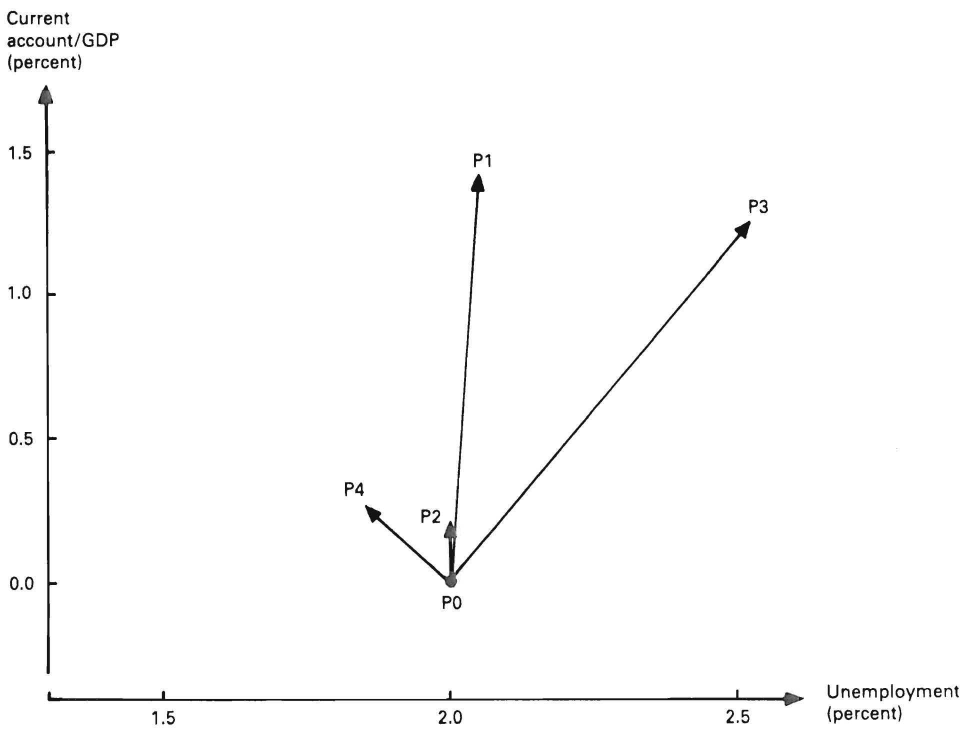


Figure 5.8 *Ten-year Effects of Policy Instruments*



P1: Upward shift of income tax scale by 0.5 percentage point every year 1981–90
P2: Depreciation of currency by 1% per year 1981–90
P3: Public consumption growth rate decreased by 0.5 percentage point for the period 1981–90
P4: Autonomous wage trend decreased by 1 percentage point for the period 1982–90

equilibrium growth. This set of parameters is called policy instruments, control parameters or the like, and they are chosen to correspond as much as possible to instruments used in actual economic political decision-making. In the present version of the model the following main instruments are available:

- parameters in the income tax function
- wage tax rate
- exchange rate
- central government consumption and grants
- autonomous wage rate increase

Needless to say the correspondence between model and real life instruments is not complete. Since the model i.a. does not explicitly include financial markets no monetary policy instruments are available.

One may of course also have reasonable doubts about how far the variables listed above are really available as policy instruments. The wish to use control variables analogous to those used in actual decision-making has in fact been an important reason for including some submodels, such as the local government model, and for the fairly detailed treatment of taxes and transfers in the household sector model. The traditional treatment of e.g. the local government sector growth as an instrument for macro policy is obviously unsatisfactory, since such an instrument simply is not available for central government decision-making. The local government submodel makes it possible to affect local growth by i.e. changing grants to local authorities. When it comes to wages some kind of control possibility is simply assumed. Note however that the wage rate control is assumed to work indirectly through the constant term in the wage function (cf. section 3.13). "Uncontrollable" short-term fluctuations in the wage rate may still occur.

Before describing the effects of different policy measures in the model a few words should be said about how they are used. Since the model is solved year by year it would be possible to change control every year. For practical as well as theoretical reasons, this is however not done. Suffice it to say that important short-term mechanisms are still lacking in the model, e.g. inventory functions, and that credit markets are not yet made explicit. A certain setup of policy variable values is instead assumed to be maintained for several years – in most applications of the model five or ten years. The use of control variables in the model should for that reason be seen as medium-term guidelines to economic policy without regard to short-term fluctuations.

Figure 5.8 shows the ten year effects of four policy instruments in terms of unemployment rate and balance on current account (as percent of GDP). The arrows show the difference in these respects between a reference simulation, PO, and simulations with different values of policy parameters. All differences are measured for the single year 1990 only. For example, the arrow P1 gives the effects in 1990 of a gradual shift upwards of the personal income tax scale during the eighties. As can be seen from the figure this will improve the current account compared to the reference simulation outcome in 1990, while leaving the unemployment rate unaffected. The immediate impact of increased taxes is to depress private consumption, which will save imports and improve the current account. The fall in domestic demand will almost totally be compensated by external demand stimulated by improved relative prices due to less pressure on the labor market and hence on wages.

Besides tax policy domestic demand can be directly affected by control of public consumption growth. The arrow P3 shows the effect of decreasing the growth rate by one half of a percentage point during the eighties compared to the reference case. Like P1, which reduces private consumption, this policy will improve the external balance while increasing unemployment. The unemployment rate will however be considerably larger given a certain improvement on current account. The reason is that the tax increase will by itself reduce imports through reduced private consumption, while public consumption variations have small direct effects on imports. In this case the whole external improvement will be indirect and caused by higher unemployment rates which reduce wages and thereby improve external competitiveness but also reduce the income and consumption of households.

