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Deadlines and Distractions

by Maria Saez-Marti and Anna Sjögren

IUI, The Research Institute of Industrial Economics
P.O. Box 55665
SE-102 15 Stockholm
Sweden

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Maria Saez-Marti and Anna Sjögren†
The Research Institute of Industrial Economics
Stockholm, Sweden

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Abstract

We analyze the effect of deadlines on timing of effort when agents are occasionaly distracted. We show that agents get started early when completion of the task is uncertain, but rather likely. Agents who are rarely distracted will always postpone effort since the risk of not completing is small. As a result, agents who are more often distracted may out perform rarely distractes agents. We further show that principals can increase the probability that a task gets done and thus achieve higher profits by setting harsh deadlines, provided that they *sometimes* grant extensions or postpone the deadline.

Keywords: Deadlines, time-consistency, timing of effort, optimal incentives.

JEL-codes D81, J22, M50.

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†Corresponding author: Anna Sjögren, The Research Institute of Industrial Economics, Box 5501, 114 83 Stockholm, Sweden, e-mail: annas@iui.se.

1 Introduction

Many situations are characterized by the existence of deadlines. Deadlines very often involve tasks that take time to complete such that failing to meet the deadline implies that all or a substantial fraction of the effort exerted up to that point is wasted. For those of us who sometimes get distracted from working, the risk of wasting effort makes deadlines discouraging. In fact, this is why we may fail to get started reading the 47 pages long "Incentives for Procrastinators" on a Friday afternoon.[1] The risk is overwhelming that we only get half way through and have to start over on Monday because a colleague asks us to come for a drink.

Distractions, whether in the form of a more acutely pressing task at work or a sick child requiring a parent to stay at home from work, put completion of the task at risk. Agents may respond in different ways to this risk. One possibility is that the risk of being distracted is so large that the agent does not find it worthwhile to get started on a task that takes time to complete. Another possibility is that the presence of distractions actually helps and encourages us to get started reading that paper right away on Friday morning, precisely because we know we better work early, in case we get distracted later if we are ever to get through before we call it a day. This paper explores how distractions affect the timing of effort and the probability of completing a time consuming task when time is limited.

Deadlines apart from setting a latest completion date, typically also impose an earliest date to get the payoff for completing the task. Very often there is a brief window of opportunity and no payoff to completing before or after the deadline.[2] A window of opportunity in the future, provides a reason for discounting individuals to postpone effort in order to complete just in time. However, agents who are aware that they may get distracted, need to trade off their desire not to work earlier than necessary against the need to get started early enough to avoid failing to meet the deadline in case they get distracted.

There are numerous examples of deadline situations involving this trade off. Grant or job applications, submission of term and conference papers, reaching of sales targets to get bonus payments or a contract to deliver a good or a service at some specific date are but a few. We can also think of tenure decisions and probation periods in employment contracts as examples of such deadline situations. In all these examples, does the task require work which takes time such that there is a risk of being only half way through when the deadline expires in case one gets distracted or prevented from working.

There is evidence that distractions and the time consuming nature of some tasks are

[1] O'Donnoghue and Rabin (1999).

[2] There is a substantial operations reseach literature dealing with optimal routing and logistics when there are delivery time windows as a result of just-in-time-management.

important determinants of when to get started. Being distracted by too many other things to do is the most frequently stated reason for academic procrastination and failure to meet deadlines according to Solomon and Ruthblum (1984). Moreover, it was found in Boice (1989) that it is because some tasks, e.g. academic writing, are perceived as requiring long periods of work that researchers view themselves too busy to get their writing going. Our reading of this evidence is that some tasks require the agents full attention for some extended period of time, i.e. that supply of effort is discrete because working halfheartedly is suboptimal due to start up costs.[3]

The first objective of this paper is to analyze how occasional distractions affect the timing of work on tasks that require the agents full attention for more than one period in the presence of deadlines. Understanding how deadlines and distractions affect how agents trade off the probability of completing against the desire to postpone effort is of interest to setters of deadlines, be it contractors, employers or grant funders who are concerned with timely completion of tasks.

We find a non-monotonic relation between distractions and the probability of completing a task: Being more easily distracted can, for some levels of distractions, actually *increase* the probability of getting the task done. Of course this is not always true. Agents who are never distracted, always carry out the task at the last moment and they always complete. Agents who are distracted most of the time, of course, never even get started. The interesting result arises for moderate likelihoods of being distracted when the risk of being distracted induces precautionary effort. By getting started early, distracted agents can improve their probability of completing the task beyond that of those who are not distracted often enough to make precautionary effort.

Distractions as a motive for precautionary effort, is of obvious relevance for employers hiring decisions and job assignments and provides a rationale for why having an active lifestyle (of course up to a limit) is regarded as a positive attribute of a job candidate, why employees with small children have a reputation of being very productive when at work, or why socially active students are sometimes better achievers than the overly dedicated. Up to a point, agents whose time is a scarce resource because they get distracted by competing activities or obligations have a stronger motive for working as a precaution than agents who are rarely distracted.

Most deadlines are not as dead as one may think. There is plenty of evidence that real world deadlines do get extended or postponed or simply broken.[4] Some deadline contracts

[3] Reasons for discrete supply of labor are discussed in Mulligan (1999) where the discrete time, indivisible labor models of Hansen (1985) and others are derived as special cases of a continuous time model of optimal work session lengths where agents trade off start-up costs and fatigue effects.

[4] For example, in April 2003, the Europeean Commission granted an extension of deadline to the CESR (Committee of European Securities Regulators) to deliver technical advice on implementing measures for a directive, and in December 2001, the US Congress extended the deadline to comply with the Health Insurance

even specify explicit conditions for the possibility of granting extensions or for postponement. Apart from examples concerning compliance with regulations, and delivery of commentaries or advice, there are also uncountable announcements of extended deadlines for submission of papers to academic congresses or submissions of candidates for various prizes or for submitting tenders.[5]

One may ask why deadlines are imposed, only to be extended, postponed or broken. Why are agents not granted maximal time at the outset? Two answers can be found in the previous literature. Toxvaerd (2004) suggests that deadlines are broken because many real world contracts are not effectively incorporating the proper incentive considerations. He also suggests how contracts could be improved, maintaining the assumption that progress on the task is verifiable. The answer provided in O'Donoghue and Rabin (1999) is that agents are time-inconsistent and that deadlines harsher than those imposed by nature are optimal if agents have time-inconsistent preferences, because tough deadlines help time-inconsistent agents overcome their self-control problems. Their claim is also that if agents are time-consistent granting maximal time at the outset is optimal. A limitation of their analysis is that it rests on the assumption that once an agent gets started he also gets the job done.

