THE RESEARCH INSTITUTE OF INDUSTRIAL ECONOMICS

Working Paper No. 589, 2003

Ethnic Diversity and Civil War

by Thomas P. Tangerås and Nils-Petter Lagerlöf

IUI, The Research Institute of Industrial Economics
P.O. Box 5501
SE-114 85 Stockholm
Sweden

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Thomas P. Tangerås[†] and Nils-Petter Lagerlöf[‡]

October 14, 2002

Abstract

We construct a model in which a number of equally powerful ethnic groups compete for power by engaging in civil war. In non-redistributive equilibrium, ethnically homogeneous and ethnically diverse countries face a lower probability of civil war than countries with a moderate degree of ethnic diversity. The likelihood of conflict is maximized when there are two ethnic groups. When rent-extraction possibilities are not too big and society sufficiently ethnically homogeneous, there also exists a pacific equilibrium path sustained by redistribution from the ruling group to the out-of-power groups.

Keywords: Civil war, ethnic diversity, redistribution, dynamic game.

JEL classification codes: H56, J15, K42, N40, N47

*We would like to thank Antoine Faure-Grimaud and Jean Tirole as well as participants at the 16th Annual Congress of the European Economic Association in Lausanne and seminar participants at IIES (Stockholm University), IUI, University of Oslo, Stockholm School of Economics and Université des Sciences Sociales (Toulouse) for their helpful comments. Thomas Tangerås acknowledges financial support from the Jan Wallander and Tom Hedelius Research Foundation.

†(Corresponding author) The Research Institute of Industrial Economics (IUI), Box 5501, 11485 Stockholm, Sweden. *Mailto*: thomas.tangeras@iui.se.

‡Department of Economics, Concordia University, 1455 de Maisonneuve Blvd W., Montreal, QC, H3G 1M8, Canada. *Mailto*: nippe@alcor.concordia.ca.

1. Introduction

Since 1960, 79 civil wars have erupted globally (Collier and Hoeffler, 2000), some of them ongoing still today. In addition to the human sacrifice and suffering caused by these wars,[1] they have had a devastating effect on the economic performance of the countries involved. Destruction of human and physical capital, infrastructure, schooling and health services, dwindling tourism and foreign investment lead to an average *shrinkage* in GDP of 0.05% per year in the suffering countries (Collier and Hoeffler, 2000, Table 2). Chad, a country of permanent tension and unrest ever since state formation, had 1993 a 20% lower GDP per capita than when granted independence in 1960, despite a substantial inflow of foreign aid during the later years (Azam et.al, 1999). Armed conflict and war seem to be a fundamental obstacle to growth and prosperity in many third-world countries.

This paper presents a theoretical study into the causes of civil war. The starting point of our analysis is that many developing countries are organized along ethnic or tribal lines, in particular in Africa. In the words of Azam (2001, p.429): "Ethnic [affiliation] ensures to most African people the provision of many services that a modern state has taken over in rich countries." Power automatically implies the control over foreign aid, economic rents from natural resources and over public spending and investment, which allows the state leader to favourise his own ethnic group or tribe at the expense of out-of-power groups. In Uganda for example, the dictator Idi Amin and his successor Milton Obote indulged in "massive favoritism to the benefit of their minority ethnic group from the north,

[1] According to the standard definition (Singer and Small, 1982, 1994) a conflict qualifies as civil war if and only if it involves a number of battle deaths exceeding 1000 per year. Azam (2002) presents estimates on 8 civil wars on the African continent in the period 1956-92 indicating that the number of casualties, including civilian, exceeded 3 million.

and to the detriment of the majority Baganda group" Azam (2001, p.430). Chad during the sixties provides another example of a discriminating regime with most public expenditure on schooling and infrastructure going to the south, mainly populated by the ruling Saras, at the expense of the Toubos in the north, although everybody paid the same tax (Azam et.al, 1999). This way of organizing society creates a tension between the current ruler's favoured group and the others. Add to this a non-democratic governance structure, a common feature of conflict-ridden countries, in which society is ruled by a king, emperor or president in a one-party system, and the stage is set for armed conflict across ethnic lines over resources and spending.[2]

The key question we address is: "in a society within which the ruling group has the power to abuse other ethnic groups economically, how does the degree of ethnic diversity (the number of ethnic groups) affect the likelihood of civil war?" To analyse this question, we build a model in which an exogenous number of equally sized ethnic groups are involved in a dynamic conflict game against each other. At any given point in time one of the ethnic groups (the *incumbent* or *ruler*) is in power, which enables it to enrichen itself at the expense of the other groups (the *outsiders*). The only way for outsiders to challenge power is by means of rebellion. A successful insurrection or defense implies the right to rule the country the subsequent period. Failure implies being an outsider the subsequent period.

[2] Political tension and exploitation does not necessarily imply conflict. Repressed groups must be able to organize themselves into a fighting group in order to rebel. For example, the urban community has been systematically favoured by the government over the rural community in a number of African countries. However, "in general, small farmers offer little resistance to governmental policies" (Azam et.al., 1999, p.22). The *collective action problem* constituted by raising arms may explain why many observed conflicts are ethnic, in the first place. It is perhaps easier for ethnic groups to organize a military resistance than for peasants, owing to a common history, language, culture and religion.

The ethnically homogeneous society (a single ethnic group) is *politically stable*, since there is no disagreement over the distribution of resources (this is true by assumption). Increasing the number of ethnic groups may or may not create instability, i.e. conflict. If the potential benefit of holding office is negligible compared to the cost of conflict or if the future is discounted heavily, no outsider will ever rebel. In the case of a *politically unstable* country, where the value of holding office is high, increasing the degree of ethnic diversity (the number of ethnic groups), has two countervailing effects. Holding constant the probability that each group rebels, increasing the number of ethnic groups leads to an increase in the probability of civil war – a *direct effect*. However, the expected amount of resources invested in conflict increases with the number of ethnic groups, reducing the probability of successful rebellion for each group. This makes insurrection a less attractive policy option – a *strategic effect*. It is shown that the strategic effect dominates whenever society consists of two or more ethnic groups. Taken together, these results imply that the likelihood of civil war is maximized in a country with two ethnic groups. This fits well with the existing evidence: the risk of civil war is relatively low both in societies with high and low degrees of ethnic diversity. (Collier and Hoeffler, 1998, 2000, 2002; Collier, Hoeffler and Söderbom, 1999; Elbadawi and Sambanis, 2000).[3]

Although all countries on the African continent are ethnically divided, only a fraction of them have experienced civil war. This observation has lead Azam (2001) to the conclusion that the core of the problem is a failure of the state to reconcile differences, not ethnic diversity in itself. Our results lend support to this

[3]Lately this has also been put forward in more popular contexts, such as Amoako's (1999) address to the Organization of African Unity.

view. We find that pacific equilibria sometimes co-exist with conflict equilibria. In pacific equilibrium the incumbent reduces tension by redistributing income to all ethnic groups. However, pacific equilibria are of a reputational nature, they will prevail only if the ethnic groups expect the others to behave decently in the future. In case of distrust there is no redistribution and maximization of short-term rent only, with periods of conflict being the inevitable result. Pacific equilibria exist if and only if society is sufficiently homogeneous (there are only a few ethnic groups) and the value of holding office is sufficiently low compared to the cost of conflict. Otherwise there are too many groups that have to be paid off or the value of the rent is so high that one ethnic group or the other will always have an incentive to deviate from any tacit agreement to remain peaceful.