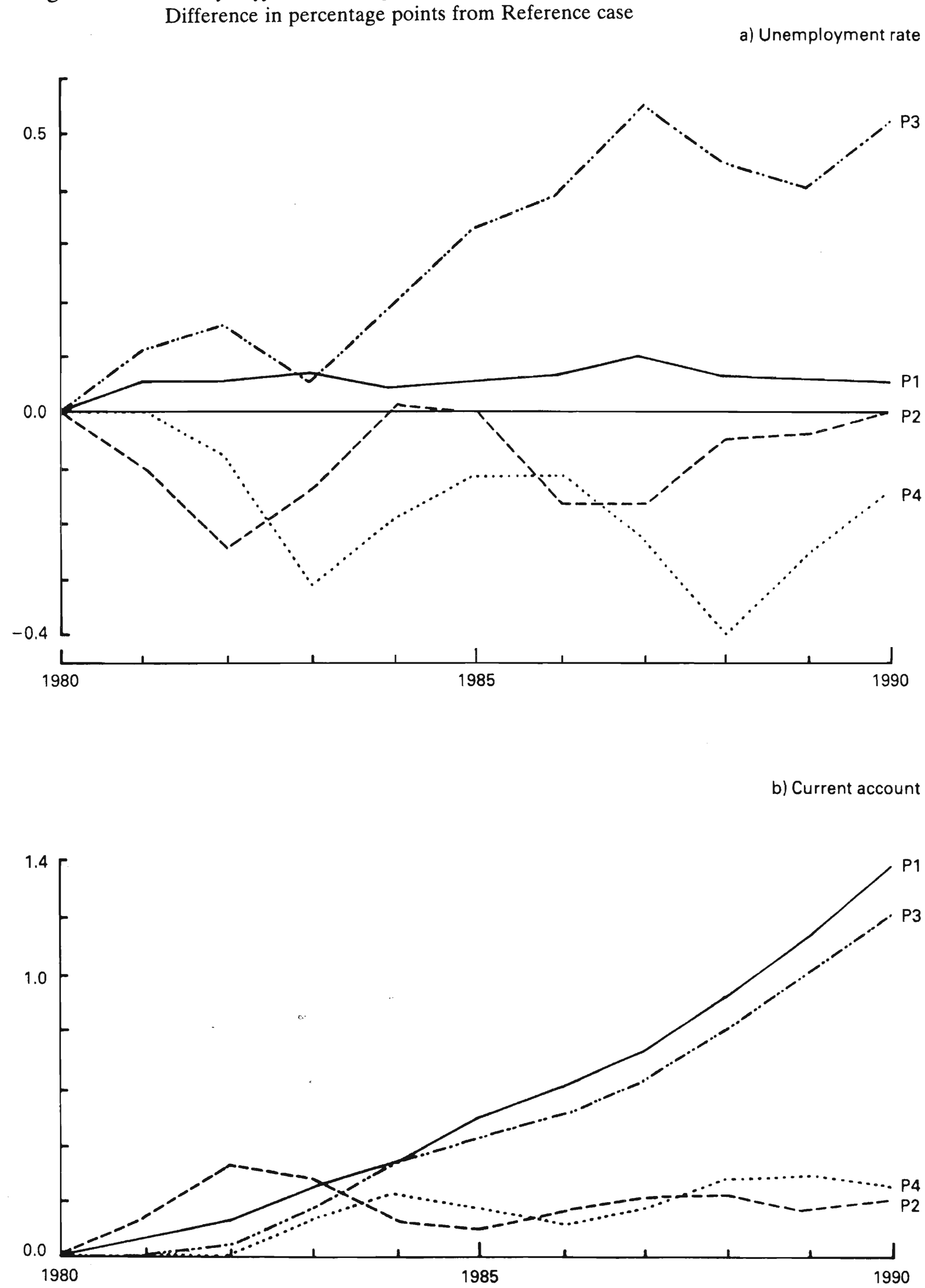
The two measures P2 and P4 are aimed directly at relative wage and cost conditions. With the wage function used (cf. sections 3.14 and 4.2) devaluations of the currency will be of little use in affecting the external or internal balance since it will only produce compensatory inflation responses. Reducing the "autonomous" wage growth will, on the other hand, obviously affect current account as well as employment positively as shown by P4.

Some words of caution should be added regarding the interpretation of the "effect-arrows" in Figure 5.8. To begin with, since ISAC is a dynamic model the effects of policy measures of the kind discussed above will not be constant over time. What is needed for our interpretation and use of these parameters as control variables is however only that, after some years, the effects should stabilize around some fixed level or rate of change. Figure 5.9 shows the development over the eighties of the four policy measures shown in Figure 5.8. Although there are evident cycles in some variables, the levels seem fairly stable and the amplitudes are small. We may therefore safely pick a single year to represent the effects of different policy measures. As shown in Figure 5.9 however there may occur shifts in the effects between policy instruments even after the initial impact phase. In the particular simulations shown in the figure this relates primarily to exchange and wage policy, respectively. Most of the crossings of the effect curves P2 and P4 have however other causes. The wage rate is exogenous in 1981, for technical reasons implying that wage policy is in fact not initiated before 1982. An appropriate picture of the stability of policy effects is obtained if the wage policy curve, P4, is shifted one year backwards in both diagrams.

A second point to bear in mind in connection with the policy effects as shown in Figure 5.8 is the difficulty to compare the power of different policy measures. Generally, the length of the arrows can not be used for such comparisons since one also has to consider the policy parameter changes behind them. Strict evaluation requires a social loss function to be specified. However, one kind of conclusion as to the usefulness of different measures can be drawn. Assume that the model economy initially is out of balance in 1990. To move it back to an equilibrium solution in terms of the exchange and labor markets, generally two of the control variables must be used. From Figure 5.8 it is obvious that changes in income tax and in the exchange rate are rather ineffective means to influence employment levels. A policy packet consisting of only these two measures can not therefore cope with excess demand in the labor market as the model is specified. Figure 5.8 also indicates that a combination of wage policy and public consumption control may be the most effective policy packet. The fact that their effect arrows run almost perpendicular makes it possible to reach an arbitrary shift in unemployment and current account with lesser changes in policy parameters.

Finally, as have been indicated several times before, there is no outright optimization procedure used in controlling the model. The rough method used to measure the effects of e.g. different world market conditions is to steer the model to an equilibrium solution in the exchange and labor markets, checking off the

Figure 5.9 *Policy Effects during the Eighties*
Difference in percentage points from Reference case



P1: Upward shift of income tax scale by 0.5 percentage point every year 1981–90
P2: Depreciation of currency by 1 % per year 1981–90
P3: Public consumption growth rate decreased by 0.5 percentage point for the period 1981–90
P4: Autonomous wage trend decreased by 1 percentage point for the period 1982–90

capacity gap, and then to read off the differences over time in relevant variables including the policy parameters. Of course this introduces some degree of arbitrariness into the analysis but seems nevertheless to be the most practical way to use such a large model.

5.4 Structural Stability

The concept of structural stability is a more tricky notion than it looks at first sight. To make sensitivity tests of structural coefficients around a given simulation path is a standard procedure for any conscientious modeler. In the case of ISAC it is intuitively obvious that the model is structurally stable in the narrow sense that the absence of explicit discontinuities in the model guarantees that small isolated changes in the coefficients will only result in small displacements of the simulated growth path.

We will illustrate this statement by three tests where some strategic parameters are changed. These relates to (A) the price setting behavior of manufacturing companies, (B) the price responsiveness of foreign demand for Swedish goods and services and, finally, (C) the extent of compensation for past inflation in wage formation.

In the first test (A) manufacturing companies are assumed to pay increased attention to domestic production costs relative to world market price developments in setting their prices. More precisely the elasticity of prices with respect to unit costs is increased from 0.5 to 0.75 for manufacturing branches and the elasticity with respect to world market prices correspondingly reduced from 0.5 to 0.25 (excl. branch no. 12, cf. section 3.12 and Table 4.4). As can be seen from Figure 5.10 the effects on the equilibrium growth path in terms of excess demands are rather insignificant. The initial impact will be a small deterioration in the utilization of resources compared to standard assumptions since the initial unfavorable cost development will to a larger extent push up export prices and thus weaken real foreign demand. However, when wage and cost increases are slowed down after a couple of years this will affect export prices and competitiveness positively, leading to reduced unemployment and an improved external balance. Deviations from the standard case will persist through the whole simulation period but will remain small.

In the second structural stability test, (B), the immediate foreign demand response to Swedish export prices is increased. More specifically the short-run elasticity of export with respect to relative price is set on average to 2.2 compared to 1.2 in the standard version. The result is shown in Figure 5.11. Excess demands are also in this case confined to a fairly narrow region around the reference equilibrium growth path, although fluctuations seem to increase somewhat. This is what should be expected since small changes in prices will tranform into a larger change in real demand (from abroad) than in the standard setup.