In many situations these conditions are not met. First, getting started reading a paper is, as we all know, no guarantee for getting through the conclusions. The reason, suggested in this paper, is that people sometimes get distracted half way through lengthy tasks. Second, effort and progress are often unobservable or too costly to monitor for a principal who will thus never know if an agent failed to complete a task because he got distracted or because he never even got started. This is why principals are left with setting deadlines and paying for completed tasks regardless of when the job was done and why agents have to bare all the risk of wasting effort.

The second objective of this paper is to show that when there is a risk that the agent gets distracted half way through a project, deadlines harsher than the natural deadline can be optimal also when agents are time consistent. We also provide an explanation for why harsh deadlines are often broken. Our analysis shows that imposing tough deadlines with a possibility of extension or postponement, is in fact optimal for principals who face time consistent agents who *sometimes* get distracted. The reason is that the tough deadline may encourage the agent to get started early, while the possibility of an extension or a postponement makes the deadline less discouraging and allows that more started projects are completed.

Other papers related to ours are Fischer (2001) and Toxvaerd (2004). Fischer studies the effect of deadlines on how agents optimally allocate time between effort and leisure when tasks take time to complete. An important difference between her paper and ours is that

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Portability and Accountability Act (HIPAA) Transactions and Code Sets requirements. See also Toxvaerd (2004) for a review of evidence on more or less systematic time over runs in a number of industries.

[5] In fact, searching Yahoo.com for "Extension of the deadline" gave some 52000 hits on March 30, 2004.

we assume indivisibilities in effort such that the agent at any period can only work *or* do something else - be distracted in our framework. Toxvaerd (2004) considers tasks which consist of a number of subtasks which can be completed one after the other, but that each subtask requires the full attention of the agent to be successfully carried out. This model differs from ours in that the principal can both pay the agent along the way and choose a deadline. Inability to compensate the agent conditional on when the work is carried out, is argued to be crucial for the optimality of *sometimes extending* or *postponing* deadlines found in our paper.

The paper proceeds as follows. In section 2 we introduce a simple decision theoretic model of an agent working against a deadline. We analyze how timing of effort and probability of completion depend on the likelihood of being distracted and on the length of the deadline. In section 3, we change the perspective and in a principal-agent framework, we let the deadline regime be chosen by a principal who faces an agent who sometimes gets distracted. We discuss the conditions under which setting and extending or postponing deadlines is the optimal way to influence agent behaviour. Section 4 concludes.

2 The model

In order to study the effect of deadlines and distractions on timing of effort and probability of completing a task, we introduce a discrete time model where a principal has a project that needs to be completed within a deadline. The agent who can do the job has a fluctuating opportunity cost of time. We assume that the project takes time such that the agent needs to work more than once if he is going to complete it. This assumption captures an important aspect of many real world situations where agents doubt getting started on a time consuming project because they are afraid of wasting effort in case they fail to complete on time. It is because agents know they are occasionally distracted from working that getting started on a time consuming projects with a deadline becomes a risky endeavour.

Consider a principal and an agent who both make relation-specific investments which have no return outside their relation. A completed venture generates a total value $W > 0$ which can be realized at time $\hat{T}$, which is also when it is first verifiable by outsiders. A completed project can be stored at zero cost until its value is realized at time $\hat{T}$. In the first period the principal commissions the project to an agent who needs to spend two full time periods working on it. The costs associated with monitoring and verifying the agents progress are preventively high and the principal can only commit to rewarding the agent V for a task completed within a stipulated deadline T. At any time period, the agent's opportunity cost is high, $\overline{u}$, with probability p and low, $\underline{u}$, with probability $(1 - p)$. The source of fluctuation could be either that agents are occasionally offered other more lucrative projects or that agents are unable to work on the project in question because they fall ill or because they are otherwise prevented from working.

2.1 The agent's decision

For an agent to meet a deadline at T and get the reward V, the last opportunity to complete the project is on the eve of T, i.e. at $t = T-1$. Therefore, in each period $t = 0, 1, 2, ..., T-1$, before the deadline, the agent decides whether to work or not after observing the realization of his opportunity cost. The decision to work depends whether the agent has worked before or not, on the realization of the opportunity cost $u_t \in \{\underline{u}, \overline{u}\}$ and on the number of periods left before the deadline expires $(T-t)$. We assume that the individual is forward looking and maximizes his expected utility, discounting the future exponentially. We solve the model backwards.

Decision at $T-1$.

On the eve of the deadline, the relevant decision for the agent is whether to complete the project or not. Getting started is not an issue, since there is not enough time left. The agent with a history of working once, will complete the project if the discounted payoff to meeting the deadline outweighs the opportunity cost of working:

$$\delta V \geq u_{T-1}, \tag{1}$$

where $u_{T-\mathbf{1}} \in \{\underline{u}, \overline{u}\}$, and $\delta \in (0, 1]$ is the agent's discount factor.

Since the purpose of the analysis is to illustrate timing of effort, and the probability of completing a deadlined task when there is competition for the agents time, we will assume that this competition is fierce in one particular respect: The agent never works on the deadlined task when the opportunity cost of time is high. However, we assume that the project is worth completing when the opportunity cost is low. Throughout the paper we will therefore maintain the following assumption:

Assumption 1: $\delta V \in [\underline{u}, \overline{u})$.

Under this assumption the payoff of the task only motivates effort to complete the task when the opportunity cost is low and it can be shown that for such V's the agent never ever works when the opportunity cost is high. In what follows, we therefore analyze the agent's decision to work conditional on the opportunity cost being low.

Decision at $T-2$.

First, assume that the agent has never worked before. Under *Assumption 1*, the agent knows that if he gets started at $T-2$ he will complete the task at $T-1$ provided he does not get distracted. The agent will thus start at $t = T-2$ if

$$\delta^2 (1-p) V + \delta p \bar{u} \geq \underline{u} + \delta(p\overline{u} + (1-p)\underline{u})$$

Note that, by assumption 1, the agent's behaviour when the opportunity cost is high does not depend on what the agent did in the past or on what he plans to do in the future and hence, all the working conditions we will derive will only depend on p, δ and $\underline{u}$, since all terms in $\bar{u}$ cancel out. The condition above can be rewritten as

$$V \geq \frac{\underline{u}}{\delta^2} \frac{1 + (1-p)\delta}{(1-p)} = s_2(p). \tag{2}$$

The condition for starting when the deadline is two periods away, $s_2(p)$, is increasing in p. Moreover, $s_2(p) > \underline{u}/\delta$ for all p and $\lim_{p \to 1} s_2(p) = \infty$. Since $s_2(p) > \underline{u}/\delta$, the agent will only start at $T-2$ if he is certain to complete at $T-1$ provided that the opportunity cost is low. Henceforth we denote the relevant *starting* conditions at time t when the deadline is at T by s_{T-t}. The subscript $T-t$ indicates how many periods are left before the deadline expires.