There is a large theoretical literature on conflict and appropriation, starting with Hirschleifer (1988, 1989, 1995), Grossman (1991) and Skaperdas (1991, 1992). Our approach differs from these papers in at least one of the three following respects: (*i*) we allow for *more than two groups*;[4] (*ii*) the decision to engage in conflict is endogenous,[5] and (*iii*) we use a *dynamic setting*, which allows us to study the circumstances under which self-enforcing redistribution can be used to reduce the threat of conflict.[6]

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 derives the main results, first assuming no transfers, and then allowing for transfers. Section 4 concludes. All proofs are in the appendix.

[4] This complicates the analysis, but is necessary for our purpose of analyzing the effects of changing the number of ethnic groups. Neary (1997) and Hirschleifer (1995) also allow for the number of players to vary.

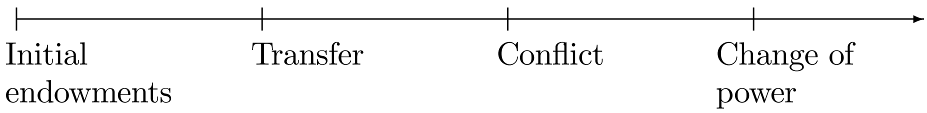
[5] In the standard conflict model it is costly to invest in arms, but having undertaken that investment there is no cost of engaging in conflict. Thus (at least in the most common interpretation), this is a setting which predicts perpetual conflict. Our model can predict war with probability less than one. See also Neary (1996) and Grossman and Kim (1995).

[6] Azam (1995) considers a two-group model with exogenous enforcement of transfers.

2. The Model

We consider a dynamic game between $N+1$ identical ethnic groups. In any given period one of them is in power, the *incumbent* or *ruler*. The $N \geq 1$ other groups are the *outsiders*. The timing of the stage game is depicted in the figure below.

Figure 1:



At the beginning of each period the incumbent receives an initial endowment θ, the rent from holding power. It stems from such things as controlling foreign aid distribution and the rents from natural resources like oil fields and diamond mines. The incumbent may or may not choose to share parts of θ with the other ethnic groups. If the ruler chooses to do so, an equal amount $x \geq 0$ is transferred to each outsider group at total cost $Nx \leq \theta$ to the ruler. Considering that ethnic groups often are geographically segregated,[7] one can think of redistribution as regional expenditures on infrastructure, health and schooling financed by the central government.

This is not a democracy: no outsider has any direct saying in the amount x to be distributed. The only way to challenge power is by force. We model this by assuming that outsiders in every period decide simultaneously and non-

[7] Chad, for example, is inhabited by the Toubos in the north, black Arabs and Fulanis in the middle and the Saras in the south (Azam et.al, 1999).

cooperatively whether to rebel.[8] The decision to rebel is taken after that period's transfer x has been distributed (and consumed). Each belligerent group (including the incumbent, who cannot choose whether to fight or not) incurs disutility K. K includes the alternative cost of military expenditures, the human sacrifice and so forth associated with war. Finally, the change of power, if any, takes place. All decision-makers are assumed risk-neutral.

What determines the probability of winning a conflict? For the incumbent, this is relatively straightforward. The more ethnic groups that are involved in a rebellion, the more resources are invested into the conflict and the more difficult it becomes to survive. Hence, one would expect the likelihood of the incumbent's survival to be decreasing in the number of groups involved in rebellion. Things are more complicated for the outsiders. The more other outsiders are involved, the easier it probably becomes to fight the incumbent since he has to divide his resources to fight more challengers. However, it does not suffice to beat the incumbent to become the ruler, a challenger must even beat the other fighting groups. It is not clear how all of this should be resolved, but we make the standard assumption that any belligerent group's likelihood of winning a conflict depends on the amount of resources invested by that group relative to the amount invested by all rebel groups. All groups are identical, hence it is not unnatural to assume that they invest equally much in conflict. Suppose an outsider has decided to engage in conflict, and there are M other outsiders who have made the same decision. In this case the relative amount of resources invested by each ethnic

[8] Under the simultaneity assumption we avoid assigning arbitrary first-mover or second-mover advantages to groups. When it comes to the assumption that groups move non-cooperatively, this seems at odds with the casual observation that ethnic groups from time to time manage to form coalitions in a rebellion against the sitting government. However, such coalitions tend to break down after a while. Hence, coalitions do not appear to be stable in the long run. Our assumption is that coalitions are unstable even in the short run.

group is $1/(M+2)$, which then is the probability of winning the conflict.[9] The more groups are involved, the lower is the likelihood that each wins. The number of belligerents is stochastic. An outsider does not know at the time of rebellion how many challengers he is going to face. Since outsiders take the decision non-cooperatively and simultaneously and all are equally likely to participate, the number of belligerents is binomially distributed. Thus, the probability of facing M other outsiders for an ethnic group that has decided to rebel is:

$$b(q, M, N-1) = \frac{(N-1)!q^M(1-q)^{N-1-M}}{M!(N-1-M)!}, \tag{2.1}$$

with q the probability of rebellion and $N-1$ the total number of other outsiders. The expected probability $p(q, N)$ of winning the conflict then equals:

$$p(q, N) = \sum_{M=0}^{N-1} \frac{(N-1)!q^M(1-q)^{N-1-M}}{M!(N-1-M)!} \frac{1}{M+2}. \tag{2.2}$$

This expression is difficult to work with. However (the proof is in appendix A.1):

Lemma 2.1.

$$p(q, N) = \frac{(N+1)q - (1-(1-q)^{N+1})}{(N+1)Nq^2}. \tag{2.3}$$

$p(q, N)$ has the following properties (subscripts denote partial derivatives throughout): *(i)* $p(q, 1) = 1/2$; *(ii)* $p(1, N) = 1/(N+1)$; *(iii)* $p(0, N) = 1/2$; *(iv)* $p_q(q, N) < 0$ for $N > 1$ and $q > 0$; *(v)* $p_N(q, N) < 0$ for $q > 0$.

These results make intuitive sense: if all $N+1$ groups partake in the conflict with certainty ($q = 1$), each has probability $1/(N+1)$ of winning. Similarly, if

[9] This corresponds to Tullock's (1980) Contest Power Function.

only one outsider partakes ($q = 0$ or $N = 1$), $p = 1/2$, i.e., the ruler and the sole participating outsider each wins the conflict with equal probability. The comparative statics results are equally intuitive: the higher is the perceived probability that the other outsiders engage in conflict (the higher is q) or the more ethnic groups might potentially participate (the higher is N), the more resources are on average deployed into conflict and the lower is the expected probability of winning for each of the belligerents.