In the last structural test (C) the coefficient variation will, on the other hand, have a stabilizing effect. In this test the degree of compensation for (past) inflation

Figure 5.10 *Increased Cost Influence in Price Setting (A)*
Standard assumption given by dashed lines

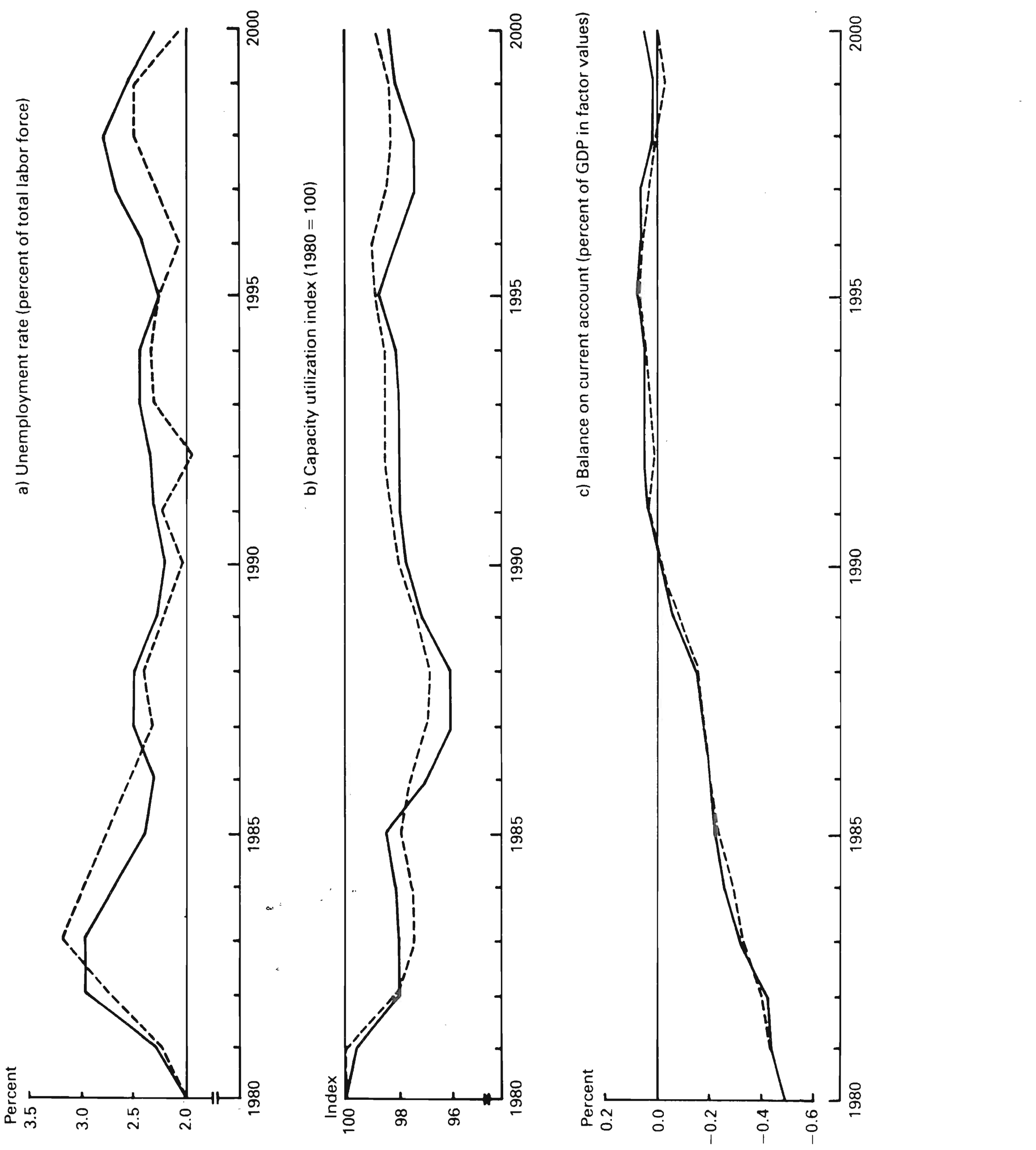
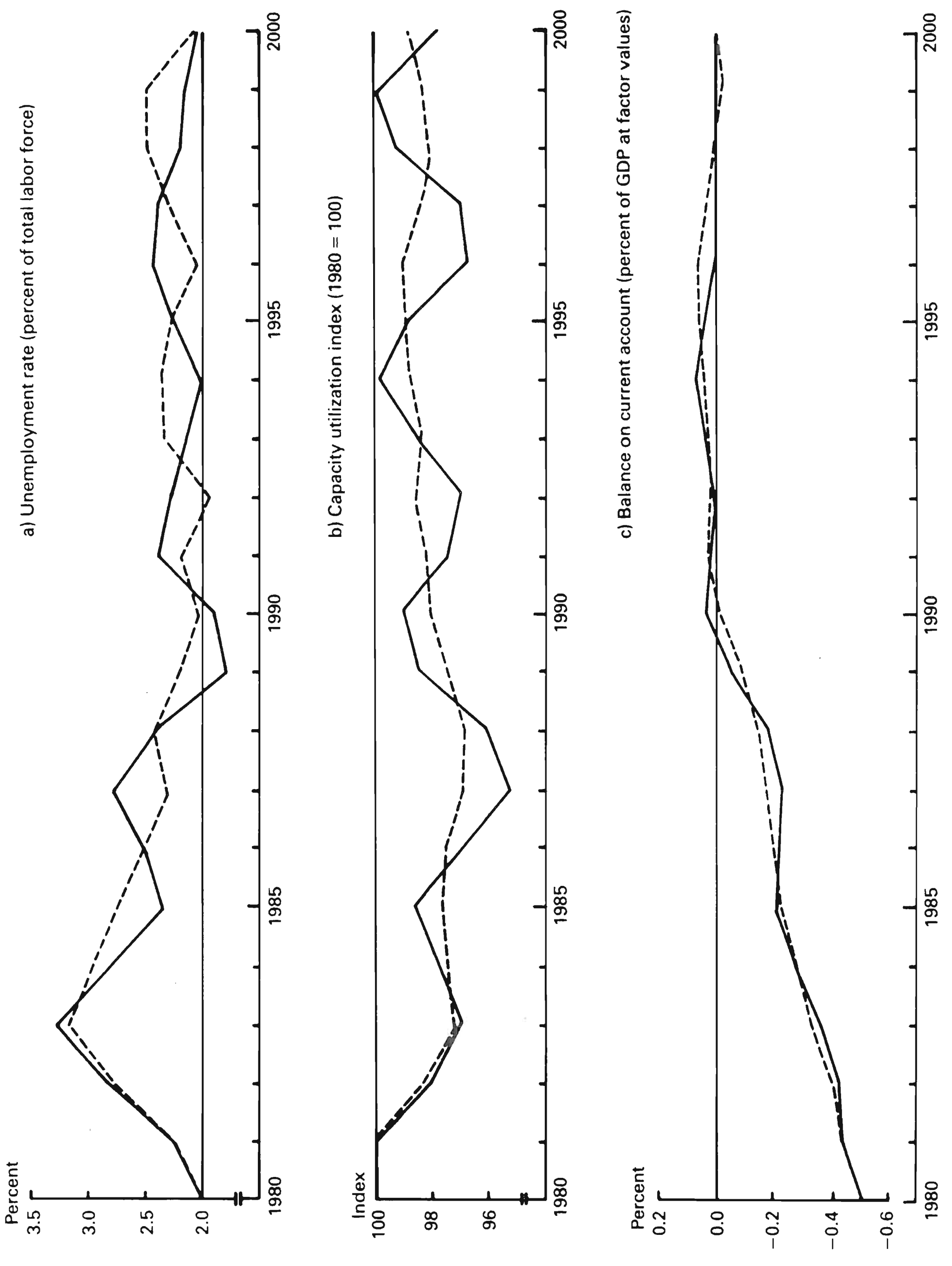


Figure 5.11 *Increased Price Response in Foreign Markets (B)*
Standard assumption given by dashed lines



in the wage function was reduced from 1.0 to 0.75 (cf. section 4.2). This will reduce the rate of unemployment "needed" to break the tendency to self-sustaining wage inflation implied by the specification of the wage formation in the model. As is seen in Figure 5.12 the reduced compensation also permits the economy to increase the long-run employment level through reduced real wages and improved competitiveness on world markets. Still, the corresponding growth path is stable.