It is now easy to obtain the probability of meeting the deadline for an agent who faces a deadline two periods away, but who has not yet got started, $P_2(p, V)$. Such a project will be undertaken and completed only if the agent has two consecutive periods of low opportunity cost of time and the payoff, V, is high enough, namely

$$P_2(p, V) = \begin{cases} 0 \text{ if } V < s_2(p) \\ (1-p)^2 \text{ if } V \geq s_2(p). \end{cases} \tag{3}$$

Assume now that the deadline is two periods away, but that the agent has already got started on the project. If the opportunity cost of time is low, the agent will decide to complete the project ahead of the deadline when[6]

$$\delta(1-p)\underline{u} + \delta^2 V \geq \underline{u} + (1-p)\, \delta^2 V.$$

This condition can be rearranged to obtain the following *completion* condition:

$$V \geq \frac{\underline{u}}{\delta^2} \frac{1 - (1-p)\delta}{p} = c_2(p). \tag{4}$$

The reward required by the agent to complete two periods ahead of the deadline, $c_2(p)$, is decreasing in p and always stricter than the condition for completing on the eve of the deadline, $c_2(p) > \underline{u}/\delta$ for all p. This implies that the more likely it is that an agent gets distracted, the more willing is he to complete ahead of the deadline. An agent who is never distracted is never willing to complete ahead of the deadline: $\lim_{p \to 0} c_2(p) = \infty$. Henceforth, the conditions for completing a task $T-t$ periods ahead of the deadline are denoted by c_{T-t}.

[6] From now onwards we will not include the payoffs obtained when the opportunity cost is high since they always cancel out.

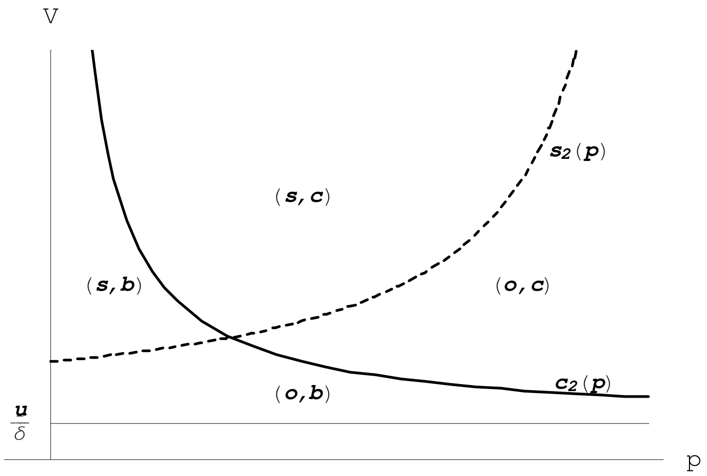

Figure 1: Decisions at T-2

In Figure 1, we depict the starting and completing conditions when the deadline is two periods away, $s_2(p)$ and $c_2(p)$. Optimal strategies are noted as a tuplet in brackets. The first letter in the bracket denotes the optimal action when the agent has never worked, namely whether to start on the task (s) or not (o) and the second refers to the action of completing (c) or to taking a break (b). For instance (s,b) means that the parameters (p,V) are such that the agent's optimal strategy at $T-2$, is to get started on the project, but to take a break if the project is already under way.

Decisions at $\mathbf{T-k}$.

Moving further from the deadline, we now consider the decisions to start and complete a task when there are k periods left before the deadline expires. The decision to complete a task which is already under way, $c_k(p)$, depends on the agents willingness to complete in later periods i.e. $c_j(p)$, $j<k$. The decision to get started, $s_k(p)$, however, depends both on future completing and starting conditions, since the agent needs to compare the consequences of getting started immediately with the consequences of postponing start taking into account future behavior.

In the appendix, we derive the starting and continuing conditions for any $k\geq 3$. The following proposition summarizes our findings. We illustrate the main results by means of Figure 2, where we have depicted the starting conditions for $k=2,3,4,5$, and the completion conditions for the the corresponding $k=1,2,3,4$.

Proposition 1 *Assume the deadline is $k\geq 3$ periods away then,*
(i) $c_k(p)>c_{k-1}(p)$ for all $p, c_k'(p)<0, \lim_{p\to 0} c_k(p)=\infty$ and $c_k(1)=\underline{u}/\delta^k$;
(ii) $s_k(p)$ is $U-$shaped, $\lim_{p\to 0} s_k(p)=\infty$ and $\lim_{p\to 1} s_k(p)=\infty$;
(iii) $s_k(p)>s_{k-1}(p)$ for all p if $k>1+1/(1-\delta)$, otherwise $s_k(p)<s_{k-1}(p)$ for large enough p's;

(iv) $s_k(p) > c_{k-1}(p)$ *for all* p*'s if* $k < 2 + 1/\delta$*, otherwise* $s_k(p) < c_{k-1}(p)$ *for small enough* p*'s; and*
(v) $s_k(p) > c_{k-2}(p)$ *for all* p*.*