Whenever the players have a decision to make, the *action* they choose is a function of the game's *history*: the ruler determines the size of the transfer and outsiders randomize between rebelling and remaining peaceful. The history is the vector of choices all ethnic groups have made in the past. A player's *strategy* is a plan that to every period assigns which action to take as a function of every conceivable history. The equilibrium concept applied is that of *Subgame-Perfection*. A Subgame-Perfect Equilibrium (SPE) is a vector of strategies, one strategy for each player, that has the following property: at no point in time and for no history can any player profitably deviate from the action prescribed by the equilibrium strategy, given that the player expects (*i*) all other players to play their equilibrium strategies today and (*ii*) all players, including herself, to play the equilibrium strategy in the future. In order to keep the analysis tractable, we restrict our attention to *symmetric and time-invariant* equilibria; the amount of redistribution is constant, and all outsiders rebel with the same probability at every point in time along the equilibrium path.

3. Equilibrium analysis

Consider first the expected value V^O of being an outsider along a symmetric and time-invariant equilibrium path:

$$V^O = x + (1-q)\delta V^O + q[p\delta V^I + (1-p)\delta V^O - K]. \tag{3.1}$$

First, the outsider consumes the equilibrium transfer x this period. Subsequently the group stays peaceful with probability $1-q$ and rebels with probability q. In the first event, the group remains an outsider even the next period, which has value V^O discounted by $\delta \in (0,1)$. In case of conflict, the belligerent incurs disutility K with certainty, expects to win and gain power with probability $p = p(q, N)$, the discounted value of which being δV^I, and to lose and remain an outsider with probability $(1-p)$, the discounted value of which being δV^O.

The value V^I of being an incumbent along the same equilibrium path is

$$V^I = \theta - Nx + z\delta V^I + (1-z)[p\delta V^I + (1-p)\delta V^O - K]. \tag{3.2}$$

The ruler keeps $\theta - Nx$ for himself and his group and distributes the rest of the rent to the outsiders. Subsequently peace prevails with probability $z = (1-q)^N$. In this case the incumbent remains in power even the next period, which has discounted value δV^I. War breaks out with probability $1-z$, the incumbent group is ousted to become an outsider with probability $1-p$ and remains in power with probability p, the discounted value being δV^O respective δV^I in the two cases. The disutility K of war is incurred with certainty.

(3.1) and (3.2) constitute two linear equations in two unknowns V^O and V^I.

They can be solved in order to explicitly obtain the equilibrium value functions:

$$V^O(q,x) = \frac{\delta pq(\theta - Nx) + (1-\delta(z+(1-z)p))x - q(1-\delta z)K}{(1-\delta)(1-\delta(p(1-q)+(1-p)z))}, \tag{3.3}$$

$$V^I(q,x) = \frac{(1-\delta(1-pq))(\theta - Nx) + (1-z)(\delta((1-p)x+(1-q)K) - K)}{(1-\delta)(1-\delta(p(1-q)+(1-p)z))}. \tag{3.4}$$

3.1. Equilibrium without redistribution

This section describes and analyses the equilibrium of the conflict game for which the incumbent group keeps the entire rent for itself, i.e. $x = 0$. Let q^{nr} be the equilibrium probability of rebellion by each group (this is shown below to be unique) in non-redistributive equilibrium. Write $V^{Inr} = V^I(q^{nr}, 0)$ and $V^{Onr} = V^O(q^{nr}, 0)$ the two equilibrium value functions. Denote by $z^{nr} = (1-q^{nr})^N$ the equilibrium probability of peace and by $p^{nr} = p(q^{nr}, N)$ the equilibrium expected probability of winning a conflict.

Consider first the incumbent's incentives for transferring rents to out-of-power groups. Suppose the outsiders are believed to be unresponsive to transfers, i.e. $q(x) = q^{nr}$ for all $x \in [0, \theta/N]$. The value $\widehat{V}^I(x)$ of making a transfer x to each outsider is then given by:

$$\widehat{V}^I(x) = \theta - Nx + z^{nr}\delta V^{Inr} + (1-z^{nr})[p^{nr}\delta V^{Inr} + (1-p^{nr})\delta V^{Onr} - K].$$

Increasing the amount of resdistribution marginally yields net loss

$$-\widehat{V}^I_x(x) = N.$$

If the incumbent believes that he cannot affect the outsiders' actions, there is no point in redistributing anything to the other groups. Consider next an outsider's incentives. Suppose each outsider believes all other outsiders to be unresponsive to transfers. Every group chooses the probability $\gamma \in [0,1]$ of rebelling so as to maximize

$$\widehat{V}^O(\gamma) = x + \gamma[p^{nr}\delta V^{Inr} + (1-p^{nr})\delta V^{Onr} - K] + (1-\gamma)\delta V^{Onr}.$$

The net benefit of rebellion is

$$\widehat{V}^O_\gamma(\gamma) = p^{nr}\delta[V^{Inr} - V^{Onr}] - K.$$

The first term is the marginal benefit of conflict. It is the discounted value of the difference between ruling and being an outsider the subsequent period times the probability of winning the conflict. From this is to be subtracted the cost K of rebellion. Observe that the optimal choice of γ is independent of x under the current set of beliefs, hence $q(x) = q^{nr}$ for all $x \in [0, \theta/N]$ constitutes a consistent set of beliefs.

Set $x = 0$ and $q = q^{nr}$ in (3.3) and (3.4), substitute into the expression above and simplify so as to obtain:

$$\widehat{V}^O_\gamma(\gamma) = \frac{\delta p^{nr}\theta - (1-\delta z^{nr})K}{1 - \delta[p^{nr}(1-q^{nr}) + (1-p^{nr})z^{nr}]}. \tag{3.5}$$

The denominator is positive for all values of q, hence the sign of the marginal incentive for engaging in conflict depends entirely on the sign of the numerator of (3.5). By the properties of $p(\cdot, N)$, the numerator is decreasing in q. The more likely rebellion by other groups in this and future periods is perceived to be, the more resources are expected to be invested into conflict. Consequently, the expected likelihood of becoming the ruler and being able to maintain that position is decreasing in q. This makes insurrection a less attractive policy option for each ethnic group the higher is the estimated likelihood that the other groups will rebel. Decisions are *strategic substitutes.* Note also that if the value of holding office is small [large] compared to the cost of conflict (θ/K is small [large]), rebellion becomes relatively less [more] appealing. Furthermore, if outsiders are very impatient (δ is small) conflict will never occur since the cost of conflict is realized today and the benefits in the future. In sum (the proof is in appendix A.2):

Proposition 3.1. *In symmetric and time-invariant SPE without redistribution: (i) there is perpetual peace ($q^{nr} = 0$) if the period benefit of holding office is small compared to the period cost of engaging in war or if outsiders discount the future heavily ($\delta\theta \leq 2(1-\delta)K$); (ii) there is perpetual civil war ($q^{nr} = 1$) if the discounted period value of holding office outweighs the maximal period disutility of war ($\delta\theta \geq (N+1)K$); (iii) each outsider goes to war with probability $q^{nr} \in (0,1)$ implicitly given by (3.6) otherwise.*[10]

$$\delta p(q^{nr}, N)\theta = (1 - \delta(1 - q^{nr})^N)K \tag{3.6}$$

Having characterized the potential equilibria of the game, we move on to the main purpose of this section: to study how the likelihood of conflict varies with the number of ethnic groups. The equilibrium probability of civil war in any given period is:

[10]Strategies are functions of all the possible histories of the game, even off the equilibrium path. There are infinitely many action profiles that sustain $x = 0$ and q^{nr} as the equilibrium probability of conflict, hence there are infinitely many SPE. However, q^{nr} is uniquely defined by the properties of p, hence there is a unique symmetric and time-invariant equilibrium path in the non-redistributive case. If we restrict attention to SPE in strategies that are allowed to depend on pay-off relevant state variables only, so-called Markov-Perfect Equilibria (MPE), the SPE with $x = 0$ and probablity q^{nr} of rebellion is the unique symmetric MPE.

$$y^{nr} = 1 - (1 - q^{nr})^N. \tag{3.7}$$

If the incumbent's ability to extract rent is small ($\delta\theta \leq 2(1-\delta)K$), there is never conflict - irrespective of the number of ethnic groups (part (i) of proposition 3.1). The country is *politically stable*. Below we consider the more interesting case of a *politically unstable* country. This is a country for which rent extraction possibilities are so big ($\delta\theta > 2(1-\delta)K$) that insurrections occur regularly ($q^{nr} > 0$ from parts (ii) and (iii) of proposition 3.1). To see how the likelihood of conflict in an unstable country depends on the number of ethnic groups, differentiate y^{nr} with respect to N:

$$\frac{dy^{nr}}{dN} = -z^{nr}\ln(1-q^{nr}) + N(1-q^{nr})^{N-1}\frac{dq^{nr}}{dN}. \tag{3.8}$$

In an interior equilibrium an increase in the number of ethnic groups has two effects. Holding fixed the probability that each group rebels, increasing the number of ethnic groups leads to an increase in the probability of civil war. This *direct effect* is captured by the first term in (3.8). However, increasing the number of ethnic groups affects each group's incentive for starting conflict since the probability of winning changes. This *strategic effect* is captured by the second term.

In general, one would expect the rent to holding office and the disutility of conflict both to depend on N, i.e. $\theta = \theta(N)$ and $K = K(N)$. By utilising (3.6), one obtains

$$\frac{dq^{nr}}{dN} = \frac{p_N^{nr}\theta + z^{nr}K\ln(1-q^{nr}) + K\frac{d}{dN}[\frac{\theta}{K}]}{N(1-q^{nr})^{N-1}K - p_q^{nr}\theta}. \tag{3.9}$$

The sign of the strategic effect is ambiguous in general. For constant population size, an increased number of ethnic groups implies smaller-sized groups and thus larger gain of holding office. Hence, $\theta'(N) > 0$ seems likely. The expected outcome of conflict for an ethnic group is a function of the effort and resources deployed into winning by each individual belonging to that group. Since a significant share of the cost of warfare is carried by individuals, whereas the proceeds of victory are spread between all group members, the individual only internalizes a fraction of his own contribution, hence contributes to little from the group's point of view. As is well known (see e.g. Olson, 1965), the severity of this *collective action problem* tends to increase in group size. Thus, $K'(N) > 0$ seems realistic too. Depending on the relative magnitude of the rent and cost effects, the strategic effect can be negative or positive. We have no a priori views on how θ/K would be expected to vary as a function of N. In order to avoid biasing our results in any direction, we choose to ignore that effect in the remainder of the analysis, i.e. assume $\frac{d}{dN}[\frac{\theta(N)}{K(N)}] = 0$.[11] Under this assumption, the strategic effect becomes unambiguously negative.

Varying the number of ethnic groups in this model is like varying the number of firms under Cournot competition. Increasing the number of firms has a direct effect for given firm output: lowering prices through increased supply. However, there is a strategic effect working in the opposite direction: firms reduce output as a response to increased competition. It is well known that the effect of entry on prices in Cournot equilibrium is ambiguous in general.[12] In this model, however, the strategic effect dominates the direct effect (the proof is in appendix A.3):

Proposition 3.2. *In the politically unstable country without redistribution, the*

[11] If I is total population size and rent θ is evenly distributed across the group members, $\theta(N) = \theta N/I$. If $K(N) = KN/I$, we have $\theta(N)/K(N) = \theta/K$. More generally, all our results carry through for $\frac{d}{dN}[\frac{\theta(N)}{K(N)}] \leq 0$.

[12] See Amir and Lambson (2000) for a state-of-the-art analysis.

probability of conflict is decreasing in the degree of ethnic diversity along the symmetric, time-invariant equilibrium path. The likelihood of conflict is maximized when there are 2 ethnic groups.

This is consistent with the findings by Collier and Hoeffler (1998), namely that the likelihood of civil war is the smallest in very homogeneous or heterogeneous societies.[13] Their regressions predict that, holding other explanatory variables constant at their sample means, the probability of civil war reaches a maximum at a degree of ethnic diversity that corresponds to approximately two equally strong ethnic groups.

For the other parameters of the model, aggregate behaviour is captured by studying individual behaviour. In the interior equilibrium we derive:

$$\frac{dq^{nr}}{d\theta/K} = \frac{p^{nr}K}{N(1-q^{nr})^{N-1}K - p_q^{nr}\theta} > 0,$$

$$\frac{dq^{nr}}{d\delta} = \frac{p^{nr}\theta + z^{nr}K}{\delta[N(1-q^{nr})^{N-1}K - p_q^{nr}\theta]} > 0.$$

It immediately follows from this and from $dy^{nr} = N(1-q^{nr})^{N-1}dq^{nr}$ that:

Proposition 3.3. *In the politically unstable country without redistribution, the probability of conflict is increasing in the rent of holding office θ, decreasing in the cost of conflict K and increasing in the discount rate δ along the symmetric, time-invariant equilibrium path.*

The intuition is straightforward: a higher θ/K makes it more attractive to become a ruler relative to the cost of conflict, which induces outsiders to rebel. So

[13] By assumption there is no potential for intra-group conflict in our model. Hence, a perfectly homogeneous society ($N = 0$) would trivially be pacific.

does a high δ, since it implies that the future pay-offs to insurrections are given a high weight.

Collier and Hoeffler (1998) identify wealth as a significant determinant of conflict. The wealthier are countries, the lower is the likelihood of civil war. This analysis invites a more nuanced view. It is not per capita wealth that matters, but rather the distribution of it. A country can be rich and conflict-ridden if wealth is distributed unequally (high θ but no redistribution) or poor and stable if wealth is distributed equally across ethnic groups (this becomes more evident in the next section). This interpretation is in line with Azam et.al. (1999: p.19) who list "[g]reat inequality in resource distribution among ethnic groups..." and "[g]reat inequality in the distribution of public expenditure and of taxation..." among their factors of conflict.

3.2. Equilibrium with redistribution

Conflict leads to costly waste of resources by all involved parties. Clearly, all ethnic groups can be made better off if the incumbent can somehow compensate the outsider groups for remaining peaceful. The problem is that the incumbent cannot commit to redistribution and outsiders cannot commit not to engage in conflict. Contracts between the government and the outsiders are not enforceable by court, they must be self-enforcing. Somehow, the outsiders must find a way to punish the ruler for failing to meet his obligations and there must be a mechanism by which outsiders are kept in line. The way we think of it here, opportunistic behaviour either by the ruler or some of the outsiders throws the country into turmoil: the outsiders return to a strategy of non-cooperation (the equilibrium analysed in the previous section). Let a *Pacific Transfer Equilibrium* (PTE) refer to an SPE in which civil war does not break out in equilibrium and where deviations from the equilibrium path are punished by reversion to the non-redistributive equilibrium derived in the previous section.[14] This section derives the circumstances under which redistribution can be used to uphold peace and specifically analyses the effect of ethnic diversity on the existence of PTE. Logically, we confine the analysis to the case of a politically unstable country.