Given the likely structural stability of the model one may wish to penetrate further into the effects of different coefficient assumptions. This will be done below based on the three types of parameter changes discussed above. The coefficient assumptions will be compared with respect to the development of private consumption, the effects of policy parameters and, finally, the resilience of the system.

Even though structural coefficients may not affect the stability of the system, the resulting growth paths may differ when evaluated in terms of some social welfare or loss function. In Figure 5.13 the three simulations A–C are compared to the reference case simply in terms of level of private consumption. Although differences to the standard growth path are small – within +/−0.5 % – for all cases some pattern is discernible. For A and B the consumption path is more or less a mirror image of employment (cf. Figures 5.10a and 5.11a), i.e. higher employment leads to higher consumption and vice versa with no differences in real income per wage-earner compared to the reference case. In (C) however, with less of inflation compensated wages real income per wage-earner will be reduced leading to lower total private consumption despite increased employment. Since aggregate productivity is almost equal between the simulations and public consumption is the same by definition, the external balance must improve as showed in Figure 5.12c.

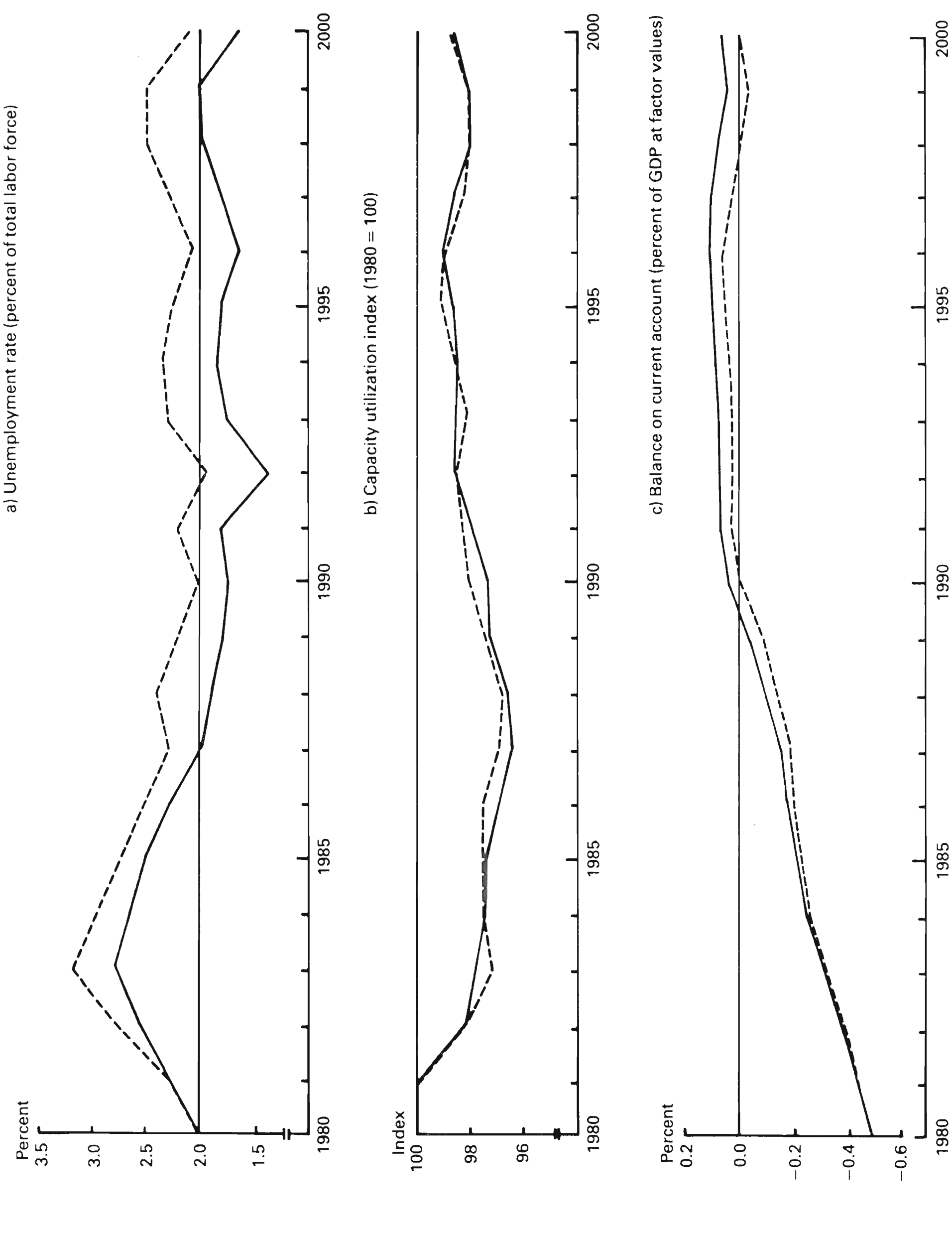
The consumption effects of variations in the coefficients may seem small relative to the rather significant changes that were studied. The reason for this lies partly in the model specification. Since wages are assumed to respond quickly and strongly to labor market conditions, employment levels close to normal will almost always be realized. Secondly, aggregate productivity does not change much unless investments are substantially increased. Finally, public consumption is exogenous in the version of the model used for these experiments. These factors together imply that increased private consumption can only be explained by increased employment, increased external deficit or improved terms-of-trade. The model specification, however, to a great extent rules out any significant contributions from the employment effect.

One might nevertheless have expected fairly large effects from especially the change in export price elasticities. Amplified price responses should lead to improvements in the external balance and/or to a smaller deterioration in terms-of-trade since foreign market demand could be enlarged without large decreases in relative export prices.

That no strong effects materialize in our experiments may be due less to model specification than to some other general problems encountered in interpreting and analyzing sensitivity tests.

A first type of problem is connected with the obvious fact that the results of

Figure 5.12 *Reduced Inflation Compensation in Wage Formation (C)*
Standard assumption given by dashed lines