Proof. See appendix. ■

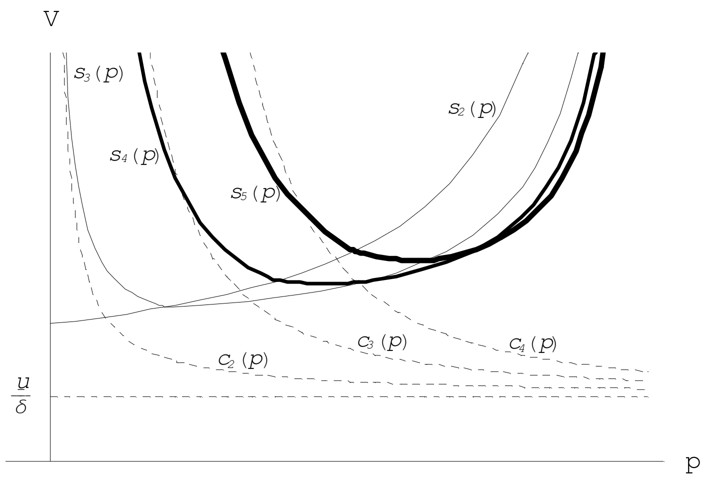

Figure 2: Decisions at k=5,4,3,2

First, we find that the agent's willingness to complete a task which is already half way through is higher the closer is the deadline and the more distracted the agent. The reason is that delaying completion puts meeting of the deadline at risk. Second, agents who are never distracted always postpone effort until the last minute, while agents who get distracted most all the time, never even start. Hence, for a given deadline, the rarely distracted and the often distracted are least willing to get started. The rarely distracted are reluctant to start because they prefer to postpone working since they are sure to complete anyway, while the very distracted are reluctant to start because the risk of failing to meet the deadline is high. Similarly, because they are almost sure they will complete the task, the further away is the deadline, the lower is the willingness to get started right away for those who are rarely distracted. The very distracted, however, are more willing to get started on the task the further away is the deadline, provided they are patient enough.

A somewhat surprising result is that moderately distracted agents may get started early with the idea of taking a break when the deadline is far ahead. For such an agent, getting started early is an investment in the probability of completing, but since the deadline is still far ahead and he is distracted with low probability, he is willing to grant himself a break since this does not compromise the probability of completing enough to outweigh the earlier enjoyment of the low opportunity cost of time, $\underline{u}$. Agents who are slightly more distracted will also get started early, but they do not break.

2.2 Getting the task done

We next turn to the issue of who gets a task done. We can compute the probability of actually completing a task when the deadline is T periods away, $P_T(p,V)$, by considering when the agent would be willing to start and complete, provided he is not distracted. The agent will get started on the task in the first period if the opportunity cost is low and $V \geq s_T(p)$. In this case, the probability of completion, depends on whether the agent will take a break or not. The agent completes at the first opportunity if $V \geq c_{k-1}(p)$ and he takes a break when $V < c_{k-1}(p)$. If the the opportunity cost happens to be high when he is about to get started, or if $V < s_T(p)$, then the probability of finishing when the deadline is T periods away is the same as the one obtained with a deadline one period closer, i.e. $P_T(p,V) = P_{T-1}(p,V)$. Hence, we can write

$$P_T(p,V) = \begin{cases} P_{T-1}(p,V) & \text{if } V < s_T(p) \\ p\ P_{T-1}(p,V) + (1-p)P^c_{T-1}(p,V) & \text{if } V \geq s_T(p) \end{cases} \tag{5}$$

for $T \geq 3$, when $P_2(p,V)$ is given by Equation (3), $P_1(p,V) = 0$, and the probability that the agent completes a task, conditional on having started, is given by

$$P^c_{T-1}(p,V) = \begin{cases} 1 - p^{T-2} & \text{if } V < c_{T-1}(p) \\ 1 - p^{T-1} & \text{if } V \geq c_{T-1}(p) \end{cases} .$$

Proposition 2 characterizes how the probability of completion in Equation (5), depends on the payoff to the task, V, the probability of being distracted, p, and on the length of the deadline, T.

Proposition 2 *The probability of completion $P_T(p,V)$ is discontinuous in p and V, non-decreasing in V, and if $V \geq \min_p s_3(p)$ non-monotonic in p. Moreover, $P_T(p,V) \geq P_{T-1}(p,V)$ for any T.*

Increasing the length of the deadline will only have a positive affect on the probability of completion if the agent is willing to get started on the task at once. The effect on the probability of completion of increasing the pay off is similar to that of making the deadline longer, the probability of completing will be equal when the higher V does not encourage more effort, and higher when it does. It is interesting to note that agents with substantially higher level of distraction can have higher success rates than those who are rarely distracted. This is evident in Figure 3, which shows, for a given V, how the probability of completion P_T changes with the length of the deadline, T. The dashed line is P_2, the thin, solid line is P_3 and the gray line is P_4. It is clear, that adding time, makes no difference to those who are only rarely distracted. They never start working until the last minute anyway. It is for individuals who are moderately distracted that granting longer deadlines matter. When an agent is distracted enough to get started early, he outcompetes less distracted agents, whose behaviour is unaffected by adding an extra period.

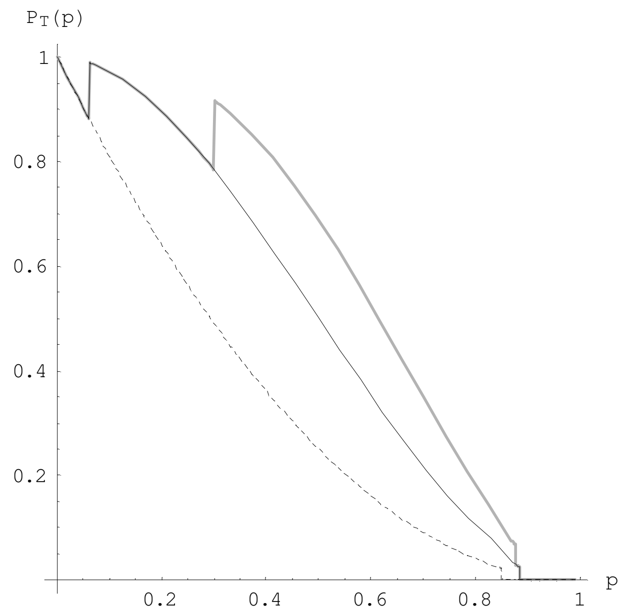

Figure 3: Getting the task done

3 The principal's decision

We now turn to the principal's decision. Assume that the value W of a completed project can only be verified and realized at $\hat{T}$. At that time, W is split after a bargaining process. Let $\hat{V}$ be the resulting payment to the agent, and $W - \hat{V}$ the principal's surplus

Assume that the principal hires an agent to carry out the project and sets a deadline at $T \leq \hat{T}$. At T, an agent who can anticipate the outcome of expost bargaining at $\hat{T}$, will accept any payment $V_T \geq \delta^{\hat{T}-T}\hat{V}$ for a completed project. With a lower payment, the agent would rather wait and deliver at $\hat{T}$. The principal, however, is only willing to pay V_T for delivery at T if $\delta^{\hat{T}-T}W - V_T \geq \delta^{\hat{T}-T}(W - \hat{V})$, since if he were to pay more for early delivery he would rather tell the agent to come back later and deliver at $\hat{T}$. Because the value of the project can be verified only at $\hat{T}$, the only payment schedule contractible upon is therefore $V_T = \delta^{\hat{T}-T}\hat{V}$. With this payment schedule, the principal's expected profit of setting a deadline at T, $\Pi(T)$ is:

$$\Pi(T) = (\delta^{\hat{T}}W - \delta^T V_T)P_T(p, V_T) = \delta^{\hat{T}}(W - \hat{V})P_T(p, \delta^{\hat{T}-T}\hat{V}). \tag{6}$$