Suppose the incumbent makes a period transfer x^r to each outsider. If all outsiders stay peaceful, each group receives x^r with certainty in every period from now on to eternity. The value V^{Or} to an outsider of staying peaceful in such an equilibrium is given by

$$V^{Or} = V^{O}(0, x^r) = \frac{x^r}{1-\delta}. \tag{3.10}$$

If the ruler wants to stay in power with certainty, the outsiders must prefer peace to war. The expected value of being an outsider whenever x^r is played is

$$\widehat{V}^{O}(\gamma) = x^r + \gamma[\frac{\delta}{2}[V^{Inr} + V^{Onr}] - K] + (1-\gamma)\delta V^{Or}.$$

By attacking, the outsider group surrenders K, but wins the conflict with probability 1/2 since it only has to fight the incumbent. In the next period cooperation breaks down, and the game reverts back to the non-cooperative state analysed in the previous section. If the outsider group remains peaceful, on the other hand, it gets utility δV^{Or}. A necessary and sufficient condition for the outsider to prefer

[14]This begs the question of how much cooperation can be achieved, i.e. whether optimal punishments exist. The model considered here is dynamic in the sense that each player's action set is history-dependent. Specifically, the action set depends on whether the player is a ruler or outsider. This means that the results obtained by Abreu (1986) and others cannot be utilised since they apply to infinitely repeated games. At this stage we have not been able to verify that reversion to non-redistributive stationary and symmetric equilibrium does in fact constitute an optimal punishment.

peace is:

$$\widehat{V}^O_\gamma(\gamma) = \frac{\delta}{2}[V^{Inr} + V^{Onr}] - [K + \delta V^{Or}] \leq 0.$$

Given (3.10), this equation shows that there exists a lower bound $\underline{x}$ on transfers required by the outsider in order to be willing to stay peaceful.

How much is the incumbent willing pay for peace? The incumbent has period income $\theta - Nx^r$ if he gets to rule in peace, which generates the value

$$V^{Ir} = V^I(0, x^r) = \frac{\theta - Nx^r}{1-\delta}. \tag{3.11}$$

along the pacific equilibrium path. Consider the expected value $\widehat{V}^I(x)$ to the ruler of deviating from cooperative play and paying the rent x instead:

$$\widehat{V}^I(x) = \theta - Nx + z^{nr}\delta V^{Inr} + (1 - z^{nr})[p^{nr}\delta V^{Inr} + (1 - p^{nr})\delta V^{Onr} - K].$$

Since cooperation breaks down for any deviation from x^r, the incumbent optimally sets $x = 0$ conditional on deviating. A deviation is unprofitable if and only if $V^{Ir} \geq \widehat{V}^I(0) = V^{Inr}$. This creates an upper bound $\overline{x}$ on the amount of redistribution in which the incumbent is willing to indulge, in order to preserve peace. Any $x^r \in [\underline{x}, \overline{x}]$ is sufficiently large to keep outsiders in line and constitutes a sufficiently small price for the incumbent to pay for peace. Civil war breaks out for any transfer outside this region, either because the incumbent chooses to grab the instantaneous rent or because power is challenged by one or more outsiders. Obviously, PTE exist if and only if $\overline{x} \geq \underline{x}$.

Regarding existence of pacific equilibria, consider first the effect of the rent to holding office versus the disutility of warfare. From proposition 3.3 we know that the higher is the rent relative to the cost of conflict, the more favourable it becomes to control the resource allocation and the higher is the likelihood of conflict, all other things equal. On the other hand, a high rent also leaves the incumbent a lot of room for redistribution. Owing to the fact that the incumbent pockets some of the increased rent for himself, there exists a point at which the rent of office becomes so high that peace cannot be sustained in equilibrium (the proof is in appendix A.4):

Proposition 3.4. *When the period value θ of holding office is sufficiently large relative to the disutility K of conflict ($\frac{\theta}{K} > \frac{2(1+\delta)}{\delta(1-\delta)}$), there exists no PTE.*

Clearly then, for a PTE to exist, it is necessary that rent not be too high. Consider next what happens as the number of ethnic groups changes. As N increases, the incumbent is forced to bribe more and more groups in order to maintain peace.[15] Also the probability of insurrection is decreasing in N. Deviating becomes an increasingly attractive policy option for the incumbent as N increases since the punishment threat becomes weaker and weaker and the cost savings larger and larger. Consider next the incentives of an outsider. A belligerent outsider wins the subsequent conflict with probability $1/2$ irrespective of the number of outsiders, since the only other party involved in conflict is the incumbent. Once the conflict is won, the new ruler is unlikely to be replaced if there is a lot of ethnic diversity, since the probability of subsequent war is low. Hence, each outsider must receive a large period transfer in order to remain peaceful when the number of ethnic groups is large. In sum, the transfer demands of the outsiders

[15] Holding constant the amount x transferred to each group, per capita transfer increases as the number of ethnic groups increases, all other things held equal. That is, per capita transfer is xN/I, where I is total population size. This would tend to push down the transfer demands of each individual group. Under the assumption that disutility of conflict increases proportionally in N, i.e. $K(N) = KN/I$ and that economic rent θ is evenly distributed across the ruling group, i.e. $\theta(N) = \theta N/I$ under the non-redistributive equilibrium, this moderating effect of N on transfer demands vanishes.

and the ruler's willingness to pay are incompatible for N sufficiently high (the proof is in appendix A.5):

Proposition 3.5. *In the politically unstable country for which the period value of holding office is not too large ($\theta/K < \frac{2(1+\delta)}{\delta(1-\delta)}$), there exists an $\overline{N} > 1$ such that a PTE exists if and only if $N \leq \overline{N}$.*[16]

The proposition states that pacific equilibria exist only in countries for which the rent to holding office does not overshadow the cost of warfare and where, at the same time, the degree of ethnic diversity is not too large. If this is not the case, either the benefit of holding office is too large or there are too many groups to bribe to be able to reach a pacific equilibrium. By combining parts (*ii*) and (*iii*) of proposition 3.1 with proposition 3.5 we prove the existence of multiple equilibria and hence provide an example of the policy failure alluded to by Azam (2001).