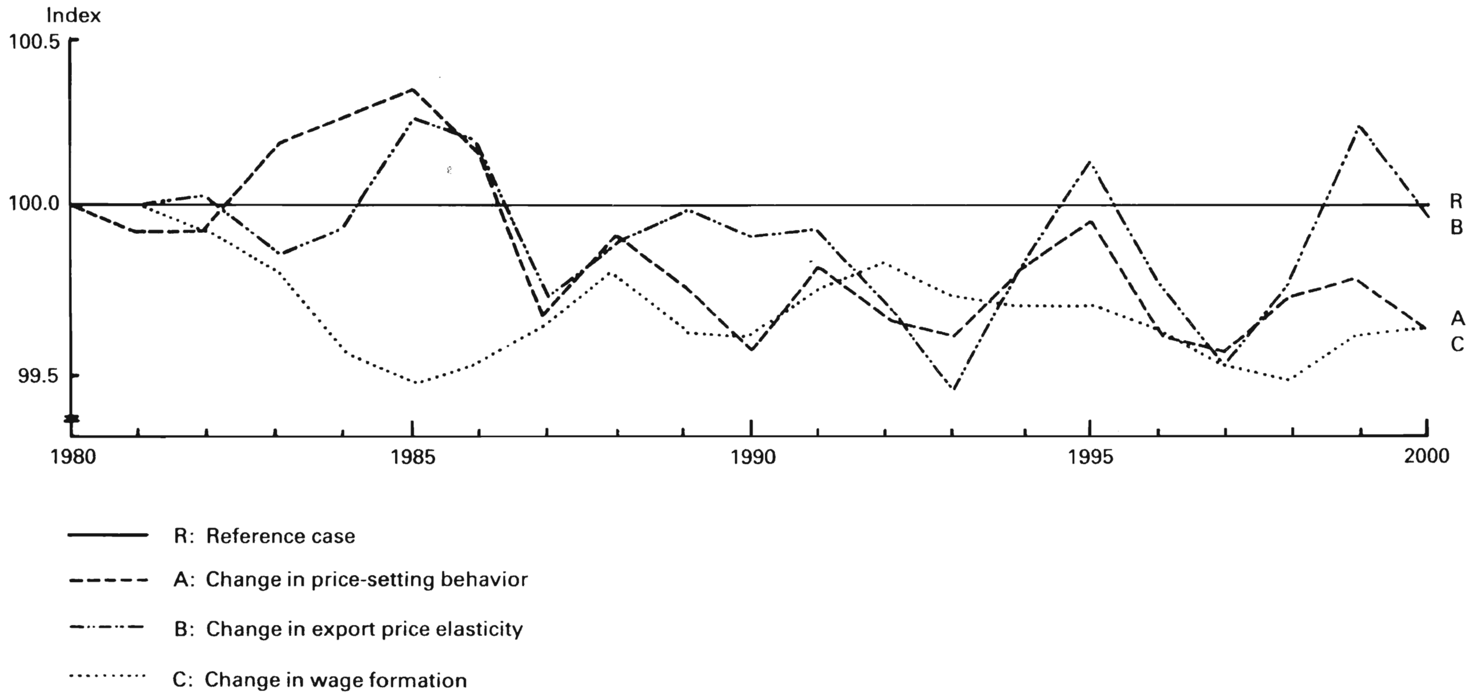


Figure 5.13 *Effects on Private Consumption*
Reference case = 100.0

different coefficient variation will not be independent of each other, i.e. cannot be simply added. The effects of changing export or import price-elasticities will, e.g., in general vary with variations in the income elasticity of imports. The positive effect, in a trade-deficit situation, of a more price-elastic export market is thus reduced by a higher propensity to use marginal income additions for import.

Also, the effects of changed coefficients generally depend on the standard growth path used as reference point. A specific change in foreign trade price elasticities will, e.g., affect the growth path more or less depending on the degree of relative price change assumed in the reference simulation. If, for instance, world trade growth is so generous as to permit external balance without reduced relative export prices, the effects of changes in price elasticities will of course be insignificant. If, on the other hand, we had started with assuming significant relative price changes, the price elasticities would no doubt have turned out to be of strategic importance.

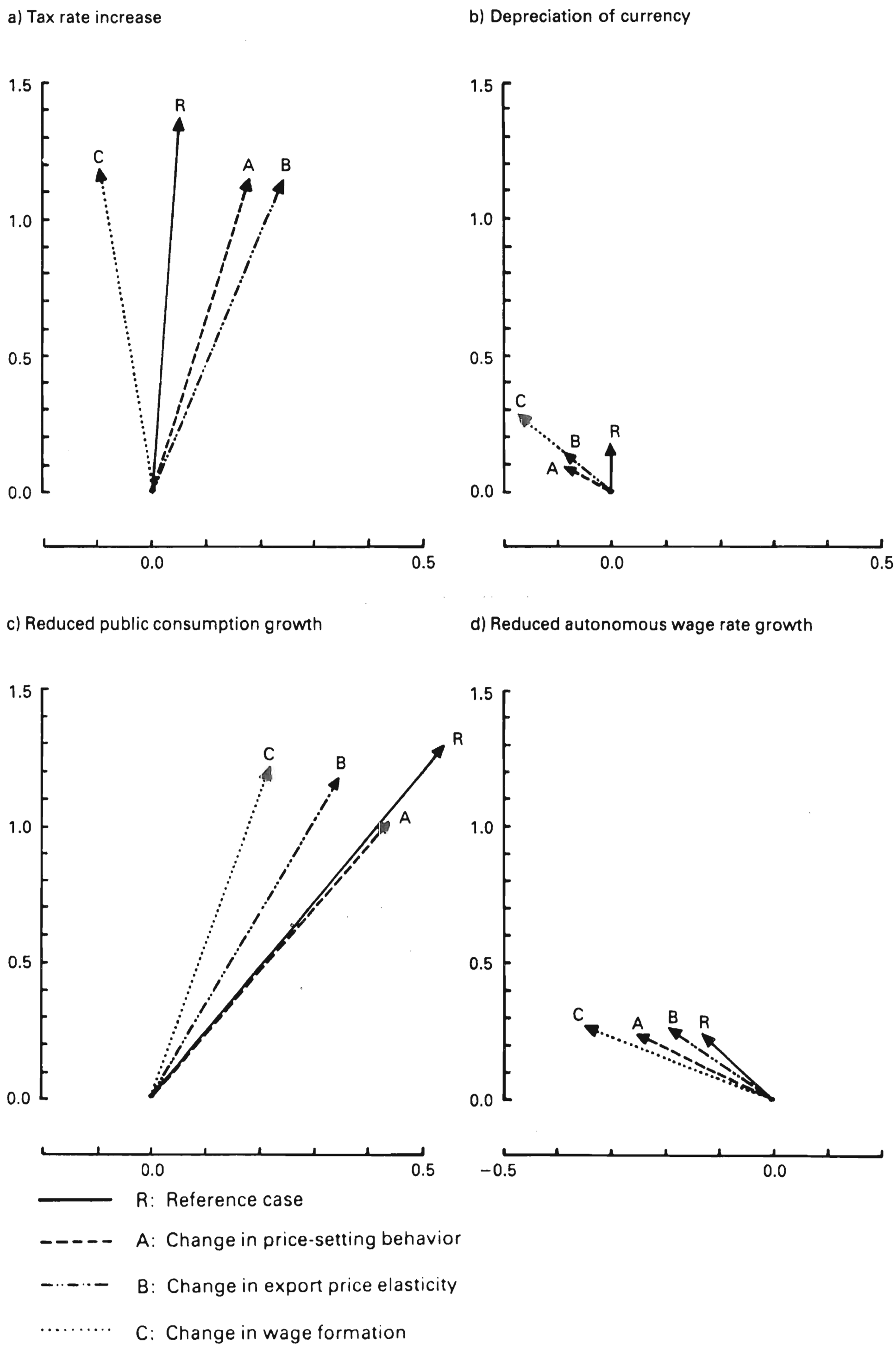
To illustrate the fact that also policy effects may differ significantly under different coefficient assumptions the same policy instruments as in section 5.3, summarized in Figure 5.8, can be used. From Figure 5.9 it is evident that the relations between policy effects in terms of unemployment and external balance will change with time but will tend to stabilize after some years. (Note the one-year delayed execution of wage policy). This relative stability is necessary in order to discuss ten-year (or long-run) effects as in Figure 5.8. Changing structural coefficients may however also change cyclical patterns in the model economy. In that case the relative effects of a change in a specific policy instrument under different coefficient assumptions may be unstable. Despite these problems of interpretation the ten-year effects of each of the four policy instruments are indicated in Figure 5.14 for the reference simulation (R) and the three coefficient variations (A, B and C) described above.

Each of the four subdiagrams are constructed in the following way. For the standard coefficient setup the model is run through the eighties with and without the specific policy change. The 1990 difference in unemployment rate, x-axis, and current account (as percentage of GDP), y-axis, is measured by the arrow R. This procedure is repeated for each of the chosen coefficient variations resulting in arrows A–C.

Correcting for cyclical effects tax rate increase will have almost identical long-run effects on unemployment and external balance irrespective of the choice of coefficients. The effect will be close to the reference case (R) in Figure 5.14a.