It follows immediately from Proposition 2, that , $\Pi(T)$ is maximized by setting the deadline at $T = \hat{T}$, since allowing for maximal time, maximizes the probability of completion for any

p. This result should come as no surprise to those who successfully completed "Incentives for Procrastinators." O'Donoghue and Rabin (1999), indeed conclude that time-consistent agents should be granted maximal time. Yet, the non-monotonicity in the probability of completion found in the previous section, suggests that it would be possible to improve the probability of completion further if the principal could somehow convince an agent to get started earlier. In what follows, we show how principals who face distracted agents can increase their expected profits by introducing uncertainty about the length of the deadline, and hence, by departing from granting maximal time.

3.1 Stochastic deadlines

Assume that the principal can create uncertainty concerning the length of the deadline. This could be done by initially setting a *harsh* deadline, which is later either postponed or extended with some positive probability. We say that a deadline is *stochastic at* $\hat{T}$ if a harsh deadline which is set at $\hat{T}-1$ is postponed or extended to $\hat{T}$ with probability $q > 0$. We assume that the uncertainty about the true deadline is resolved at $\hat{T}-1$, such that a task under way may not be completed if the true deadline turns out to be harsh, but that agents may consider getting started at $\hat{T}-2$, if the chances are high enough that the deadline does get postponed or extended. With an *extended* deadline, early completers are paid at $\hat{T}-1$ and, in the event that extra time is granted, the additional completers get paid at $\hat{T}$. Under a *postponed* deadline, those who complete within the initial harsh deadline wait for payment until $\hat{T}$. Given the payment schedule discussed above, agents are indifferent to the timing of payments and the effect on behaviour of extended and postponed deadlines is the same. We now derive the agent's conditions for getting started and for completing the task under a stochastic deadline.

Decision at $\hat{\mathbf{T}}-\mathbf{1}$.

If the true deadline is realized at $\hat{\mathbf{T}}-\mathbf{1}$, the agent can do nothing. However, if the deadline is extended or postponed, the relevant conditions for working at $\hat{T}-1$ are the same as those the agent faces at $\hat{T}-1$ when the deadline is *deterministic at* $\hat{T}$. The agent will only consider working if he worked once and the actual deadline is $\hat{T}$. That is,

$$\delta\hat{V} \geq \underline{u}. \tag{7}$$

Decisions at $\hat{\mathbf{T}}-2$

First, assume that the agent has not worked before. The agent knows that if he gets started at $\hat{T}-2$ he will have a chance to complete the task at $\hat{T}-1$ only if the deadline is extended or postponed to $\hat{T}$ (and the opportunity cost is low). The agent will thus decide to get started at $\hat{T}-2$ when

$$(1-p)q\delta^2\hat{V} + (1-p)(1-q)\delta\underline{u} \geq \underline{u} + \delta(1-p)\underline{u},$$

which can be re-written as

$$\hat{V} \geq \frac{\underline{u}}{\delta^2} \frac{1 + (1-p)q\delta}{(1-p)q} = s_2^s(p,q).$$

Hereafter, we denote the working conditions at t, when the deadline is *stochastic* at T, by $c^s_{T-t}(p,q)$ and $s^s_{T-t}(p,q)$ to distinguish them from those obtained under the *deterministic* regime. The agent obviously never starts at $\hat{T}-2$ if he has not worked before and the deadline is certain to be *harsh*, i.e., $q=0$: $\lim_{q\to 0} s^s_2(p,q) = \infty$. Moreover, it is easily verified that $s^s_2(p,q)$ declines in q, such that the agent is more willing to get started at $\hat{T}-2$ the higher is the likelihood that the deadline is extended or postponed. Since $s^s_2(p,q) > \underline{u}/\delta$, the agent will only start at $\hat{T}-2$ if he has a chance to complete at $\hat{T}-1$.

Let us now turn to the agent's condition for completing a task at $t = \hat{T}-2$, which may turn out to be the last chance to meet the deadline. The agent completes if

$$\delta((1-p)\underline{u} + \delta\hat{V}) \geq \underline{u} + (1-p)q\delta^2\hat{V} + (1-p)(1-q)\delta\underline{u},$$

which can be re-written as

$$\hat{V} \geq \frac{\underline{u}}{\delta^2} \frac{1 - (1-p)q\delta}{(1-q)+pq} = c_2^s(p,q).$$

It is easy to see that $c^s_2(p,q) < c_2(p)\ \forall\ q<1$. The threat of a harsh deadline makes the agent less willing to get started on the task at $\hat{T}-2$, but that it increases the agent's willingness to complete.

We can now compute the probability that an agent who has not yet started, gets a task done when he faces a stochastic deadline at $\hat{T}=2$:

$$P_2^s(p,q,V) = \begin{cases} 0 \text{ if } \hat{V} < s_2^s(p,q) \\ q(1-p)^2 \text{ if } \hat{V} \geq s_2^s(p,q). \end{cases} \tag{8}$$

It should be obvious that a stochastic deadline two periods away gives a lower probability of completion than a deterministic deadline of the same length.

Decisions at $\hat{T}-k$.

We now extend the analysis to cover decisions to work when there is a stochastic deadline k periods away. As the horizon is extended beyond 2, the effect of the stochastic deadline is still to make the agent more willing to complete early. This increased willingness to complete has implications for the decision to get started. For low levels of distraction, the treat of a harsh deadline will in fact make the agent more willing to get started. However, for high levels of distraction, chances of completing are low and the threat of a harsh deadline make them even worse and hence, the agent is discouraged from getting started altogether. As before, we provide formal derivations in the appendix, and present the results in the following proposition. Figure 4 illustrates the main results .