Corollary 3.1. *In politically unstable societies with limited degree of ethnic diversity ($N \in [1, \overline{N}]$) and with limited possibilities for rent-extraction ($\theta/K < \frac{2(1+\delta)}{\delta(1-\delta)}$), there exists, in addition conflict equilibria without redistribution, peaceful equilibria with redistribution.*

In relatively homogeneous countries the ruler sometimes has both the possibility and the incentive for using transfers to avoid civil war. This does not mean that peace will necessarily prevail. Peace can be accomplished only if the groups out of power expect the ruler to honour his agreement and redistribute wealth in their favour, and the ruler expects groups to remain peaceful. A country may equally

[16] In the knife-edge case $\theta/K = \frac{2(1+\delta)}{\delta(1-\delta)}$ there exists a PTE if and only if $N = 1$.

well be caught up in an equilibrium of distrust and conflict for which outsiders rebel in order to gain influence, and the insider takes full advantage of being in power by enriching himself and his peers. Hence, two societies identical in terms of ethnic diversity and wealth may have experienced totally different histories of conflict - one being stable, with political participation and redistribution from the state to all ethnic groups, the other characterized by systematic favourisation of the ethnic group of the current ruler, with political instability and frequent uprisings as a result.

The Ivory Coast provides a striking example of the Pacific Transfer Equilibrium. According to Azam (1995) two features were instrumental in maintaining peace during the long reign of the late Houpouët-Boigny, president from independence in 1961 until his death in 1993: shared political power with his political opponents and heavy redistribution from rich ethnic groups, including his own, to poorer regions. In contrast, neighbours Liberia and Burkina Faso, with which the Ivory Coast shares major ethnic groups, were politically unstable during the same period, experiencing frequent coups d'etat and sometimes civil war. The case of Uganda (Azam, 1995) provides anecdotal evidence on the existence of multiple equilibria. Uganda remained essentially peaceful under the regime of Museveni who included in his government representatives from other ethnic groups. However, under his predecessor Obote, who favoured his own ethnic group and its close allies, "Uganda witnesses one of the worst slaughters ever..." (Azam, 1995: 175).

4. Conclusion

This paper has studied the determinants of civil war. We have focused on ethnic conflicts to see if we can find an explanation to what seems to be an empirical regularity (Collier and Hoeffler 1998, 2000, 2002; Collier, Hoeffler and Söderbom 1999; Elbadawi and Sambanis 2000): countries with moderate ethnic diversity seem to be most at risk of civil war, whereas more homogeneous and more ethnically diverse societies both face lower risks. We have constructed a model with an exogenous number of ethnic groups who play a dynamic conflict game against each other. At any given point in time one of the ethnic groups (the incumbent) is in power, which enables it to enrichen itself at the expense of the other groups (the outsiders). The only way for outsiders to challenge power is by means of insurrection.

In an ethnically homogeneous society (a single ethnic group) there is never conflict since there is no disagreement over the distribution of resources (this is true by assumption). Increasing the degree of ethnic diversity (the number of ethnic groups), has two countervailing effects. Holding fixed the probability that each group rebels, increasing the number of ethnic groups leads to an increase in the probability of civil war − a *direct effect*. However, competition for power increases with the number of ethnic groups, which makes insurrection a less attractive policy option for each separate group − a *strategic effect*. We show that the strategic effect dominates whenever society consists of two or more ethnic groups. Taken together, these results imply that the likelihood of civil war is maximized in a society with two ethnic groups. Further, the likelihood of civil war is increasing in period rent of holding power and in the discount rate and decreasing in the disutility of war.

There exist multiple equilibria in societies that are not too ethnically diverse and where the rent-extraction possibilities are not too big. In addition to non-cooperative, conflict equilibria there exits a continuum of equilibria in which the ruler distributes income to all outsiders to keep them pacific. The existence of multiple equilibria has implications for empirical testing of the relationship between civil war and ethnic diversity: what we can expect to observe empirically should depend on which equilibrium we believe is more likely to occur. Since pacific equilibria are associated with redistributive policies from the ruling ethnic group to the outsiders (which could have effects on growth and development in other frameworks), our model might lend insights to an expanding empirical literature on the relationship between ethnic fractionalisation, policies, political institutions, and growth (See Mauro, 1995; Lian and Oneal, 1997; Easterly and Levine, 1997; Burnside and Dollar, 2000; Annett, 2000; Easterly 2000).

Does the model provide any insights into how the danger of civil war can be reduced in ethnically diverse countries? Given that borders can only occasionally be redrawn to create ethnically homogenous societies, the solution seems to be either to increase the cost of warfare or affect governmental redistribution possibilities. Many of the world's most conflict-ridden countries are major recipients of foreign aid. This provides a tool for the international community to influence domestic policies. Investments in infrastructure, health and education tailored directly to the various ethnic groups in a country increases their wealth and simultaneously the cost of warfare. Moreover, it reduces rent-extraction possibilities by channelling foreign aid outside the central government directly to the recipients. Finally, democratization would lead to an increase in political participation by ethnic minorities and probably to a composition of government that approximates the ethnic composition of society. This would reduce tension between ethnic groups and lead to a fairer distribution of resources, which in turn might reduce the risk of war.

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A. Appendix

A.1. Proof of lemma 2.1

Multiply and divide each element in the summand of (2.2) by $M+1$ to obtain:

$$p(q,N)=\sum_{M=0}^{N-1}\frac{(M+1)(N-1)!q^M(1-q)^{N-1-M}}{(M+2)!(N-1-M)!}. \tag{A.1}$$

Define the new variables $S=M+2$ and $B=N+1$ and perform a change of variables on (A.1):

$$\begin{aligned}
p(q,B-1) &= \sum_{S=2}^{B}\frac{(S-1)(B-2)!q^{S-2}(1-q)^{B-S}}{S!(B-S)!} \\
&= \frac{1}{B(B-1)q^2}\sum_{S=2}^{B}\frac{(S-1)B!q^S(1-q)^{B-S}}{S!(B-S)!} \\
&= \frac{1}{B(B-1)q^2}\sum_{S=2}^{B}b(q,S,B)(S-1).
\end{aligned}$$

The second equality follows from multiplying and dividing through by $B(B-1)q^2$ in $p(q,B-1)$ and the third equality from the definition of $b(q,\cdot,\cdot)$, see equation

(2.1). Note that $b(q,1,B)(1-1)=0$ and $b(q,0,B)(0-1)=-(1-q)^B$. Thus:

$$p(q,B-1)=\frac{1}{B(B-1)q^2}[(1-q)^B+\sum_{S=0}^{B} b(q,S,B)(S-1)].$$

Since

$$Bq=\sum_{S=0}^{B} b(q,S,B)S \text{ and } 1=\sum_{S=0}^{B} b(q,S,B)$$

by the properties of the binomial distribution, we get:

$$p(q,B-1)=\frac{Bq-[1-(1-q)^B]}{B(B-1)q^2}.$$

Substitute $B=N+1$ into this expression to obtain (2.3).

Properties (i) and (ii) follow directly from inserting respectively $N=1$ and $q=1$ into (2.3) and simplifying.

Property (iii): Define $p(0,N)=\lim_{q\to 0} p(q,N)$.

$$\lim_{q\to 0} p(q,N)=\lim_{q\to 0}\frac{1-(1-q)^N}{2Nq}=\lim_{q\to 0}\frac{(1-q)^{N-1}}{2}=\frac{1}{2},$$

where we have applied L'Hôpital's rule twice.