The effects of a gradual currency depreciation do not seem to depend very much either on different coefficient values. It may be interesting to note, however, that the long run "efficiency" of the instrument (the length of the arrow) is insignificant in all but one coefficient setup. The exception, of course, is C, i.e. the reduced inflation compensation in wage formation. With total compensation, the potential improvements in international competitiveness due to a depreciation of the currency will not materialize, since imported price increases will be compensated for, and again push up domestic costs and prices. The same general remarks are valid for the instruments shown in the last two subdiagrams. Thus, the conclusion is that

Figure 5.14 *Consequences for Policy Parameter Effects of Changed Structural Coefficients*
(See text for explanation of symbols)



(long-run) policy effects seem to be fairly unaffected by, at least, those coefficient variations exemplified above.

Finally, the effects of coefficient variations on the resilience of the model should be briefly discussed. The testing procedure is analogous to the one used in the policy effect experiments above. The model is run with different coefficient configurations and tested for the effects of world market disturbances. These disturbances are designed as sudden 5 % increases in 1981/1982 in world market prices and world trade respectively above the level assumed in the reference case. The higher levels of prices and traded volumes are assumed to persist (cf. section 5.2). The results of these experiments are shown in Figures 5.15–16 in terms of differences in unemployment rates between disturbed and undisturbed simulations for each coefficient assumption.

To start with the price disturbance shown in Figure 5.15, the resilience of the system seems to be preserved although there are perceivable differences between different coefficient assumptions. With less weight for world market prices assumed in price setting (case A) the external price shock initially yields a considerable lowering of relative prices. Through increased net foreign demand this price differential will become transformed into a reduction of unemployment that is twice as large as in the reference case (R). After the first full cycle the further development will however stick fairly close to the path given by the standard coefficient setup (R).

With increased price responsiveness of foreign demand (case B) cycle amplitude in terms of unemployment differences will be still enhanced. Moreover, although the amplitude of the oscillations seems to decrease somewhat, with this choice of coefficients they will be more than four times as large as in the other cases at the end of the period. In case C, finally, the limited inflation compensation can be seen to increase the model's ability to absorb an external price shock as expected.

Turning now to the effects of a trade growth shock, Figure 5.16 tells pretty much the same story. The repercussions of this disturbance will be strong throughout the simulation period in the case of increased export demand price elasticity (case B). Although the simulation path does not seem to "explode" there are no signs of dampening of the unemployment cycle generated by the initial disturbance (note the difference in scale from Figure 5.15).

Taken together these experiments show that the resilience of the model, although seemingly rather strong, is by no means always guaranteed.

Figure 5.15 *Effects on Unemployment Rates of 5 % World Price Increase in 1981/82 for Different Parameter Assumptions*
Difference in percentage points from undisturbed cases

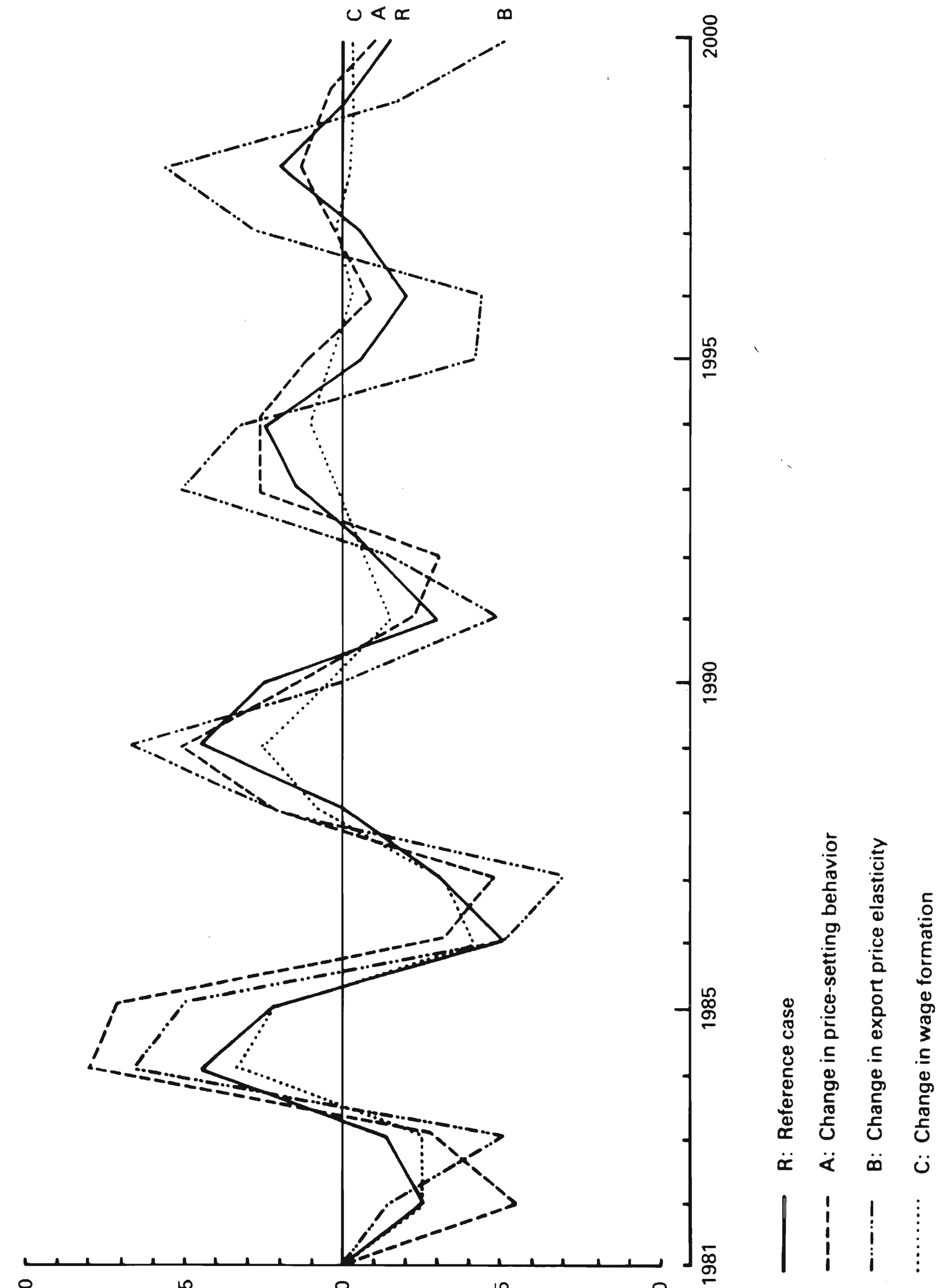
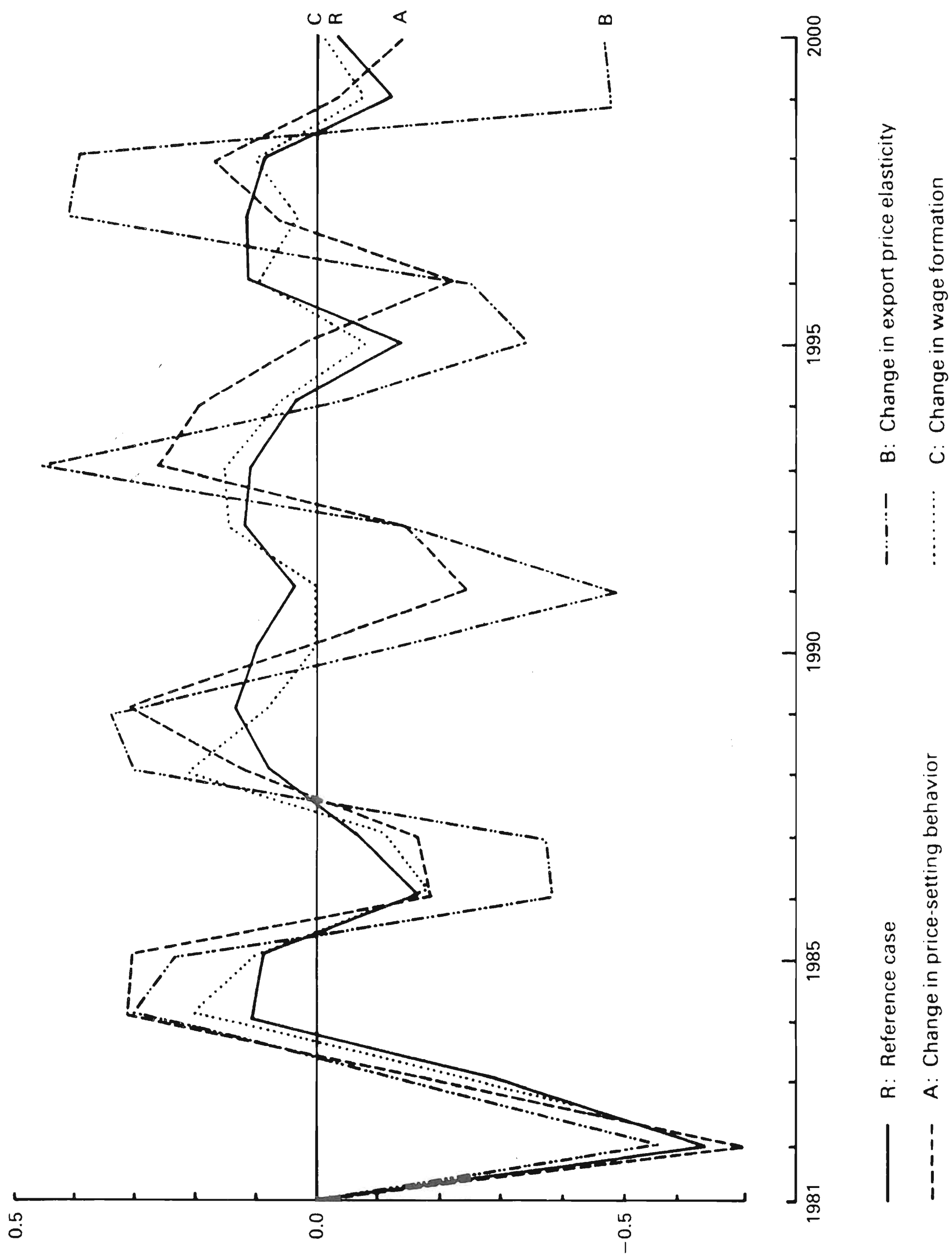