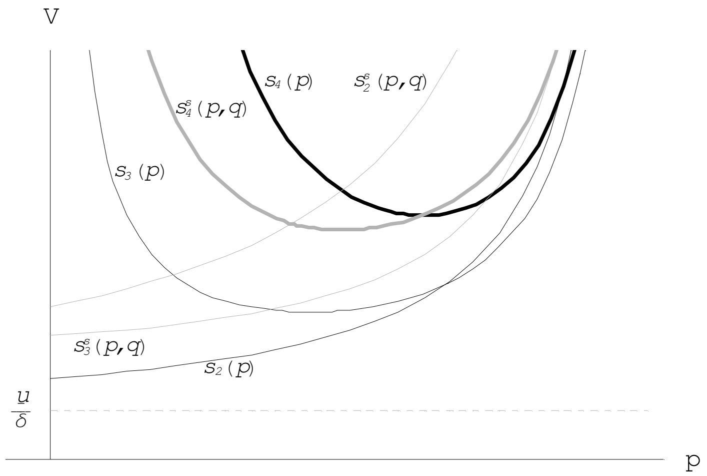

Figure 4: Stochastic deadlines

Proposition 3 *A stochastic deadline at $\hat{T}$ makes the agent more willing to start for small p's, but discourages start for high p's provided $\hat{T} \geq 3$. The agent is more willing to complete the task under stochastic deadlines.*

Proof. See appendix. ■

3.2 Maximizing profits

It is clear from Equation (6) that the principal's profits are maximized when the probability of completion is maximized. When the principal can set q, his profits are

$$\Pi(\hat{T}, q) = (W - \hat{V})P_{\hat{T}}(p, q, \hat{V}), \tag{9}$$

where $P_{\hat{T}}(p, q, \hat{V})$ is the probability of completion under a stochastic deadline at $\hat{T}$.

Figure 5 depicts the probabilities of completion under a deterministic deadline at $\hat{T} = 4$ (solid line) and a stochastic deadline (grey line) at $\hat{T}$. The stochastic deadline induces a higher probability of completion for small and intermediate values of p. This is the range of the likelihood of distraction that make the agent willing to start one period ahead with the stochastic deadline. In particular the agent will start at $\hat{T} - 3$ when p is small and at $\hat{T} - 4$ for intermediate p's. A general result, stated in the following proposition, is that a stochastic deadline may actually achieve a higher probability of completion than a deterministic one. Although the stochastic deadline is, by construction, a convex combination of two deterministic regimes, its outcome is not. The reason is straightforward. The stochastic deadline does not crowd out early effort to the same extent as the deterministic, but long deadline, nor does it leave as many started tasks uncompleted as does the harsh deadline: For the range of p's that it does not crowd out early effort, the stochastic deadline improves on the shorter, since agents that got started early, but failed to complete because they were distracted, have a chance to complete in the event that the deadline is the period after. It also improves on the long one, since it does not discourage early undertaking.

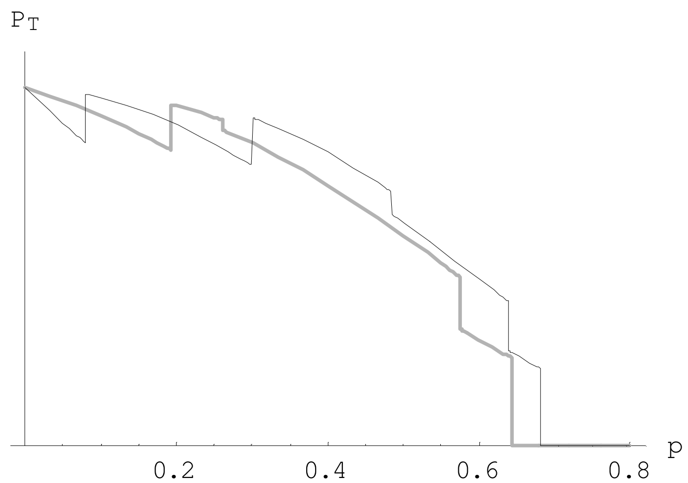

Figure 5: Probability of completion

Proposition 4 *Assume that* $\hat{T} \geq 3$. $P^S_{\hat{T}}(p, V, q) > P_{\hat{T}}(p, V)$ *whenever the stochastic deadline induces earlier start than the deterministic deadline.*
Proof. *See appendix* ■

Corollary 1 *Profits,* $\Pi(\hat{T}, q)$, *are maximized at the* $q \in (0, 1)$ *which solves* $V = s^s_k(p, q)$, *for* k *such that* $s_k(p) > V > s_{k-1}(p)$.

Figure 6 illustrates the optimality of stochastic deadlines from the principal's point of view. Note that profits are maximized for an interior q. In this example, the agent will not work at all when the deadline is harsh with high probability, i.e. when q is small. Granting maximal time, $q = 1$, is dominated by some stochastic deadlines because the latter induce earlier start.

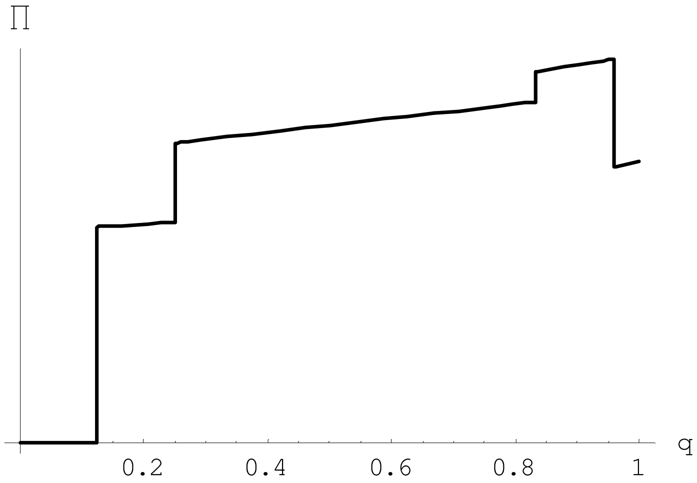

Figure 6: The optimality of stochastic deadlines.

4 Conclusion

We have studied the timing of effort and the probability of getting a task done for time-consistent agents who are occasionally distracted from working. Our first result is that, given the deadline regime, agents who are more frequently distracted may out-perform agents who are rarely distracted, the reason being that the risk of being distracted induces the agent to get started early.

Furthermore, our analysis has shown that although stochastic deadlines are by construction a convex combination of deterministic deadlines, the resulting probability of completion is not. The probability of completion with stochastic deadlines dominates both harsh and extended deadlines whenever the risk of a shorter deadline encourages the agent to get started earlier. In the light of this result, there is no need to invoke time-inconsistency or inefficient contracts in order to understand why many real world deadlines get pushed ahead.