Property (iv):

$$p_q(q,N)=\frac{2[1-(1-q)^{N+1}]-(N+1)[1+(1-q)^N]q}{(N+1)Nq^3}. \tag{A.2}$$

The denominator is positive for $q>0$, hence the sign of $p_q(q,N)$ depends on the sign of the numerator, which we define as $A(q)$. A has the following properties:

$$A'(q)=(N+1)[N(1-q)^{N-1}q+(1-q)^N-1] \tag{A.3}$$

$$A''(q)=-(N+1)N(N-1)(1-q)^{N-2}q \tag{A.4}$$

Since $A''(q)<0$ for $N>1$ and $q\in(0,1)$, and $A'(0)=0$, $A(q)$ is maximized at $q=0$. $A(0)=0$, hence $A(q)<0$ for all $q>0$, which establishes the result.

Property (v): Differentiate to obtain:

$$p_N(q,N)=\frac{2N+1-(N+1)^2q-[2N+1-(N+1)N\ln(1-q)](1-q)^{N+1}}{(N+1)^2N^2q^2}.$$

$p_N(q, N)$ is negative if the numerator is negative, since the denominator is positive for $q > 0$. Define the numerator as $B(q)$. Note that:

$$B'(q) = (N+1)^2[(1 - N\ln(1-q))(1-q)^N - 1] \tag{A.5}$$

$$B''(q) = (N+1)^2 N^2 \ln(1-q)(1-q)^{N-1} \tag{A.6}$$

$B''(q) < 0$ for all $q \in (0, 1)$, which implies $B(q) < 0$ for all $q > 0$ by $B'(0) = 0$ and $B(0) = 0$. ■

A.2. Proof of proposition 3.1

Define the function

$$H(s, N) = \delta p(s, N)\theta - (1 - \delta(1-s)^N)K. \tag{A.7}$$

By definition, the numerator of (3.5) is equal to $H(q^{nr}, N)$. If it is positive (negative), the outsider will rebel (remain peaceful) for sure, if it is zero he is indifferent between war and peace. By the properties of $p(\cdot, N)$ (see lemma 2.1), it is straightforward to verify that $H_s < 0$ for all $s \in [0, 1]$.

Suppose $H(0, N) \leq 0$ or equivalently (recalling from lemma 2.1 that $p(0, N) = 1/2$):

$$\delta\theta \leq 2(1-\delta)K.$$

In this case the net benefit of conflict is always non-positive and strictly negative so long as the outsider expects the other groups to rebel with positive probability ($q > 0$). No outsider can ever benefit from rebellion. This means that the unique symmetric and time-invariant equilibrium without redistribution in this case is that all groups remain peaceful.

Consider next the case with $H(1, N) \geq 0$ or equivalently (recalling from lemma 2.1 that $p(1, N) = 1/(N+1)$):

$$\delta\theta \geq (N+1)K.$$

Now the opposite holds. The net benefit of conflict is always non-negative and strictly positive whenever the other outsiders remain peaceful with positive probability ($q < 1$). No outsider can ever benefit from staying peaceful. There is perpetual civil war along the symmetric and time-invariant equilibrium path.

In the intermediate case $2(1-\delta)K < \delta\theta < (N+1)K$, war may or may not break out in equilibrium. $H(0, N) > 0$, $H(1, N) < 0$ and $H_s(s, N) < 0$ yield a unique $q^{nr} \in (0, 1)$ given by (3.6) such that $H(q^{nr}, N) = 0$. $q > [<]q^{nr}$ cannot be a symmetric equilibrium since each firm individually would prefer to remain peaceful (set $\gamma = 0$) [rebel (set $\gamma = 1$)] in that case. Hence, the unique symmetric and time-invariant mixed strategy equilibrium is the solution to (3.6). ■

A.3. Proof of proposition 3.2

We first prove an intermediate result:

Lemma A.1. *$dy^{nr}/\, dN < 0$ for all $q^{nr} \in (0,1)$.*

Proof. Substitute (3.9) into (3.8), recalling the assumption $\frac{d}{dN}[\frac{\theta}{K}] = 0$, and simplify to obtain (for simplicity superscript nr used to indicate equilibrium is dropped):

$$\frac{dy}{dN} = \frac{(1-q)\ln(1-q)p_q(q,N) + Np_N(q,N)}{N(1-q)^{N-1}K - p_q(q,N)\theta}\theta(1-q)^{N-1}.$$

In the notation of the previous proof, $p_q(q,N) = A(q)/\,(N+1)Nq^3$ and $p_N(q,N) = B(q)/(N+1)^2N^2q^2$. Use this to rewrite dy/dN as:

$$\frac{dy}{dN} = \frac{(N+1)A(q)(1-q)\ln(1-q) + B(q)q}{(N+1)Nq^3[N(1-q)^{N-1}K - p_q(q,N)\theta]}\theta(1-q)^{N-1}.$$

The sign of this expression depends on the sign of the numerator of the fraction since the denominator and the second term are both positive for $q \in (0,1)$. Define the function $C(s)$:

$$C(s) = (N+1)A(s)(1-s)\ln(1-s) + B(s)s.$$

By definition, $dy/dN < 0 \Longleftrightarrow C(q) < 0$. $C(\cdot)$ has the following properties:

$$\begin{aligned} C'(s) &= (N+1)[A'(s)(1-s)\ln(1-s) - A(s)\ln(1-s) - A(s)] \\ &\quad +B'(s)s + B(s). \end{aligned}$$

$$\begin{aligned} C''(s) &= [(N+1)A''(s)(1-s)\ln(1-s) + B''(s)s] \\ &\quad +2[B'(s) - (N+1)A'(s)] \\ &\quad -2(N+1)A'(s)\ln(1-s) + \frac{(N+1)A(s)}{1-s}. \end{aligned} \tag{A.8}$$

By substituting (A.4) for $A''(\cdot)$ and (A.6) for $B''(\cdot)$ in the first line of (A.8) and (A.5) for $B'(\cdot)$ and (A.3) for $A'(\cdot)$ in the second line of (A.8), one obtains (after simplification):

$$\begin{aligned} C''(s) &= (N+1)^2N(1-s)^{N-1}s\ln(1-s) \\ &\quad -2(N+1)^2N[s + (1-s)\ln(1-s)](1-s)^{N-1} \\ &\quad -2(N+1)A'(s)\ln(1-s) + \frac{(N+1)A(s)}{1-s}. \end{aligned}$$

The two first lines of this expression and the first term on the third line are negative for all $s \in (0,1)$. The last term on the third line vanishes for $N = 1$ and is negative for all $s \in (0,1)$ and $N > 1$. Thus, $C''(s) < 0$ for all $s \in (0,1)$. This, along with $C'(0) = C(0) = 0$, implies $C(s) < 0$ for all $s > 0$. $q \in (0,1)$ by assumption, hence $C(q) < 0$ and the result follows. ■

For $\delta\theta \leq 2(1-\delta)K$, $q^{nr} = 0$ for all $N \geq 1$. For $\delta\theta \in (2(1-\delta)K, 2K)$, $\delta\theta < (N+1)K$ and thus $q^{nr} \in (0,1)$ for all $N \geq 1$, hence the result follows directly from lemma A.1. For $\delta\theta \geq 2K$, $q^{nr} = 1$ for all $N \in [1, (\delta\theta - K)/K]$, and $q^{nr} \in (0,1)$ for all $N > (\delta\theta - K)/K$. Hence, q^{nr} is non-increasing in N even in this case, which completes the proof. ■