Figure 5.16 *Effects on Unemployment Rates of 5 % World Trade Increase in 1981/ 82 for Different Parameter Assumptions*
Difference in percentage points from undisturbed cases



5.5 Adaptivity or the Limits of Macromodels

A major problem in making long-term projections with a model, whose structure has been estimated from the experience of past decades, is that we do not know to what extent the attitudes and the behavior reflected in the estimated structure will be relevant also in the future. Has the stagflation and the stabilization problems following in the wake of the '73-crisis caused such changes in the behavior or structure of the economy that we can no longer trust past experience and econometric estimates? (Cf. e.g. Lucas 1976). The increasing tax-consciousness, the tendencies towards more cautious pricing and investment-reactions to advantageous cost-developments, the growing dominance of public employees in the wage formation process, the increasing extent of inflation-indexing, the new currency regimes and shifts in corporative ownership and financing are points in question for the Swedish economy.

That these changes do not come out of the blue, but have to be explained in terms of the economic events in the past, seems evident. An even more ambitious task would be to study to what extent these institutional and behavioral changes tend to make the economy more stable or more manageable. Does, e.g., the increased tax-consciousness and tax-adjustment make local authorities more sensitive to income changes and by that less counter-cyclical in spending behavior? Has the increased price uncertainty also increased the risk-aversion in the firms, adding new inertia to the economy? To what degree is the economy adaptive – or the opposite?

There is obviously no way in which we could even attempt to answer these large questions, crucial as they may be for any hope of basing policy on econometric projections. We have, anyhow, probably the wrong kind of model to even start exploring them. The structural coefficients of a macromodel, like ISAC, should always be thought of as rough estimates of the aggregate outcome of underlying probabilistic micro-processes. The aggregate demand curves we estimate, e.g., sum the net effects of a number of oligopolistic firms trying to capture new markets and customers – or retain old ones – by new prices and more differentiated products and attempting to get the message across to the customers who in their turn are searching for new and better price-product combinations. The degree of flexibility in market supply and demand, and with that the resilience and manageability of the economy will then, in the end, depend on the production, consumption and search strategies of the involved firms and households. An explanation of structural change and adaptivity will, in most cases, have to start at the micro-level. When we come up against questions of adaptivity we have reached one – of the many – limits to macromodels.

Appendix A. The System of Equations

n = 23
m = 14
u = 20

Bloc	Number of variables	Number of equations
Commodity balances	33n	3n
Production capacity etc.	(2u + 15) n	(2u + 39) n
Foreign trade	n	2n
Private consumption	n + 2m + 2	n + 2
Government production etc.	0.5m + 4	n + 1.5m + 4
Prices	n + 1	4n + 1
Wages	n + 3m + 4	2n + 4m + 4
Total	(52 + 2u) · n + + 5,5m + 11 = 2204	(52 + 2u) · n + + 5,5m + 11 = 2204

Commodity balances

n = 23
m = 14

Equations	Name of endogenous variables not counted before	Number of variables	Number of equations
$m + x = h + e$	m, x, h, e	4n	n
$\hat{m} \cdot \theta p^w + \hat{x} \cdot p^x = \hat{h} \cdot p^h + \hat{e} \cdot p^e$	p^x, p^h, p^e	3n	n
$h = Ax + inv + pc + pu + ds$	A, inv, pc, pu	26n	n
Total		33n	3n

Production capacity and production technique.

n = 23
m = 14
u = 20

Equations	Name of endogenous variable not counted before	Number of variables	Number of equations
$Q = \sum_{v=t-u}^{t} Q^v \otimes \hat{\bar{x}}^{-1} \bar{x}^v$	$Q, Q^t, \bar{x}, \bar{x}^v$	$(u+11)n$	5n
$Q^t = F_Q[p^h(t-1), w(t-1), t]$	–	–	5n
$\bar{x}^t = (\hat{Q}_5^t)^{-1} g$	g	n	n
$d^v = F_d[p^x(t-1), p^h(t-1), w(t-1),$ $Q^v, \bar{x}^v(t-1)]; \quad v = t-u, \ldots, t-1$	d^v	$(u-1)n$	$(u-1)n$
$\hat{\bar{x}}^v = (I - \hat{d}^v)\hat{\bar{x}}^v(t-1); \quad v = t-u, \ldots, t-1$	–	–	$(u-1)n$
$\bar{x} = \sum_{v=t-u}^{t} \bar{x}^v$	–	–	n
$ur = \hat{\bar{x}}^{-1} x$	ur	n	n
$A = F_A(Q)$	–	–	23n
$g = F_g[ep(t-s), ur(t-1), k(t-1)];$ $s = 1, \ldots, 4$	ep	n	n
$\hat{e}p = (\hat{p}^x - \hat{p}^h A - \hat{w}\hat{\ell} - O\hat{\imath})(\hat{p}^k k)^{-1}$	k	n	n
$p^k = F_p(p^h, r_w)$	p^k	n	n
$k = \hat{Q}_5 \cdot \bar{x}$	–	n	–
$inv = G \cdot g$	–	–	n
Total		$(2u+15)n$	$(2u+39)n$

Note: a) Only 20 different vintages are distinguished in each branch. ⊗ denotes the inner product.
b) Q_5 is a vector of capital output coefficients.
c) Most of these variables are exogenous for branches outside the manufacturing sector.