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5 Appendix:

Proof to proposition 1. We derive the continuation condition when the deadline is k periods ahead. Assume that $V \geq c_{k-i}(p)$ for $i = 1, 2, ..., k-1$.The agent will complete if

$$\sum_{i=1}^{k-1} \delta^i (1-p)\underline{u} + \delta^k V \geq \underline{u} + \sum_{i=0}^{k-2} p^i (1-p) \sum_{n=i+2}^{k-1} \delta^n (1-p)\underline{u} + (1-p^{k-1})\delta^k V$$

which can be re-written as

$$V \geq \frac{\underline{u}}{\delta^k} \frac{1 + (\sum\limits_{i=0}^{k-2} p^i (1-p) \sum\limits_{n=i+2}^{k-1} \delta^n - \sum\limits_{i=1}^{k-1} \delta^i)(1-p)}{p^{k-1}} = c_k(p).$$

Note that $c_k(p) > c_{k-1}(p)$ for all k.
Next, consider the starting condition. Assume first that there are k periods before the deadline and that the opportunity cost is low. Assume further that the agent does not ever plan to take a break, i.e. $V \geq c_{k-1}(p)$. The expected value of starting (net of the payoffs

when the opportunity cost is high), $S_k(p,V)$, for an agent who has probability of distraction p is,

$$S(p,k,V) = (1-p^{k-1})\delta^k V + \sum_{i=0}^{k-2} p^i(1-p) \sum_{n=i+2}^{k-1} \delta^n (1-p)\underline{u}.$$

The agent has $k-1$ periods to complete, and he will do so the first time the opportunity cost is low. The total probability of finishing is $(1-p^{k-1})$.Assume now that the agent does not start immediately but that he is willing to start in any of the $k-j$ subsequent periods i.e., $V \geq s_i(p)$ for $i = k-1, k-2,...,j$. The expected value (net of the payoffs when the opportunity cost is high) of waiting one period to start when the last time he is willing to start is j $(2 \leq j \leq k)$ periods ahead of the deadline is

$$\begin{aligned} W(p,k,j,V) &= \underline{u} + p^{k-j} \sum_{n=k-j+1}^{k-1} \delta^n \underline{u}(1-p) + \delta^k V \sum_{i=0}^{k-j-1} p^i(1-p)(1-p^{k-i-2}) + \\ & \sum_{i=0}^{k-j-1} p^i(1-p) \sum_{s=0}^{k-i-3} p^s(1-p) \sum_{n=i+s+3}^{k-1} \delta^n (1-p)\underline{u}. \end{aligned}$$

The agent gets $\underline{u}$ the first period and has $k-j$ subsequent periods to get started. If the opportunity cost happens to be high those first $k-j$ periods, the agent will never start. This will happen with probability p^{k-j}. However, as long as the agent does get started, which happens with probability $\sum_{i=0}^{k-j-1} p^i(1-p)$, the agent will finish at the first opportunity. This will happen with probability $\sum_{s=0}^{k-i-3} p^s(1-p)$. Whenever the agent completes ahead of the deadline he will enjoy $\underline{u}$ when the opportunity cost of time is low.$W(p,k,j,V)$ and $S(p,k,V)$ are linearly increasing in V with $W(p,k,j,0) > S(p,k,0)$ and, $\partial W(p,k,j,V)/\partial V < \partial S(p,k,V)/\partial V$. There is a unique V^* which solves

$$S(p,k,V) = W(p,k,j,V).$$

Let $V^* = s(p,k,j)$:

$$\frac{\underline{u} + \left(p^{k-j} \sum_{n=k-j+1}^{k-1} \delta^n + (1-p)^2 \sum_{i=0}^{k-j-1} p^i \sum_{s=0}^{k-i-3} p^s \sum_{n=i+s+3}^{k-1} \delta^n - \sum_{i=0}^{k-2} p^i(1-p) \sum_{n=i+2}^{k-1} \delta^n \right)(1-p)\underline{u}}{((1-p^{k-1}) - \sum_{i=0}^{k-j-1} p^i(1-p)(1-p^{k-i-2}))\delta^k}.$$

For all those $V's$ and p' such that $V \geq s_i(p)$ for $i = k-1, k-2,..j$, and $V \geq c_{k-1}(p)$, $W(p,k,j,V) > W(p,k,j',V)$ and, $s(p,k,j) > s(p,k,j')$ for all $j\prime \neq j$. The agent will work, provided that $V \geq c_{k-1}(p)$, whenever

$$V \geq \max\{s(p,k,2), s(p,k,3)...., s(p,k,k)\} = s_k^c(p).$$

Note that:

i) for $k \geq 3$, $s^c_k(p)$ is U-shaped, $\lim_{p\to 0} s^c_k(p) = \lim_{p\to 0} s(p,k,2) = \infty$ and $\lim_{p\to 1} s^c_k(p) = \infty$.

ii)$\lim_{p\to 0} s^c_k(p)/s^c_{k-1}(p) = \infty$ and $\lim_{p\to 1} s^c_k(p)/s^c_{k-1}(p) = (k-2)/((k-1)\delta)$.

iii) $\lim_{p\to 0} s^c_k(p)/c_{k-1}(p) = (1+\delta)/((k-1)\delta)$.

For $k > 2+1/\delta > 3$ we need consider the case with $c_{k-2}(p) < V < c_{k-1}(p)$ and $V > s(p,k,2)$ i.e., the agent plans a break if he starts k periods ahead of the deadline. The agent expected profits (net of the payoffs when the opportunity cost is high) are

$$S^b(p,k,V) = \delta(1-p)\underline{u} + (1-p^{k-2})\delta^k V + \sum_{i=1}^{k-2} p^{i-1}(1-p) \sum_{n=i+2}^{k-1} \delta^n (1-p)\underline{u}.$$