A.4. Proof of proposition 3.4

By definition $\underline{x}$ is given by

$$\frac{\delta\underline{x}}{1-\delta} = \frac{\delta}{2}[V^{Inr} + V^{Onr}] - K$$

and $\overline{x}$ by

$$\frac{\theta - N\overline{x}}{1-\delta} = \theta + z^{nr}\delta V^{Inr} + (1-z^{nr})[p^{nr}\delta V^{Inr} + (1-p^{nr})\delta V^{Onr} - K].$$

Substitute (3.3) for V^{Onr}, (3.4) for V^{Inr}, letting $q = q^{nr}$ and $x = 0$, into the two expressions above and simplify to obtain

$$\underline{x} = \frac{[1-\delta(1-2p^{nr}q^{nr})](\theta + z^{nr}K) + (1-q^{nr})[1-\delta+2\delta(1-z^{nr})(1-p^{nr})]K}{2(1-\delta(p^{nr}(1-q^{nr}) + (1-p^{nr})z^{nr})} - \frac{K}{\delta}, \tag{A.9}$$

$$\overline{x} = \frac{\delta(1-p^{nr})\theta + (1-\delta(1-q^{nr}))K}{1-\delta(p^{nr}(1-q^{nr}) + (1-p^{nr})z^{nr})}\frac{1-z^{nr}}{N}. \tag{A.10}$$

In the proof of this proposition and the next we utilise the following lemma:

Lemma A.2. *(i) $\overline{x} - \underline{x}$ is strictly decreasing in N; (ii) $\lim_{N\to\infty}(\overline{x} - \underline{x}) < 0$.*

Proof. Part (i): There are two cases to consider: (a) $q^{nr} \in (0,1)$; (b) $q^{nr} = 1$.

Case (a): From part (iii) of proposition 3.1 q^{nr} is implicitly given by the solution to (3.6), which can be rewritten as $K = \delta p^{nr}\theta/(1-\delta z^{nr})$. Substitute this for K in (A.10) and simplify to obtain:

$$\overline{x} = \frac{\delta\theta}{N}\frac{1-z^{nr}}{1-\delta z^{nr}}. \tag{A.11}$$

Differentiating wrt N yields:

$$\overline{x}_N = -\frac{\delta\theta}{N(1-\delta z^{nr})}\left[\frac{1-z^{nr}}{N} + \frac{1-\delta}{1-\delta z^{nr}}\frac{dz^{nr}}{dN}\right] < 0.$$

It is negative since $dz^{nr}/dN > 0$ (see lemma A.1). Rewrite (3.6) as $\theta = (1 - \delta z^{nr})K/\delta p^{nr}$, substitute this expression for θ in (A.9) and simplify to:

$$\underline{x} = \frac{(1-\delta)K}{2\delta}\frac{1-2p^{nr}}{p^{nr}}. \tag{A.12}$$

Differentiating $\underline{x}$ wrt N yields:

$$\underline{x}_N = -\frac{(1-\delta)K}{2\delta}\frac{1}{(p^{nr})^2}\frac{dp^{nr}}{dz^{nr}}\frac{dz^{nr}}{dN} > 0.$$

$\underline{x}_N > 0$ by $dz^{nr}/dN > 0$ and $dp^{nr} = -(K/\theta)dz^{nr}$(see (**??**)). $\overline{x}_N < 0$ and $\underline{x}_N > 0$ imply $\overline{x} - \underline{x}$ strictly decreasing in N for $q^{nr} \in (0, 1)$.

Case (b): Set $q^{nr} = 1$ and $z^{nr} = 0$ in (A.10) respective (A.9) and use $p^{nr} = 1/(N + 1)$ to obtain:

$$\overline{x} = \frac{\delta\theta}{N+1} + \frac{K}{N} \text{ respective } \underline{x} = \frac{\theta}{2}\left[1 - \delta\frac{N-1}{N+1}\right] - \frac{K}{\delta}.$$

Subtracting $\underline{x}$ from $\overline{x}$ yields:

$$\overline{x} - \underline{x} = \frac{K}{N} + \frac{K}{\delta} - \frac{(1-\delta)\theta}{2}.$$

This difference is clearly decreasing in N, which establishes the result.

Part (ii) For N large $q^{nr} \in (0, 1)$ since $\delta\theta < (N + 1)K$ for N large (see (iii) of proposition 3.1). N large, $q^{nr} \in (0, 1)$ and part (iv) of lemma 2.1 imply $p^{nr} < 1/2$ and thus $\underline{x} > 0$ by (A.12). This, and $\underline{x}_N > 0$ imply that $\underline{x}$ is bounded away from zero for all N sufficiently large. From (A.11) and $z^{nr} \in (0, 1)$ for all N sufficiently large, we have

$$0 < \overline{x} < \frac{\delta\theta}{N} \text{ for all } N \text{ sufficiently large.}$$

Thus, $\lim_{N\to\infty} \overline{x} = 0$, which along with $\underline{x}$ bounded away from zero for all N sufficiently large, completes the proof. ■

Suppose $N = 1$ and $\theta/K > 2(1 + \delta)/\delta(1 - \delta)$. This implies $\delta\theta > 2K(1 + \delta)/(1 - \delta) > 2K = 2(N + 1)$, hence $q^{nr} = 1$ by part (ii) of proposition 3.1. Insert

$N = 1$, $z^{nr} = 0$, $q^{nr} = 1$ and $p^{nr} = 1/2$ into (A.10) and (A.9) to get the two critical transfer levels:

$$\overline{x} = \frac{\delta\theta + 2K}{2} \text{ and } \underline{x} = \frac{\delta\theta - 2K}{2\delta}.$$

The difference is:

$$\overline{x} - \underline{x} = \frac{\delta(1-\delta)K}{2\delta}\left[\frac{2(1+\delta)}{\delta(1-\delta)} - \frac{\theta}{K}\right] < 0. \tag{A.13}$$

$\overline{x} - \underline{x} < 0$ for $N = 1$ combined with part (i) of lemma A.2 yields the result. ■

A.5. Proof of proposition 3.5

Suppose $N = 1$ and $\theta/K \in [2/\delta, 2(1+\delta)/\delta(1-\delta))$. For all θ/K in this range we still have $z^{nr} = 0$, $q^{nr} = 1$ and $p^{nr} = 1/2$, but now $\overline{x} - \underline{x} > 0$ from (A.13). Suppose next $N = 1$, but $\theta/K \in (2(1-\delta)/\delta, 2/\delta)$. For all θ/K in this range we have $q^{nr} \in (0, 1)$. Now $\underline{x} = 0$, since $p^{nr} = 1/2$ and $\underline{x}$ is given by (A.12). $\overline{x} > 0$ for all $q^{nr} > 0$ by (A.11). Hence, $\overline{x} - \underline{x} > 0$ for $N = 1$ for all $\theta/K \in (2(1-\delta)/\delta, 2(1+\delta)/\delta(1-\delta))$. This, combined with lemma A.2, implies the existence of a unique $\overline{N} > 1$ such that $\overline{x} - \underline{x} \geq 0$ if and only if $N \leq \overline{N}$. ■