Foreign trade

n = 23
m = 14

Equations	Name of endogenous variable not counted before	Number of variables	Number of equations
$e = F(p^e/\theta p^w(t-s), wm); \quad s = 0, 1$	–	–	n
$m = F_m(p^{xh}/\theta p^w(t-s), h); \quad s = 0, 1$	p^{xh}	n	n
Total		n	2n

Private consumption

n = 23
m = 14

Equations	Name of endogenous variable not counted before	Number of variables	Number of equations
$pc = F_{pc}(y_{11}, pc(t-1), p^h, p^h(t-1), besk)$	y_{11}, besk	2	n
$y_{11} = w' \cdot \ell + w^{p'} \cdot \ell^p$	ℓ, w^p, ℓ^p	n + 2m	1
$besk = F_{besk}(y_{11})$	–	–	1
Total		n + 2m + 2	n + 2

Note: This block shows the actual model equations in a highly abbreviated form. For details see section 3.7.

Government production and investment

n = 23
m = 4

Equations	Name of endogenous variable not counted before	Number of variables	Number of equations
$pu = (SG, LG) \cdot xp + (sg, \ell g)' \cdot ip$	ip	2	n
$xp = (xs', x\ell')'$	$x\ell$	$0.5 \cdot m$	m
$ip = (is', i\ell')'$	i_s, i_ℓ	2	2
$is = F_{is}(xs)$	–	–	1
$x\ell = F_{x\ell}(besk, utd, w^\ell, p^h)$	–	–	$0,5 \cdot m$
$i_\ell = F_{i\ell}(x\ell)$	–	–	1
Total		0.5m + 4	n + 1.5m + 4

Note: Cf. note to private consumption bloc. For details see section 3.9.

Prices

n = 23
m = 14

Equations	Name of endogenous variable not counted before	Number of variables	Number of equations
$p^e = F_p e(\theta p^w, uc, ur(t-1))$	uc	n	n
$p^{xh} = F_p xh(\theta p^w, uc, ur(t-1))$	–	–	n
$\hat{uc} = A'\hat{p}^h + \hat{Q}_4\hat{w} + \hat{Q}_5\hat{p}^k$	–	–	n
$\hat{p}^x = [\hat{e} \cdot \hat{p}^e + (\hat{x} - \hat{e}) \cdot \hat{p}^{xh}] \cdot \hat{x}^{-1}$	–	–	n
$cpi = p^{h'} \cdot pc / \sum_{i=1}^{n} pc_i$	cpi	1	1
Total		n + 1	4n + 1

Employment and Wages

n = 23
m = 14

Equations	Name of endogenous variable not counted before	Number of variables	Number of equations
$\ell = \hat{Q}_4 \cdot x$	ℓ	n	n
$q^p = (q^{s'}, q^{\ell'})'$	q^p	m	m
$\ell^p = \hat{q}^p \cdot xp$	ℓ^p	m	m
$u = (\ell_s - x' \cdot Q_4 - xp' \cdot q^p)/\ell_s$	u	1	1
$\dot{w}_o = F_w[c\dot{p}i(t-1), u, \Pi^m(t-1), \dot{Q}_4^m(t-1)]$	Π^m, Q_4^m, w_o	3	1
$\hat{\dot{w}} = \dot{w}_o \cdot I$	–	–	n
$w^p = (w^{s'}, w^{\ell'})'$	w^p	m	m
$\hat{\dot{w}}^p = \dot{w}_o(t-1) \cdot I$	–	–	m
$\Pi^m = \sum_{i=1}^{14}[p_i^x - (A'p^h)_i - i_i]x_i / \sum_{i=1}^{14} x_i$	–	–	1
$Q_4^m = \sum_{i=1}^{14} Q_{4,i} \cdot x_i / \sum_{i=1}^{14} x_1$	–	–	1
Total		n + 3m + 4	2n + 4m + 4

Appendix B. List of symbols

Endogenous variables are marked by *. Parantheses denote that only part of the vector or matrix is endogenous.

Symbol	Meaning	Dimension
	Commodity Balances	
*x	Domestic production	23
*h	Domestic absorption	23
*m	Imports	23
*e	Exports	23
(*)A	I/O coefficients	23×23
(*)inv	Demand for investment goods	23
*pc	Private consumption demand	23
(*)pu	Public sector demand for intermediate goods	23
ds	Change in inventory stocks	23
*p^x	Price of domestic production	23
p^m	Price of imports	23
*p^h	Price of domestic absorption	23
*p^e	Price of exports	23
	The I/O Matrix	
*$Q^v_{m,i}$	Aggregate input coefficient m of vintage v in manufacturing[1] branch i	$m = 5$, $i = 14$
*$\bar{x}^v_i$	Production capacity of vintage v in branch i	$i = 14$, $v = 20$
*d^v_i	Rate of scrapping of vintage v in branch i	$i = 14$, $v = 20$
*ur_i	Capacity utilization ratio in branch i	$i = 14$
*$q^f_{j,i}$	Fuel input coefficients in business sectors	$i = 23$, $j = 2$
	Investments in the Business Sector	
(*)g	Gross investments in the business sector	23
*ep	"Excess profits" in manufacturing sector	14
*k	Capital stock in manufacturing sector	14
*va	Value added at factor cost	23
*p^k	Cost of capital in manufacturing sector	14
i	Indirect taxes, net	23
r_w	External rate of interest	1
	Foreign Trade	
p^w	World market prices in foreign currency	23
θ	Exchange rate	1
wm	World market volume index	23
*p^{xh}	Domestic producers' prices on domestic markets	23

[1] There are 14 branches in the manufacturing subsector of the business sector (cf. section 4.1).

Symbol	Meaning	Dimension
	Disposable Income and Consumption Expenditures in Household Sector	
$^*y_{11}$	Total wages	1
*cpi	Consumption price index	1
*besk	Taxable household income	1
*utd	Local tax rate	1
$^*p^c$	Consumption goods prices	14
	Government Sector (s = central, ℓ = local)	
(*)xs, xℓ	Production	7
(*)cs, cℓ	Consumption	7
(*)fs, fℓ	Demand for commodities from business sector	23
(*)is, iℓ	Gross investments	1
	Stock Building	
ds_a	Change in total stocks	1
s_a	Total stock demand	1
	Prices	
*uc	Average costs incl. capital cost	23
$^*p^x$	Price of domestic production	23
$^*p^e$	Price of exports	23
$^*p^{xh}$	Domestic producers' prices on domestic markets	23
$^*p^h$	Price of domestic absorption	23
p^w	World market prices in foreign currency	23
p^m	Price of imports	23
	Wages and Employment	
$^*\dot{w}_0$	Wage rate increase	1
*w	Wage rates in business sector	23
$^*w^s, w^\ell$	Wage rates in central (s) and local (ℓ) government sector	7
$^*\ell$	Employment in business sector	23
$(^*)\ell^s, \ell^\ell$	Employment in central (s) and local (ℓ) government sector	17
ℓ_s	Labor supply	1
*u	Unemployment rate	1
$^*\Pi^m$	Gross profit share of value added in manufacturing sector	1
Q_4^m	Labor input coefficient in the manufacturing sector	1

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