The agent will work with the idea of taking a break when

$$S^b(p,k,V) \geq W(p,k,2,V).$$

Since $S^b(p,k,V)$ and $W(p,k,2,V)$ are linear in V, $S^b(p,k,0) < W(p,k,2,0)$ and $\partial W(p,k,2,V)/\partial V < \partial S^b(p,k,V)/\partial V$, there is a unique V^* which solves $S^b(p,k,V) = W(p,k,2,V)$ let $s^b(p,k,2)$ be the solution. Note that $s^b(p,k,2) \lesseqqgtr s(p,k,2)$ whenever $S^b(p,k,V) \gtreqqless S(p,k,V)$;i.e. $V \lesseqqgtr c_{k-1}(p)$. Since $\lim_{p\to 0} s^b(p,k,2)/c_{k-2}(p) = \infty$, the agent will never start when he plans to take two breaks. We can now write the condition for working k periods ahead of the deadline as

$$V \geq \min\{s^b(p,k,2), s^c_k(p)\} = s_k(p).$$

■

Proof to proposition 2. Let $V > \min_p s_3(p)$ and let $\underline{p}(V)$ and $\overline{p}(V)$, be the solutions in $(0,1)$ to

$$V = s_3(p).$$

where $0 < \underline{p}(V) < \overline{p}(V) < 1$. Only for $p \in [\underline{p}(V), \overline{p}(V)]$, will the agent's optimal strategy be to get started on the task at $t = T-3$. $\lim_{\epsilon\to 0} P_3(\underline{p}(V)-\epsilon, V) < P_3(\underline{p}(V), V)$, where $P_3(\underline{p}(V)-\epsilon, V) = P_2(p,V)$ and $P_3(\underline{p}(V), V) = pP_2(p,V) + (1-p)(1-p^2)$. The discontinuity of $P_k(p,V)$ follows directly from (5). It is clear that $P_T(p,V) \geq P_{T-1}(p,V)$, the inequality being strict for all p's such that $V \geq s_T(p)$, since $P_{T-1}(p,V) \leq 1-p^{T-2} \leq 1-p^{T-1}$. ■

Proof to proposition 3. Let us start first with the decision to continue when the deadline is stochastic. Let $c_{k-1}(p,q)$ denote the completing condition. Assume that $V > c_{k-1}(p,q)$. The agent will complete k periods ahead of the deadline when

$$(1-p)\sum_{i=1}^{k-1} \delta^i \underline{u} + \delta^k V \geq \underline{u} + \sum_{i=0}^{k-2} p^i (1-p) \sum_{n=i+2}^{k-1} \delta^n (1-p)\underline{u} + p^{k-1}(1-p)(1-q)\delta^k \underline{u} + (1-p^{k-2} + p^{k-2}(1-p)q)\delta^k V$$

which can be re-written as $V \geq c_k(p,q)$:

$$V \geq \frac{\underline{u}}{\delta^k} \frac{1+(\sum_{i=0}^{k-2} p^i(1-p) \sum_{n=i+2}^{k-1} \delta^n + p^{k-1}(1-q)\delta^k - \sum_{i=1}^{k-1} \delta^i)(1-p)}{p^{k-2} - p^{k-2}(1-p)q}.$$

$c_k(p,q) > c_{k-1}(p,q)$ and $\partial c_k(p,q)/\partial q > 1$. The agent is more willing to complete when the deadline is stochastic. Assume now that the deadline is stochastic at $\hat{T}$ and that in the first decision the opportunity cost is low. Assume that there are $k \leq \hat{T}$ periods before the deadline expires and that the agent does not ever plan to take a break; i.e. $V \geq c_{k-1}(p,q)$. The expected value of starting (net of the payoffs when the opportunity cost is high), for an agent who has probability of distraction p is,

$$\begin{aligned} S(p,k,V,q) &= (1-p^{k-2}+p^{k-2}(1-p)q)\delta^k V + \sum_{i=0}^{k-2} p^i(1-p) \sum_{n=i+2}^{k-1} \delta^n (1-p)\underline{u} + \\ & p^{k-2}(1-p)(1-q)\delta^{k-1}u. \end{aligned}$$

Assume now that the agent does not start immediately but that he is willing to start in any of the $k-j$ subsequent periods i.e., $V \geq s_i(p)$ for $i = k-1, k-2,...,j$. The expected value (net of the payoffs when the opportunity cost is high) of waiting one period to start when the last time he is willing to start is j $(2 \leq j \leq k)$ periods ahead of the deadline $W(p,k,j,V,q)$ is

$$\begin{aligned} &\underline{u} + \delta^k V \sum_{i=0}^{k-j-1} p^i(1-p)(1-p^{k-i-3}+p^{k-i-3}(1-p)q) + \\ &p^{k-j} \sum_{n=\hat{T}-j+1}^{k-1} \delta^k \underline{u}(1-p) + \sum_{i=0}^{k-j-1} p^i(1-p) \sum_{s=0}^{k-i-3} p^s(1-p) \sum_{n=i+s+3}^{k-1} \delta^n (1-p)\underline{u} + \\ &\sum_{i=0}^{k-j-1} p^i(1-p)(p^{k-i-3}(1-p)(1-q)\delta^{k-1}\underline{u}. \end{aligned}$$

Let $V^* = s^c(p,k,j,q)$ be the unique solution to $W(p,k,j,V,q) = S(p,k,V,q)$.We also obtain the condition for working when the agent plans to take a break. In this case, the value of starting, $S^b(p,k,V,q)$, is

$$\delta(1-p)\underline{u}+(1-p^{k-3}+p^{k-3}(1-p)q)\delta^k V+\sum_{i=2}^{k-2} p^{i-2}(1-p) \sum_{n=i+2}^{k-1} \delta^n(1-p)\underline{u}+p^{k-3}(1-p)(1-q)\delta^{k-1}u.$$

Solving $S^b(p,k,V,q) = W(p,k,j,V,q)$ we get $s^b(p,k,j,q)$. The condition for working is then

$$V \geq \min\{s^b(p,k,2,q), s^c_k(p,q)\} = s^s_k(p,q).$$

Note that $s^s_k(p,1) = s_k(p)$, for $k > 3$, $s^s_k(p,q)$ is U-shaped, $s^s_k(p,1)/s^s_k(p,q) = \infty$, and $\lim_{p\to 1} s^s_k(p,q)/s^s_k(p,1) = (k-1)/(k-2+q) > 1$ ■

Proof to proposition 4. Assume the first period the agent is willing to start with a deterministic deadline at $\hat{T}$ is $t = \hat{T} - k > 0$, i.e. $s_{k+1}(p) > V > s_k(p)$. Let the last period he is willing to get started be $t = \hat{T} - j$, for $j < k$. The total probability of completion with a deterministic deadline at $\hat{T}$ is

$$P_{\hat{T}}(p, V) = \sum_{i=0}^{k-j} p^i(1-p)(1-p^{k-i}).$$

If the stochastic deadline motivates early undertaking, the total probability of finishing, $P^s_{\hat{T}}(p, V, q)$, is at least

$$\sum_{i=0}^{k+1-(j+1)} p^i(1-p)(1-p^{k-i}+p^{k-i}q(1-p)) > P_{\hat{T}}(p, V) \;\; \forall q > 0.$$

■